Exercise 11C

Question 1:

$\angle BDC = \angle BAC = 40°$ [angles in the same segment]

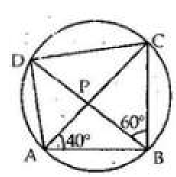
In ΔBCD, we have

$\angle BCD + \angle BDC + \angle DBC = 180°$

$\therefore \quad \angle BCD + 40° + 60° = 180°$

$\Rightarrow \quad \angle BCD = 180° - 100° = 80°$

$\therefore \quad \angle BCD = 80°$



(ii) Also $\quad \angle CAD = \angle CBD$ [angles in the same segment]

$\therefore \quad \angle CAD = 60°$ [$\because \angle CBD = 60°$]

Question 2:

In cyclic quadrilateral PQRS

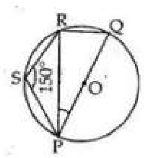
$\angle PSR + \angle PQR = 180°$

$\Rightarrow \quad 150° + \angle PQR = 180°$

$\Rightarrow \quad \angle PQR = 180° - 150° = 30° \quad ......(i)$

Also, $\quad \angle PRQ = 90° \quad ......(ii)$

[angle ina semi circle]



Now in ΔPRQ we have

$\angle PQR + \angle PRQ + \angle RPQ = 180°$

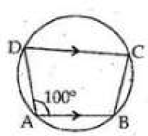
$\Rightarrow \quad 30° + 90° + \angle RPQ = 180°$ [from (i)and(ii)]

$\Rightarrow \quad \angle RPQ = 180° - 120° = 60°$

$\therefore \quad \angle RPQ = 60°$

Question 3:

In cyclic quadrilateral ABCD, AB $\parallel$ DC and $\angle BAD = 100°$



(i) $\quad \angle BCD + \angle BAD = 180°$

$\Rightarrow \quad \angle BCD + 100° = 180°$

$\Rightarrow \quad \angle BCD = 180° - 100° = 80°$

(ii) Also, $\quad \angle ADC = \angle BCD = 80°$

$\therefore \quad \angle ADC = 80°$

(iii) $\quad \angle ABC = \angle BAD = 100°$

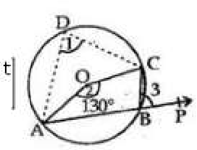
$\therefore \quad \angle ABC = 100°$

Question 4:

Take a point D on the major arc CA and join AD and DC

$\therefore \quad \angle 2 = 2\angle 1$

[Angle subtended by an arc is twice the angle subtended by it on the circumference in the alternate segment.]



$\therefore \quad 130° = 2\angle 1$

$\Rightarrow \quad \angle 1 = 65° \quad ....(i)$

$\angle PBC = \angle 1$

[$\because$ exterior angle of a cyclic quadrilateral interior opposite angle]

$\therefore \quad \angle PBC = 65°$

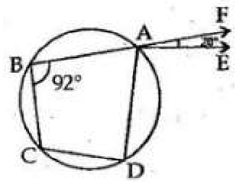
Question 5:

ABCD is a cyclic quadrilateral

$\therefore \quad \angle ABC + \angle ADC = 180^\circ$

$\Rightarrow \quad 92^\circ + \angle ADC = 180^\circ$

$\Rightarrow \quad \angle ADC = 180^\circ - 92^\circ = 88^\circ$



Also, $\quad AE \parallel CD$

$\therefore \quad \angle EAD = \angle ADC = 88^\circ$

$\therefore \quad \angle BCD = \angle DAF$

[$\because$ exterior angle of a cyclic quadrilateral = int.opp.angle]

$\therefore \quad \angle BCD = \angle EAD + \angle EAF$

$= 88^\circ + 20^\circ \quad [\because \angle FAE = 20^\circ(\text{given})]$

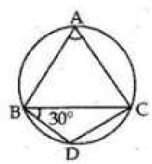
$= 108^\circ$

$\therefore \quad \angle BCD = 108^\circ$

Question 6:

$BD = DC$

$\therefore \quad \angle BCD = \angle CBD = 30^\circ$



In ΔBCD, we have

$\angle BCD + \angle CBD + \angle CDB = 180^\circ$

$\Rightarrow \quad 30^\circ + 30^\circ + \angle CDB = 180^\circ$

$\Rightarrow \quad \angle CDB = 180^\circ - 60^\circ$

$= 120^\circ$

The opposite angles of a cyclic quadrilateral are supplementary.

ABCD is a cyclic quadrilateral and thus,

$\angle CDB + \angle BAC = 180^\circ$

$= 180^\circ - 120^\circ [\because \angle CDB = 120^\circ]$

$= 60^\circ$

$\therefore \quad \angle BAC = 60^\circ$

Question 7:

Angle subtended by an arc is twice the angle subtended by it on the circumference in the alternate segment.

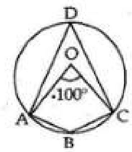
Here arc ABC makes $\angle AOC = 100°$ at the centre of the circle and $\angle ADC$ on the circumference of the circle

$\therefore \angle AOC = 2\angle ADC$

$\Rightarrow \angle ADC = \frac{1}{2}(\angle AOC)$

$\Rightarrow \quad = \frac{1}{2} \times 100° \quad [\angle AOC = 100°]$

$\Rightarrow \angle ADC = 50°$



The opposite angles of a cyclic quadrilateral are supplementary

ABCD is a cyclic quadrilateral and thus,

$\angle ADC + \angle ABC = 180°$

$= 180° - 50° \, [\because \angle ADC = 50°]$

$= 130°$

$\therefore \quad \angle ABC = 130°$

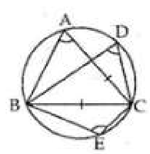
$\therefore \angle ADC = 50°$ and $\angle ABC = 130°$

Question 8:

Δ ABC is an equilateral triangle.

$\therefore$ Each of its angle is equal to 60°

$\Rightarrow \angle BAC = \angle ABC = \angle ACB = 60°$



(i) Angles in the same segment of a circle are equal.

$\therefore \angle BDC = \angle BAC$

$= 60° \quad [\because \angle BAC = 60°]$

$\Rightarrow \angle BDC = 60°$

(ii) The opposite angles of a cyclic quadrilateral are supplementary

ABCE is a cyclic quadrilateral and thus,

$\angle BAC + \angle BEC = 180°$

$\angle BEC = 180° - 60° \, [\because \angle BAC = 60°]$

$= 120°$

$\Rightarrow \quad \angle BEC = 120°$

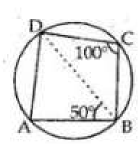
Question 9:

ABCD is a cyclic quadrilateral.

$\therefore \quad \angle A + \angle C = 180° \quad$ [opp. angle of a cyclic quadrilateral are supplementary]

$\Rightarrow \quad \angle A + 100° = 180°$

$\Rightarrow \quad \angle A = 180° - 100° = 80°$



Now in ΔABD, we have

$\angle A + \angle ABD + \angle ADB = 180°$

$\Rightarrow \quad 80° + 50° + \angle ADB = 180°$

$\Rightarrow \quad \angle ADB = 180° - 130° = 50°$

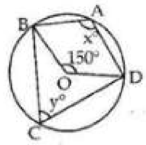
$\therefore \quad \angle ADB = 50°$

Question 10:

O is the centre of the circle and $\angle BOD = 150°$

$\therefore$ Reflex $\angle BOD = (360° - \angle BOD)$

$= (360° - 150°) = 210°$



Now, $x = \frac{1}{2}(\text{reflex} \angle BOD)$

$= \frac{1}{2} \times 210° = 105°$

$\therefore$ $x = 105°$

Again, $x + y = 180°$

$\Rightarrow$ $105° + y = 180°$

$\Rightarrow$ $y = 180° - 105° = 75°$

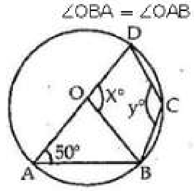
$\therefore$ $y = 75°$

Question 11:

O is the centre of the circle and $\angle DAB = 50°$

$OA = OB$ [Radii]

$\Rightarrow$ $\angle OBA = \angle OAB = 50°$



In $\triangle OAB$ we have

$\angle OAB + \angle OBA + \angle AOB = 180°$

$\Rightarrow 50° + 50° + \angle AOB = 180°$

$\Rightarrow$ $\angle AOB = 180° - 100° = 80°$

Since, AOD is a straight line,

$\therefore$ $x = 180° - \angle AOB.$

$= 180° - 80° = 100°$

$\therefore$ $x = 100°$

The opposite angles of a cyclic quadrilateral are supplementary.

ABCD is a cyclic quadrilateral and thus,

$\angle DAB + \angle BCD = 180°$

$\angle BCD = 180° - 50°$ [$\because \angle DAB = 50°$, given]

$= 130°$

$\Rightarrow$ $y = 130°$

Thus, $x = 100°$ and $y = 130°$

Question 12:

ABCD is a cyclic quadrilateral.

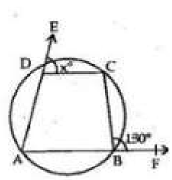
We know that in a cyclic quadrilateral exterior angle = interior opposite angle.

$\therefore$ $\angle CBF = \angle CDA = (180° - x)$

$\Rightarrow$ $130° = 180° - x$

$\Rightarrow$ $x = 180° - 130° = 50°$

$x = 50°$



Question 13:

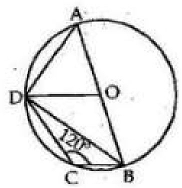
AB is a diameter of a circle with centre O and DO ∥ CB, $\angle BCD = 120°$

(i) Since ABCD is a cyclic quadrilateral

$\therefore \quad \angle BCD + \angle BAD = 180°$

$\Rightarrow \quad 120° + \angle BAD = 180°$

$\Rightarrow \quad \angle BAD = 180° - 120° = 60°$



(ii) $\angle BDA = 90°$ [angle in a semi circle]

In ΔABD we have

$\angle BDA + \angle BAD + \angle ABD = 180°$

$\Rightarrow \quad 90° + 60° + \angle ABD = 180°$

$\Rightarrow \quad \angle ABD = 180° - 150° = 30°$

(iii) OD = OA.

$\Rightarrow \quad \angle ODA = \angle OAD = \angle BAD = 60°$

$\therefore \quad \angle ODB = 90° - \angle ODA$

$= 90° - 60° = 30°$

Since DO ∥ CB, alternate angles are equal

$\Rightarrow \quad \angle CBD = \angle ODB$

$= 30°$

(iv) $\angle ADC = \angle ADB + \angle CDB$

$= 90° + 30° = 120°$

Also, in ΔAOD, we have

$\angle ODA + \angle OAD + \angle AOD = 180°$

$\Rightarrow \quad 60° + 60° + \angle AOD = 180°$

$\Rightarrow \quad \angle AOD = 180° - 120° = 60°$

Since all the angles of ΔAOD are of 60° each

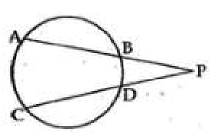
$\therefore \quad \Delta$ AOD is an equilateral triangle.

Question 14:

AB and CD are two chords of a circle which interect each other at P, outside the circle. AB = 6cm, BP = 2 cm and PD = 2.5 cm

Therefore, AP x BP = CP x DP

Or, 8 x 2 = (CD + 2.5) x 2.5 cm [as CP = CD + DP]



Let x = CD

Thus, $8 \times 2 = (x + 2.5) \times 2.5$

$\Rightarrow \quad 16 \text{ cm} = 2.5x + 6.25 \text{ cm}$

$\Rightarrow \quad 2.5x = (16 - 6.25) \text{cm}$

$\Rightarrow \quad 2.5x = 9.75 \text{cm}$

$\Rightarrow \quad x = \frac{9.75}{2.5} = 3.9 \text{cm}$

$\therefore \quad x = 3.9 \text{ cm}$

Therefore, CD = 3.9 cm

Question 15:

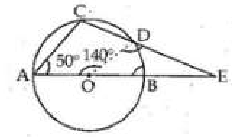
O is the centre of a circle having $\angle AOD = 140°$ and $\angle CAB = 50°$

(i) $\angle BOD = 180° - \angle AOD$

$= 180° - 140° = 40°$

$OB = OD$

$\therefore \quad \angle OBD = \angle ODB$



In ΔOBD, we have

$\angle BOD + \angle OBD + \angle ODB = 180°$

$\Rightarrow \angle BOD + \angle OBD + \angle OBD = 180° \quad [\because \angle OBD = \angle ODB]$

$\Rightarrow 40° + 2\angle OBD = 180° \quad [\because \angle BOD = 40°]$

$\Rightarrow 2\angle OBD = 180° - 40° = 140°$

$\Rightarrow \angle OBD = \angle ODB = \frac{140}{2} = 70°$

Also, $\angle CAB + \angle BDC = 180° \quad [\because ABCD \text{ is cyclic}]$

$\Rightarrow \angle CAB + \angle ODB + \angle ODC = 180°$

$\Rightarrow 50° + 70° + \angle ODC = 180°$

$\Rightarrow \angle ODC = 180° - 120° = 60°$

$\therefore \angle ODC = 60°$

$\therefore \angle EDB = 180° - (\angle ODC + \angle ODB)$

$= 180° - (60° + 70°)$

$= 180° - 130° = 50°$

(ii) $\angle EBD = 180° - \angle OBD$

$= 180° - 70° = 110°$

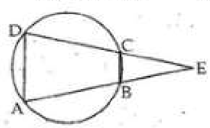
Question 16:

Consider the triangles, ΔEBC and ΔEDA

Side AB of the cyclic quadrilateral ABCD is produced to E

$\therefore \quad \angle EBC = \angle CDA$

$\Rightarrow \quad \angle EBC = \angle EDA \quad .....(i)$



Again, side DC of the cyclic quadrilateral ABCD is produced to E.

$\therefore \quad \angle ECB = \angle BAD$

$\Rightarrow \quad \angle ECB = \angle EAD \quad .....(ii)$

and $\quad \angle BEC = \angle DEA \quad$ [each equal to $\angle E$]....(iii)

Thus from (i), (ii) and (iii), we have

$\therefore \quad \Delta EBC \cong \Delta EDA$

Question 17:

ΔABC is an isosceles triangle in which $AB = AC$ and a circle passing through B and C intersects AB and AC at D and E.

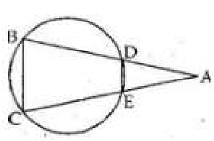
Since $\quad AB = AC$

$\therefore \quad \angle ACB = \angle ABC$

So, ext. $\quad \angle ADE = \angle ACB = \angle ABC$

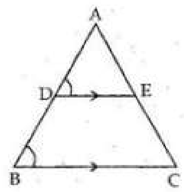
$\therefore \quad \angle ADE = \angle ABC$

$\Rightarrow \quad DE \parallel BC.$



Question 18:

ΔABC is an isosceles triangle in which AB = AC. D and E are the mid points of AB and AC respectively.



$\therefore$	$DE \parallel BC$	
$\Rightarrow$	$\angle ADE = \angle ABC$	....(i)
Also,	$AB = AC$	[Given]
$\Rightarrow$	$\angle ABC = \angle ACB$	.....(ii)
$\therefore$	$\angle ADE = \angle ACB$	[From (i) and (ii)]

Now, $\angle ADE + \angle EDB = 180°$ [$\because$ ADB is a straight line]

$\therefore$ $\angle ACB + \angle EDB = 180°$

$\Rightarrow$ The opposite angles are supplementary.

$\Rightarrow$ D, B, C and E are concyclic

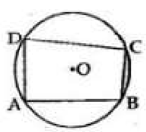
i.e. D, B, C and E is a cyclic quadrilateral.

Question 19:

Let ABCD be a cyclic quadrilateral and let O be the centre of the circle passing through A, B, C, D.

Then each of AB, BC, CD and DA being a chord of the circle, its right bisector must pass through O.

$\therefore$ the right bisectors of AB, BC, CD and DA pass through and are concurrent.



Question 20:

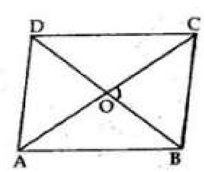
ABCD is a rhombus.

Let the diagonals AC and BD of the rhombus ABCD intersect at O.

But, we know, that the diagonals of a rhombus bisect each other at right angles.

So, $\angle BOC = 90°$

$\therefore$ $\angle BOC$ lies in a circle.

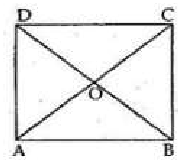


Thus the circle drawn with BC as diameter will pass through O

Similarly, all the circles described with AB, AD and CD as diameters will pass through O.

Question 21:

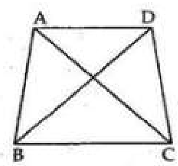
ABCD is a rectangle.
Let O be the point of intersection of the diagonals AC and BD of rectangle ABCD.



Since the diagonals of a rectangle are equal and bisect each other.
$\therefore OA = OB = OC = OD$
Thus, O is the centre of the circle through A, B, C, D.

Question 22:

Let A, B, C be the given points.
With B as centre and radius equal to AC draw an arc.
With C as centre and AB as radius draw another arc, which cuts the previous arc at D.



Then D is the required point BD and CD.
In ΔABC and ΔDCB

$AB = DC$
$AC = DB$
$BC = CB$ [common]
$\therefore \Delta ABC \cong \Delta DCB$ [by SSS]
$\Rightarrow \angle BAC = \angle CDB$ [C.P.C.T]

Thus, BC subtends equal angles, $\angle BAC$ and $\angle CDB$ on the same side of it.

$\therefore$ Points A, B, C, D are concyclic.

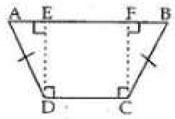
Question 23:

ABCD is a cyclic quadrilateral

$\angle B - \angle D = 60^\circ$ ......(i)
and $\angle B + \angle D = 180^\circ$ ....(ii)
Adding (i) and (ii) we get,
$2\angle B = 240^\circ$
$\therefore \angle B = \frac{240}{2} = 120^\circ$
Substituting the value of $\angle B = 120^\circ$ in (i) we get
$120^\circ - \angle D = 60^\circ$
$\Rightarrow \angle D = 120^\circ - 60^\circ = 60^\circ$
The smaller of the two angles i.e. $\angle D = 60^\circ$

Question 24:

ABCD is a quadrilateral in which $AD = BC$ and $\angle ADC = \angle BCD$
Draw $DE \perp AB$ and $CF \perp AB$



Now, in ΔADE and ΔBCF, we have

$\angle AED = \angle BFC$ [each equal to 90°]

$\angle ADE = \angle ADC - 90^\circ = \angle BCD - 90^\circ = \angle BCF$

$AD = BC$ [given]

Thus, by Angle-Angle-Side criterionof congruence, we have

$\therefore \quad \Delta ADE \cong \Delta BCF$ [by AAS congruence]

The corresponding parts of the congruent triangles are equal.

$\therefore \quad \angle A = \angle B$

Now, $\angle A + \angle B + \angle C + \angle D = 360^\circ$

$\Rightarrow \quad 2\angle B + 2\angle D = 360^\circ$

$\Rightarrow \quad 2(\angle B + \angle D) = 360^\circ$

$\Rightarrow \quad \angle B + \angle D = \frac{360}{2} = 180^\circ$

$\therefore$ ABCD is a cyclic quadrilateral.

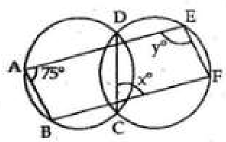
Question 25:

If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

$\Rightarrow \quad \angle BAD = \angle DCF = 75^\circ$

$\therefore \quad \angle DCF = x = 75^\circ$

$\therefore \quad x = 75^\circ$



The opposite angles of the opposite angles of a cyclic quadrilateral is 180°

$\Rightarrow \quad \angle DCF + \angle DEF = 180^\circ$

$\Rightarrow \quad 75^\circ + \angle DEF = 180^\circ$

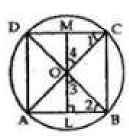
$\Rightarrow \quad \angle DEF = 180^\circ - 75^\circ = 105^\circ$

As $\quad \angle DEF = y^\circ = 105^\circ$

$\therefore \quad x = 75^\circ$ and $y = 105^\circ$

Question 26:

Given: Let ABCD be a cyclic quadrilateral whose diagonals AC and BD intersect at O at right angles.
Let $OL \perp AB$ such that LO produced meets CD at M.



To Prove: $CM = MD$

Proof: $\angle 1 = \angle 2$ [angles in the same segment]

$\angle 2 + \angle 3 = 90^\circ$ [$\because \angle OLB = 90^\circ$]

$\angle 3 + \angle 4 = 90^\circ$ [$\because$ LOM is a straight line and $\angle BOC = 90^\circ$]

$\therefore \quad \angle 2 + \angle 3 = \angle 3 + \angle 4$

$\Rightarrow \quad \angle 2 = \angle 4$

Thus, $\quad \angle 1 = \angle 2$

and $\quad \angle 2 = \angle 4$

$\Rightarrow \quad \angle 1 = \angle 4$

$\therefore \quad OM = CM$

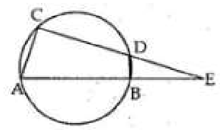
Similarly, $\quad OM = MD$

Hence, $\quad CM = MD$.

Question 27:

Chord AB of a circle is produced to E.
If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

$\therefore$ Ext. $\angle BDE = \angle BAC = \angle EAC$ ....(1)



Chord CD of a circle is produced to E

$\therefore$ Ext. $\angle DBE = \angle ACD = \angle ACE$.....(2)

Consider the triangles ΔEDB and ΔEAC.

$\angle BDE = \angle CAE$ [from(1)]

$\angle DBE = \angle ACE$ [from(2)]

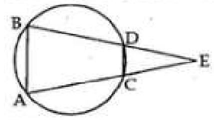
$\angle E = \angle E$ [common]

$\therefore$ $\Delta EDB \sim \Delta EAC$.

Question 28:

Given: AB and CD are two parallel chords of a circle BDE and ACE are straight lines which intersect at E.
If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

$\therefore$ Ext$\angle EDC = \angle A$ and, Ext$\angle DCE = \angle B$



Also, AB ∥ CD

$\Rightarrow$ $\angle EDC = \angle B$

and $\angle DCE = \angle A$

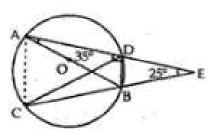
$\therefore$ $\angle A = \angle B$

$\therefore$ Δ AEB is isosceles.

Question 29:

AB is a diameter of a circle with centre O. ADE and CBE are straight lines, meeting at E, such that $\angle BAD = 35°$ and $\angle BED = 25°$.

Join BD and AC.



(i) Now, $\angle BDA = 90° = \angle EDB$ [angle in a semi circle]

$\Rightarrow$ $\angle EBD = 180° - (\angle EDB + \angle BED)$

$= 180° - (90° + 25°)$

$= 180° - 115° = 65°$

$\therefore$ $\angle DBC = (180° - \angle EBD)$

$= 180° - 65° = 115°$

$\therefore$ $\angle DBC = 115°$

(ii) Again, $\angle DCB = \angle BAD$ [angle in the same segment]

Since, $\angle BAD = 35°$

$\therefore$ $\angle DCB = 35°$

(iii) $\angle BDC = 180° - (\angle DBC + \angle DCB)$

$= 180° - (\angle DBC + \angle BAD)$

$= 180° - (115° + 35°)$

$= 180° - 150° = 30°$

$\therefore$ $\angle BDC = 30°$