RD SHARMA
Solutions
Class 9 Maths
Chapter 2
Ex *2.1*

1. Simplify the following:

(i) $3(a^4 b^3)^{10} \times 5 (a^2 b^2)^3$

Solution:

$= 3(a^{40} b^{30}) \times 5 (a^6 b^6)$

$= 15 (a^{46} b^{36})$

(ii) $(2x^{-2} y^3)^3$

Solution:

$= (2^3 x^{-2\times 3} y^{3\times 3})$

$= 8x^{-6} y^9$

(iii) $\frac{(4\times 10^7)(6\times 10^{-5})}{8\times 10^4}$

Solution:

$\frac{(4\times 10^7)(6\times 10^{-5})}{8\times 10^4}$

$= \frac{(24\times 10^7\times 10^{-5})}{8\times 10^4}$

$= \frac{(24\times 10^{7-5})}{8\times 10^4}$

$= \frac{(24\times 10^2)}{8\times 10^4}$

$= \frac{(3\times 10^2)}{10^4}$

$= \frac{3}{100}$

(iv) $\frac{4ab^2(-5ab^3)}{10a^2b^2}$

Solution:

$= \frac{-20a^2b^5}{10a^2b^2}$

$= -2b^3$

(v) $(\frac{x^2y^2}{a^2b^3})^n$

Solution:

$= \frac{x^{2n}y^{2n}}{a^{2n}b^{3n}}$

(vi) $\frac{(a^{3n-9})^6}{a^{2n-4}}$

Solution:

$= \frac{a^{18n-54}}{a^{2n-4}}$

$= a^{18n-2n-54+4}$

$= a^{16n-50}$

2. If a = 3 and b = -2, find the values of:

(i) $a^a + b^b$

(ii) $a^b + b^a$

(iii) $a^b + b^a$

Solution:

(i) We have,

$a^a + b^b$

$= 3^3 + (-2)^{-2}$

$= 3^3 + (-\frac{1}{2})^2$

$= 27 + \frac{1}{4}$

$= \frac{109}{4}$

(ii) $a^b + b^a$

$= 3^{-2} + (-2)^3$

$= (\frac{1}{3})^2 + (-2)^3$

$= \frac{1}{9} - 8$

$= -\frac{71}{9}$

(iii) We have,

$a^b + b^a$

$= (3 + (-2))^{3(-2)}$

$= (3 - 2))^{-6}$

$= 1^{-6}$

= 1

3.Prove that:

(i) $(\frac{x^a}{x^b})^{a^2+ab+b^2} \times (\frac{x^b}{x^c})^{b^2+bc+c^2} \times (\frac{x^c}{x^a})^{c^2+ca+a^2}$ **= 1**

(ii) $\left(\frac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \times \left(\frac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \times \left(\frac{x^c}{x^{-a}}\right)^{c^2-ca+a^2}$ **=** $x^{2(a^3+b^3+c^3)}$

(iii) $\left(\frac{x^a}{x^b}\right)^c \times \left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b$ **= 1**

Solution:

(i) To prove

$\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \times \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2} = 1$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{x^{a^3+a^2b+ab^2}}{x^{a^2b+ab^2+b^3}} \times \frac{x^{b^3+b^2c+bc^2}}{x^{b^2c+bc^2+c^3}} \times \frac{x^{c^3+c^2a+ca^2}}{x^{c^2a+ca^2+a^3}}$$
$$x^{a^3+a^2b+ab^2-(b^3+a^2b+ab^2)} \times x^{b^3+b^2c+bc^2-(c^3+b^2c+bc^2)} \times x^{c^3+c^2a+ca^2-(a^3+c^2a+ca^2)}$$
$$x^{a^3-b^3} \times x^{b^3-c^3} \times x^{c^3-a^3}$$
$$x^{a^3-b^3+b^3-c^3+c^3-a^3}$$
$$x^0$$
$$1$$

Or,

Therefore, LHS = RHS

Hence proved

(ii) To prove,

$\left(\frac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \times \left(\frac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \times \left(\frac{x^c}{x^{-a}}\right)^{c^2-ca+a^2} = x^{2(a^3+b^3+c^3)}$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$x^{(a+b)(a^2-ab+b^2)} \times x^{(b+c)(b^2-bc+c^2)} \times x^{(c+a)(c^2-ca+a^2)}$$
$$x^{a^3+b^3} \times x^{b^3+c^3} \times x^{c^3+a^3}$$
$$x^{a^3+b^3+b^3+c^3+c^3+a^3}$$
$$x^{2(a^3+b^3+c^3)}$$

Therefore, LHS = RHS

Hence proved

(iii) To prove,

$\left(\frac{x^a}{x^b}\right)^c \times \left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b = 1$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\left(\frac{x^{ac}}{x^{bc}}\right) \times \left(\frac{x^{ba}}{x^{ca}}\right) \times \left(\frac{x^{bc}}{x^{ab}}\right)$$

$x^{ac-bc} \times x^{ba-ca} \times x^{bc-ab}$
$x^{ac-bc+ba-ca+bc-ab}$
x^0
1

Therefore, LHS = RHS

Hence proved

4. Prove that:

(i) $\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}}$ **= 1**

(ii) $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}}$

Solution:

(i) $\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}} = 1$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{1}{1+\frac{x^a}{x^b}} + \frac{1}{1+\frac{x^b}{x^a}}$$
$$\frac{x^b}{x^b+x^a} + \frac{x^a}{x^a+x^b}$$
$$\frac{x^b+x^a}{x^a+x^b}$$
1

Therefore, LHS = RHS

Hence proved

(ii) $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}}$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}} + \frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}}$$
$$\frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^b+x^a+x^c} + \frac{x^c}{x^c+x^b+x^a}$$
$$\frac{x^a+x^b+x^c}{x^a+x^b+x^c}$$
1

Therefore, LHS = RHS

Hence proved

5.Prove that:

(i) $\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}$ **= abc**

(ii) $(a^{-1} + b^{-1})^{-1}$

Solution:

(i) To prove,

$\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}$ = abc

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{a+b+c}{\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}}$$

$$\frac{a+b+c}{\frac{a+b+c}{abc}}$$

abc

Therefore, LHS = RHS

Hence proved

(ii) To prove,

$(a^{-1} + b^{-1})^{-1} = \frac{ab}{a+b}$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{1}{(a^{-1}+b^{-1})}$$

$$\frac{1}{(\frac{1}{a}+\frac{1}{b})}$$

$$\frac{1}{(\frac{a+b}{ab})}$$

$$\frac{ab}{a+b}$$

Therefore, LHS = RHS

Hence proved

6. If abc = 1, show that $\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}$ **= 1**

Solution:

To prove,

$\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}$ = 1

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{1}{1+a+\frac{1}{b}} + \frac{1}{1+b+\frac{1}{c}} + \frac{1}{1+c+\frac{1}{a}}$$

$$\frac{b}{b+ab+1} + \frac{c}{c+bc+1} + \frac{a}{a+ac+1} \quad ....(1)$$

We know abc = 1

$$c = \frac{1}{ab}$$

By substituting the value c in equation (1), we get

$$\frac{b}{b+ab+1} + \frac{\frac{1}{ab}}{\frac{1}{ab}+b(\frac{1}{ab})+1} + \frac{a}{a+a(\frac{1}{ab})+1}$$

$$\frac{b}{b+ab+1} + \frac{\frac{1}{ab}\times ab}{1+b+ab} + \frac{ab}{1+ab+b}$$

$$\frac{b}{b+ab+1} + \frac{1}{1+b+ab} + \frac{ab}{1+ab+b}$$

$$\frac{1+ab+b}{b+ab+1}$$

1

Therefore, LHS = RHS

Hence proved

7. Simplify:

(i) $\frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$

Solution:

$= \frac{3^n \times 9^n \times 9}{\frac{3^n}{3} \times \frac{9^n}{9}}$

= 9 x 3 x 9

= 243

(ii) $\frac{(5\times 25^{n+1})(25\times 5^{2n})}{(5\times 5^{2n+3})-(25)^{n+1}}$

Solution:

$= \frac{(5\times 25^n \times 25)-(25\times 25^n)}{(5\times 25^n \times 125)(25^n \times 25)}$

$= \frac{25^n \times 25(5-1)}{25^n \times 25(25-1)}$

$= \frac{4}{24}$

$= \frac{1}{6}$

(iii) $\frac{(5^{n+3})-(6\times 5^{n+1})}{(9\times 5^n)-(2^2\times 5^n)}$

Solution:

$= \frac{(5^{n+3})-(6\times5^{n+1})}{(9\times5^{n})-(2^{2}\times5^{n})}$

$= \frac{(5^{n}\times5^{3})-(6\times5^{n}\times5)}{(9\times5^{n})-(2^{2}\times5^{n})}$

$= \frac{5^{n}(125-30)}{5^{n}(9-4)}$

$= \frac{95}{5}$

= 19

(iv) $\frac{(6\times8^{n+1})+(16\times2^{3n-2})}{(10\times2^{3n+1})-7\times(8)^{n}}$

Solution:

$= \frac{(6\times8^{n}\times8)+(16\times8^{n}\times\frac{1}{4})}{(10\times8^{n}\times2)-(7\times(8)^{n})}$

$= \frac{8^{n}(48+4)}{8^{n}(20-7)}$

$= \frac{52}{13}$

= 4

Level 2

8. Solve the following equations for x:

(i) 7^{2x+3} **= 1**

(ii) $2^{x+1} = 4^{x-3}$

(iii) $2^{5x+3} = 8^{x+3}$

(iv) $4^{2x} = \frac{1}{32}$

(v) $4^{x-1} \times (0.5)^{3-2x} = (\frac{1}{8})^{x}$

(vi) 2^{3x-7} **= 256**

Solution:

(i) We have,

=> 7^{2x+3} = 1

=> 7^{2x+3} = 7^{0}

=> 2x + 3 = 0

=> 2x = -3

=> x = $-\frac{3}{2}$

(ii) We have,

$2^{x+1} = 4^{x-3}$

$2^{x+1} = 2^{2x-6}$

x + 1 = 2x − 6

x = 7

(iii) We have,

$2^{5x+3} = 8^{x+3}$

$2^{5x+3} = 2^{3x+9}$

5x + 3 = 3x + 9

2x = 6

x = 3

(iv) We have,

$4^{2x} = \frac{1}{32}$

$2^{4x} = \frac{1}{2^5}$

$2^{4x} = 2^{-5}$

4x = -5

x = $\frac{-5}{4}$

(v) We have,

$4^{x-1} \times (0.5)^{3-2x} = (\frac{1}{8})^x$

$2^{2x-2} \times (\frac{1}{2})^{3-2x} = (\frac{1}{2})^{3x}$

$2^{2x-2} \times 2^{2x-3} = (\frac{1}{2})^{3x}$

$2^{2x-2+2x-3} = (\frac{1}{2})^{3x}$

$2^{4x-5} = 2^{-3x}$

4x-5 = -3x

7x = 5

x = $\frac{5}{7}$

(vi) 2^{3x-7} = 256

2^{3x-7} = 2^8

3x − 7 = 8

3x = 15

x = 5

9. Solve the following equations for x:

(i) $2^{2x} - 2^{x+3} + 2^4 = 0$

(ii) $3^{2x+4} + 1 = 2 \times 3^{x+2}$

Solution:

(i) We have,

=> $2^{2x} - 2^{x+3} + 2^4 = 0$

=> $2^{2x} + 2^4 = 2^x.2^3$

=> Let $2^x = y$

=> $y^2 + 2^4 = y \times 2^3$

=> $y^2 - 8y + 16 = 0$

=> $y^2 - 4y - 4y + 16 = 0$

=> y(y-4) -4(y-4) = 0

=> y = 4

=> $x^2 = 2^2$

=> x = 2

(ii) We have,

$3^{2x+4} + 1 = 2 \times 3^{x+2}$
$(3^{x+2})^2 + 1 = 2 \times 3^{x+2}$
Let 3^{x+2} = y
$y^2 + 1 = 2y$
$y^2 - 2y + 1 = 0$
$y^2 - y - y + 1 = 0$
$y(y - 1) - 1(y - 1) = 0$
$(y - 1)(y - 1) = 0$
y = 1

10. If 49392 = $a^4b^2c^3$, find the values of a, b and c, where a, b and c, where a, b, and c are different positive primes.

Solution:

Taking out the LCM , the factors are $2^4, 3^2 and 7^3$

$a^4b^2c^3$ = $2^4, 3^2 and 7^3$

a = 2, b = 3 and c = 7 [Since, a, b and c are primes]

11. If 1176 = $2^a \times 3^b \times 7^c$, Find a, b, and c.

Solution:

Given that 2, 3 and 7 are factors of 1176.

Taking out the LCM of 1176, we get

$2^3 \times 3^1 \times 7^2$ = $2^a \times 3^b \times 7^c$

By comparing, we get

a = 3, b = 1 and c = 2.

12. Given 4725 = $3^{a} \times 5^{b} \times 7^{c}$, find

(i) The integral values of a, b and c

Solution:

Taking out the LCM of 4725, we get

$3^{3} \times 5^{2} \times 7^{1}$ = $3^{a} \times 5^{b} \times 7^{c}$

By comparing, we get

a = 3, b = 2 and c = 1.

(ii) The value of $2^{-a} \times 3^{b} \times 7^{c}$

Solution:

\(2^{-a} \times 3^{b}\times 7^{c} = [latex]2^{-3}\times 3^{2}\times 7^{1}\)\(2^{-3}\times 3^{2}\times 7^{1}\]”>
[latex]2^{-3}\times 3^{2}\times 7^{1}\) = $\frac{1}{8} \times 9 \times 7$
$\frac{63}{8}$

13. If a = xy^{p-1}, b = xy^{q-1} and c = xy^{r-1}, prove that $a^{q-r}b^{r-p}c^{p-q}$ = 1

Solution:

Given,

a = xy^{p-1}, b = xy^{q-1} and c = xy^{r-1}

To prove, $a^{q-r}b^{r-p}c^{p-q}$ = 1

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

= $a^{q-r}b^{r-p}c^{p-q}$ ……(i)

By substituting the value of a, b and c in equation (i), we get

= $(xy^{p-1})^{q-r}(xy^{q-1})^{r-p}(xy^{r-1})^{p-q}$

= $xy^{pq-pr-q+r}xy^{qr-pq-r+p}xy^{rp-rq-p+q}$

= $xy^{pq-pr-q+r+qr-pq-r+p+rp-rq-p+q}$

= xy^{0}

= 1