Exercise 13.1

Question 1: Evaluate the Given limit: $\lim_{x\to 3} x+3$

Solution 1: $\lim_{x\to 3} x+3 = 3 + 3 = 6$

Question 2: Evaluate the Given limit: $\lim_{x\to \pi}\left(x-\frac{22}{7}\right)$

Solution 2: $\lim_{x\to \pi}\left(x-\frac{22}{7}\right)=\left(\pi-\frac{22}{7}\right)$

Question 3: Evaluate the Given limit: $\lim_{r\to 1} \pi r^2$

Solution 3: $\lim_{r\to 1} \pi r^2 = \pi(1^2) = \pi$

Question 4: Evaluate the Given limit: $\lim_{x\to 1}\frac{4x+3}{x-2}$

Solution 4: $\lim_{x\to 1}\frac{4x+3}{x-2}=\frac{4(4)+3}{4-2}=\frac{16+3}{2}=\frac{19}{2}$

Question 5: Evaluate the Given limit: $\lim_{x\to -1}\frac{x^{10}+x^5+1}{x-1}$

Solution 5: $\lim_{x\to -1}\frac{x^{10}+x^5+1}{x-1}=\frac{(-1)^{10}+(-1)^5+1}{-1-1}=\frac{1-1+1}{-2}=-\frac{1}{2}$

Question 6: Evaluate the Given limit: $\lim_{x\to 0}\frac{(x+1)^5-1}{x}$

Solution 6: $\lim_{x\to 0}\frac{(x+1)^5-1}{x}$

Put x + 1 = y so that y ⟶ 1 as x ⟶0.

Accordingly, $\lim\limits_{x\to 0}\dfrac{(x+1)^5-1}{x}=\lim\limits_{x\to 1}\dfrac{(y)^5-1}{y-1}$

$=5.1^{5-1}$ $\left[\lim\limits_{x\to a}\dfrac{x^n-a^n}{x-a}=na^{n-1}\right]$

= 5

$\therefore \lim\limits_{x\to 0}\dfrac{(x+1)^5-1}{x}=5$

Question 7: Evaluate the Given limit: $\lim\limits_{x\to 2}\dfrac{3x^2-x-10}{x^2-4}$

Solution 7: At x = 2, the value of the given rational function takes the form $\dfrac{0}{0}$

$\therefore \lim\limits_{x\to 2}\dfrac{3x^2-x-10}{x^2-4}=\lim\limits_{x\to 2}\dfrac{(x-2)(3x+5)}{(x-2)(x+2)}$

$=\lim\limits_{x\to 2}\dfrac{3x+5}{x+2}$

$=\dfrac{3(2)+5}{2+2}$

$=\dfrac{11}{4}$

Question 8: Evaluate the Given limit: $\lim\limits_{x\to 3}\dfrac{x^4-81}{2x^2-5x-3}$

Solution 8: At x = 2, the value of the given rational function takes the form $\dfrac{0}{0}$

$\therefore \lim\limits_{x\to 3}\dfrac{x^4-81}{2x^2-5x-3}=\lim\limits_{x\to 3}\dfrac{(x-3)(x+3)(x^2+9)}{(x-3)(2x+1)}$

$=\lim\limits_{x\to 3}\dfrac{(x+3)(x^2+9)}{(2x+1)}$

$=\dfrac{(3+3)(3^2+9)}{2(3)+1}$

$= \dfrac{6\times18}{7}$

$= \dfrac{108}{7}$

Question 9: Evaluate the Given limit: $\lim\limits_{x\to0}\dfrac{ax+b}{cx+1}$.

Solution 9:

$$\lim_{x\to0}\frac{ax+b}{cx+1}=\frac{a(0)+b}{c(0)+1}=b$$

Question 10: Evaluate the Given limit: $\lim\limits_{z\to1}\dfrac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}$

Solution 10: $\lim\limits_{z\to1}\dfrac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}$

At z = 1, the value of the given function takes the form $\dfrac{0}{0}$

Put $z^{\frac{1}{6}}$ = x so that z →1 as x →1.

Accordingly, $\lim\limits_{z\to1}\dfrac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}=\lim\limits_{x\to1}\dfrac{x^2-1}{x-1}$

$= \lim\limits_{x\to1}\dfrac{x^2-1}{x-1}$

$= 2.1^{2-1}$ $\left[\lim\limits_{x\to a}\dfrac{x^n-a^n}{x-a}=na^{n-1}\right]$

= 2

$$\lim_{z\to1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}=2$$

Question 11: Evaluate the Given limit: $\lim_{x \to 1} \frac{ax^2 + bx + c}{cx^2 + bx + a}$, a + b + c ≠ 0

Solution 11: $\lim_{x \to 1} \frac{ax^2 + bx + c}{cx^2 + bx + a} = \frac{a(1)^2 + b(1) + c}{c(1)^2 + b(1) + a}$

$= \frac{a+b+c}{a+b+c}$

= 1 [a + b + c ≠ 0]

Question 12: Evaluate the given limit: $\lim_{x \to -2} \frac{\frac{1}{x} + \frac{1}{2}}{x+2}$

Solution 12: $\lim_{x \to -2} \frac{\frac{1}{x} + \frac{1}{2}}{x+2}$

At x = -2, the value of the given function takes the form $\frac{0}{0}$

Now, $\lim_{x \to -2} \frac{\frac{1}{x} + \frac{1}{2}}{x+2} = \lim_{x \to -2} \frac{\left(\frac{2+x}{2x}\right)}{x+2}$

$= \lim_{x \to -2} \frac{1}{2x}$

$= \frac{1}{2(-2)} = \frac{-1}{4}$

Question 13: Evaluate the Given limit: $\lim_{x \to 0} \frac{\sin ax}{bx}$

Solution 13: $\lim_{x \to 0} \frac{\sin ax}{bx}$

At x = 0, the value of the given function takes the form $\frac{0}{0}$

Now, $\lim_{x \to 0} \frac{\sin ax}{bx} = \lim_{x \to 0} \frac{\sin ax}{ax} \times \frac{ax}{bx}$

$$= \lim_{x \to 0}\left(\frac{\sin ax}{ax}\right) \times \frac{a}{b}$$

$$= \frac{a}{b}\lim_{ax \to 0}\left(\frac{\sin ax}{ax}\right) \qquad [x \to 0 \Rightarrow ax \to 0]$$

$$= \frac{a}{b} \times 1 \qquad \left[\lim_{x \to 0}\left(\frac{\sin y}{y}\right)\right]$$

$$= \frac{a}{b}$$

Question 14: Evaluate the given limit: $\lim_{x \to 0}\frac{\sin ax}{\sin bx}, a, b \neq 0$

Solution 14: $\lim_{x \to 0}\frac{\sin ax}{\sin bx}, a, b \neq 0$

At x = 0, the value of the given function takes the form $\frac{0}{0}$

Now, $\lim_{x \to 0}\frac{\sin ax}{\sin bx} = \lim_{x \to 0}\frac{\left(\frac{\sin ax}{ax}\right) \times ax}{\left(\frac{\sin bx}{ax}\right) \times bx}$

$$= \frac{a}{b} \times \frac{\lim_{ax \to 0}\left(\frac{\sin ax}{ax}\right)}{\lim_{bx \to 0}\left(\frac{\sin bx}{ax}\right)} \qquad \left[\begin{array}{l} x \to 0 \Rightarrow ax \to 0 \\ and\ x \to 0 \Rightarrow bx \to 0 \end{array}\right]$$

$$= \frac{a}{b} \times \frac{1}{1} \qquad \left[\lim_{x \to 0}\left(\frac{\sin y}{y}\right) = 1\right]$$

$$= \frac{a}{b}$$

Question 15: Evaluate the Given limit: $\lim_{x \to \pi}\frac{\sin(\pi - x)}{\pi(\pi - x)}$

Solution 15: $\lim_{x \to \pi}\frac{\sin(\pi - x)}{\pi(\pi - x)}$

It is seen that $x \to \pi \Rightarrow (\pi - x) \to 0$

$$\therefore \lim_{x\to\pi}\frac{\sin(\pi-x)}{\pi(\pi-x)}=\frac{1}{\pi}\lim_{(\pi-x)\to 0}\frac{\sin(\pi-x)}{(\pi-x)}$$

$$=\frac{1}{\pi}\times 1 \qquad \left[\lim_{y\to 0}\frac{\sin y}{y}=1\right]$$

$$=\frac{1}{\pi}$$

Question 16: Evaluate the given limit: $\lim_{x\to 0}\frac{\cos x}{\pi-x}$

Solution 16: $\lim_{x\to 0}\frac{\cos x}{\pi-x}=\frac{\cos 0}{\pi-0}=\frac{1}{\pi}$

Question 17: Evaluate the Given limit: $\lim_{x\to 0}\frac{\cos 2x-1}{\cos x-1}$

Solution 17: $\lim_{x\to 0}\frac{\cos 2x-1}{\cos x-1}$

At x = 0, the value of the given function takes the form $\frac{0}{0}$

Now, $\lim_{x\to 0}\frac{\cos 2x-1}{\cos x-1}=\lim_{x\to 0}\frac{1-2\sin^2 x-1}{1-2\sin^2\frac{x}{2}-1}$ $\qquad \left[\cos x=1-2\sin^2\frac{x}{2}\right]$

$$=\lim_{x\to 0}\frac{\sin^2 x}{\sin^2\frac{x}{2}}=\lim_{x\to 0}\frac{\left(\frac{\sin^2 x}{x^2}\right)\times x^2}{\left(\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right)\times\frac{x^2}{4}}$$

$$=4\frac{\lim_{x\to 0}\left(\frac{\sin^2 x}{x^2}\right)}{\lim_{x\to 0}\left(\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right)}$$

$$= 4\frac{\left(\lim_{x\to 0}\frac{\sin^2 x}{x^2}\right)^2}{\left(\lim_{x\to 0}\frac{\sin\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right)^2} \qquad \left[x\to 0 \Rightarrow \frac{x}{2}\to 0\right]$$

$$= 4\frac{1^2}{1^2} \qquad \left[\lim_{y\to 0}\frac{\sin y}{y}=1\right]$$

= 4

Question 18: Evaluate the given limit: $\lim_{x\to 0}\frac{ax + x\cos x}{b\sin x}$

Solution 18: $\lim_{x\to 0}\frac{ax + x\cos x}{b\sin x}$

At x = 0, the value of the given function takes the form $\frac{0}{0}$

Now, $\lim_{x\to 0}\frac{ax + x\cos x}{b\sin x} = \frac{1}{b}\lim_{x\to 0}\frac{x(a+\cos x)}{\sin x}$

$$= \frac{1}{b}\lim_{x\to 0}\left(\frac{x}{\sin x}\right)\times\lim_{x\to 0}\left(a+\cos x\right)$$

$$= \frac{1}{b}\left(\frac{1}{\lim_{x\to 0}\left(\frac{\sin x}{x}\right)}\right)\times\lim_{x\to 0}\left(a+\cos x\right)$$

$$= \frac{1}{b}\times\left(a+\cos 0\right) \qquad \left[\lim_{y\to 0}\frac{\sin x}{x}=1\right]$$

$$= \frac{a+1}{b}$$

Question 19: Evaluate the given limit: $\lim_{x\to 0} x\sec x$

Solution 19: $\lim_{x\to 0} x\sec x = \lim_{x\to 0} \frac{x}{\cos x} = \frac{0}{\cos 0} = \frac{0}{1} = 0$

Question 20: Evaluate the given limit: $\lim_{x\to 0} \frac{\sin ax + bx}{ax + \sin bx}$ a, b, a + b$\neq 0$

Solution 20: At x = 0, the value of the given function takes the form $\frac{0}{0}$

Now, $\lim_{x\to 0} \frac{\sin ax + bx}{ax + \sin bx}$

$$= \lim_{x\to 0} \frac{\left(\frac{\sin ax}{ax}\right) ax + bx}{ax + bx\left(\frac{\sin bx}{bx}\right)}$$

$$= \frac{\left(\lim_{x\to 0} \frac{\sin ax}{ax}\right) \times \lim_{x\to 0}(ax) + \lim_{x\to 0}(bx)}{\lim_{x\to 0} ax + \lim_{x\to 0} bx \left(\lim_{x\to 0} \frac{\sin bx}{bx}\right)}$$ [As x $\to 0 \Rightarrow$ ax $\to 0$ and bx $\to 0$]

$$= \frac{\lim_{x\to 0}(ax) + \lim_{x\to 0} bx}{\lim_{x\to 0} ax + \lim_{x\to 0} bx}$$ $\left[\lim_{y\to 0} \frac{\sin x}{x} = 1\right]$

$$= \frac{\lim_{x\to 0}(ax + bx)}{\lim_{x\to 0}(ax + bx)}$$

$$= \lim_{x\to 0}(1)$$

= 1

Question 21: Evaluate the given limit: $\lim_{x\to 0}(\operatorname{cosec} x - \cot x)$

Solution 21: At x = 0, the value of the given function takes the form $\infty - \infty$

Now, $\lim_{x\to 0}(\operatorname{cosec} x - \cot x)$

$$= \lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)$$

$$= \lim_{x\to 0}\left(\frac{1 - \cos x}{\sin x}\right)$$

$$= \lim_{x\to 0} \frac{\left(\frac{1 - \cos x}{x}\right)}{\left(\frac{\sin x}{x}\right)}$$

$$= \frac{\lim_{x \to 0} \frac{1-\cos x}{x}}{\lim_{x \to 0} \frac{\sin x}{x}}$$

$$= \frac{0}{1} \qquad \left[\lim_{y \to 0} \frac{1-\cos x}{x} = 0 \text{ and } \lim_{y \to 0} \frac{\sin x}{x} = 1\right]$$

$= 0$

Question 22: Evaluate the given limit: $\lim_{x \to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$

Solution 22: $\lim_{x \to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$

At x = $\frac{\pi}{2}$, the value of the given function takes the form $\frac{0}{0}$

Now, put So that x – $\frac{\pi}{2}$ = y so that x → $\frac{\pi}{2}$, y → 0

$$\therefore \lim_{x \to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}} = \lim_{y \to 0} \frac{\tan 2\left(y + \frac{\pi}{2}\right)}{y}$$

$$= \lim_{y \to 0} \frac{\tan(\pi + 2y)}{y}$$

$$= \lim_{y \to 0} \frac{\tan 2y}{y} \qquad [\tan(\pi + 2y) = \tan 2y]$$

$$= \lim_{y \to 0} \frac{\sin 2y}{y \cos 2y}$$

$$= \lim_{y \to 0} \left(\frac{\sin 2y}{2y} \times \frac{2}{\cos 2y}\right)$$

$$= \left(\lim_{y \to 0} \frac{\sin 2y}{2y}\right) \times \lim_{y \to 0} \left(\times \frac{2}{\cos 2y}\right) \qquad [y \to 0 \Rightarrow 2y \to 0]$$

$$= 1 \times \frac{2}{\cos 0} \qquad \left[\lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= 1 \times \frac{2}{1}$$

$= 2$

Question 23: Find $\lim_{x \to 0} f(x)$ and $\lim_{x \to 1} f(x)$, where f(x) = $\begin{cases} 2x+3, & x \le 0 \\ 3(x+1), & x > 0 \end{cases}$

Solution 23: The given function is

$$f(x) = \begin{cases} 2x+3, & x \le 0 \\ 3(x+1), & x > 0 \end{cases}$$

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0} [2x + 3] = 2(0) + 3 = 3$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0} 3(x + 1) = 3(0 + 1) = 3$

$\therefore \lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0} f(x) = 3$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1} (x + 1) = 3(1 + 1) = 6$

$\therefore \lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1} f(x) = 6$

Question 24: Find $\lim\limits_{x\to 1} f(x)$, when $f(x) = \begin{cases} x^2-1, & x \le 1 \\ -x-1, & x > 1 \end{cases}$

Solution 24:

The given function is

$$f(x) = \begin{cases} x^2-1, & x \le 1 \\ -x-1, & x > 1 \end{cases}$$

$\therefore \lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1} \left[x^2-1\right] = 1^2 - 1 = 1 - 1 = 0$

It is observed that $\lim\limits_{x\to 1^-} f(x) \ne \lim\limits_{x\to 1^+} f(x)$.

Hence, $\lim\limits_{x\to 1} f(x)$ does not exist.

Question 25: Evaluate $\lim\limits_{x\to 0} f(x)$, where $f(x) = \begin{cases} \dfrac{|x|}{x}, & x \ne 0 \\ 0, & x = 0 \end{cases}$

Solution 25: The given function is

$$f(x) = \begin{cases} \dfrac{|x|}{x}, & x \ne 0 \\ 0, & x = 0 \end{cases}$$

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-}\left[\dfrac{|x|}{x}\right]$

$= \lim\limits_{x\to 0}\left(\dfrac{-x}{x}\right)$ [When x is negative, $|x|$ = -x]

$= \lim\limits_{x\to 0}(-1)$

= -1

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left[\frac{|x|}{x}\right]$$

$= \lim_{x\to 0}\left(\frac{x}{x}\right)$ [When x is positive, $|x|$ = x]

$= \lim_{x\to 0}(1)$

= 1

It is observed that $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$.

Hence, $\lim_{x\to 0} f(x)$ does not exist.

Question 26: Find $\lim_{x\to 0} f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Solution 26: The given function is

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\left[\frac{x}{|x|}\right]$$

$= \lim_{x\to 0}\left(\frac{x}{-x}\right)$ [When x < 0, $|x|$ = -x]

$= \lim_{x\to 0}(-1)$

= -1

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left[\frac{x}{|x|}\right]$$

$= \lim_{x\to 0}\left(\frac{x}{x}\right)$ [When x > 0, $|x|$ = x]

$= \lim_{x\to 0}(1)$

= 1

It is observed that $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$.

Hence, $\lim_{x\to 0} f(x)$ does not exist.

Question 27: Find $\lim_{x\to 5} f(x)$, where f(x) = $|x|$ - 5

Solution 27: The given function is f(x) = $|x|$ - 5

$$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^-}(|x| - 5)$$

$= \lim_{x\to 5}(x - 5)$ [When x > 0, $|x|$ = x]

= 5- 5

= 0

$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} (|x| - 5)$

$= \lim_{x \to 5} (x - 5)$ [When x > 0, $|x|$ = x]

= 5 – 5

= 0

$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x) = 0$

Hence, $\lim_{x \to 5} f(x) = 0$

Question 28: Suppose f(x) = $\begin{cases} a + bx, & x < 0 \\ 4, & x = 1 \\ b - ax, & x > 1 \end{cases}$ and if $\lim_{x \to 1} f(x) = f(1)$ what are possible values of a and b?

Solution 28: The given function is

$$f(x) = \begin{cases} a + bx, & x < 0 \\ 4, & x = 1 \\ b - ax, & x > 1 \end{cases}$$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} (a + bx) = a + b$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} (b - ax) = b - a$

f(1) = 4

It is given that $\lim_{x \to 1} f(x) = f(1)$.

$\therefore \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = \lim_{x \to 1} f(x) = f(1)$

$\Rightarrow$ a + b = 4 and b – a = 4

On solving these two equations, we obtain a = 0 and b = 4.

Thus, the respective possible values of a and b are 0 and 4.

Question 29: Let $a_1, a_2, \ldots, a_n$ be fixed real numbers and define a function

$f(x) = (x - a_1)(x - a_2)\ldots(x - a_n)$

What is $\lim_{x \to a_1} f(x)$? For some $a \neq a_1, a_2, \ldots, a_n$. Compute $\lim_{x \to a} f(x)$.

Solution 29: The given function is $f(x) = (x - a_1)(x - a_2)\ldots(x - a_n)$

$\lim_{x \to a_1} f(x) = \lim_{x \to a_1} [(x - a_1)(x - a_2)\ldots(x - a_n)]$

$= (a_1 - a_1)(a_1 - a_2)\ldots(a_1 - a_n) = 0$

$\therefore \lim_{x \to a_1} f(x) = 0$

Now, $\lim_{x \to a} f(x) = \lim_{x \to a} [(x - a_1)(x - a_2)\ldots(x - a_n)]$

$= (a - a_1)(a - a_2)\ldots\ldots(a - a_n)$

$\therefore \lim_{x\to a} f(x) = (a - a_1)(a - a_2)\ldots\ldots(a - a_n)$

Question 30: If f(x) = $\begin{cases} |x|+1, & x<0 \\ 0, & x=0 \\ |x|-1, & x>1 \end{cases}$.

For what value (s) of does $\lim_{x\to a} f(x)$ exists?

Solution 30: The given function is

If f(x) = $\begin{cases} |x|+1, & x<0 \\ 0, & x=0 \\ |x|-1, & x>1 \end{cases}$.

When a = 0,

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(|x|+1)$

$= \lim_{x\to 0}(-x+1)$ [If x < 0, $|x|$ = –x]

= 0 + 1

= 1

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(|x|+1)$

$= \lim_{x\to 0}(x-1)$ [If x > 0, $|x|$ = –x]

= 0 – 1

= –1

Here, it is observed that $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$.

$\therefore \lim_{x\to 0} f(x)$ does not exist.

When a < 0,

$\lim_{x\to a^-} f(x) = \lim_{x\to a^-}(|x|+1)$

$= \lim_{x\to a}(-x+1)$ [x < a < 0 $\Rightarrow$ $|x|$ = –x]

= –a + 1

$\lim_{x\to a^+} f(x) = \lim_{x\to a^+}(|x|+1)$

$= \lim_{x\to a}(-x+1)$ [a < x < 0 $\Rightarrow$ $|x|$ = –x]

= –a + 1

$\therefore \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = -a + 1$

Thus, limit of f(x) exists at x = a, where a < 0.
When a > 0

$\lim_{x \to a^-} f(x) = \lim_{x \to a^-} (|x| + 1)$

$= \lim_{x \to a} (-x - 1)$ [0 < x < a $\Rightarrow$ $|x|$ = x]

= a – 1

$\lim_{x \to a^+} f(x) = \lim_{x \to a^+} (|x| - 1)$

$= \lim_{x \to a} (-x - 1)$ [0 < x < a $\Rightarrow$ $|x|$ = x]

= a – 1

$\therefore \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = a - 1$

Thus, limit of f(x) exists at x = a, where a > 0.
Thus, $\lim_{x \to a} f(x)$ exists for all a $\neq 0$.

Question 31: If the function f(x) satisfies, $\lim_{x \to 1} \frac{f(x) - 2}{x^2 - 1} = \pi$, evaluate $\lim_{x \to 1} f(x)$.

Solution 31: $\lim_{x \to 1} \frac{f(x) - 2}{x^2 - 1} = \pi$

$$\Rightarrow \frac{\lim_{x \to 1} \left(f(x) - 2 \right)}{\lim_{x \to 1} \left(x^2 - 1 \right)} = \pi$$

$$\Rightarrow \lim_{x \to 1} \left(f(x) - 2 \right) = \pi \lim_{x \to 1} \left(x^2 - 1 \right)$$

$$\Rightarrow \lim_{x \to 1} \left(f(x) - 2 \right) = \pi \left(1^2 - 1 \right)$$

$$\Rightarrow \lim_{x \to 1} \left(f(x) - 2 \right) = 0$$

$$\Rightarrow \lim_{x \to 1} f(x) - \lim_{x \to 1} 2 = 0$$

$$\Rightarrow \lim_{x \to 1} f(x) - 2 = 0$$

$$\therefore \lim_{x \to 1} f(x) = 2$$

Question 32: If f(x) = $\begin{cases} mx^2+n, & x<0 \\ nx+m, & 0\le x\le 1. \\ nx^3+m, & x>1 \end{cases}$

For what integers m and n does $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$ exists?

Solution 32: f(x) = $\begin{cases} mx^2+n, & x<0 \\ nx+m, & 0\le x\le 1.. \\ nx^3+m, & x>1 \end{cases}$

$\lim_{x\to 0} f(x) = \lim_{x\to 0}(mx^2+n)$

= $m(0)^2+n$

= n

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0}(nx+m)$

= $n(0)+m$

= m

Thus, $\lim_{x\to 0^+} f(x)$ exists if m = n.

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1}(nx+m)$

= n (1) + m

= m + n

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1}(nx^3+m)$

= n $(1)^3$ + m

= m + n

$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1} f(x)$.

Thus, $\lim_{x\to 1} f(x)$ exists for any internal value of m and n.