

Exercise 9B

1.

Sol:

We prepare the cumulative frequency table, as shown below:

Age (in years)	Number of patients (f_i)	Cumulative Frequency (cf)
0 – 15	5	5
15 – 30	20	25
30 – 45	40	65
45 – 60	50	115
60 – 75	25	140
Total	$N = \sum f_i = 140$	

Now, $N = 140 \Rightarrow \frac{N}{2} = 70$.

The cumulative frequency just greater than 70 is 115 and the corresponding class is 45 – 60.

Thus, the median class is 45 – 60.

$\therefore l = 45, h = 15, f = 50, N = 140$ and $cf = 65$.

Now,

$$\begin{aligned}\text{Median} &= l + \left(\frac{\frac{N}{2} - cf}{f} \right) \times h \\ &= 45 + \left(\frac{\frac{140}{2} - 65}{50} \right) \times 15 \\ &= 45 + \left(\frac{70 - 65}{50} \right) \times 15 \\ &= 45 + 1.5 \\ &= 46.5\end{aligned}$$

Hence, the median age is 46.5 years.

2.

Sol:

Class	Frequency (f)	Cumulative Frequency (cf)
0 – 7	3	3
7 – 14	4	7
14 – 21	7	14
21 – 28	11	25
28 – 35	0	25
35 – 42	16	41
42 – 49	9	50
	$N = \sum f = 50$	

Now, $N = 50 \Rightarrow \frac{N}{2} = 25$.

The cumulative frequency just greater than 25 is 41 and the corresponding class is 35 – 42. Thus, the median class is 35 – 42.

$\therefore l = 35, h = 7, f = 16, cf = \text{c.f. of preceding class} = 25$ and $\frac{N}{2} = 25$.

Now,

$$\begin{aligned}\text{Median} &= l + \left(\frac{\frac{N}{2} - cf}{f} \right) \times h \\ &= 35 + 7 \times \left(\frac{25 - 25}{16} \right) \\ &= 35 + 0 \\ &= 35\end{aligned}$$

Hence, the median age is 46.5 years.

3.

Sol:

Class	Frequency (f)	Cumulative Frequency (cf)
0 – 100	40	40
100 – 200	32	72
200 – 300	48	120
300 – 400	22	142
400 – 500	8	150
	$N = \sum f = 150$	

Now, $N = 150$

$$\Rightarrow \frac{N}{2} = 75.$$

The cumulative frequency just greater than 75 is 120 and the corresponding class is 200 – 300.

Thus, the median class is 200 – 300.

$$\therefore l = 200, h = 100, f = 48, cf = \text{c.f. of preceding class} = 72 \text{ and } \frac{N}{2} = 75.$$

Now,

$$\begin{aligned} \text{Median, } M &= l + \left\{ h \times \left(\frac{\frac{N}{2} - cf}{f} \right) \right\} \\ &= 200 + \left\{ 100 \times \left(\frac{75 - 72}{48} \right) \right\} \\ &= 200 + 6.25 \\ &= 206.25 \end{aligned}$$

Hence, the median daily wage income of the workers is Rs 206.25.

4.

Sol:

Class	Frequency (f)	Cumulative Frequency (cf)
5 – 10	5	5
10 – 15	6	11
15 – 20	15	26
20 – 25	10	36
25 – 30	5	41
30 – 35	4	45
35 – 40	2	47
40 – 45	2	49
	$N = \sum f = 49$	

Now, $N = 49$

$$\Rightarrow \frac{N}{2} = 24.5.$$

The cumulative frequency just greater than 24.5 is 26 and the corresponding class is 15 – 20.

Thus, the median class is 15 – 20.

$$\therefore l = 15, h = 5, f = 15, cf = \text{c.f. of preceding class} = 11 \text{ and } \frac{N}{2} = 24.5.$$

Now,

$$\begin{aligned} \text{Median, } M &= l + \left\{ h \times \left(\frac{\frac{N}{2} - cf}{f} \right) \right\} \\ &= 15 + \left\{ 5 \times \left(\frac{24.5 - 11}{15} \right) \right\} \end{aligned}$$

$$= 15 + 4.5$$

$$= 19.5$$

Hence, the median = 19.5.

5.

Sol:

Class	Frequency (f)	Cumulative Frequency (cf)
65- 85	4	4
85 – 105	5	9
105 – 125	13	22
125 – 145	20	42
145 – 165	14	56
165 – 185	7	63
185 – 205	4	67
	$N = \sum f = 67$	

Now, $N = 67$

$$\Rightarrow \frac{N}{2} = 33.5.$$

The cumulative frequency just greater than 33.5 is 42 and the corresponding class is 125 - 145.

Thus, the median class is 125 – 145.

$\therefore l = 125, h = 20, f = 20, cf = \text{c.f. of preceding class} = 22$ and $\frac{N}{2} = 33.5$.

Now,

$$\begin{aligned} \text{Median, } M &= l + \left\{ h \times \left(\frac{\frac{N}{2} - cf}{f} \right) \right\} \\ &= 125 + \left\{ 20 \times \left(\frac{33.5 - 22}{20} \right) \right\} \\ &= 125 + 11.5 \\ &= 136.5 \end{aligned}$$

Hence, the median = 136.5.

6.

Sol:

Class	Frequency (f)	Cumulative Frequency (cf)
135 – 140	6	6
140 – 145	10	16
145 – 150	18	34
150 – 155	22	56
155 – 160	20	76
160 – 165	15	91
165 – 170	6	97
170 – 175	3	100
	$N = \sum f = 100$	

Now, $N = 100$

$$\Rightarrow \frac{N}{2} = 50.$$

The cumulative frequency just greater than 50 is 56 and the corresponding class is 150 - 155.

Thus, the median class is 150 – 155.

$$\therefore l = 150, h = 5, f = 22, cf = \text{c.f. of preceding class} = 34 \text{ and } \frac{N}{2} = 50.$$

Now,

$$\begin{aligned} \text{Median, } M &= l + \left\{ h \times \left(\frac{\frac{N}{2} - cf}{f} \right) \right\} \\ &= 150 + \left\{ 5 \times \left(\frac{50 - 34}{22} \right) \right\} \\ &= 150 + 3.64 \\ &= 153.64 \end{aligned}$$

Hence, the median = 153.64.

7.

Sol:

Class	Frequency (f _i)	Cumulative Frequency (cf)
0 – 10	5	5
10 – 20	25	30
20 – 30	x	x + 30
30 – 40	18	x + 48
40 – 50	7	x + 55

Median is 24 which lies in 20 – 30

\therefore Median class = 20 – 30

Let the unknown frequency be x.

Here, $l = 20$, $\frac{n}{2} = \frac{x+55}{2}$, c.f. of the preceding class = c.f = 30, $f = x$, $h = 10$

Now,

$$\text{Median, } M = l + \frac{\frac{n}{2} - cf}{f} \times h$$

$$\Rightarrow 24 = 20 + \frac{\frac{x+55}{2} - 30}{x} \times 10$$

$$\Rightarrow 24 = 20 + \frac{x+55-60}{x} \times 10$$

$$\Rightarrow 24 = 20 + \frac{x-5}{2x} \times 10$$

$$\Rightarrow 24 = 20 + \frac{5x-25}{x}$$

$$\Rightarrow 24 = \frac{20+5x-25}{x}$$

$$\Rightarrow 24x = 25x - 25$$

$$\Rightarrow -x = -25$$

$$\Rightarrow x = 25$$

Hence, the unknown frequency is 25.

8.

Sol:

Class	Frequency (f)	Cumulative Frequency (cf)
0 – 5	12	12
5 – 10	a	12 + a
10 – 15	12	24 + a
15 – 20	15	39 + a
20 – 25	b	39 + a + b
25 – 30	6	45 + a + b
30 – 35	6	51 + a + b
35 – 40	4	55 + a + b
Total	$N = \sum f_i = 70$	

Let a and b be the missing frequencies of class intervals 5 – 10 and 20 – 25 respectively.

$$\text{Then, } 55 + a + b = 70 \Rightarrow a + b = 15 \dots (1)$$

Median is 16, which lies in 15 – 20. So, the median class is 15 – 20.

$$\therefore l = 15, h = 5, N = 70, f = 15 \text{ and } cf = 24 + a$$

Now,

$$\text{Median, } M = l + \left(\frac{\frac{N}{2} - cf}{f} \right) \times h$$

$$\Rightarrow 16 = 15 + \left(\frac{\frac{70}{2} - (24 + a)}{15} \right) \times 5$$

$$\Rightarrow 16 = 15 + \left(\frac{35 - 24 - a}{3} \right)$$

$$\Rightarrow 16 = 15 + \left(\frac{11 - a}{3} \right)$$

$$\Rightarrow 16 - 15 = \frac{11 - a}{3}$$

$$\Rightarrow 1 \times 3 = 11 - a$$

$$\Rightarrow a = 11 - 3$$

$$\Rightarrow a = 8$$

$$\therefore b = 15 - a \quad [\text{From (1)}]$$

$$\Rightarrow b = 15 - 8$$

$$\Rightarrow b = 7$$

Hence, $a = 8$ and $b = 7$.

9.

Sol:

We prepare the cumulative frequency table, as shown below:

Runs scored	Number of batsman (f_i)	Cumulative Frequency (cf)
2500 – 3500	5	5
3500 – 4500	x	5 + x
4500 – 5500	y	5 + x + y
5500 – 6500	12	17 + x + y
6500 – 7500	6	23 + x + y
7500 – 8500	2	25 + x + y
Total	$N = \sum f_i = 60$	

Let x and y be the missing frequencies of class intervals 3500 – 4500 respectively. Then,

$$25 + x + y = 60 \Rightarrow x + y = 35 \quad \dots\dots(1)$$

Median is 5000, which lies in 4500 – 5500. So, the median class is 4500 – 5500.

$$\therefore l = 4500, h = 1000, N = 60, f = y \text{ and } cf = 5 + x$$

Now,

$$\text{Median, } M = l + \left(\frac{\frac{N}{2} - cf}{f} \right) \times h$$

$$\Rightarrow 5000 = 4500 + \left(\frac{\frac{60}{2} - (5+x)}{y} \right) \times 1000$$

$$\Rightarrow 5000 - 4500 = \left(\frac{30-5-x}{y} \right) \times 1000$$

$$\Rightarrow 500 = \left(\frac{25-x}{y} \right) \times 1000$$

$$\Rightarrow y = 50 - 2x$$

$$\Rightarrow 35 - x = 50 - 2x \quad [\text{From (1)}]$$

$$\Rightarrow 2x - x = 50 - 35$$

$$\Rightarrow x = 15$$

$$\therefore y = 35 - x \quad [\text{From (1)}]$$

$$\Rightarrow y = 35 - 15$$

$$\Rightarrow y = 20$$

Hence, $x = 15$ and $y = 20$.

10.

Sol:

Class	Frequency (f)	Cumulative Frequency (cf)
0 - 10	f_1	f_1
10 - 20	5	$f_1 + 5$
20 - 30	9	$f_1 + 14$
30 - 40	12	$f_1 + 26$
40 - 50	f_2	$f_1 + f_2 + 26$
50 - 60	3	$f_1 + f_2 + 29$
60 - 70	2	$f_1 + f_2 + 31$
	$N = \sum f = 40$	

$$\text{Now, } f_1 + f_2 + 31 = 40$$

$$\Rightarrow f_1 + f_2 = 9$$

$$\Rightarrow f_2 = 9 - f_1$$

The median is 32.5 which lies in 30 - 40.

Hence, median class = 30 - 40

$$\text{Here, } l = 30, \frac{N}{2} = \frac{40}{2} = 20, f = 12 \text{ and } cf = 14 + f_1$$

Now, median = 32.5

$$\Rightarrow l + \left(\frac{\frac{N}{2} - cf}{f} \right) \times h = 32.5$$

$$\Rightarrow 30 + \left(\frac{20 - (14 + f_1)}{12} \right) \times 10 = 32.5$$

$$\Rightarrow \frac{6 - f_1}{12} \times 10 = 2.5$$

$$\Rightarrow \frac{60 - 10f_1}{12} = 2.5$$

$$\Rightarrow 60 - 10f_1 = 30$$

$$\Rightarrow 10f_1 = 30$$

$$\Rightarrow f_1 = 3$$

From equation (i), we have:

$$f_2 = 9 - 3$$

$$\Rightarrow f_2 = 6$$

11.

Sol: First, we will convert the data into exclusive form.

Class	Frequency (f)	Cumulative Frequency (cf)
18.5 – 25.5	35	35
25.5 – 32.5	96	131
32.5 – 39.5	68	199
39.5 – 46.5	102	301
46.5 – 53.5	35	336
53.5 – 60.5	4	340
	$N = \sum f = 340$	

Now, $N = 340$

$$\Rightarrow \frac{N}{2} = 70.$$

The cumulative frequency just greater than 70 is 131 and the corresponding class is 25.5 – 32.5.

Thus, the median class is 25.5 – 32.5.

$\therefore l = 25.5, h = 7, f = 68, cf = \text{c.f. of preceding class} = 131 \text{ and } \frac{N}{2} = 70.$

$$\begin{aligned} \therefore \text{Median, } M &= l + \left\{ h \times \left(\frac{\frac{N}{2} - cf}{f} \right) \right\} \\ &= 25.5 + \left\{ 7 \times \left(\frac{70 - 131}{68} \right) \right\} \\ &= 25.5 + 4.01 \\ &= 36.51 \end{aligned}$$

Hence, the median = 36.51.

12.

Sol:

Class	Frequency (f)	Cumulative Frequency (cf)
60.5 – 70.5	5	5
70.5 – 80.5	15	20
80.5 – 90.5	20	40
90.5 – 100.5	30	70
100.5 – 110.5	20	90
110.5 – 120.5	8	98
	$N = \sum f = 98$	

Now, $N = 98$

$$\Rightarrow \frac{N}{2} = 49.$$

The cumulative frequency just greater than 49 is 70 and the corresponding class is 90.5 – 100.5.

Thus, the median class is 90.5 – 100.5.

Now, $l = 90.5$, $h = 10$, $f = 30$, $cf = \text{c.f. of preceding class} = 40$ and $\frac{N}{2} = 49$.

$$\begin{aligned} \therefore \text{Median, } M &= l + \left\{ h \times \left(\frac{\frac{N}{2} - cf}{f} \right) \right\} \\ &= 90.5 + \left\{ 10 \times \left(\frac{49 - 40}{30} \right) \right\} \\ &= 90.5 + 3 \\ &= 93.5 \end{aligned}$$

Hence, median wages = Rs. 93.50.

13.

Sol:

Converting into exclusive form, we get:

Class	Frequency (f)	Cumulative Frequency (cf)
0.5 – 5.5	7	7
5.5 – 10.5	10	17
10.5 – 15.5	16	33
15.5 – 20.5	32	65
20.5 – 25.5	24	89
25.5 – 30.5	16	105
30.5 – 35.5	11	116

35.5 – 40.5	5	121
40.5 – 45.5	2	123
	$N = \sum f = 123$	

Now, $N = 123$

$$\Rightarrow \frac{N}{2} = 61.5.$$

The cumulative frequency just greater than 61.5 is 65 and the corresponding class is 15.5 – 20.5.

Thus, the median class is 15.5 – 20.5.

$\therefore l = 15.5, h = 5, f = 32, cf = \text{c.f. of preceding class} = 33$ and $\frac{N}{2} = 61.5$.

$$\begin{aligned} \therefore \text{Median, } M &= l + \left\{ h \times \left(\frac{\frac{N}{2} - cf}{f} \right) \right\} \\ &= 15.5 + \left\{ 5 \times \left(\frac{61.5 - 33}{32} \right) \right\} \\ &= 15.5 + 4.45 \\ &= 19.95 \end{aligned}$$

Hence, median = 19.95.

14.

Sol:

Class	Cumulative frequency (cf)	Frequency (f)
0 – 10	12	12
10 – 20	32	20
20 – 30	57	25
30 – 40	80	23
40 – 50	92	12
50 – 60	116	24
60 – 70	164	48
70 – 80	200	36
		$N = \sum f = 200$

Now, $N = 200$

$$\Rightarrow \frac{N}{2} = 100.$$

The cumulative frequency just greater than 100 is 116 and the corresponding class is 50 – 60.

Thus, the median class is 50 – 60.

$\therefore l = 50, h = 10, f = 24, cf = \text{c.f. of preceding class} = 92$ and $\frac{N}{2} = 100$.

$$\begin{aligned}\therefore \text{Median, } M &= l + \left\{ h \times \left(\frac{\frac{N}{2} - cf}{f} \right) \right\} \\ &= 50 + \left\{ 10 \times \left(\frac{100 - 92}{24} \right) \right\} \\ &= 50 + 3.33 \\ &= 53.33\end{aligned}$$

Hence, median = 53.33.

