# Exercise – 8A

**1.**

**Sol:**

(i) $LHS = (1 - \cos^2 \theta)\ cosec^2\theta$

$= \sin^2 \theta\ \ cosec^2\theta \qquad (\because \cos^2 \theta + \sin^2 \theta = 1)$

$= \frac{1}{cosec^2\theta} \times cosec^2\theta$

$= 1$

Hence, LHS = RHS

(ii) $LHS = (1 + \cot^2 \theta) \sin^2 \theta$

$= cosec^2\theta \sin^2 \theta \qquad (\because cosec^2 \theta - \cot^2 \theta = 1)$

$= \frac{1}{\sin^2 \theta} \times \sin^2 \theta$

$= 1$

Hence, LHS = RHS

**2.**

**Sol:**

(i) $LHS = (\sec^2 \theta - 1) \cot^2 \theta$

$= \tan^2 \theta \times \cot^2 \theta \qquad (\because sec^2\theta - \tan^2 \theta = 1)$

$= \frac{1}{\cot^2 \theta} \times \cot^2 \theta$

$= 1$

= RHS

(ii) $LHS = (\sec^2 \theta - 1)(cosec^2\theta - 1)$

$= \tan^2 \theta \times \cot^2 \theta \qquad (\because \sec^2 \theta - \tan^2 \theta = 1 and\ cosec^2 \theta - \cot^2 \theta = 1)$

$= \tan^2 \theta \times \frac{1}{\tan^2 \theta}$

$= 1$

=RHS

(iii) $LHS = (1 - \cos^2 \theta) \sec^2 \theta$

$= \sin^2 \theta \times \sec^2 \theta \qquad (\because \sin^2 \theta + \cos^2 \theta = 1)$

$= \sin^2 \theta \times \frac{1}{\cos^2 \theta}$

$= \frac{\sin^2 \theta}{\cos^2 \theta}$

$= \tan^2 \theta$

= RHS

**3.**

**Sol:**

(i) $LHS = \sin^2\theta + \frac{1}{(1+\tan^2\theta)}$

$= \sin^2\theta + \frac{1}{\sec^2\theta}$ $\quad (\because \sec^2\theta - \tan^2\theta = 1)$

$= \sin^2\theta + \cos^2\theta$

$= 1$

$=$ RHS

(ii) $LHS = \frac{1}{(1+\tan^2\theta)} + \frac{1}{(1+\cot^2\theta)}$

$= \frac{1}{\sec^2\theta} + \frac{1}{cosec^2\theta}$

$= \cos^2\theta + \sin^2\theta$

$= 1$

$=$ RHS

**4.**

**Sol:**

(i) $LHS = (1+\cos\theta)(1-\cos\theta)(1+\cot^2\theta)$

$= (1-\cos^2\theta)cosec^2\theta$

$= \sin^2\theta \times cosec^2\theta$

$= \sin^2\theta \times \frac{1}{\sin^2\theta}$

$= 1$

$=$ RHS

(ii) $LHS = cosec\theta(1+\cos\theta)(cosec\,\theta - \cot\theta)$

$= (cosec\,\theta + cosec\,\theta \times \cos\theta)(cosec\,\theta - \cot\theta)$

$= \left(cosec\theta + \frac{1}{\sin\theta} \times \cos\theta\right)(cosec\,\theta - \cot\theta)$

$= (cosec\,\theta + \cot\theta)(cosec\theta - \cot\theta)$

$= cosec^2\theta - \cot^2\theta$ $\quad (\because cosec^2\theta - \cot^2\theta = 1)$

$= 1$

$=$ RHS

**5.**

**Sol:**

(i) $LHS = \cot^2\theta - \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta - 1}{\sin^2\theta}$

$= \frac{-\sin^2\theta}{\sin^2\theta}$

= -1

=RHS

(ii) $LHS = \tan^2\theta - \frac{1}{\cos^2\theta}$

$= \frac{\sin^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$

$= \frac{\sin^2\theta - 1}{\cos^2\theta}$

$= \frac{-\cos^2\theta}{\cos^2\theta}$

= -1

= RHS

(iii) $LHS = \cos^2\theta + \frac{1}{(1+\cot^2\theta)}$

$= \cos^2\theta + \frac{1}{cosec^2\theta}$

$= \cos^2\theta + \sin^2\theta$

=1

= RHS

**6.**

**Sol:**

$LHS = \frac{1}{(1+\sin\theta)} + \frac{1}{(1-\sin\theta)}$

$= \frac{(1-\sin\theta)+(1+\sin\theta)}{(1+\sin\theta)\,(1-\sin\theta)}$

$= \frac{2}{1-\sin^2\theta}$

$= \frac{2}{\cos^2\theta}$

$= 2\sec^2\theta$

= RHS

**7.**

**Sol:**

(i) $LHS = \sec\theta(1-\sin\theta)(\sec\theta+\tan\theta)$

$= (\sec\theta - \sec\theta\sin\theta)(\sec\theta+\tan\theta)$

$=(\sec\theta - \frac{1}{\cos\theta}\times\sin\theta)(\sec\theta+\tan\theta)$

$= (\sec\theta-\tan\theta)(\sec\theta+\tan\theta)$

$= \sec^2\theta - \tan^2\theta$

$= 1$

= RHS

(ii) $LHS = \sin\theta(1+\tan\theta)+\cos\theta(1+\cot\theta)$

$=\sin\theta+\sin\theta\times\frac{\sin\theta}{\cos\theta}+\cos\theta+\cos\theta\times\frac{\cos\theta}{\sin\theta}$

$=\frac{\cos\theta\sin^2\theta+\sin^3\theta+\cos^2\theta\sin\theta+\cos^3\theta}{\cos\theta\sin\theta}$

$=\frac{(\sin^3\theta+\cos^3\theta)+(\cos\theta\sin^2\theta+\cos^2\theta\sin\theta)}{\cos\theta\sin\theta}$

$=\frac{(\sin\theta+\cos\theta)(\sin^2\theta-\sin\theta\cos\theta+\cos^2\theta)+\sin\theta\cos\theta(\sin\theta+\cos\theta)}{\cos\theta\sin\theta}$

$=\frac{(\sin\theta+\cos\theta)(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta+\sin\theta\cos\theta)}{\cos\theta\sin\theta}$

$=\frac{(\sin\theta+\cos\theta)\ (1)}{\cos\theta\sin\theta}$

$=\frac{\sin\theta}{\cos\theta\sin\theta}+\frac{\cos\theta}{\cos\theta\sin\theta}$

$=\frac{1}{\cos\theta}+\frac{1}{\sin\theta}$

$= \sec\theta + cosec\ \theta$

=RHS



**8.**

**Sol:**

(i) $LHS = 1+\frac{\cot^2\theta}{(1+cosec\theta)}$

$= 1+\frac{(cosec^2\theta-1)}{(cosec\theta+1)}$ $\quad (\because cosec^2\theta - \cot^2\theta = 1)$

$= 1+\frac{(cosec\theta+1)(cosec\ \theta-1)}{(cosec\ \theta+1)}$

$=1+(cosec\ \theta-1)$

$= cosec\ \theta$

= RHS

(ii) $LHS = 1 + \frac{\tan^2\theta}{(1+\sec\theta)}$

$= 1 + \frac{(\sec^2\theta - 1)}{(\sec\theta + 1)}$

$= 1 + \frac{(\sec\theta+1)(\sec\theta-1)}{(\sec\theta+1)}$

$= 1 + (\sec\theta - 1)$

$= \sec\theta$

= RHS

**9.**

**Sol:**

$LHS = \frac{(1+\tan^2\theta)\cot\theta}{cosec^2\theta}$

$= \frac{\sec^2\theta\cot\theta}{cosec^2\theta}$

$= \frac{\frac{1}{\cos^2\theta}\times\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin^2\theta}}$

$= \frac{1}{\cos\theta\sin\theta}\times\sin^2\theta$

$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$

=RHS

Hence, LHS = RHS

**10.**

**Sol:**

LHS $= \frac{\tan^2\theta}{(1+\tan^2\theta)} + \frac{\cot^2\theta}{(1+\cot^2\theta)}$

$= \frac{\tan^2\theta}{\sec^2\theta} + \frac{\cot^2\theta}{cosec^2\theta}$ $(\because \sec^2\theta - \tan^2\theta = 1 and\ cosec^2\theta - \cot^2\theta = 1)$

$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} + \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}}$

$= \sin^2\theta + \cos^2\theta$

= 1

= RHS

Hence, LHS = RHS

**11.**

**Sol:**

LHS $= \frac{\sin\theta}{(1+\cos\theta)} + \frac{(1+\cos\theta)}{\sin\theta}$

$= \frac{\sin^2\theta+(1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$

$= \frac{\sin^2\theta+1+\cos^2\theta+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{1+1+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{2+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$

$= \frac{2}{\sin\theta}$

$= 2\text{cosec}\ \theta$

= RHS

Hence, L.H.S = R.H.S.

**12.**

**Sol:**

LHS $= \frac{\tan\theta}{(1-\cot\theta)} + \frac{\cot\theta}{(1-\tan\theta)}$

$= \frac{\tan\theta}{(1-\frac{\cos\theta}{\sin\theta})} + \frac{\cot\theta}{(1-\frac{\sin\theta}{\cos\theta})}$

$= \frac{\sin\theta\tan\theta}{(\sin\theta-\cos\theta)} + \frac{\cos\theta\cot\theta}{(\cos\theta-\sin\theta)}$

$= \frac{\sin\theta\times\frac{\sin\theta}{\cos\theta}-\cos\theta\times\frac{\cos\theta}{\sin\theta}}{(\sin\theta-\cos\theta)}$

$= \frac{\frac{\sin^2\theta}{\cos\theta}-\frac{\cos^2\theta}{\sin\theta}}{(\sin\theta-\cos\theta)}$

$= \frac{\sin^3\theta-\cos^3\theta}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$

$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$

$= \frac{1+\sin\theta\cos\theta}{\cos\theta\sin\theta}$

$= \frac{1}{\cos\theta\sin\theta} + \frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}$

$= \frac{1}{\cos\theta\sin\theta} + \frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}$

$= \sec\theta cosec\ \theta + 1$

$=1 + \sec\theta cosec\ \theta$

= RHS

**13.**

**Sol:**

$$\frac{\cos^2\theta}{(1-\tan\theta)} + \frac{\sin^3\theta}{(\sin\theta-\cos\theta)} = (1+\sin\theta\cos\theta)$$

$$LHS = \frac{\cos^2\theta}{(1-\tan\theta)} + \frac{\sin^3\theta}{(\sin\theta-\cos\theta)}$$

$$= \frac{\cos^2\theta}{(1-\frac{\sin\theta}{\cos\theta})} + \frac{\sin^3\theta}{(\sin\theta-\cos\theta)}$$

$$= \frac{\cos^3\theta}{(\cos\theta-\sin\theta)} + \frac{\sin^3\theta}{(\sin\theta-\cos\theta)}$$

$$= \frac{\cos^3\theta-\sin^3\theta}{(\cos\theta-\sin\theta)}$$

$$= \frac{(\cos\theta-\sin\theta)(\cos^2\theta+\cos\theta sin+\sin^2\theta)}{(\cos\theta-\sin\theta)}$$

$$= (\sin^2\theta + \cos^2\theta + \cos\theta\sin\theta)$$

$$= (1+\sin\theta\cos\theta)$$

= RHS

Hence, L.H.S = R.H.S.

**14.**

**Sol:**

$$\text{LHS} = \frac{\cos\theta}{(1-\tan\theta)} - \frac{\sin^2\theta}{(\cos\theta-\sin\theta)}$$

$$= \frac{\cos\theta}{\left(1-\frac{\sin\theta}{\cos\theta}\right)} - \frac{\sin^2\theta}{(\cos\theta-\sin\theta)}$$

$$= \frac{\cos^2\theta}{(\cos\theta-\sin\theta)} - \frac{\sin^2\theta}{(\cos\theta-\sin\theta)}$$

$$= \frac{\cos^2\theta-\sin^2\theta}{(\cos\theta-\sin\theta)}$$

$$= \frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{(\cos\theta-\sin\theta)}$$

$$= (\cos\theta+\sin\theta)$$

= RHS

Hence, LHS = RHS

**15.**

**Sol:**

$$LHS = (1+\tan^2\theta)\,(1+\cot^2\theta)$$

$$= \sec^2\theta . cosec^2\theta \qquad (\because \sec^2\theta - \tan^2\theta = 1 \text{ } and \text{ } cosec^2 - \cot^2\theta = 1)$$

$= \frac{1}{\cos^2\theta.\sin^2\theta}$

$= \frac{1}{(1-\sin^2\theta)\sin^2\theta}$

$= \frac{1}{\sin^2\theta-\sin^4\theta}$

=RHS

Hence, LHS = RHS

**16.**

**Sol:**

$LHS = \frac{\tan\theta}{(1+\tan^2\ \theta\ )^2} + \frac{\cot\theta}{(1+\cot^2\theta)^2}$

$= \frac{\tan\theta}{(\sec^2\theta)^2} + \frac{\cot\theta}{(cosec^2\theta)^2}$

$= \frac{\tan\theta}{\sec^4\theta} + \frac{\cot\theta}{cosec^4\theta}$

$= \frac{\sin\theta}{\cos\theta}\times\cos^4\theta + \frac{\cos\theta}{\sin\theta}\times\sin^4\theta$

$= \sin\theta\cos^3\theta + \cos\theta\sin^3\theta$

$= \sin\theta\cos\theta(\cos^2\theta + \sin^2\theta)$

$= \sin\theta\cos\theta$

= RHS

**17.**

**Sol:**

(i) $LHS = \sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta)$

$= 1\times\{(\sin^2\theta)^2 + 2\sin^2\theta\cos^2\theta + (\cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta\}$

$= (\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta$

$= (1)^2 - 3\sin^2\theta\cos^2\theta$

$= 1 - 3\sin^2\theta\cos^2\theta$

= RHS

Hence, LHS = RHS

(ii)$LHS = \sin^2\theta + \cos^4\theta$

$= \sin^2\theta + (\cos^2\theta)^2$

$= \sin^2\theta + (1-\sin^2\theta)^2$

$= \sin^2\theta + 1 - 2\sin^2\theta + \sin^4\theta$

$= 1 - \sin^2\theta + \sin^4\theta$

$= \cos^2\theta + \sin^4\theta$

= RHS

Hence, LHS = RHS

(iii) $LHS = cosec^4\theta - cosec^2\theta$

$= cosec^2\theta(cosec^2\theta - 1)$

$= cosec^2\theta \times \cot^2\theta$ $\quad (\because cosec^2\theta - \cot^2\theta = 1)$

$= (1 + \cot^2\theta) \times \cot^2\theta$

$= \cot^2\theta + \cot^4\theta$

= RHS

Hence, LHS = RHS

**18.**

**Sol:**

(i) $LHS = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$

$= \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}$

$= \frac{\cos^2\theta-\sin^2\theta}{1}$

$= \cos^2\theta - \sin^2\theta$

= RHS

(ii) $LHS = \frac{1-\tan^2\theta}{\cot^2\theta-1}$

$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}-1}$

$= \frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta-\sin^2\theta}{\sin^2\theta}}$

$= \frac{\sin^2\theta}{\cos^2\theta}$

$= \tan^2\theta$

= RHS

**19.**

**Sol:**

(i) $LHS = \frac{\tan\theta}{(\sec\theta-1)} + \frac{\tan\theta}{(\sec\theta+1)}$

$= \tan\theta\,\{\frac{\sec\theta+1+\sec\theta-1}{(\sec\theta-1)(\sec\theta+1)}\}$

$= \tan\theta\,\{\frac{2\sec\theta}{(\sec^2\theta-1)}\}$

$= \tan\theta\times\frac{2\sec\theta}{\tan^2\theta}$

$= 2\frac{\sec\theta}{\tan\theta}$

$= 2\frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$

$= 2\frac{1}{\sin\theta}$

$= 2cosec\ \theta$

= RHS

Hence, LHS = RHS

(ii) $LHS = \frac{\cot\theta}{(cosec\ \theta+1)} + \frac{(cosec\theta+1)}{\cot\theta}$

$= \frac{\cot^2\theta+(cosec\ \theta+1)^2}{(cosec\ \theta+1)\cot\theta}$

$= \frac{\cot^2\theta+cosec^2\theta+2cosec\ \theta+1}{(cosec\ \theta+1)\cot\theta}$

$= \frac{\cot^2\theta+cosec^2\theta+2cosec\ \theta+cosec^2\theta-\cot^2\theta}{(cosec\theta+1)\cot\theta}$

$= \frac{2cosec^2\theta+2cosec\ \theta}{(cosec\ \theta+1)\cot\theta}$

$= \frac{2cosec\theta(cosec\theta+1)}{(cosec\theta+1)\cot\theta}$

$= \frac{2cosec\theta}{\cot\theta}$

$= 2\times\frac{1}{\sin\theta}\times\frac{\sin\theta}{\cos\theta}$

$= 2\sec\theta$

= RHS

Hence, LHS = RHS

**20.**

**Sol:**

(i) $LHS = \dfrac{\sec\theta - 1}{\sec\theta + 1}$

$= \dfrac{\frac{1}{\cos\theta} - 1}{\frac{1}{\cos\theta} + 1}$

$= \dfrac{\frac{1-\cos\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}$

$= \dfrac{1-\cos\theta}{1+\cos\theta}$

$= \dfrac{(1-\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1+\cos\theta)}$ $\left\{\begin{matrix}\text{Dividing the numerator and} \\ \text{denominator by} (1+\cos\theta)\end{matrix}\right\}$

$= \dfrac{1-\cos^2\theta}{(1+\cos\theta)^2}$

$= \dfrac{\sin^2\theta}{(1+\cos\theta)^2}$

$=$ RHS

(ii) $LHS = \dfrac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta}$

$= \dfrac{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$

$= \dfrac{\frac{1-\sin\theta}{\cos\theta}}{\frac{1+\sin\theta}{\cos\theta}}$

$= \dfrac{1-\sin\theta}{1+\sin\theta}$

$= \dfrac{(1-\sin\theta)(1+\sin\theta)}{(1+\sin\theta)(1+\sin\theta)}$ $\left\{\begin{matrix}\text{Dividing the numerator and} \\ \text{denominator by} (1+\cos\theta)\end{matrix}\right\}$

$= \dfrac{(1-\sin^2\theta)}{(1+\sin\theta)^2}$

$= \dfrac{\cos^2\theta}{(1+\sin\theta)^2}$

$=$ RHS

**21. Sol:**

(i) $LHS = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$

$= \sqrt{\frac{(1+\sin\theta)}{(1-\sin\theta)} \times \frac{(1+\sin\theta)}{(1+\sin\theta)}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$

$= \frac{1+\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$= (\sec\theta + \tan\theta)$

= RHS

(ii) $LHS = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$= \sqrt{\frac{(1-\cos\theta)}{(1+\cos\theta)} \times \frac{(1-\cos\theta)}{(1-\cos\theta)}}$

$= \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$

$= \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$

$= \frac{1-\cos\theta}{\sin\theta}$

$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$

$= (cosec\ \theta - \cot\theta)$

= RHS

(iii) $LHS = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$= \sqrt{\frac{(1+\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}} + \sqrt{\frac{(1-\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}}$

$= \sqrt{\frac{(1+\cos\theta)^2}{(1-\cos^2\theta)}} + \sqrt{\frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}}$

$= \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}} + \sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$

$= \frac{(1+\cos\theta)}{\sin\theta} + \frac{(1-\cos\theta)}{\sin\theta}$

$= \frac{1+\cos\theta+1-\cos\theta}{\sin\theta}$

$= \frac{2}{\sin\theta}$

$= 2\cos ec\theta$

= RHS

**22.**

**Sol:**

$$LHS = \frac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta} + \frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$$

$$= \frac{(\cos\theta+\sin\theta)(\cos^2\theta-\cos\theta\sin\theta+\sin^2\theta)}{(\cos\theta+\sin\theta)} + \frac{(\cos\theta-\sin\theta)(\cos^2\theta+\cos\theta\sin\theta+\sin^2\theta)}{(\cos\theta-\sin\theta)}$$

$$= (\cos^2\theta + \sin^2\theta - \cos\theta\sin\theta) + (\cos^2\theta + \sin^2\theta + \cos\theta\sin\theta)$$

$$= (1 - \cos\theta\sin\theta) + (1 + \cos\theta\sin\theta)$$

$= 2$

$=$ RHS

Hence, LHS = RHS

**23.**

**Sol:**

$$LHS = \frac{\sin\theta}{(\cot\theta+cosec\,\theta)} - \frac{\sin\theta}{(\cot\theta-cosec\,\theta)}$$

$$= \sin\theta\left\{\frac{(\cot\theta-cosec\,\theta)-(\cot\theta+cosec\,\theta)}{(\cot\theta+cosec\,\theta)(\cot\theta-cosec\,\theta)}\right\}$$

$$= \sin\theta\left\{\frac{-2cosec\theta}{-1}\right) \quad (\because cosec^2\theta - \cot^2\theta = 1)$$

$$= \sin\theta.2\cos ec\theta$$

$$= \sin\theta \times 2 \times \frac{1}{\sin\theta}$$

$= 2$

$=$ RHS

**24.**

**Sol:**

(i) $$LHS = \frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta} + \frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}$$

$$= \frac{(\sin\theta-\cos\theta)^2+(\sin\theta+\cos\theta)^2}{(\sin\theta+cos\,\theta)(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta}{\sin^2\theta-\cos^2\theta}$$

$$= \frac{1+1}{\sin^2\theta-(1-\sin^2\theta)} \quad (\because \sin^2\theta + \cos^2\theta = 1)$$

$$= \frac{2}{\sin^2\theta-1+\sin^2\theta}$$

$$= \frac{2}{\sin^2\theta-1}$$

$=$ RHS

(ii) $LHS = \frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} + \frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$

$= \frac{(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2}{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}$

$= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta}{(\sin^2\theta-\cos^2\theta)}$

$= \frac{1+1}{(1-\cos^2\theta)-\cos^2\theta}$ $(\because \sin^2\theta + \cos^2\theta = 1)$

$= \frac{2}{1-2\cos^2\theta}$

= RHS

**25.**

**Sol:**

$LHS = \frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{(1+\cos\theta)-(1-\cos^2\theta)}{\sin\theta(1+\cos\theta)}$

$= \frac{\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$

= RHS

Hence, L.H.S. = R.H.S.

**26.**

**Sol:**

(i) Here, $\frac{cosec\theta+\cot\theta}{cosec\theta-\cot\theta}$

$= \frac{(cosec\theta+\cot\theta)(cosec\theta+\cot\theta)}{(cosec\theta-\cot\theta)(cosec\theta+\cot\theta)}$

$= \frac{(cosec\theta+\cot\theta)^2}{(cosec^2\theta-\cot^2\theta)}$

$= \frac{(cosec\theta+\cot\theta)^2}{1}$

$= (cosec\theta + \cot\theta)^2$

Again, $(cosec\theta + \cot\theta)^2$

$= cosec^2\theta + \cot^2\theta + 2cosec\theta\cot\theta$

$= 1 + \cot^2\theta + \cot^2\theta + 2cosec\theta \cot\theta \quad (\because cosec^2\theta - \cot^2\theta = 1)$

$= 1 + 2\cot^2\theta + 2cosec\theta \cot\theta$

(ii) Here, $\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}$

$= \frac{(\sec\theta+\tan\theta)(\sec\theta+\tan\theta)}{(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)}$

$= \frac{(\sec\theta+\tan\theta)^2}{\sec^2\theta-\tan^2\theta}$

$= \frac{(\sec\theta+\tan\theta)^2}{1}$

$= (\sec\theta + \tan\theta)^2$

Again, $(\sec\theta + \tan\theta)^2$

$= \sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta$

$= 1 + \tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta$

$= 1 + 2\tan^2\theta + 2\sec\theta\tan\theta$

**27.**

**Sol:**

(i) LHS $= \frac{1+\cos\theta+\sin\theta}{1+\cos\theta-sin\theta}$

$= \frac{\{(1+\cos\theta)+\sin\theta\}\{(1+\cos\theta)+\sin\theta\}}{\{(1+\cos\theta)-\sin\theta\}\{(1+\cos\theta)+\sin\theta\}}$ $\left\{\begin{array}{l}\text{Multiplying the numerator and} \\ \text{denominator by } (1+\cos\theta+\sin\theta)\end{array}\right\}$

$= \frac{\{(1+\cos\theta)+\sin\theta\}^2}{\{(1+\cos\theta)^2-\sin^2\theta\}}$

$= \frac{1+\cos^2\theta+2\cos\theta+\sin^2\theta+2\sin\theta(1+\cos\theta)}{1+\cos^2\theta+2\cos\theta-\sin^2\theta}$

$= \frac{2+2\cos\theta+2\sin\theta(1+\cos\theta)}{1+\cos^2\theta+2\cos\theta-(1-\cos^2\theta)}$

$= \frac{2(1+\cos\theta)+2\sin\theta(1+\cos\theta)}{2\cos^2\theta+2\cos\theta}$

$= \frac{2(1+\cos\theta)(1+\sin\theta)}{2\cos\theta(1+\cos\theta)}$

$= \frac{1+\sin\theta}{\cos\theta}$

= RHS

(ii)$LHS = \frac{\sin\theta+1\cos\theta}{\cos\theta-1+\sin\theta}$

$= \frac{(\sin\theta+1-\cos\theta)(\sin\theta+\cos\theta+1)}{(\cos\theta-1+\sin\theta)(\sin\theta+\cos\theta+1)}$ $\left\{\begin{array}{l}\text{Multiplying the numerator and} \\ \text{denominator by } (1+\cos\theta+\sin\theta)\end{array}\right\}$

$= \frac{(\sin\theta+1)^2-\cos^2\theta}{(\sin\theta+\cos\theta)^2-1^2}$

$= \frac{\sin^2\theta+1+2\sin\theta-\cos^2\theta}{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}$

$$= \frac{\sin^2\theta+\sin^2\theta+\cos^2\theta+2\sin\theta-\cos^2\theta}{2\sin\theta\cos\theta}$$
$$= \frac{2\sin^2\theta+2sin\ \theta}{2\sin\theta\cos\theta}$$
$$= \frac{2\sin\theta(1+\sin\theta)}{2\sin\theta\cos\theta}$$
$$= \frac{1+\sin\theta}{\cos\theta}$$
$= \text{RHS}$

**28.**

**Sol:**

$$LHS = \frac{\sin\theta}{(\sec\theta+\tan\theta-1)} + \frac{\cos\theta}{(cosec\ \theta+\cot\theta-1)}$$
$$= \frac{\sin\theta\cos\theta}{1+\sin\theta-\cos\theta} + \frac{\cos\theta\sin\theta}{1+\cos\theta-\sin\theta}$$
$$= \sin\theta\cos\theta\ [\frac{1}{1+(\sin\theta-\cos\theta)} + \frac{1}{1-(\sin\theta-\cos\theta)}]$$
$$= \sin\theta\cos\theta\left[\frac{1-(\sin\theta-\cos\theta)+1+(\sin\theta-\cos\theta)}{\{1+(\sin\theta-\cos\theta)\}\{1-(\sin\theta-\cos\theta)\}}\right]$$
$$= \sin\theta\cos\theta[\frac{1-\sin\theta+\cos\theta+1+\sin\theta-\cos\theta}{1-(\sin\theta-\cos\theta)^2}$$
$$= \frac{2\sin\theta cos\ \theta}{1-(\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta)}$$
$$= \frac{2\sin\theta\cos\theta}{2\sin\theta\cos\theta}$$
$= 1$

$= \text{RHS}$

Hence, LHS = RHS

**29.**

**Sol:**

We have $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} + \frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$
$$= \frac{(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2}{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}$$
$$= \frac{\sin^2\theta+\cos^2\theta+2sin\theta\cos\theta+\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta}{\sin^2\theta-\cos^2\theta}$$
$$= \frac{1+1}{\sin^2\theta-\cos^2\theta}$$
$$= \frac{2}{\sin^2\theta-\cos^2\theta}$$

Again, $\frac{2}{\sin^2\theta-\cos^2\theta}$
$$= \frac{2}{\sin^2\theta-(1-\sin^2\theta)}$$
$$= \frac{2}{2\sin^2\theta-1}$$

**30.**

**Sol:**

$$LHS = \frac{\cos\theta cosec\theta - \sin\theta sec\theta}{\cos\theta + \sin\theta}$$

$$= \frac{\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}}{\cos\theta + \sin\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta\sin\theta(\cos\theta + \sin\theta)}$$

$$= \frac{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}{\cos\theta\sin\theta(\cos\theta + \sin\theta)}$$

$$= \frac{(\cos\theta - \sin\theta)}{\cos\theta\sin\theta}$$

$$= \frac{1}{\sin\theta} - \frac{1}{\cos\theta}$$

$$= cosec\,\theta - \sec\theta$$

= RHS

Hence, LHS = RHS

**31.**

**Sol:**

$$LHS = (1 + \tan\theta + \cot\theta)(\sin\theta - \cos\theta)$$

$$= \sin\theta + tan\theta\sin\theta + \cot\theta\sin\theta - \cos\theta - \tan\theta\cos\theta - \cot\theta\cos\theta$$

$$= \sin\theta + \tan\theta\sin\theta + \frac{\cos\theta}{\sin\theta} \times \sin\theta - \cos\theta - \frac{\sin\theta}{\cos\theta} \times \cos\theta - \cot\theta\cos\theta$$

$$= \sin\theta + \tan\theta\sin\theta + \cos\theta - \cos\theta - \sin\theta - \cot\theta\cos\theta$$

$$= \tan\theta\sin\theta - \cot\theta\cos\theta$$

$$= \frac{\sin\theta}{\cos\theta} \times \frac{1}{cosec\theta} - \frac{\cos\theta}{\sin\theta} \times \frac{1}{\sec\theta}$$

$$= \frac{1}{cosec\theta} \times \frac{1}{cosec\theta} \times \sec\theta - \frac{1}{\sec\theta} \times \frac{1}{\sec\theta} \times cosec\theta$$

$$= \frac{\sec\theta}{cosec^2\theta} - \frac{cosec\theta}{\sec^2\theta}$$

= RHS

Hence, LHS = RHS

**32.**

**Sol:**

$$LHS = \frac{\cot^2\theta(\sec\theta - 1)}{(1 + \sin\theta)} + \frac{\sec^2\theta(\sin\theta - 1)}{(1 + \sec\theta)}$$

$$= \frac{\frac{\cos^2\theta}{\sin^2\theta}(\frac{1}{\cos\theta} - 1)}{(1 + \sin\theta)} + \frac{\frac{1}{\cos^2\theta}(\sin\theta - 1)}{(1 + \frac{1}{\cos\theta})}$$

$$=\frac{\frac{\cos^2\theta}{\sin^2\theta}\left(\frac{1-\cos\theta}{\cos\theta}\right)}{(1+\sin\theta)}+\frac{\frac{(\sin\theta-1)}{\cos^2\theta}}{\left(\frac{\cos\theta+1}{\cos\theta}\right)}$$
$$=\frac{\cos^2\theta(1-\cos\theta)}{\sin^2\theta\cos\theta(1+\sin\theta)}+\frac{(\sin\theta-1)\cos\theta}{(\cos\theta+1)\cos^2\theta}$$
$$=\frac{\cos\theta(1-\cos\theta)}{(1-\cos^2\theta)(1+\sin\theta)}+\frac{(\sin\theta-1)\cos\theta}{(\cos\theta+1)(1-\sin^2\theta)}$$
$$=\frac{\cos\theta(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)(1+\sin\theta)}+\frac{-(1\sin\theta)\cos\theta}{(\cos\theta+1)(1-\sin\theta)(1+\sin\theta)}$$
$$=\frac{\cos\theta}{(1+\cos\theta)(1+\sin\theta)}-\frac{\cos\theta}{(\cos\theta+1)(1+\sin\theta)}$$
$$=\theta$$
$$=\text{RHS}$$

**33.**

**Sol:**

$$LHS=\left\{\frac{1}{\sec^2\theta-\cos^2\theta}+\frac{1}{cosec^2\theta-\sin^2\theta}\right\}(\sin^2\theta\cos^2\theta)$$
$$=\left\{\frac{\cos^2\theta}{1-\cos^4\theta}+\frac{\sin^2\theta}{1-\sin^4\theta}\right\}(\sin^2\theta\cos^2\theta)$$
$$=\left\{\frac{\cos^2\theta}{(1-\cos^2\theta)(1+\cos^2\theta)}+\frac{\sin^2\theta}{(1-\sin^2\theta)(1+\sin^2\theta)}\right\}(\sin^2\theta\cos^2\theta)$$
$$=\left[\frac{\cot^2\theta}{1+\cos^2\theta}+\frac{\tan^2\theta}{1+\sin^2\theta}\right]\sin^2\theta\cos^2\theta$$
$$=\frac{\cos^4\theta}{1+\cos^2\theta}+\frac{\sin^4\theta}{1+\sin^2\theta}$$
$$=\frac{(\cos^2\theta)^2}{1+\cos^2\theta}+\frac{(\sin^2\theta)^2}{1+\sin^2\theta}$$
$$=\frac{(1-\sin^2\theta)}{1+\cos^2\theta}+\frac{(1-\cos^2\theta)^2}{1+\sin^2\theta}$$
$$=\frac{(1-\sin^2\theta)^2(1+\sin^2)+(1-\cos^2\theta)^2(1+\cos^2\theta)}{(1+\sin^2\theta)(1+\cos^2\theta)}$$
$$=\frac{\cos^4\theta(1+\sin^2\theta)+\sin^4\theta(1+\cos^2\theta)}{1+\sin^2\theta+\cos^2\theta+\sin^2\theta\cos^2\theta}$$
$$=\frac{\cos^4\theta\cos^4\theta\sin^2\theta+\sin^4\theta+\sin^4\theta\cos^2\theta}{1+1\sin^2\theta\cos^2\theta}$$
$$=\frac{\cos^4\theta+\sin^4\theta+\sin^2\theta\cos^2\theta(\sin^2\theta+\cos^2\theta)}{2+\sin^2\theta\cos^2\theta}$$
$$=\frac{(\cos^2\theta)^2+(\sin^2\theta)^2+\sin^2\theta\cos^2\theta(1)}{2+\sin^2\theta\cos^2\theta}$$
$$=\frac{(\cos^2\theta+\sin^2\theta)^2-2\sin^2\theta\cos^2\theta+\sin^2\theta\cos^2\theta(1)}{2+\sin^2\theta\cos^2\theta}$$
$$=\frac{1^2+\cos^2\theta\sin^2\theta-2\cos^2\theta\sin^2\theta}{2+\sin^2\theta\cos^2\theta}$$
$$=\frac{1-\cos^2\theta\sin^2\theta}{2+\sin^2\theta\cos^2\theta}$$
$$=\text{RHS}$$

**34.**

**Sol:**

$LHS = \frac{(sinA-sinB)}{(cosA+cosB)} + \frac{(cosA-cosB}{(sinA+SinB)}$

$= \frac{(sinA-sinB)(sinA+sinB)+(cosA-cosB)(cosA-cosB)}{(cosA+cosB)(sinA+sinB)}$

$= \frac{\sin^2 A-\sin^2 B+\cos^2 A-\cos^2 B}{(cosA+cosB)(sinA+sinB)}$

$= \frac{0}{(cosA+cosB)(sinA+sinB)}$

= 0

= RHS

**35.**

**Sol:**

$LHS = \frac{tanA+tanB}{cotA+cotB}$

$= \frac{tanA+tanB}{\frac{1}{tanA}+\frac{1}{tanB}}$

$= \frac{tanA+tanB}{\frac{tanA+tanB}{tanA\ tanB}}$

$= \frac{tanA\ tanB(tanA+tanB)}{(\tan A+\tan B)}$

$= tanA\ tanB$

= RHS

Hence, LHS = RHS

**36.**

**Sol:**

$(i) \cos^2\theta + \cos\theta = 1$

$LHS = \cos^2\theta + \cos\theta$

$=1 - \sin^2\theta + \cos\theta$

$= 1 - (\sin^2\theta - \cos\theta)$

Since LHS ≠ RHS, this not an identity.

(ii) $\sin^2\theta + \sin\theta = 1$

$LHS = \sin^2\theta + \sin\theta$

$= 1 - \cos^2\theta + \sin\theta$

$= 1 - (\cos^2\theta - \sin\theta)$

Since LHS ≠ RHS, this is not an identity.

(iii) $\tan^2\theta + \sin\theta = \cos^2\theta$

$LHS = \tan^2\theta + \sin\theta$

$= \frac{\sin^2\theta}{\cos^2\theta} + \sin\theta$

$= \frac{1-\cos^2\theta}{\cos^2\theta} + \sin\theta$

$= \sec^2\theta - 1 + \sin\theta$

Since LHS ≠ RHS, this is not an identity.

**37.**

**Sol:**

$RHS = (2\cos^3\theta - \cos\theta)\tan\theta$

$= (2\cos^2\theta - 1)\cos\ \theta \times \frac{\sin\theta}{\cos\theta}$

$= [2(1 - \sin^2\theta) - 1]\sin\ \theta$

$= (2 - 2\sin^2\theta - 1)\sin\ \theta$

$= (1 - 2\sin^2\theta)\sin\theta$

$= (\sin\theta - 2\sin^3\theta)$

= LHS