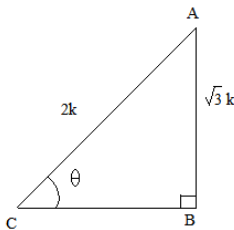


1. If $\sin \theta = \frac{\sqrt{3}}{2}$, find the value of all T-ratios of θ

Sol:

Let us first draw a right $\triangle ABC$, right angled at B and $\angle C = \theta$

Now, we know that $\sin \theta = \frac{\text{Perpendicular}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$



So, if $AB = \sqrt{3}k$, then $AC = 2k$, where k is a positive number.

Now, using Pythagoras theorem, we have:

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow BC^2 = AC^2 - AB^2 = (2k)^2 - (\sqrt{3}k)^2$$

$$\Rightarrow BC^2 = 4k^2 - 3k^2 = k^2$$

$$\Rightarrow BC = k$$

Now, finding the other T-ratios using their definitions, we get:

$$\cos \theta = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\tan \theta = \frac{AB}{BC} = \frac{\sqrt{3}k}{k} = \sqrt{3}$$

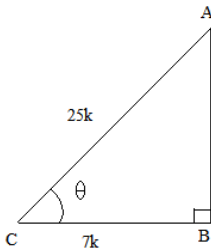
$$\therefore \cot \theta = \frac{1}{\tan \theta} = \frac{1}{\sqrt{3}}, \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{2}{\sqrt{3}} \text{ and } \sec \theta = \frac{1}{\cos \theta} = 2$$

2. If $\cos \theta = \frac{7}{25}$, find the value of all T-ratios of θ

Sol:

Let us first draw a right $\triangle ABC$, right angled at B and $\angle C = \theta$.

Now, we know that $\cos \theta = \frac{\text{Base}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{7}{25}$



So, if $BC = 7k$, then $AC = 25k$, where k is a positive number.

Now, using Pythagoras theorem, we have:

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AB^2 = AC^2 - BC^2 = (25k)^2 - (7k)^2$$

$$\Rightarrow AB^2 = 625k^2 - 49k^2 = 576k^2$$

$$\Rightarrow AB = 24k$$

Now, finding the trigonometric ratios using their definitions, we get:

$$\sin \theta = \frac{AB}{AC} = \frac{24k}{25k} = \frac{24}{25}$$

$$\tan \theta = \frac{AB}{BC} = \frac{24k}{7k} = \frac{24}{7}$$

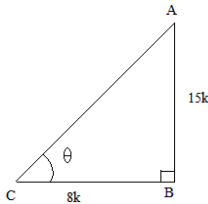
$$\therefore \cot \theta = \frac{1}{\tan \theta} = \frac{7}{24}, \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{25}{24} \text{ and } \sec \theta = \frac{1}{\cos \theta} = \frac{25}{7}$$

3. If $\tan \theta = \frac{15}{8}$, find the values of all T-ratios of θ

Sol:

Let us first draw a right $\triangle ABC$, right angled at B and $\angle C = \theta$

Now, we know that $\tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AB}{BC} = \frac{15}{8}$



So, if $BC = 8k$, then $AB = 15k$ where k is positive number.

Now, using Pythagoras theorem, we have:

$$AC^2 = AB^2 + BC^2 = (15k)^2 + (8k)^2$$

$$\Rightarrow AC^2 = 225k^2 + 64k^2 = 289k^2$$

$$\Rightarrow AC = 17k$$

Now, finding the other T-ratios using their definitions, we get:

$$\sin \theta = \frac{AB}{AC} = \frac{15k}{17k} = \frac{15}{17}$$

$$\cos \theta = \frac{BC}{AC} = \frac{8k}{17k} = \frac{8}{17}$$

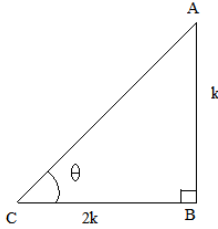
$$\therefore \cot \theta = \frac{1}{\tan \theta} = \frac{8}{15}, \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{17}{15} \text{ and } \sec \theta = \frac{1}{\cos \theta} = \frac{17}{8}$$

4. If $\cot \theta = 2$ find all the values of all T-ratios of θ

Sol:

Let us first draw a right $\triangle ABC$, right angled at B and $\angle C = \theta$

Now, we know that $\cot \theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{BC}{AB} = 2$



So, if $BC = 2k$, then $AB = k$, is a positive number.

Now, using Pythagoras theorem, we have:

$$AC^2 = AB^2 + BC^2 = (2k)^2 + (k)^2$$

$$\Rightarrow AC^2 = 4k^2 + k^2 = 5k^2$$

$$\Rightarrow AC = \sqrt{5}k$$

Now, finding the other T-ratios using their definitions, we get:

$$\sin \theta = \frac{AB}{AC} = \frac{k}{\sqrt{5}k} = \frac{1}{\sqrt{5}}$$

$$\cos \theta = \frac{BC}{AC} = \frac{2k}{\sqrt{5}k} = \frac{2}{\sqrt{5}}$$

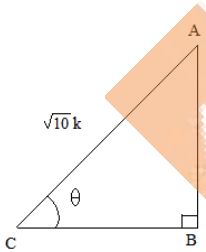
$$\therefore \tan \theta = \frac{1}{\cot \theta} = \frac{1}{2}, \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \sqrt{5} \text{ and } \sec \theta = \frac{1}{\cos \theta} = \frac{\sqrt{5}}{2}$$

5. If $\operatorname{cosec} \theta = \sqrt{10}$ find all the values of all T-ratios of θ

Sol:

Let us first draw a right $\triangle ABC$, right angled at B and $\angle C = \theta$

Now, we know that $\operatorname{cosec} \theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{AC}{AB} = \frac{\sqrt{10}}{1}$



So, if $AC = (\sqrt{10})k$, then $AB = k$ is a positive number.

Now, by using Pythagoras theorem, we have:

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow BC^2 = AC^2 - AB^2$$

$$\Rightarrow BC^2 = 9k^2$$

$$\Rightarrow BC = 3k$$

Now, finding the other T-ratios using their definitions, we get:

$$\tan \theta = \frac{AB}{BC} = \frac{k}{3k} = \frac{1}{3}$$

$$\cos \theta = \frac{BC}{AC} = \frac{3k}{\sqrt{10}k} = \frac{3}{\sqrt{10}}$$

$$\therefore \sin \theta = \frac{1}{\operatorname{cosec} \theta} = \frac{1}{\sqrt{10}}, \cot \theta = \frac{1}{\tan \theta} = 3 \text{ and } \sec \theta = \frac{1}{\cos \theta} = \frac{\sqrt{10}}{3}$$

6. If $\sin \theta = \frac{a^2 - b^2}{a^2 + b^2}$ find all the values of all T-ratios of θ

Sol:

$$\text{We have } \sin \theta = \frac{a^2 - b^2}{a^2 + b^2}$$

As,

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$= 1 - \left(\frac{a^2 - b^2}{a^2 + b^2} \right)^2$$

$$= \frac{1}{1} - \frac{(a^2 - b^2)^2}{(a^2 + b^2)^2}$$

$$= \frac{(a^2 + b^2)^2 - (a^2 - b^2)^2}{(a^2 + b^2)^2}$$

$$= \frac{[(a^2 + b^2) - (a^2 - b^2)][(a^2 + b^2) + (a^2 - b^2)]}{(a^2 + b^2)^2}$$

$$= \frac{[a^2 + b^2 - a^2 + b^2][a^2 + b^2 + a^2 - b^2]}{(a^2 + b^2)^2}$$

$$= \frac{[2b^2][2a^2]}{(a^2 + b^2)^2}$$

$$\Rightarrow \cos^2 \theta = \frac{4a^2b^2}{(a^2 + b^2)^2}$$

$$\Rightarrow \cos \theta = \frac{\sqrt{4a^2b^2}}{\sqrt{(a^2 + b^2)^2}}$$

$$\Rightarrow \cos \theta = \frac{2ab}{(a^2 + b^2)}$$

Also,

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$= \frac{\left(\frac{a^2 - b^2}{a^2 + b^2} \right)}{\left(\frac{2ab}{a^2 + b^2} \right)}$$

$$= \frac{a^2 - b^2}{2ab}$$

Now,

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$= \frac{1}{\left(\frac{a^2 - b^2}{a^2 + b^2} \right)}$$

$$= \frac{a^2 + b^2}{a^2 - b^2}$$

Also,

$$\sec \theta = \frac{1}{\cos \theta}$$

$$= \frac{1}{\left(\frac{2ab}{a^2 + b^2} \right)}$$

$$= \frac{a^2 + b^2}{2ab}$$

And,

$$\begin{aligned} \cot\theta &= \frac{1}{\tan\theta} \\ &= \frac{1}{\left(\frac{a^2-b^2}{2ab}\right)} \\ &= \frac{2ab}{a^2-b^2} \end{aligned}$$

7. If $15 \cot A = 8$ find all the values of $\sin A$ and $\sec A$

Sol:

We have,

$$15 \cot A = 8$$

$$\Rightarrow \cot A = \frac{8}{15}$$

As,

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$= 1 + \left(\frac{8}{15}\right)^2$$

$$= 1 + \frac{64}{225}$$

$$= \frac{225+64}{225}$$

$$\Rightarrow \operatorname{cosec}^2 A = \frac{289}{225}$$

$$\Rightarrow \operatorname{cosec} A = \sqrt{\frac{289}{225}}$$

$$\Rightarrow \operatorname{cosec} A = \frac{17}{15}$$

$$\frac{1}{\sin A} = \frac{17}{15}$$

$$\sin A = \frac{15}{17}$$

Also,

$$\cos^2 A = 1 - \sin^2 A$$

$$= 1 - \left(\frac{15}{17}\right)^2$$

$$= 1 - \frac{225}{289}$$

$$= \frac{289-225}{289}$$

$$\Rightarrow \cos^2 A = \frac{64}{289}$$

$$\Rightarrow \cos A = \sqrt{\frac{64}{289}}$$

$$\Rightarrow \cos A = \frac{8}{17}$$

$$\Rightarrow \frac{1}{\sec A} = \frac{8}{17}$$

$$\Rightarrow \sec A = \frac{17}{8}$$

8. If $\sin A = \frac{9}{41}$ find all the values of $\cos A$ and $\tan A$

Sol:

We have $\sin A = \frac{9}{41}$

As,

$$\cos^2 A = 1 - \sin^2 A$$

$$= 1 - \left(\frac{9}{41}\right)^2$$

$$= 1 - \frac{81}{1681}$$

$$= \frac{1681-81}{1681}$$

$$\Rightarrow \cos^2 A = \frac{1600}{1681}$$

$$\Rightarrow \cos A = \sqrt{\frac{1600}{1681}}$$

$$\Rightarrow \cos A = \frac{40}{41}$$

Also,

$$\tan A = \frac{\sin A}{\cos A}$$

$$= \frac{\left(\frac{9}{41}\right)}{\left(\frac{40}{41}\right)}$$

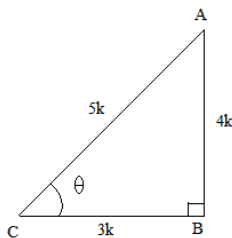
$$= \frac{9}{40}$$

9. If $\cos \theta = 0.6$ show that $(5 \sin \theta - 3 \tan \theta) = 0$

Sol:

Let us consider a right $\triangle ABC$ right angled at B.

Now, we know that $\cos \theta = 0.6 = \frac{BC}{AC} = \frac{3}{5}$



So, if $BC = 3k$, then $AC = 5k$, where k is a positive number.

Using Pythagoras theorem, we have:

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AB^2 = AC^2 - BC^2$$

$$\Rightarrow AB^2 = (5k)^2 - (3k)^2 = 25k^2 - 9k^2$$

$$\Rightarrow AB^2 = 16k^2$$

$$\Rightarrow AB = 4k$$

Finding out the other T-ratios using their definitions, we get:

$$\sin \theta = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

$$\tan \theta = \frac{AB}{BC} = \frac{4k}{3k} = \frac{4}{3}$$

Substituting the values in the given expression, we get:

$$5 \sin \theta - 3 \tan \theta$$

$$\Rightarrow 5 \left(\frac{4}{5} \right) - 3 \left(\frac{4}{3} \right)$$

$$\Rightarrow 4 - 4 = 0 = RHS$$

i.e., LHS = RHS

Hence, Proved.

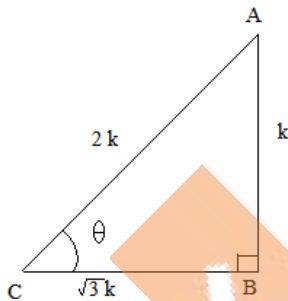
10. If $\operatorname{cosec} \theta = 2$ show that $\left(\cot \theta + \frac{\sin \theta}{1 + \cos \theta} \right) = 2$

Sol:

Let us consider a right $\triangle ABC$, right angled at B and $\angle C = \theta$.

Now, it is given that $\operatorname{cosec} \theta = 2$.

$$\text{Also, } \sin \theta = \frac{1}{\operatorname{cosec} \theta} = \frac{1}{2} = \frac{AB}{AC}$$



So, if $AB = k$, then $AC = 2k$, where k is a positive number.

Using Pythagoras theorem, we have:

$$\Rightarrow AC^2 = AB^2 + BC^2$$

$$\Rightarrow BC^2 = AC^2 - AB^2$$

$$\Rightarrow BC^2 = (2k)^2 - (k)^2$$

$$\Rightarrow BC^2 = 3k^2$$

$$\Rightarrow BC = \sqrt{3}k$$

Finding out the other T-ratios using their definitions, we get:

$$\cos \theta = \frac{BC}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\tan \theta = \frac{AB}{BC} = \frac{k}{\sqrt{3}k} = \frac{1}{\sqrt{3}}$$

$$\cot \theta = \frac{1}{\tan \theta} = \sqrt{3}$$

Substituting these values in the given expression, we get:

$$\begin{aligned}
 & \cot \theta + \frac{\sin \theta}{1 + \cos \theta} \\
 &= \sqrt{3} + \frac{\left(\frac{1}{2}\right)}{1 + \frac{\sqrt{3}}{2}} \\
 &= \sqrt{3} + \frac{\frac{1}{2}}{\frac{2 + \sqrt{3}}{2}} \\
 &= \sqrt{3} + \frac{1}{2 + \sqrt{3}} \\
 &= \frac{\sqrt{3}(2 + \sqrt{3}) + 1}{2 + \sqrt{3}} \\
 &= \frac{2\sqrt{3} + 3 + 1}{2 + \sqrt{3}} \\
 &= \frac{2(2 + \sqrt{3})}{2 + \sqrt{3}} = 2
 \end{aligned}$$

i.e., LHS = RHS

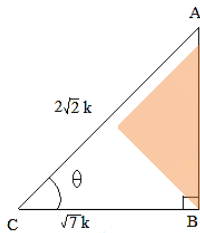
Hence proved.

11. If $\tan \theta = \frac{1}{\sqrt{7}}$ show that $\frac{(\operatorname{cosec}^2 \theta - \sec^2 \theta)}{\operatorname{cosec}^2 \theta + \sec^2 \theta} = \frac{3}{4}$

Sol:

Let us consider a right $\triangle ABC$, right angled at B and $\angle C = \theta$.

Now it is given that $\tan \theta = \frac{AB}{BC} = \frac{1}{\sqrt{7}}$



So, if $AB = k$, then $BC = \sqrt{7}k$, where k is a positive number.

Using Pythagoras theorem, we have:

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = (k)^2 + (\sqrt{7}k)^2$$

$$\Rightarrow AC^2 = k^2 + 7k^2$$

$$\Rightarrow AC = 2\sqrt{2}k$$

Now, finding out the values of the other trigonometric ratios, we have:

$$\sin \theta = \frac{AB}{AC} = \frac{k}{2\sqrt{2}k} = \frac{1}{2\sqrt{2}}$$

$$\cos \theta = \frac{BC}{AC} = \frac{\sqrt{7}k}{2\sqrt{2}k} = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\therefore \operatorname{cosec} \theta = \frac{1}{\sin \theta} = 2\sqrt{2} \text{ and } \sec \theta = \frac{1}{\cos \theta} = \frac{2\sqrt{2}}{\sqrt{7}}$$

Substituting the values of $\operatorname{cosec} \theta$ and $\sec \theta$ in the give expression, we get:

$$\begin{aligned} & \frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta} \\ &= \frac{(2\sqrt{2})^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{(2\sqrt{2})^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2} \\ &= \frac{8 - \left(\frac{8}{7}\right)}{8 + \left(\frac{8}{7}\right)} \\ &= \frac{56 - 8}{56 + 8} \\ &= \frac{48}{64} = \frac{3}{4} = RHS \end{aligned}$$

i.e., LHS = RHS

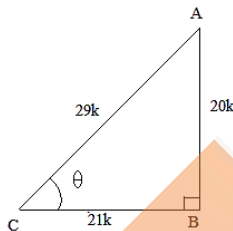
Hence proved.

12. If $\tan \theta = \frac{20}{21}$, show that $\frac{(1 - \sin \theta + \cos \theta)}{(1 + \sin \theta + \cos \theta)} = \frac{3}{7}$

Sol:

Let us consider a right $\triangle ABC$ right angled at B and $\angle C = \theta$

Now, we know that $\tan \theta = \frac{AB}{BC} = \frac{20}{21}$



So, if $AB = 20k$, then $BC = 21k$, where k is a positive number.

Using Pythagoras theorem, we get:

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ \Rightarrow AC^2 &= (20k)^2 + (21k)^2 \\ \Rightarrow AC^2 &= 841k^2 \\ \Rightarrow AC &= 29k \end{aligned}$$

Now, $\sin \theta = \frac{AB}{AC} = \frac{20}{29}$ and $\cos \theta = \frac{BC}{AC} = \frac{21}{29}$

Substituting these values in the give expression, we get:

$$\begin{aligned} LHS &= \frac{1 - \sin \theta + \cos \theta}{1 + \sin \theta + \cos \theta} \\ &= \frac{1 - \frac{20}{29} + \frac{21}{29}}{1 + \frac{20}{29} + \frac{21}{29}} \\ &= \frac{\frac{29 - 20 + 21}{29}}{\frac{29 + 20 + 21}{29}} = \frac{30}{70} = \frac{3}{7} = RHS \end{aligned}$$

\therefore LHS = RHS

Hence proved.

13. If $\sec \theta = \frac{5}{4}$ show that $\frac{(\sin \theta - 2 \cos \theta)}{(\tan \theta - \cot \theta)} = \frac{12}{7}$

Sol:

We have,

$$\sec \theta = \frac{5}{4}$$

$$\Rightarrow \frac{1}{\cos \theta} = \frac{5}{4}$$

$$\Rightarrow \cos \theta = \frac{4}{5}$$

Also,

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$= 1 - \left(\frac{4}{5}\right)^2$$

$$= 1 - \frac{16}{25}$$

$$= \frac{9}{25}$$

$$\Rightarrow \sin \theta = \frac{3}{5}$$

Now,

$$LHS = \frac{(\sin \theta - 2 \cos \theta)}{(\tan \theta - \cot \theta)}$$

$$= \frac{(\sin \theta - 2 \cos \theta)}{\left(\frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta}\right)}$$

$$= \frac{(\sin \theta - 2 \cos \theta)}{\left(\frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta \cos \theta}\right)}$$

$$\frac{\sin \theta \cos \theta (\sin \theta - 2 \cos \theta)}{(\sin^2 \theta - \cos^2 \theta)}$$

$$= \frac{\frac{3}{5} \times \frac{4}{5} \left(\frac{3}{5} - 2 \times \frac{4}{5}\right)}{\left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2}$$

$$= \frac{\frac{12}{25} \left(\frac{3}{5} - \frac{8}{5}\right)}{\left(\frac{9}{25} - \frac{16}{25}\right)}$$

$$= \frac{\frac{12}{25} \times \left(-\frac{5}{5}\right)}{\left(-\frac{7}{25}\right)}$$

$$= \frac{12}{7}$$

$$= RHS$$

14. If $\cot \theta = \frac{3}{4}$, show that $\sqrt{\frac{\sec \theta - \operatorname{cosec} \theta}{\sec \theta + \operatorname{cosec} \theta}} = \frac{1}{\sqrt{7}}$

Sol:

$$\begin{aligned}
LHS &= \sqrt{\frac{\sec \theta - \operatorname{cosec} \theta}{\sec \theta + \operatorname{cosec} \theta}} \\
&= \sqrt{\frac{\left(\frac{1}{\cos \theta} - \frac{1}{\sin \theta}\right)}{\left(\frac{1}{\cos \theta} + \frac{1}{\sin \theta}\right)}} \\
&= \sqrt{\frac{\left(\frac{\sin \theta - \cos \theta}{\sin \theta \cos \theta}\right)}{\left(\frac{\sin \theta + \cos \theta}{\sin \theta \cos \theta}\right)}} \\
&= \sqrt{\frac{\left(\frac{\sin \theta - \cos \theta}{\sin \theta}\right)}{\left(\frac{\sin \theta + \cos \theta}{\sin \theta}\right)}} \\
&= \sqrt{\frac{\left(\frac{\sin \theta}{\sin \theta} - \frac{\cos \theta}{\sin \theta}\right)}{\left(\frac{\sin \theta}{\sin \theta} + \frac{\cos \theta}{\sin \theta}\right)}} \\
&= \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} \\
&= \sqrt{\frac{\left(1 - \frac{3}{4}\right)}{\left(1 + \frac{3}{4}\right)}} \\
&= \sqrt{\frac{\left(\frac{1}{4}\right)}{\left(\frac{7}{4}\right)}} \\
&= \sqrt{\frac{1}{7}} \\
&= \frac{1}{\sqrt{7}} \\
&= \text{RHS}
\end{aligned}$$

15. If $\sin \theta = \frac{3}{4}$, show that $\sqrt{\frac{\operatorname{cosec}^2 \theta - \cot^2 \theta}{\sec^2 \theta - 1}} = \frac{\sqrt{7}}{3}$

Sol:

$$\begin{aligned}
LHS &= \sqrt{\frac{\operatorname{cosec}^2 \theta - \cot^2 \theta}{\sec^2 \theta - 1}} \\
&= \sqrt{\frac{1}{\tan^2 \theta}} \\
&= \sqrt{\cot^2 \theta} \\
&= \cot \theta \\
&= \sqrt{\operatorname{cosec}^2 \theta - 1}
\end{aligned}$$

$$\begin{aligned}
 &= \sqrt{\left(\frac{1}{\left(\frac{3}{4}\right)}\right)^2 - 1} \\
 &= \sqrt{\left(\frac{4}{3}\right)^2 - 1} \\
 &= \sqrt{\frac{16}{9} - 1} \\
 &= \sqrt{\frac{16-9}{9}} \\
 &= \sqrt{\frac{7}{9}} \\
 &= \frac{\sqrt{7}}{3} \\
 &= \text{RHS}
 \end{aligned}$$

16. If $\sin \theta = \frac{a}{b}$, show that $(\sec \theta + \tan \theta) = \sqrt{\frac{b+a}{b-a}}$

Sol:

$$\text{LHS} = (\sec \theta + \tan \theta)$$

$$\begin{aligned}
 &= \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \\
 &= \frac{1+\sin \theta}{\cos \theta} \\
 &= \frac{1+\sin \theta}{\sqrt{1-\sin^2 \theta}} \\
 &= \frac{\left(1+\frac{a}{b}\right)}{\sqrt{1-\left(\frac{a}{b}\right)^2}} \\
 &= \frac{\left(\frac{1+a}{b}\right)}{\sqrt{1-\frac{a^2}{b^2}}} \\
 &= \frac{\left(\frac{b+a}{b}\right)}{\sqrt{\frac{b^2-a^2}{b^2}}} \\
 &= \frac{(b+a)}{\sqrt{(b+a)}\sqrt{(b-a)}} \\
 &= \frac{\sqrt{(b+a)}}{\sqrt{(b-a)}} \\
 &= \sqrt{\frac{b+a}{b-a}} \\
 &= \text{RHS}
 \end{aligned}$$

17. If $\cos \theta = \frac{3}{5}$, show that $\frac{(\sin \theta - \cot \theta)}{2 \tan \theta} = \frac{3}{160}$

Sol:

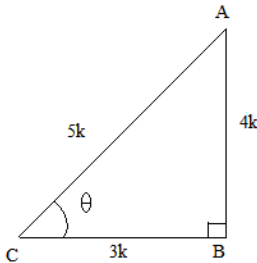
$$\begin{aligned}
 LHS &= \frac{(\sin \theta - \cot \theta)}{2 \tan \theta} \\
 &= \frac{\sin \theta - \frac{\cos \theta}{\sin \theta}}{2 \left(\frac{\sin \theta}{\cos \theta} \right)} \\
 &= \frac{\frac{\sin^2 \theta - \cos \theta}{\sin \theta}}{\left(\frac{2 \sin \theta}{\cos \theta} \right)} \\
 &= \frac{\cos \theta (\sin^2 \theta - \cos \theta)}{2 \sin^2 \theta} \\
 &= \frac{\cos \theta (1 - \cos^2 \theta - \cos \theta)}{2(1 - \cos^2 \theta)} \\
 &= \frac{\frac{3}{5} \left[1 - \left(\frac{3}{5} \right)^2 - \frac{3}{5} \right]}{2 \left[1 - \left(\frac{3}{5} \right)^2 \right]} \\
 &= \frac{\frac{3}{5} \left(1 - \frac{9}{25} - \frac{3}{5} \right)}{2 \left(1 - \frac{9}{25} \right)} \\
 &= \frac{\frac{3}{5} \left(\frac{25 - 9 - 15}{25} \right)}{2 \left(\frac{25 - 9}{25} \right)} \\
 &= \frac{\frac{3}{5} \left(\frac{1}{25} \right)}{2 \left(\frac{16}{25} \right)} \\
 &= \frac{3}{5 \times 2 \times 16} \\
 &= \frac{3}{160} \\
 &= \text{RHS}
 \end{aligned}$$

18. If $\tan \theta = \frac{4}{3}$, show that $(\sin \theta + \cos \theta) = \frac{7}{5}$

Sol:

Let us consider a right $\triangle ABC$, right angled at B and $\angle C = \theta$

Now, we know that $\tan \theta = \frac{AB}{BC} = \frac{4}{3}$



So, if $BC = 3k$, then $AB = 4k$, where k is a positive number.

Using Pythagoras theorem, we have:

$$AC^2 = AB^2 + BC^2 = (4k)^2 + (3k)^2$$

$$\Rightarrow AC^2 = 16k^2 + 9k^2 = 25k^2$$

$$\Rightarrow AC = 5k$$

Finding out the values of $\sin \theta$ and $\cos \theta$ using their definitions, we have:

$$\sin \theta = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

$$\cos \theta = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

Substituting these values in the given expression, we get:

$$(\sin \theta + \cos \theta) = \left(\frac{4}{5} + \frac{3}{5}\right) = \left(\frac{7}{5}\right) = RHS$$

i.e., LHS = RHS

Hence proved.

19. If $\tan \theta = \frac{a}{b}$, show that $\frac{(a \sin \theta - b \cos \theta)}{(a \sin \theta + b \cos \theta)} = \frac{(a^2 - b^2)}{a^2 + b^2}$

Sol:

It is given that $\tan \theta = \frac{a}{b}$

$$LHS = \frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta}$$

Dividing the numerator and denominator by $\cos \theta$, we get:

$$\frac{a \tan \theta - b}{a \tan \theta + b} \quad \left(\because \tan \theta = \frac{\sin \theta}{\cos \theta}\right)$$

Now, substituting the value of $\tan \theta$ in the above expression, we get:

$$\frac{a\left(\frac{a}{b}\right) - b}{a\left(\frac{a}{b}\right) + b}$$

$$= \frac{\frac{a^2}{b} - b}{\frac{a^2}{b} + b}$$

$$= \frac{a^2 - b^2}{a^2 + b^2} = RHS$$

i.e., LHS = RHS

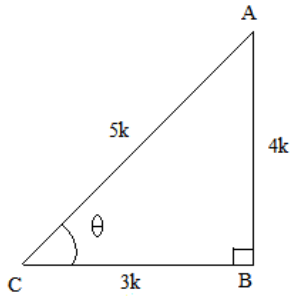
Hence proved.

20. If $3 \tan \theta = 4$, show that $\frac{(4 \cos \theta - \sin \theta)}{(4 \cos \theta + \sin \theta)} = \frac{4}{5}$

Sol:

Let us consider a right $\triangle ABC$ right angled at B and $\angle C = \theta$.

$$\text{We know that } \tan \theta = \frac{AB}{BC} = \frac{4}{3}$$



So, if $BC = 3k$, then $AB = 4k$, where k is a positive number.

Using Pythagoras theorem, we have:

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ \Rightarrow AC^2 &= 16k^2 + 9k^2 \\ \Rightarrow AC^2 &= 25k^2 \\ \Rightarrow AC &= 5k \end{aligned}$$

Now, we have:

$$\begin{aligned} \sin \theta &= \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5} \\ \cos \theta &= \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5} \end{aligned}$$

Substituting these values in the given expression, we get:

$$\begin{aligned} &\frac{4 \cos \theta - \sin \theta}{2 \cos \theta + \sin \theta} \\ &= \frac{4\left(\frac{3}{5}\right) - \frac{4}{5}}{2\left(\frac{3}{5}\right) + \frac{4}{5}} \\ &= \frac{\frac{12}{5} - \frac{4}{5}}{\frac{6}{5} + \frac{4}{5}} \\ &= \frac{\frac{12-4}{5}}{\frac{6+4}{5}} \\ &= \frac{8}{10} = \frac{4}{5} = RHS \end{aligned}$$

i.e., LHS = RHS

Hence proved.

21. If $3 \cot \theta = 2$, show that $\frac{(4 \sin \theta - 4 \cos \theta)}{(2 \sin \theta + 6 \cos \theta)} = \frac{1}{3}$

Sol:

It is given that $\cos \theta = \frac{2}{3}$

$$LHS = \frac{4 \sin \theta - 3 \cos \theta}{2 \sin \theta + 6 \cos \theta}$$

Dividing the above expression by $\sin \theta$, we get:

$$\frac{4 - 3 \cot \theta}{2 + 6 \cot \theta} \quad [\because \cot \theta = \frac{\cos \theta}{\sin \theta}]$$

Now, substituting the values of $\cot \theta$ in the above expression, we get:

$$\frac{4-3\left(\frac{2}{3}\right)}{2+6\left(\frac{2}{3}\right)}$$

$$= \frac{4-2}{2+4} = \frac{2}{6} = \frac{1}{3}$$

i.e., LHS = RHS
Hence proved.

22. If $3 \cot \theta = 4$, show that $\frac{(1 - \tan^2 \theta)}{(1 + \tan^2 \theta)} = (\cos^2 \theta - \sin^2 \theta)$

Sol:

$$LHS = \frac{(1 - \tan^2 \theta)}{(1 + \tan^2 \theta)}$$

$$= \frac{\left(1 - \frac{1}{\cot^2 \theta}\right)}{\left(1 + \frac{1}{\cot^2 \theta}\right)}$$

$$= \frac{\frac{\cot^2 \theta - 1}{\cot^2 \theta}}{\left(\frac{\cot^2 \theta + 1}{\cot^2 \theta}\right)}$$

$$= \frac{\cot^2 \theta - 1}{\cot^2 \theta + 1}$$

$$= \frac{\left(\frac{4}{3}\right)^2 - 1}{\left(\frac{4}{3}\right)^2 + 1}$$

$$\left(As, 3 \cot \theta = 4 \text{ or } \cot \theta = \frac{4}{3}\right)$$

$$= \frac{\frac{16}{9} - 1}{\frac{16}{9} + 1}$$

$$= \frac{\left(\frac{16-9}{9}\right)}{\left(\frac{16+9}{9}\right)}$$

$$= \frac{\left(\frac{7}{9}\right)}{\left(\frac{25}{9}\right)}$$

$$= \frac{7}{25}$$

$$RHS = (\cos^2 \theta - \sin^2 \theta)$$

$$= \frac{(\cos^2 \theta - \sin^2 \theta)}{1}$$

$$= \frac{\left(\frac{\cos^2 \theta - \sin^2 \theta}{\sin^2 \theta}\right)}{\left(\frac{1}{\sin^2 \theta}\right)}$$

$$= \frac{\cos^2 \theta \sin^2 \theta}{\sin^2 \theta \sin^2 \theta}$$

$$= \frac{(\cot^2 \theta - 1)}{(\cot^2 \theta + 1)}$$

$$= \frac{\left[\left(\frac{4}{3}\right)^2 - 1\right]}{\left[\left(\frac{4}{3}\right)^2 + 1\right]}$$

$$\begin{aligned}
 &= \frac{\left(\frac{16}{9} - \frac{1}{1}\right)}{\left(\frac{16}{9} + \frac{1}{1}\right)} \\
 &= \frac{\left(\frac{16-9}{9}\right)}{\left(\frac{16+9}{9}\right)} \\
 &= \frac{\left(\frac{7}{9}\right)}{\left(\frac{25}{9}\right)} \\
 &= \frac{7}{25}
 \end{aligned}$$

Since, LHS = RHS

Hence, verified.

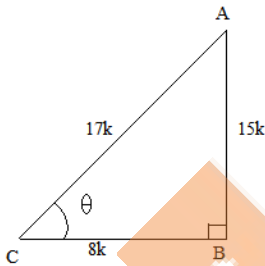
23. If $\sec \theta = \frac{17}{8}$ verify that $\frac{(3 - 4 \sin^2 \theta)}{(4 \cos^2 \theta - 3)} = \frac{(3 - \tan^2 \theta)}{(1 - \tan^3 \theta)}$

Sol:

It is given that $\sec \theta = \frac{17}{8}$

Let us consider a right $\triangle ABC$ right angled at B and $\angle C = \theta$

We know that $\cos \theta = \frac{1}{\sec \theta} = \frac{8}{17} = \frac{BC}{AC}$



So, if $BC = 8k$, then $AC = 17k$, where k is a positive number.

Using Pythagoras theorem, we have:

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AB^2 = AC^2 - BC^2 = (17k)^2 - (8k)^2$$

$$\Rightarrow AB^2 = 289k^2 - 64k^2 = 225k^2$$

$$\Rightarrow AB = 15k.$$

Now, $\tan \theta = \frac{AB}{BC} = \frac{15}{8}$ and $\sin \theta = \frac{AB}{AC} = \frac{15k}{17k} = \frac{15}{17}$

The given expression is $\frac{3 - 4 \sin^2 \theta}{4 \cos^2 \theta - 3} = \frac{3 - \tan^2 \theta}{1 - 3 \tan^2 \theta}$

Substituting the values in the above expression, we get:

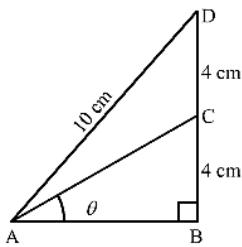
$$\begin{aligned}
 LHS &= \frac{3 - 4\left(\frac{15}{17}\right)^2}{4\left(\frac{8}{17}\right)^2 - 3} \\
 &= \frac{3 - \frac{900}{289}}{\frac{256}{289} - 3} \\
 &= \frac{867 - 900}{256 - 867} = -\frac{33}{-611} = \frac{33}{611}
 \end{aligned}$$

$$\begin{aligned}
 RHS &= \frac{3 - \left(\frac{15}{8}\right)^2}{1 - 3\left(\frac{15}{8}\right)^2} \\
 &= \frac{3 - \frac{225}{64}}{1 - \frac{675}{64}} \\
 &= \frac{192 - 225}{64 - 675} = -\frac{33}{-611} = \frac{33}{611}
 \end{aligned}$$

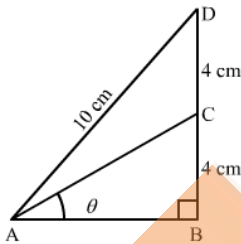
\therefore LHS = RHS

Hence proved.

24. In the adjoining figure, $\angle B = 90^\circ$, $\angle BAC = \theta^\circ$, $BC = CD = 4\text{ cm}$ and $AD = 10\text{ cm}$. find (i) $\sin \theta$ and (ii) $\cos \theta$



Sol:



In $\triangle ABD$,

Using Pythagoras theorem, we get

$$\begin{aligned}
 AB &= \sqrt{AD^2 - BD^2} \\
 &= \sqrt{10^2 - 8^2} \\
 &= \sqrt{100 - 64} \\
 &= \sqrt{36} \\
 &= 6\text{ cm}
 \end{aligned}$$

Again,

In $\triangle ABC$,

Using Pythagoras theorem, we get

$$\begin{aligned}
 AC &= \sqrt{AB^2 + BC^2} \\
 &= \sqrt{6^2 + 4^2} \\
 &= \sqrt{36 + 16} \\
 &= \sqrt{52} \\
 &= 2\sqrt{13}\text{ cm}
 \end{aligned}$$

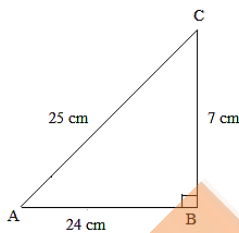
Now,

$$\begin{aligned} \text{(i) } \sin \theta &= \frac{BC}{AC} \\ &= \frac{4}{2\sqrt{13}} \\ &= \frac{2}{\sqrt{13}} \\ &= \frac{2\sqrt{13}}{13} \end{aligned}$$

$$\begin{aligned} \text{(ii) } \cos \theta &= \frac{AB}{AC} \\ &= \frac{6}{2\sqrt{13}} \\ &= \frac{3}{\sqrt{13}} \\ &= \frac{3\sqrt{13}}{13} \end{aligned}$$

25. In a $\triangle ABC$, $\angle B = 90^\circ$, $AB = 24$ cm and $BC = 7$ cm find (i) $\sin A$ (ii) $\cos A$ (iii) $\sin C$ (iv) $\cos C$

Sol:



Using Pythagoras theorem, we get:

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ \Rightarrow AC^2 &= (24)^2 + (7)^2 \\ \Rightarrow AC^2 &= 576 + 49 = 625 \\ \Rightarrow AC &= 25 \text{ cm} \end{aligned}$$

Now, for T-Ratios of $\angle A$, *base = AB and perpendicular = BC*

$$\begin{aligned} \text{(i) } \sin A &= \frac{BC}{AC} = \frac{7}{25} \\ \text{(ii) } \cos A &= \frac{AB}{AC} = \frac{24}{25} \end{aligned}$$

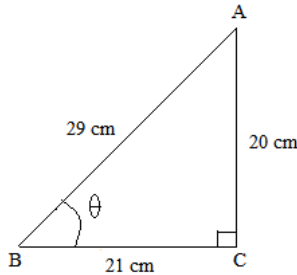
Similarly, for T-Ratios of $\angle C$, *base = BC and perpendicular = AB*

$$\begin{aligned} \text{(iii) } \sin C &= \frac{AB}{AC} = \frac{24}{25} \\ \text{(iv) } \cos C &= \frac{BC}{AC} = \frac{7}{25} \end{aligned}$$

26. In $\triangle ABC$, $\angle C = 90^\circ$, $\angle ABC = \theta^\circ$, $BC = 21$ units and $AB = 29$ units. Show that

$$(\cos^2 \theta - \sin^2 \theta) = \frac{41}{841}$$

Sol:



Using Pythagoras theorem, we get:

$$AB^2 = AC^2 + BC^2$$

$$\Rightarrow AC^2 = AB^2 - BC^2$$

$$\Rightarrow AC^2 = (29)^2 - (21)^2$$

$$\Rightarrow AC^2 = 841 - 441$$

$$\Rightarrow AC^2 = 400$$

$$\Rightarrow AC = \sqrt{400} = 20 \text{ units}$$

$$\text{Now, } \sin \theta = \frac{AC}{AB} = \frac{20}{29} \text{ and } \cos \theta = \frac{BC}{AB} = \frac{21}{29}$$

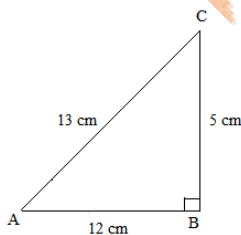
$$\cos^2 \theta - \sin^2 \theta = \left(\frac{21}{29}\right)^2 - \left(\frac{20}{29}\right)^2 = \frac{441}{841} - \frac{400}{841} = \frac{41}{841}$$

Hence proved.

27. In a $\triangle ABC$, $\angle B = 90^\circ$, $AB = 12$ cm and $BC = 5$ cm Find

(i) $\cos A$ (ii) $\operatorname{cosec} A$ (iii) $\cos C$ (iv) $\operatorname{cosec} C$

Sol:



Using Pythagoras theorem, we get:

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = 12^2 + 5^2 = 144 + 25$$

$$\Rightarrow AC^2 = 169$$

$$\Rightarrow AC = 13 \text{ cm}$$

Now, for T-Ratios of $\angle A$, base = AB and perpendicular = BC

$$(i) \cos A = \frac{AB}{AC} = \frac{12}{13}$$

$$(ii) \operatorname{cosec} A = \frac{1}{\sin A} = \frac{AC}{BC} = \frac{13}{5}$$

Similarly, for T-Ratios of $\angle C$, base = BC and perpendicular = AB

$$(iii) \cos C = \frac{BC}{AC} = \frac{5}{13}$$

$$(iv) \operatorname{cosec} C = \frac{1}{\sin C} = \frac{AC}{AB} = \frac{13}{12}$$

28. If $\sin \alpha = \frac{1}{2}$ prove that $(3 \cos \alpha - 4 \cos^2 \alpha) = 0$

Sol:

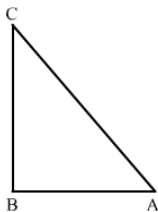
$$\begin{aligned} LHS &= (3 \cos \alpha - 4 \cos^3 \alpha) \\ &= \cos \alpha (3 - 4 \cos^2 \alpha) \\ &= \sqrt{1 - \sin^2 \alpha} [3 - 4(1 - \sin^2 \alpha)] \\ &= \sqrt{1 - \left(\frac{1}{2}\right)^2} \left[3 - 4\left(1 - \left(\frac{1}{2}\right)^2\right)\right] \\ &= \sqrt{\frac{1}{1} - \frac{1}{4}} \left[3 - 4\left(\frac{1}{1} - \frac{1}{4}\right)\right] \\ &= \sqrt{\frac{3}{4}} \left[3 - 4\left(\frac{3}{4}\right)\right] \\ &= \sqrt{\frac{3}{4}} [3 - 3] \\ &= \sqrt{\frac{3}{4}} [0] \\ &= 0 \\ &= \text{RHS} \end{aligned}$$

29. IF $\triangle ABC$, $\angle B = 90^\circ$ AND $\tan A = \frac{1}{\sqrt{3}}$. Prove that

(i) $\sin A \cdot \cos C + \cos A \cdot \sin C = 1$

(ii) $\cos A \cdot \cos C - \sin A \cdot \sin C = 0$

Sol:



In $\triangle ABC$, $\angle B = 90^\circ$,

As, $\tan A = \frac{1}{\sqrt{3}}$

$$\Rightarrow \frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

Let $BC = x$ and $AB = x\sqrt{3}$

Using Pythagoras the get

$$AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{(x\sqrt{3})^2 + x^2}$$

$$= \sqrt{3x^2 + x^2}$$

$$= \sqrt{4x^2}$$

$$= 2x$$

Now,

$$(i) \text{LHS} = \sin A \cdot \cos C + \cos A \cdot \sin C$$

$$= \frac{BC}{AC} \cdot \frac{BC}{AC} + \frac{AB}{AC} \cdot \frac{AB}{AC}$$

$$= \left(\frac{BC}{AC}\right)^2 + \left(\frac{AB}{AC}\right)^2$$

$$= \left(\frac{x}{2x}\right)^2 + \left(\frac{x\sqrt{3}}{2x}\right)^2$$

$$= \frac{1}{4} + \frac{3}{4}$$

$$= 1$$

$$= \text{RHS}$$

$$(ii) \text{LHS} = \cos A \cdot \cos C - \sin A \cdot \sin C$$

$$= \frac{AB}{AC} \cdot \frac{BC}{AC} - \frac{BC}{AC} \cdot \frac{AB}{AC}$$

$$= \frac{x\sqrt{3}}{2x} \cdot \frac{x}{2x} - \frac{x}{2x} \cdot \frac{x\sqrt{3}}{2x}$$

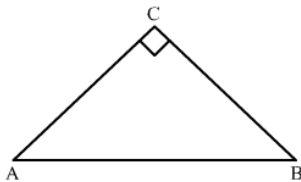
$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

$$= 0$$

$$= \text{RHS}$$

30. If $\angle A$ and $\angle B$ are acute angles such that $\sin A = \sin B$ prove that $\angle A = \angle B$

Sol:



In $\triangle ABC$, $\angle C = 90^\circ$

$$\sin A = \frac{BC}{AB} \text{ and}$$

$$\sin B = \frac{AC}{AB}$$

As, $\sin A = \sin B$

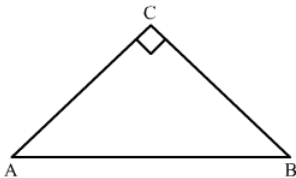
$$\Rightarrow \frac{BC}{AB} = \frac{AC}{AB}$$

$$\Rightarrow BC = AC$$

So, $\angle A = \angle B$ (Angles opposite to equal sides are equal)

31. If $\angle A$ and $\angle B$ are acute angles such that $\tan A = \tan B$ then prove that $\angle A = \angle B$

Sol:



In $\triangle ABC$, $\angle C = 90^\circ$

$$\tan A = \frac{BC}{AC} \text{ and}$$

$$\tan B = \frac{AC}{BC}$$

As, $\tan A = \tan B$

$$\Rightarrow \frac{BC}{AC} = \frac{AC}{BC}$$

$$\Rightarrow BC^2 = AC^2$$

$$\Rightarrow BC = AC$$

So, $\angle A = \angle B$ (Angles opposite to equal sides are equal)

32. If a right $\triangle ABC$, right-angled at B, if $\tan A = 1$ then verify that $2 \sin A \cdot \cos A = 1$

Sol:

We have,

$$\tan A = 1$$

$$\Rightarrow \frac{\sin A}{\cos A} = 1$$

$$\Rightarrow \sin A = \cos A$$

$$\Rightarrow \sin A - \cos A = 0$$

Squaring both sides, we get

$$(\sin A - \cos A)^2 = 0$$

$$\Rightarrow \sin^2 A + \cos^2 A - 2 \sin A \cdot \cos A = 0$$

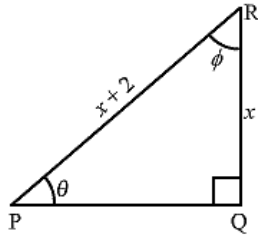
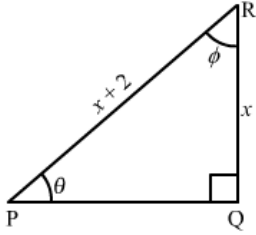
$$\Rightarrow 1 - 2 \sin A \cdot \cos A = 0$$

$$\therefore 2 \sin A \cdot \cos A = 1$$

33. In the figure of $\triangle PQR$, $\angle P = \theta^\circ$ and $\angle R = \phi^\circ$ find

(i) $\sqrt{x+1} \cot \phi$

(ii) $\sqrt{x^3 + x^2} \tan \theta$

(iii) $\cos \theta$ **Sol:**In ΔPQR , $\angle Q = 90^\circ$,

Using Pythagoras theorem, we get

$$\begin{aligned} PQ &= \sqrt{PR^2 - QR^2} \\ &= \sqrt{(x+2)^2 - x^2} \\ &= \sqrt{x^2 + 4x + 4 - x^2} \\ &= \sqrt{4(x+1)} \\ &= 2\sqrt{x+1} \end{aligned}$$

Now,

$$\begin{aligned} \text{(i)} \quad & (\sqrt{x+1}) \cot \theta \\ &= (\sqrt{x+1}) \times \frac{QR}{PQ} \\ &= (\sqrt{x+1}) \times \frac{x}{2\sqrt{x+1}} \\ &= \frac{x}{2} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad & (\sqrt{x^3 + x^2}) \tan \theta \\ &= (\sqrt{x^2(x+1)}) \times \frac{QR}{PQ} \\ &= x \sqrt{(x+1)} \times \frac{x}{2\sqrt{x+1}} \\ &= \frac{x^2}{2} \end{aligned}$$

(iii) $\cos \theta$

$$= \frac{PQ}{PR} \quad \theta = \frac{2\sqrt{x+1}}{x+2}$$

34. If $x = \operatorname{cosec} A + \cos A$ and $y = \operatorname{cosec} A - \cos A$ then prove that $\left(\frac{2}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2 = 1$

Sol:

$$\begin{aligned}
 LHS &= \left(\frac{2}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2 - 1 \\
 &= \left[\frac{2}{(\operatorname{cosec} A + \cos A) + (\operatorname{cosec} A - \cos A)}\right]^2 + \left[\frac{(\operatorname{cosec} A + \cos A) - (\operatorname{cosec} A - \cos A)}{2}\right]^2 - 1 \\
 &= \left[\frac{2}{\operatorname{cosec} A + \cos A + \operatorname{cosec} A - \cos A}\right]^2 + \left[\frac{\operatorname{cosec} A + \cos A - \operatorname{cosec} A + \cos A}{2}\right]^2 - 1 \\
 &= \left[\frac{2}{2 \operatorname{cosec} A}\right]^2 + \left[\frac{2 \cos A}{2}\right]^2 - 1 \\
 &= \left[\frac{1}{\operatorname{cosec} A}\right]^2 + [\cos A]^2 - 1 \\
 &= [\sin A]^2 + [\cos A]^2 - 1 \\
 &= \sin^2 A + \cos^2 A - 1 \\
 &= 1 - 1 \\
 &= 0 \\
 &= \text{RHS}
 \end{aligned}$$

35. If $x = \cot A + \cos A$ and $y = \cot A - \cos A$ then prove that $\left(\frac{x-y}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2 = 1$

Sol:

$$\begin{aligned}
 LHS &= \left(\frac{x-y}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2 \\
 &= \left[\frac{(\cot A + \cos A) - (\cot A - \cos A)}{(\cot A + \cos A) + (\cot A - \cos A)}\right]^2 + \left[\frac{(\cot A + \cos A) - (\cot A - \cos A)}{2}\right]^2 \\
 &= \left[\frac{\cot A + \cos A - \cot A + \cos A}{\cot A + \cos A + \cot A - \cos A}\right]^2 + \left[\frac{\cot A + \cos A - \cot A + \cos A}{2}\right]^2 \\
 &= \left[\frac{2 \cos A}{2 \cot A}\right]^2 + \left[\frac{2 \cos A}{2}\right]^2 \\
 &= \left[\frac{\cos A}{\left(\frac{\cos A}{\sin A}\right)}\right]^2 + [\cos A]^2 \\
 &= \left[\frac{\sin A \cos A}{\cos A}\right]^2 + [\cos A]^2 \\
 &= [\sin A]^2 + [\cos A]^2 \\
 &= \sin^2 A + \cos^2 A \\
 &= 1 \\
 &= \text{RHS}
 \end{aligned}$$