Answer each of the following questions either in one word or one sentence or as per requirement of the questions:

Question 1.

Define an arithmetic progression.

Solution:

A sequence a_1, a_2, a_3, …, an is called an arithmetic progression of then exists a constant d

Such that $a_2 - a_1 = d$, $a_3 - a_2 = d$, ………. $a_n - a_{n-1} = d$

and so on and d is called common difference

Question 2.

Write the common difference of an A.P. whose nth term is $a_n = 3n + 7$.

Solution:

$a_n = 3n + 7$

$a_1 = 3 \times 1 + 7 = 3 + 7 = 10$

$a_2 = 3 \times 2 + 7 = 6 + 7 = 13$

$a_3 = 3 \times 3 + 7 = 9 + 7 = 16$

$d = a_3 - a_2$ or $a_2 - a_1 = 16 - 13 = 3$ or $13 - 10 = 3$

Question 3.

Which term of the sequence 114, 109, 104, … is the first negative term ?

Solution:

Sequence is 114, 109, 104, …..

Let a_n term be negative

Then $a_n < 0$

First term $(a) = 114$, common difference

$(d) = 109 - 114$

$\Rightarrow d = -5$

$\because a_n = a + (n - 1) d$

$\Rightarrow a_n = 114 + (n - 1)(-5)$

$\Rightarrow a_n = 114 - 5n + 5$

$\Rightarrow a_n = 119 - 5n$

$\because a_n < 0$

$\therefore 119 - 5n < 0 \Rightarrow 119 < 5n$

$$\Rightarrow 5n > 119 \Rightarrow n > \frac{119}{5} \Rightarrow n > 24$$

$\therefore$ 24th term will be negative

Question 4.

Write the value of $a_{30} - a_{10}$ for the A.P. 4, 9, 14, 19, ………..

Solution:

In the A.P., 4, 9, 14, 19, ...

First term $(a) = 4$

Common difference $(d) = 9 - 4 = 5$

$\therefore\ a_n = a + (n-1)d$

$\Rightarrow a_{10} = 4 + (10-1) \times 5$

$= 4 + 9 \times 5 = 4 + 45 = 49$

and $a_{30} = 4 + (30-1) \times 5$

$= 4 + 29 \times 5 = 4 + 145 = 149$

$\therefore\ a_{30} - a_{10} = 149 - 49 = 100$

Question 5.

Write 5th term from the end of the A.P. 3, 5, 7, 9,..., 201.

Solution:

A.P. is 3, 5, 7, 9, ..., 201

Here first term $(a) = 3$

Common difference $(d) = 5 - 3 = 2$

$a_n = a + (n-1)d \Rightarrow 201 = 3 + (n-1) \times 2$

$\Rightarrow 201 = 3 + 2n - 2 \Rightarrow 201 + 2 - 3 = 2n$

$\Rightarrow 2n = 200 \Rightarrow n = 100$

Now fifth term from the end will be a_{n-4}

$\therefore\ a_{n-4} = 3 + (n - 4 - 1) \times 2$

$= 3 + (100 - 5) \times 2 = 3 + 95 \times 2$

= 3 + 190 = 193

5th term from the end = 193

Question 6.

Write the value of x for which 2x, x + 10 and 3x + 2 are in A.P.

Solution:

$\because\ 2x,\ x + 10$ and $3x + 2$ are in A.P.

$$\therefore\ x + 10 = \frac{2x + 3x + 2}{2}$$

$\Rightarrow 2(x + 10) = 5x + 2$

$\Rightarrow 2x + 20 = 5x + 2$

$\Rightarrow 5x - 2x = 20 - 2$

$$\Rightarrow 3x = 18 \Rightarrow x = \frac{18}{3} = 6$$

Question 7.
Write the nth term of an A.P. the sum of whose n terms is S_n.
Solution:
Sum of n terms = S_n
Let a be the first term and d be the common difference $a_n = S_n - S_{n-1}$

Question 8.
Write the sum of first n odd natural numbers.
Solution:
The first n odd natural number are
1, 3, 5, 7, ...
Here $a = 1$, $d = 3 - 1 = 2$

$$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 1 + (n-1) \times 2]$$

$$= \frac{n}{2}[2 + 2n - 2] = \frac{n}{2} \times 2n = n^2$$

Question 9.
Write the sum of first n even natural numbers.
Solution:
First n even natural numbers are
2, 4, 6, 8, ...
Here a = 2, d = 2

$$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 2 + (n-1) \times 2]$$

$$= \frac{n}{2}[4 + 2n - 2] = \frac{n}{2}[2 + 2n]$$

$$= \frac{n}{2} \times 2(1 + n) = n(n+1)$$

Question 10.
If the sum of n terms of an A.P. is $S_n = 3n^2 + 5n$. Write its common difference.

Solution:

$$S_n = 3n^2 + 5n$$
$$S_1 \text{ or } a_1 = 3(1)^2 + 5(1) = 3 + 5 = 8$$
$$S_2 = 3(2)^2 + 5(2) = 12 + 10 = 22$$
$$\therefore a_2 = S_2 - S_1 = 22 - 8 = 14$$
$$\therefore d = a_1 = a_1 = 14 - 8 = 6$$

Question 11.
Write the expression for the common difference of an A.P. Whose first term is a and nth term is b.
Solution:
First term of an A.P. = a
and $a_n = a + (n - 1) d = b$.
Subtracting, $b - a = (n - 1) d$
d = b–an–1
Question 12.
The first term of an A.P. is p and its common difference is q. Find its 10th term. **[CBSE 2008]**
Solution:
First term of an A.P. (a) = p
and common difference (d) = q
$a_{10} = a + (n - 1) d$
$= p + (10 - 1) q = p + 9q$

Question 13.
For what value of p are 2p + 1, 13, 5p – 3 are three consecutive terms of an A.P.? **[CBSE 2009]**
Solution:

$\because 2p + 1, 13, 5p - 3$ are consecutive terms of an A.P.

$$\therefore c.d. = 13 - 2p - 1, = 5p - 3 - 13$$
$$\Rightarrow 5p + 2p = 13 - 1 + 13 + 3$$
$$\Rightarrow 7p = 28 \Rightarrow p = \frac{28}{7} = 4$$

Hence $p = 4$

Question 14.
If 45, a, 2 are three consecutive terms of an A.P., then find the value of a.

Solution:

$\because \frac{4}{5}, a, 2$ are there consecutive terms of an A.P.

$$\therefore c.d. = a - \frac{4}{5} = 2 - a$$

$$\Rightarrow a + a = 2 + \frac{4}{5} \Rightarrow 2a = \frac{14}{5}$$

$$\Rightarrow a = \frac{14}{5 \times 2} = \frac{7}{5}$$

$$\therefore a = \frac{7}{5}$$

Question 15.

If the sum of first p term of an A.P. is $ap^2 + bp$, find its common difference.

Solution:

Sum of first p terms $= ap^2 + bp$

$\therefore S_p = ap^2 + bp$

$S_{p-1} = a(p-1)^2 + b(p-1)$

$\therefore a_p = S_p - S_{p-1}$

$= ap^2 + bk - a(p-1)^2 - b \quad (p-1)$

$= ap^2 + bk - a(p^2 - 2p + 1) - bp + b$

$= ap^2 + bp - ap^2 + 2ap - a - bp + b$

$= 2ap - a - b = 2ap - (a + b)$

Now $= 2a(1) - (a + b)$

$= 2a - a - b = a - b$

and $a_2 = 2a(2) - a - b = 4a - a - b$

$= 3a - b$

$= d = a_2 - a_1$

$d = 3a - b - a + b = 2a$

$\therefore$ Commong difference $= 2a$

Question 16.

Find the 9th term from the end of the A.P. 5, 9, 13, …, 185. **[CBSE 2016]**

Solution:

Here first term, a = 5

Common difference, d = 9 – 5 = 4

Last term, l = 185

nth term from the end = l – (n – 1) d

9th term from the end = 185 – (9 – 1) 4 = 185 – 8 x 4 = 185 – 32 = 153

Question 17.
For what value of k will the consecutive terms 2k + 1, 3k + 3 and 5k – 1 form on A.P.? **[CBSE 2016]**
Solution:
(3k + 3) – (2k + 1) = (5k – 1) – (3k + 3)
3k + 3 – 2k – 1 = 5k – 1 – 3k – 3
k + 2 = 2k – 4
2k – k = 2 + 4
k = 6

Question 18.
Write the nth term of the A.P.
1m , 1+mm , 1+2mm , ……… **[CBSE 2017]**
Solution:

Here, $a = \frac{1}{m}$

$$d = \frac{1+2m}{m} - \frac{1+m}{m} = \frac{1+2m-1-m}{m} = \frac{m}{m} = 1$$

$$T_n = a + (n-1)d$$

$$= \frac{1}{m} + (n-1) \times 1 = \frac{m(n-1)+1}{m}$$