

### Exercise 8.1

1.

**Sol:**

(i)  $x^2 + 6x - 4 = 0$

(ii)  $\sqrt{3x^2} - 2x + \frac{1}{2} = 0$

(iii)  $3x^2 - 5x + 9 = x^2 - 7x + 3$

(iv)  $x + \frac{1}{x} = 1$

(v)  $(2x + 1)(3x + 2) = 6(x - 1)(x - 2)$

(vi)  $16x^2 - 3 = (2x + 5)(5x - 3)$

(vii)  $(x + 2)^3 = x^3 - 4$

These are all quadratic equations

2.

**Sol:**

(i)  $x^2 - 3x + 2 = 0, x = 2, x = -1$

Here LHS =  $x^2 - 3x + 2$

and RHS = 0

Now, substitute  $x = 2$  in LHS

We get  $(2)^2 - 3(2) + 2 = 4 - 6 + 2$

$= 6 - 6$

$= 0$

$\Rightarrow$  RHS

Since, LHS = RHS

$x = 2$  is a solution for the given equation.

Similarly,

Now substitute  $x = -1$  in LHS

We get  $(-1)^2 - 3(-1) + 2$

$\Rightarrow 1 + 3 + 2 = 6 \neq$  RHS

Since LHS  $\neq$  RHS

$x = -1$  is not a solution for the given equation

(ii)  $x^2 + x + 1 = 0, x = 0, x = 1$

Here LHS =  $x^2 + x + 1$  and RHS = 0

Now substitute  $x = 0$  and  $x = 1$  in LHS

$\Rightarrow 0^2 + 0 + 1$  and  $(1)^2 + (1) + 1$

$\Rightarrow 1$  and  $1 + 1 + 1 = 3$

$\neq$  RHS  $\neq$  RHS

$\therefore x = 0, x = 1$  are not solutions of the given equation

(iii)  $x^2 - 3\sqrt{3}x + 6 = 0, x = \sqrt{3}, x = -8\sqrt{3}$

Here LHS =  $x^2 - 3\sqrt{3}x + 6$  and RHS = 0

Substitute  $x = \sqrt{3}$  and  $x = -2\sqrt{3}$  in LHS

$$\Rightarrow (\sqrt{3})^2 - 3\sqrt{3}(\sqrt{3}) + 6 \text{ and } (-2\sqrt{3})^2 - 3\sqrt{3}(-2\sqrt{3}) + 6$$

$$\Rightarrow 3 - 9 + 6 \text{ and } 18 + 18 + 6$$

$$\Rightarrow 0 \text{ and } 36$$

$$\Rightarrow \text{RHS} \neq \text{RHS}$$

$\therefore x = \sqrt{3}$  is a solution and  $x = -2\sqrt{3}$  is not a solution for the given equation

(iv)  $x + \frac{1}{2} = \frac{13}{6} = x = \frac{5}{6}, x = \frac{4}{3}$

Here LHS =  $x + \frac{1}{x}$  and RHS =  $\frac{13}{6}$

Substitute  $x = \frac{5}{6}$  and  $x = \frac{4}{3}$  in the LHS

$$\Rightarrow \frac{5}{6} + \frac{1}{\left(\frac{5}{6}\right)} \text{ and } \frac{4}{3} + \frac{1}{\left(\frac{4}{9}\right)}$$

$$\Rightarrow \frac{5}{6} + \frac{6}{5} \text{ and } \frac{4}{3} + \frac{3}{4}$$

$$\Rightarrow \frac{85+36}{30} \text{ and } \frac{16+9}{18}$$

$$\Rightarrow \frac{61}{30} \text{ and } \frac{85}{18}$$

$$\neq \text{RHS} \neq \text{RHS}$$

$\therefore x = \frac{5}{6}$  and  $x = \frac{4}{3}$  are not solutions of the given equation

(v)  $2x^2 - x + 9 = x^2 + 4x + 3, x = 2$  and  $x = 3$

$$\Rightarrow 2x^2 - x^2 - x - 4x + 9 - 3 = 0$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

Here, LHS =  $x^2 - 5x + 6$  and RHS = 0

Substitute  $x = 2$  and  $x = 3$  in LHS

$$\Rightarrow (2)^2 - 5(2) + 6 \text{ and } (3)^2 - 5(3) + 6$$

$$\Rightarrow 4 - 10 + 6 \text{ and } 9 - 15 + 6$$

$$\Rightarrow 10 - 10 \text{ and } 15 - 15$$

$$\Rightarrow 0 \text{ and } \Rightarrow 0$$

$$= \text{RHS} = \text{RHS}$$

$x = 3$  and  $x = 2$  are solutions of the given equation.

(vi)  $x^2 - \sqrt{2}x - 4 = 0, x = -\sqrt{2}$  and  $x = -8\sqrt{2}$

Here, LHS =  $x^2 - \sqrt{2}x - 4$  and RHS = 0

Substitute  $x = -\sqrt{2}$  and  $x = -2\sqrt{2}$  in LHS

$$\Rightarrow (-\sqrt{2})^2 - \sqrt{2}(\sqrt{2}) - 4 \text{ and } (-2\sqrt{2})^2 - \sqrt{2}(-2\sqrt{2}) - 4$$

$$\Rightarrow 2 + 2 - 4 \text{ and } 8 + 4 - 4$$

$$\Rightarrow 4 - 4 \text{ and } 8 - 4$$

$$\Rightarrow 0 \text{ and } 8$$

$$= \text{RHS} \neq \text{RHS}$$

$\therefore x = -\sqrt{2}$  is a solution and  $x = -2\sqrt{2}$  is not a solution is the given equation.

(vii)  $ax^2 - 3abx + 2b^2 = 0, x = \frac{a}{b}$  and  $x = \frac{b}{a}$

Here, LHS =  $ax^2 - 3abx + 2b^2$  and RHS = 0

Substitute  $x = \frac{a}{b}$  and  $x = \frac{b}{a}$  in LHS

$$\Rightarrow a^2 \left(\frac{a}{b}\right)^2 - 3ab \left(\frac{a}{b}\right) + 2b^2 \text{ and } a^2 \left(\frac{b}{a}\right)^2 - 3ab \left(\frac{b}{a}\right) + 2b^2$$

$$\Rightarrow a^2 \left(\frac{a^2}{b^2}\right) - 3a \times a + 2b^2 \text{ and } a^2 \times \frac{b^2}{a^2} - 3b \times b + 2b^2$$

$$\Rightarrow \frac{a^2}{b^2} - 3a^2 + 2b^2 \text{ and } b^2 - 3b^2 + 2b^2$$

$$\Rightarrow \frac{a^4}{b^2} - 3a^2 + 2b^2 \text{ and } 3b^2 - 3b^2 = 0$$

$$\Rightarrow \neq \text{RHS} = \text{RHS}$$

$\therefore x = \frac{b}{a}$  is a solution and  $x = \frac{a}{b}$  is not a solution for the given equation.

3.

**Sol:**

(i) Given that  $x = \frac{2}{3}$  is a root of the given equation

$$\Rightarrow x = \frac{2}{3} \text{ satisfies the equation}$$

$$i.e. 7 \left(\frac{2}{3}\right)^2 + k \left(\frac{2}{3}\right) - 3 = 0$$

$$\Rightarrow 7 \times \frac{4}{9} + 2 \frac{k}{3} - 3 = 0$$

$$\Rightarrow 2\frac{k}{3} = 3 - \frac{28}{9}$$

$$\Rightarrow 2\frac{k}{3} = \frac{27-28}{9}$$

$$\Rightarrow 2\frac{k}{3} = -\frac{1}{9} \Rightarrow \boxed{k = -\frac{1}{6}}$$

(ii) Given that  $x = a$  is a root of the given equation

$$x^2 - x(a+b) + k = 0$$

$\Rightarrow x = a$  Satisfies the equation

$$i \cdot e (a)^2 - a(a+b) + k = 0$$

$$\Rightarrow a^2 - a^2 - ab + k = 0 \Rightarrow -ab + k = 0$$

$$\Rightarrow \boxed{k = ab}$$

(iii) Given that  $x = \sqrt{2}$  is a root at the given equation

$$kx^2 + \sqrt{2}x - 4 = 0$$

$\Rightarrow x = \sqrt{2}$  Satisfies the equation

$$i \cdot e k(\sqrt{2})^2 + \sqrt{2}(\sqrt{2}) - 4 = 0$$

$$\Rightarrow 2k + 2 - 4 = 0$$

$$\Rightarrow 2k - 2 = 0 \Rightarrow 2k = 2$$

$$\Rightarrow \boxed{k = 1}$$

(iv) Given that  $x = -a$  is a root of the given equation  $x^2 + 3ax + k = 0$

$\Rightarrow x = -a$  Satisfies the equation

$$i \cdot e (-a)^2 + 3a(-a) + k = 0$$

$$\Rightarrow a^2 - 3a^2 + k = 0 \Rightarrow -2a^2 + k = 0$$

$$\Rightarrow \boxed{k = 2a^2}$$

4.

**Sol:**

$$a = 3, b = -6$$

5.

**Sol:**

Given to check whether 3 is a root of the equation

$$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 16}$$

$$\text{Here LHS} = \sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} \text{ and RHS} = \sqrt{4x^2 - 14x + 16}$$

Substitute  $x = 3$  in LHS

$$\Rightarrow \sqrt{3^2 - 4(3) + 3} + \sqrt{(3)^2} = 9$$

$$\Rightarrow \sqrt{9 - 18 + 3} + \sqrt{9 - 9}$$

$$\Rightarrow \sqrt{0} + \sqrt{0} \Rightarrow 0 \quad \therefore \text{LHS} = 0$$

Similarly, substitute  $x = 3$  in RHS.

$$\Rightarrow \sqrt{4(3)^2 - 14(3) + 16}$$

$$\Rightarrow \sqrt{4 \times 9 - 42 + 16} \Rightarrow \sqrt{36 - 42 + 16}$$

$$\Rightarrow \sqrt{52 - 42} \Rightarrow \sqrt{10}$$

$$\therefore \text{RHS} = \sqrt{10}$$

Now, we can observe that

$\text{LHS} \neq \text{RHS}$

$\therefore x = 3$  is not a solution or root for the equation

$$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 16}$$

## Exercise 8.2

1.

**Sol:**

Given that the smallest integer of 2 consecutive integer is denoted by  $x$

$\Rightarrow$  The two integer will be  $x$  and  $(x+1)$

Product of two integers  $\Rightarrow x(x+1)$

Given that the product is 306

$$\therefore x(x+1) = 306$$

$$\Rightarrow x^2 + x = 306 \Rightarrow x^2 + x - 306 = 0$$

$\therefore$  The required quadratic equation is  $x^2 + x - 306 = 0$

2.

**Sol:**

Given that John and Jivani together have 45 marbles and John has  $x$  marbles

$\Rightarrow$  Jivani had  $(45 - x)$  marbles

No. of marbles John had after loosing 5 marbles  $= x - 5$

No. of marbles Jivani had after loosing 5 marbles  $= (45 - x) - 5$

$$= 45 - 5 - x$$

$$= 40 - x$$

Given that product of the no of marbles they now have = 128

$$\Rightarrow (x-5)(40-x) = 128$$

$$\Rightarrow 40x - x^2 - 40 \times 5 + 5x = 128$$

$$\Rightarrow 45x - x^2 - 200 = 128 \Rightarrow x^2 - 45x + 128 + 200 = 0$$

$$\Rightarrow x^2 - 45x + 328 = 0$$

$\therefore$  The required quadratic equation is  $x^2 - 45x + 328 = 0$

3.

**Sol:**

Given that  $x$  denotes the no of toys product in a day

$\Rightarrow$  The cost of production of each by =  $55 -$  no. of toys produced in a day

$$= (55 - x)$$

Total cost of production is nothing but product of no. of toys produced in a day and cost of production of each toy

$$\Rightarrow x(55 - x)$$

But total cost of production = Rs 750

$$\Rightarrow x(55 - x) = 750$$

$$\Rightarrow 55x - x^2 = 750$$

$$\Rightarrow x^2 - 55x + 750 = 0$$

$\therefore$  The required quadratic from of the given data is  $x^2 - 55x + 750 = 0$

4.

**Sol:**

Given that in a right triangle is 7cm less than its base

Let base of the triangle be denoted by  $x$

$$\Rightarrow \text{Height of the triangle} = (x - 7) \text{ cm}$$

We have hypotenuse of the triangle = 13cm

We know that, in a right triangle

$$(\text{base})^2 + (\text{Height})^2 = (\text{Hypotenuse})^2$$

$$\Rightarrow (x)^2 + (x - 7)^2 = (13)^2$$

$$\Rightarrow x^2 + x^2 - 14x + 49 = 169$$

$$\Rightarrow 2x^2 - 14x + 49 - 169 = 0$$

$$\Rightarrow 2x^2 - 14x - 120 = 0$$

$$\Rightarrow 2(x^2 - 7x - 60) = 0$$

$$\Rightarrow x^2 - 7x - 60 = 0$$

∴ The required quadratic equation is  $x^2 - 7x - 60 = 0$

5.

**Sol:**

Let the average speed of express train be denoted by  $x$  km/hr

Given that average speed of express train is 11 km/hr more than that of the passenger train

⇒ Average speed of passenger train =  $(x - 11)$  km/hr

Total distance travelled by the train = 132 km

We know that,

$$\text{Time taken to travel} = \frac{\text{Distance travelled}}{\text{Average speed}}$$

$$\Rightarrow \text{Time taken by express train} = \frac{\text{Distance travelled}}{\text{Average speed of express train}}$$

$$= \frac{132}{x} \text{ hr}$$

$$\Rightarrow \text{Time taken by passenger train} = \frac{132}{(x-11)} \text{ hr}$$

Given that time taken by express train is 1 hour less than that of passenger train.

⇒ Time taken by passenger train - Time taken by express train = 1 hour

$$\Rightarrow \frac{132}{x-11} - \frac{132}{x} = 1$$

$$\Rightarrow 132 \left( \frac{1}{x-11} - \frac{1}{x} \right) = 1$$

$$\Rightarrow 132 \left( \frac{x - (x-11)}{x(x-11)} \right) = 1$$

$$\Rightarrow 132(x - 2 + 11) = x(x-11)$$

$$\Rightarrow 132(11) = x^2 - 11x$$

$$\Rightarrow x^2 - 11x = 1452$$

$$\Rightarrow x^2 - 11x - 1452 = 0$$

The required quadratic is  $x^2 - 11x - 1452 = 0$

6.

**Sol:**

Let Speed of train be  $x$  km / hr

Distance travelled by train = 360 km

We know that

$$\text{Time of total} = \frac{\text{Distance travelled}}{\text{Speed of the train}} = \frac{360}{x} \text{ hr}$$

If speed had been 5 km/hr more  $\Rightarrow (x+5) \text{ km/hr}$

$$\text{Time of travel} = \frac{\text{Distance travelled}}{\text{Speed of the train}} = \frac{360}{x+5} \text{ hr}$$

Give that,

Time of travel when speed is increased is 1 hour less than of the actual time of travel

$$\Rightarrow \frac{360}{x} - \frac{360}{x+5} = 1$$

$$\Rightarrow 360 \left( \frac{1}{x} - \frac{1}{x+5} \right) = 1$$

$$\Rightarrow 360 \left( \frac{x+5-x}{x(x+5)} \right) = 1$$

$$\Rightarrow 360(5) = x(x+5)$$

$$\Rightarrow x^2 + 5x = 1800$$

$$\Rightarrow x^2 + 5x + 1800 = 0$$

$\therefore$  The required quadratic equation to find the speed of the train is  $x^2 + 5x - 1800 = 0$

### Exercise 8.3

1.

**Sol:**

We have

$$(x-4)(x+2) = 0$$

$$\Rightarrow \text{either } (x-4) = 0 \text{ or } (x+2) = 0$$

$$\Rightarrow x = 4 \text{ or } x = -2$$

Thus,  $x = 4$  and  $x = -2$  are two roots of the equation  $(x-4)(x+2) = 0$

2.

**Sol:**

We have,

$$(2x+3)(3x-7) = 0$$



$$\Rightarrow (2x+3)=0 \text{ or } (3x-7)=0$$

$$\Rightarrow 2x = -3 \text{ or } 3x = 7$$

$$\Rightarrow x = \frac{-3}{2} \text{ or } x = \frac{7}{3}$$

Thus,  $x = \frac{-3}{2}$  and  $x = \frac{7}{3}$  are two roots of the equation  $(2x+3)(3x-7)=0$

3.

**Sol:**

We have  $4x^2 + 5x = 0$

$$\Rightarrow x(4x+5)=0$$

$$\Rightarrow \text{either } x=0 \text{ or } 4x+5=0$$

$$\Rightarrow x=0 \text{ or } 4x=-5$$

$$\Rightarrow x=0 \text{ or } x = \frac{-5}{4}$$

Thus,  $x=0$  and  $x = \frac{-5}{4}$  are two roots of equation  $4x^2 + 5x = 0$

4.

**Sol:**

We have  $9x^2 - 3x - 2 = 0$

$$\Rightarrow 9x^2 - 6x + 3x - 2 = 0$$

$$\Rightarrow 3x(3x-2) + 1(3x-2) = 0$$

$$\Rightarrow (3x-2)(3x+1) = 0$$

$$\Rightarrow \text{either } 3x-2=0 \text{ or } 3x+1=0$$

$$\Rightarrow 3x=2 \text{ or } 3x=-1$$

$$\Rightarrow x = \frac{2}{3} \text{ or } x = -\frac{1}{3}$$

Thus,  $x = \frac{2}{3}$  and  $x = -\frac{1}{3}$  are two roots of the equation  $9x^2 - 3x - 2 = 0$

5.

**Sol:**

We have  $6x^2 - x - 2 = 0$

$$\Rightarrow 6x^2 + 3x - 4x - 2 = 0$$

$$\Rightarrow 3x(2x+1) - 2(2x+1) = 0$$

$$\Rightarrow (2x+1)(3x-2) = 0$$

$$\Rightarrow \text{either } 2x+1=0 \text{ or } 3x-2=0$$

$$\Rightarrow 2x=-1 \text{ or } 3x=2$$

$$\Rightarrow x=-\frac{1}{2} \text{ or } x=\frac{2}{3}$$

Thus,  $x=-\frac{1}{2}$  and  $x=\frac{2}{3}$  are two roots of the equation  $6x^2-x-2=0$

6.

**Sol:**

We have

$$6x^2+11x+3=0$$

$$\Rightarrow 6x^2+9x+2x+3=0$$

$$\Rightarrow 3x(2x+3)+1(2x+3)=0$$

$$\Rightarrow (2x+3)(3x+1)=0$$

$$\Rightarrow 2x+3=0 \text{ or } x=-\frac{1}{3}$$

Thus,  $x=-\frac{3}{2}$  and  $x=-\frac{1}{3}$  are the two roots of the given equation.

7.

**Sol:**

We have,

$$5x^2-3x-2=0$$

$$\Rightarrow 5x^2-5x+2(x-1)=0$$

$$\Rightarrow 5x(x-1)+2(x-1)=0$$

$$\Rightarrow (x-1)(5x+2)=0$$

$$\Rightarrow (x-1)=0 \text{ or } 5x+2=0$$

$$\Rightarrow x=1 \text{ or } x=-\frac{2}{5}$$

$\therefore x=1$  and  $x=-\frac{2}{5}$  are the two roots of the given equation.

8.

**Sol:**

We have

$$48x^2-13x-1=0$$

$$\Rightarrow 48x^2-16x+3x-1=0$$

$$\Rightarrow 16x(3x-1)+1(3x-1)=0$$

$$\Rightarrow (3x-1)(16x+1)=0$$

$$\Rightarrow 3x - 1 = 0 \text{ or } 16x + 1 = 0$$

$$\Rightarrow x = \frac{1}{3} \text{ or } x = -\frac{1}{16}$$

$\therefore x = -\frac{1}{16}$  and  $x = \frac{1}{3}$  are the two roots of the given equation.

9.

**Sol:**

We have

$$3x^2 = -11x - 10$$

$$\Rightarrow 3x^2 + 11x + 10 = 0$$

$$\Rightarrow 3x^2 + 6x + 5x + 10 = 0$$

$$\Rightarrow 3x(x+2) + 5(x+2) = 0$$

$$\Rightarrow (x+2)(3x+5) = 0$$

$\Rightarrow (x+2) = 0$  or  $x = -\frac{5}{3} \therefore x = 2$  and  $x = -\frac{5}{3}$  are the two roots at the quadratic equation

$$3x^2 = -11x - 10$$

10.

**Sol:**

We have

$$(x+1) = -4$$

$$\Rightarrow (25x) \times x + (25x) \times 10 - 4$$

$$\Rightarrow 25x^2 + 25x + 4 = 0 \quad [25 \times 4 = 100 \Rightarrow 25 = 20 + 5 \Rightarrow 100 = 20 \times 5]$$

$$\Rightarrow 25x^2 + 20x + 5x + 4 = 0$$

$$\Rightarrow 5x(5x+4) + 1(5x+4) = 0$$

$$\Rightarrow (5x+4)(5x+1) = 0$$

$$\Rightarrow 5x+4 = 0 \text{ or } 5x+1 = 0$$

$$\Rightarrow x = -\frac{4}{5} \text{ or } x = -\frac{1}{5}$$

$\therefore x = -\frac{4}{5}$  and  $x = -\frac{1}{5}$  are the two solutions of the quadratic equation  $25x(x+1) = -4$

11.

**Sol:**

We have

$$\begin{aligned}
10x - \frac{1}{x} &= 3 \\
\Rightarrow \frac{10x^2 - 1}{x} &= 3 \\
\Rightarrow 10x^2 - 1 &= 3x \\
\Rightarrow 10x^2 - 3x - 1 &= 0 \quad [10x - 1 = -10 \Rightarrow -10 = -5 \times 2 \text{ and } -3 = -5 + 2] \\
\Rightarrow 10x^2 - 5x^2 + 2x - 1 &= 0 \\
\Rightarrow 5x(2x - 1) + 1(2x - 1) &= 0 \\
\Rightarrow (2x - 1)(5x + 1) &= 0 \\
\Rightarrow 2x - 1 = 0 \text{ or } 5x + 1 &= 0 \\
\Rightarrow x = \frac{1}{2} \text{ or } x = -\frac{1}{5} \\
\therefore x = \frac{1}{2} \text{ and } x = -\frac{1}{5} &\text{ are the two roots of the given equation}
\end{aligned}$$

12.

**Sol:**

We have,

$$\begin{aligned}
\frac{2}{2^2} - \frac{5}{x} + 2 &= 0 \\
\Rightarrow \frac{2 - 5x + 2x^2}{x^2} &= 0 \\
\Rightarrow 2x^2 - 5x + 2 &= 0 \\
[2 \times 2 = 4 \Rightarrow 4 = -4 \times -1 \Rightarrow -5 = -4 + 1] \\
\Rightarrow 2x^2 - 4x - x + 8 &= 0 \\
\Rightarrow 2x(x - 2) - 1(x - 2) &= 0 \\
\Rightarrow (x - 2)(2x - 1) &= 0 \\
\Rightarrow x - 2 = 0 \text{ or } 2x - 1 &= 0 \\
\Rightarrow x = 2 \text{ or } x = \frac{1}{2} \\
\therefore x = 2 \text{ and } x = \frac{1}{2} &\text{ are the two roots at the given quadratic equation}
\end{aligned}$$

13.

**Sol:**

We have,

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$

$$\left[ 4\sqrt{3} \times 2\sqrt{3} = -8 \times 3 = -24 \Rightarrow -24 = -8 \times 3 = -3 \times 8 \Rightarrow 5 = -3 + 8 \right]$$

$$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$$

$$\Rightarrow 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$$

$$\Rightarrow (4x - \sqrt{3})(\sqrt{3}x + 2) = 0$$

$$\Rightarrow 4x - \sqrt{3} = 0 \text{ or } \sqrt{3}x = -2$$

$$\Rightarrow x = \frac{\sqrt{3}}{4} \text{ or } x = -\frac{2}{\sqrt{3}}$$

$$\therefore x = \frac{\sqrt{3}}{4} \text{ and } x = -\frac{2}{\sqrt{3}} \text{ are the two roots of the given quadratic equation}$$

14.

**Sol:**

We have,

$$\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$$

$$\left[ \sqrt{2} \times -2\sqrt{2} = -2 \times 2 = -4 \Rightarrow -4 = -4 \times 1 \Rightarrow -3 = -4 + 1 \right]$$

$$\Rightarrow \sqrt{2}x^2 - 4x + x - 2\sqrt{2} = 0$$

$$\Rightarrow \sqrt{2}x^2 - (2\sqrt{2}x - \sqrt{2})x + x - 2\sqrt{2} = 0$$

$$\Rightarrow \sqrt{2}x - (x - 2\sqrt{2}) + 1(x - 2\sqrt{2}) = 0$$

$$\Rightarrow (x - 2\sqrt{2})(\sqrt{2}x + 1) = 0$$

$$\Rightarrow x - 2\sqrt{2} = 0 \text{ or } \sqrt{2}x + 1 = 0$$

$$\Rightarrow x = 2\sqrt{2} \text{ or } x = \frac{-1}{\sqrt{2}}$$

$$\therefore x = -\frac{1}{\sqrt{2}} \text{ and } x = 2\sqrt{2} \text{ are the two roots of the given quadratic equation.}$$

15.

**Sol:**

We have,

$$a^2x^2 - 30bx + 2b^2 = 0$$

$$\Rightarrow a^2x^2 - abx - 2abx + 2b^2 = 0$$

$$\left[ a^2 \times 2b^2 = 2a^2b^2 \Rightarrow 2a^2b^2 = 2ab \times ab = -2ab \times -ab \Rightarrow -3ab = -2ab - ab \right]$$

$$\Rightarrow ax(ax - b) - 2b(ax - b) = 0$$

$$\Rightarrow (ax - 2b)(ax - b) = 0$$

$$\Rightarrow ax - 2b = 0 \text{ or } ax - b = 0$$

$$\Rightarrow ax = 2b \text{ or } ax = b$$

$$\Rightarrow x = \frac{2b}{a} \text{ or } x = \frac{b}{a}$$

$\therefore x = \frac{b}{a}$  and  $x = \frac{2b}{a}$  are the two roots of the given quadratic equation

16.

**Sol:**

We have,

$$x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$$

$$\Rightarrow x^2 - \sqrt{2}x - 1 \times x + \sqrt{2} = 0$$

$$[1 \times \sqrt{2} = \sqrt{2} \Rightarrow \sqrt{2} = -\sqrt{2} \times -1]$$

$$\Rightarrow x^2 - \sqrt{2}x - x + \sqrt{2} = 0$$

$$\Rightarrow x(x - \sqrt{2}) - 1(x - \sqrt{2}) = 0$$

$$\Rightarrow (x - \sqrt{2})(x - 1) = 0$$

$$\Rightarrow x = \sqrt{2} = 0 \text{ or } x - 1 = 0$$

$$\Rightarrow x = \sqrt{2} \text{ or } x = 1$$

$\therefore x = 1$  and  $x = \sqrt{2}$  are the roots of the given quadratic equation

17.

**Sol:**

We have,

$$x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$$

$$\Rightarrow x^2 - \sqrt{3}x - 1 \times x + \sqrt{3} = 0$$

$$[\sqrt{3} \times 1 = \sqrt{3} \Rightarrow \sqrt{3} = -\sqrt{3} \times -1 \Rightarrow (\sqrt{3} + 1) = -\sqrt{3} - 1]$$

$$\Rightarrow x(x - \sqrt{3}) - 1(x - \sqrt{3}) = 0$$

$$\Rightarrow (x - \sqrt{3})(x - 1) = 0$$

$$\Rightarrow x - \sqrt{3} = 0 \text{ or } x - 1 = 0$$

$$\Rightarrow x = \sqrt{3} \text{ or } x = 1$$

$\therefore x = 1$  and  $x = \sqrt{3}$  are the two roots of the given quadratic equation

18.

**Sol:**

We have,

$$4x^2 + 4bx - (a^2 - b^2) = 0$$

$$\left[ 4x - (a^2 - b^2) = 4x - (a - b)(a + b) = -2(a - b) \times 2(a + b) = 2(b - a) \times 2(a + b) \Rightarrow 4b = 2b + 2b = 8(b - a) + \right.$$

$$\Rightarrow 4x^2 + (2(b - a) + 2(a + b))x - (a - b)(a + b) = 0$$

$$\Rightarrow 2x^2 + (2x + b - a) + (a + b)(2x + (b - a)) = 0$$

$$\Rightarrow (2x + b - a) \text{ or } 2x + a + b = 0$$

$$\Rightarrow 2x = a - b \text{ or } 2x = -a - b$$

$$\Rightarrow x = \frac{a - b}{2} \text{ or } 2x = -(a + b) \Rightarrow x = -\frac{(a + b)}{2}$$

$\therefore x = -\frac{(a + b)}{2}$  and  $x = \frac{a - b}{2}$  are the two roots of the given quadratic equation

19.

**Sol:**

We have,

$$ax^2 + (4a^2 - 3b)x - 12ab = 0$$

$$\left[ a \times 12ab = -12a^2b^2 = 4a^2 \times -3b \right]$$

$$\Rightarrow ax^2 + 4a^2x - 3bx + (4a \times (-3b)) = 0$$

$$\Rightarrow ax(x + 4a) - 3b(x + 4a) = 0$$

$$\Rightarrow (a + 4a)(ax - 3b) = 0$$

$$\Rightarrow (x + 4a) = 0 \text{ or } (ax - 3b) = 0$$

$$\Rightarrow x = -4a \text{ or } x = \frac{3b}{a}$$

$\therefore x = \frac{3b}{a}$  and  $x = -4a$  are the two roots of the given equations

20.

**Sol:**

We have,

$$\begin{aligned} \left(x - \frac{1}{2}\right)^2 &= 4 \\ \Rightarrow \left(x - \frac{1}{2}\right)^2 - 4 &= 0 \\ \Rightarrow \left(x - \frac{1}{2}\right)^2 - (2)^2 &= 0 \\ \Rightarrow \left[\left(x - \frac{1}{2}\right) + 2\right] \left[\left(x - \frac{1}{2}\right) - 2\right] &= 0 \quad [\because a^2 - b^2 = (a+b)(a-b)] \\ \Rightarrow \left(x - \frac{1}{2} + 2\right) = 0 \text{ or } \left(x - \frac{1}{2} - 2\right) &= 0 \\ \Rightarrow x = 2 - \frac{1}{2} \text{ or } x = 2 - \frac{1}{2} \\ \Rightarrow x = \frac{4-1}{2} \text{ or } x = \frac{4+1}{2} \\ \Rightarrow x = \frac{3}{2} \text{ or } x = \frac{5}{2} \\ \Rightarrow x = \frac{3}{2} \text{ and } x = \frac{5}{2} \text{ are the two roots at the given equations} \end{aligned}$$

21.

**Sol:**

We have,

$$x^2 - 4\sqrt{2}x + 6 = 0$$

$$\left[1 \times 6 = 6 \Rightarrow 6 = -3\sqrt{2}x - \sqrt{2} \text{ and } -4\sqrt{2} = -3\sqrt{2} - \sqrt{2}\right]$$

$$\Rightarrow x^2 - 3\sqrt{2}x - \sqrt{2}x + (-3\sqrt{2} \times -2) = 0$$

$$\Rightarrow x(x - 3\sqrt{2})\sqrt{2}(x - 3\sqrt{2}) = 0$$

$$\Rightarrow (x - 3\sqrt{2})(x - \sqrt{2}) = 0$$

$$\Rightarrow x = 3\sqrt{2} = 0 \text{ or } x - \sqrt{2} = 0$$

$$\Rightarrow x = 3\sqrt{2} \text{ or } x = \sqrt{2}$$

$\therefore x = 3\sqrt{2}$  and  $x = \sqrt{2}$  are the two roots of the given equation.

22.



**Sol:**

$$\text{We have, } \frac{x+3}{x+2} = \frac{3x-7}{2x-3}$$

$$\Rightarrow (x+3)(2x-3) = (x+2)(3x-7)$$

$$\Rightarrow 2x^2 - 3x + 6x - 9 = 3x^2 - x - 14$$

$$\Rightarrow 2x^2 + 3x - 9 = 3x^2 - x - 14$$

$$\Rightarrow x^2 - 3x - x - 14 + 9 = 0$$

$$\Rightarrow x^2 - 4x - 5 = 0$$

$$[1x - 5 = -5 - 4 = -5 + 1]$$

$$\Rightarrow x^2 - 5x + x - 5 = 0$$

$$\Rightarrow x(x-5) + 1(x-5) = 0$$

$$\Rightarrow (x-5)(x+1) = 0$$

$$\Rightarrow x-5 = 0 \text{ or } x+1 = 0$$

$$\Rightarrow x = 5 \text{ or } x = -1$$

$\therefore x = 5$  and  $x = -1$  are the two roots of the given quadratic equation.

23.

**Sol:**

$$\text{We have, } \frac{2x}{x-4} + \frac{2x-5}{x-3} = \frac{25}{3}$$

$$\Rightarrow \frac{2x(x-3) + (2x-5)(x-4)}{(x-4)(x-3)} = \frac{25}{3}$$

$$\Rightarrow \frac{2x^2 - 6x + 2x^2 - 5x - 5x + 20}{x^2 - 4x - 3x + 12} = \frac{25}{3}$$

$$\Rightarrow \frac{4x^2 - 19x + 20}{x^2 - 7x + 12} = \frac{25}{3}$$

$$\Rightarrow 3(4x^2 - 19x + 20) = 25(x^2 - 7x + 12)$$

$$\Rightarrow 12x^2 - 57x + 60 = 25x^2 - 175x + 300$$

$$\Rightarrow 25x^2 - 12x^2 - 175x + 57x + 300 - 60 = 0$$

$$\Rightarrow 13x^2 - 118x + 240 = 0$$

$$\Rightarrow 13x^2 - 78x - 40x + 240 = 0$$

$$[\because 13 \times 240 = 3120 \Rightarrow 3180 = -78 \times 40 \text{ and } -118 = -78 - 40]$$

$$\Rightarrow 13x(x-6) - 40(x-6) = 0$$

$$\Rightarrow (x-6)(13x-40) = 0$$

$$\Rightarrow x - 6 = 0 \text{ or } 13x - 40 = 0$$

$$\Rightarrow x = 6 \text{ or } x = \frac{40}{13}$$

$\therefore x = 6$  and  $x = \frac{40}{13}$  are the two roots of the given equation.

24.

**Sol:**

We have,

$$\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$$

$$\Rightarrow \frac{x(x+3) - (x-2)(1-x)}{x(x-2)} = \frac{17}{4}$$

$$\Rightarrow \frac{x^2 + 3x - (x - x^2 - 2 + 2x)}{x^2 - 2x} = \frac{17}{4}$$

$$\Rightarrow \frac{x^2 + 3x - x + x^2 + 2 - 2x}{x^2 - 2x} = \frac{17}{4}$$

$$\Rightarrow \frac{2x^2 + 2}{x^2 - 2x} = \frac{17}{4}$$

$$\Rightarrow 4(2x^2 + 2) = 17(x^2 - 2x)$$

$$\Rightarrow 8x^2 + 8 = 17x^2 - 34x$$

$$\Rightarrow 8x^2 + 8 = 17x^2 - 34x$$

$$\Rightarrow (17-8)x^2 - 34x - 8 = 0$$

$$\Rightarrow 9x^2 - 34x - 8 = 0$$

$$[9 \times -8 = -72 \Rightarrow -72 = -36 \times 2 \text{ and } -34 = -36 + 2]$$

$$\Rightarrow 9x^2 - 36x + 2x - 8 = 0$$

$$\Rightarrow 9x(x-4) + 2(x-4) = 0$$

$$\Rightarrow (x-4)(9x+2) = 0$$

$$\Rightarrow (x-4) = 0 \text{ or } 9x+2 = 0$$

$$\Rightarrow x = 4 \text{ or } x = -\frac{2}{9}$$

$\therefore x = 4$  and  $x = -\frac{2}{9}$  are the two roots of the given equations

25.

**Sol:**

$$-4, \frac{9}{4}$$

26.

**Sol:**

We have,  $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}, x \neq 0$

$$\Rightarrow \frac{(x+1)+1(x-2)}{(x-2)(x-1)} = \frac{6}{x}$$

$$\Rightarrow \frac{x-1+2x-4}{x^2-2x-x+2} = \frac{6}{x}$$

$$\Rightarrow \frac{3x-5}{x^2-3x+2} = \frac{6}{x}$$

$$\Rightarrow x(3x-5) = 6(x^2-3x+2)$$

$$\Rightarrow 3x^2 - 5x = 6x^2 - 16x + 12$$

$$\Rightarrow 3x^2 - 18x + 5x + 18 = 0$$

$$\Rightarrow 3x^2 - 13x + 18 = 0$$

$$[\because 3 \times 18 = 36 \Rightarrow -9x - 4 \text{ and } -13 = -9 - 4]$$

$$\Rightarrow 3x^2 - 9x - 4x + 12 = 0$$

$$\Rightarrow 3x(x-3) - 4(x-3) = 0$$

$$\Rightarrow (x-3)(3x-4) = 0$$

$$\Rightarrow x-3 = 0 \text{ or } 3x-4 = 0$$

$$\Rightarrow x = 3 \text{ or } x = \frac{4}{3}$$

$\therefore x = 3$  and  $x = \frac{4}{3}$  are the two roots of the given equation

27.

**Sol:**

We have

$$\frac{x+1}{x-1} - \frac{x-1}{x+1} = \frac{5}{6}, x \neq 1 \text{ and } x \neq -1$$

$$\Rightarrow \frac{(x+1)(x+1) - (x-1)(x-1)}{(x-1)(x+1)} = \frac{5}{6}$$

$$\Rightarrow \frac{(x+1)^2 - (x-1)^2}{x^2 - 1^2} = \frac{5}{6}$$

$$\begin{aligned}
\Rightarrow \frac{4 \times x \times 1}{x^2 - 1} &= \frac{5}{6} && \left[ \because (x+b)^2 - (a-b)^2 = 4ab \text{ and } (a-b)(a+b) = a^2 - b^2 \right] \\
\Rightarrow 6(4x) &= 5(x^2 - 1) \\
\Rightarrow 24x &= 5x^2 - 5 \\
\Rightarrow 5x^2 - 5 - 24x &= 0 \\
\Rightarrow 5x^2 - 24x - 5 &= 0 && \left[ \because 5x - 5 = -25 \Rightarrow -25 = -25 \times 1 - 24 = -25 + 1 \right] \\
\Rightarrow 5x^2 - 25x + x - 5 &= 0 \\
\Rightarrow 5x(x-5) + 1(x-5) &= 0 \\
\Rightarrow (x-5)(5x+1) &= 0 \\
\Rightarrow x-5 = 0 \text{ or } 5x+1 = 0 \\
\Rightarrow x = 5 \text{ or } x = -\frac{1}{5} \\
\therefore x = 5 \text{ and } x = -\frac{1}{5} &\text{ are the two roots of the given equation.}
\end{aligned}$$

28.

**Sol:**

We have

$$\begin{aligned}
\frac{x-1}{2x+1} - \frac{2x+1}{x-1} &= \frac{5}{2}, x \neq -\frac{1}{2}, 1 \\
\Rightarrow \frac{(x-1)(x-1) - (2x+1)(2x+1)}{(2x+1)(x-1)} &= \frac{5}{2} \\
\Rightarrow \frac{(x-1)^2 - (2x+1)^2}{2x^2 - 2x + x - 1} &= \frac{5}{2} \\
\Rightarrow \frac{x^2 - 2x + 1 + 4x^2 + 4x + 1}{2x^2 - x - 1} &= \frac{5}{2} && \left[ \because (a+b)^2 = a^2 + b^2 + 2ab, (a-b)^2 = a^2 + b^2 - 2ab \right] \\
\Rightarrow \frac{5x^2 + 2x + 2}{2x^2 - x - 1} &= \frac{5}{2} \\
\Rightarrow 2(5x^2 + 2x + 2) &= 5(2x^2 - x - 1) \\
\Rightarrow 10x^2 + 4x + 4 &= 10x^2 - 5x - 5 \\
\Rightarrow 4x + 5x + 4 + 5 &= 0 \\
\Rightarrow 9x + 9 &= 0 \\
\Rightarrow 9x &= -9 \\
\Rightarrow \boxed{x = -1}
\end{aligned}$$

$\therefore x = -1$  is the only root for the given equation

29.

**Sol:**

We have,  $3x^2 - 14x - 5 = 0$

$$\Rightarrow 3x^2 - 15x + x - 5 = 0$$

$$\Rightarrow 3x(x-5) + 1(x-5) = 0 \quad [\because 3 \times -5 = -15 \Rightarrow -15 = -15 \times 1 \text{ and } +4 = +5 + 1]$$

$$\Rightarrow (x-5)(3x+1) = 0$$

$$\Rightarrow x-5=0 \text{ or } 3x+1=0$$

$$\Rightarrow x=5 \text{ or } x=-\frac{1}{3}$$

$\therefore x=5$  and  $x=-\frac{1}{3}$  are the two roots of the given quadratic equation

30.

**Sol:**

We have given,

$$\frac{m}{n}x^2 + \frac{n}{m} = 1 - 2x$$

$$\Rightarrow \frac{m^2x^2 + n^2}{mn} = 1 - 2x$$

$$\Rightarrow m^2x^2 + 2mnx + (n^2 - mn) = 0$$

Now we solve the above quadratic equation using factorization method.

Therefore,

$$\left[ m^2x^2 + mnx + m\sqrt{mnx} \right] + \left[ mnx - m\sqrt{mnx} + (n + \sqrt{mn})(n - \sqrt{mn}) \right] = 0$$

$$\Rightarrow \left[ m^2x^2 + mnx + m\sqrt{mnx} \right] + \left[ (mx)(n - \sqrt{mn}) + (n + \sqrt{mn})(n - \sqrt{mn}) \right] = 0$$

$$\Rightarrow (mx)(mx + n + \sqrt{mn}) + (n - \sqrt{mn})(mx + n + \sqrt{mn}) = 0$$

$$\Rightarrow (mx + n + \sqrt{mn})(mx + n - \sqrt{mn}) = 0$$

Now, one of the products must be equal to zero for the whole product to be zero. Hence we equate both the product to zero. In order to find the value of  $x$ . Therefore,

$$mx + n + \sqrt{mn} = 0$$

$$\Rightarrow mx = -n - \sqrt{mn}$$

$$\Rightarrow x = \frac{-n - \sqrt{mn}}{m}$$

Or

$$mx + n - \sqrt{mn} = 0$$

$$\Rightarrow mx = -n + \sqrt{mn}$$

$$\Rightarrow x = \frac{-n + \sqrt{mn}}{m}$$

$$\text{Hence } x = \frac{-n - \sqrt{mn}}{m} \text{ or } n = \frac{-n + \sqrt{mn}}{m}.$$

31.

**Sol:**

We have,

$$\frac{x-a}{x-b} + \frac{x-b}{x-a} = \frac{a}{b} + \frac{b}{a}$$

$$\Rightarrow \frac{(x-a)(x-a) + (x-b)(x-b)}{(x-b)(x-a)} = \frac{a^2 + b^2}{ab}$$

$$\Rightarrow \frac{(x-a)^2 + (x-b)^2}{x^2 - ax - bx + ab} = \frac{a^2 + b^2}{ab}$$

$$\Rightarrow \frac{x^2 - 2ax + a^2 + x^2 - 2bx + b^2}{x^2 - (a+b)x + ab} = \frac{a^2 + b^2}{ab}$$

$$\Rightarrow (2x^2 - 2x(a+b) + a^2 + b^2)ab = (a^2 + b^2)(x^2 - (a+b)x + ab)$$

$$\Rightarrow 2abx^2 - 2abx(a+b) + ab(a^2 + b^2) = (a^2 + b^2)x^2 - (a^2 + b^2)(a+b)x + ab(a^2 + b^2)$$

$$\Rightarrow (a^2 + b^2 - 2ab)x^2 - (a+b)(a^2 + b^2 - 2ab)x = 0$$

$$\Rightarrow (a-b)^2 x^2 - (a+b)(a-b)^2 x = 0$$

$$\Rightarrow (a-b)^2 (x - (a+b)) = 0$$

$$\Rightarrow x(x - (a+b)) = 0$$

$$\Rightarrow x = 0 \text{ or } x - (a+b) = 0 \Rightarrow x = a+b$$

$\therefore x = 0$  and  $x = (a+b)$  are the two roots of the equation

32.

**Sol:**

We have,

$$\begin{aligned}
& \frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} + \frac{1}{(x-3)(x-4)} = \frac{1}{6} \\
& \Rightarrow \frac{(x-3)(x-4) + (x-1)(x-4) + (x-1)(x-8)}{(x-1)(x-2)(x-3)(x-4)} = \frac{1}{6} \\
& \Rightarrow \frac{(x-3)(x-4) + (x-1)[(x-4) + (x-8)]}{(x-1)(x-2)(x-3)(x-4)} = \frac{1}{6} \\
& \Rightarrow \frac{(x-3)(x-4) + (x-1)(2x-6)}{(x-1)(x-2)(x-3)(x-4)} = \frac{1}{6} \\
& \Rightarrow \frac{(x-3)(x-4) + (x-1) \times 2(x-3)}{(x-1)(x-2)(x-3)(x-4)} = \frac{1}{6} \\
& \Rightarrow \frac{(x-3)[x-4+2x-2]}{(x-1)(x-2)(x-3)(x-4)} = \frac{1}{6} \\
& \Rightarrow \frac{(x-3)(3x-6)}{(x-1)(x-2)(x-3)(x-4)} = \frac{1}{6} \\
& \Rightarrow \frac{(x-3)(x-2) \times 3}{(x-1)(x-2)(x-3)(x-4)} = \frac{1}{6} \\
& \Rightarrow \frac{3}{(x-1)(x-4)} = \frac{1}{6} \\
& \Rightarrow (x-1)(x-4) = 3 \times 6 \\
& \Rightarrow x^2 - 4x - x + 4 = 18 \\
& \Rightarrow x^2 - 5x - 14 = 0 \quad [\because -14 = -7 \times 8 \text{ and } -5 = -7 + 8] \\
& \Rightarrow x^2 - 7x + 8x - 14 = 0 \\
& \Rightarrow x(x-7) + 8(x-7) = 0 \\
& \Rightarrow (x-7)(x+8) = 0 \\
& \Rightarrow x-7 = 0 \text{ or } x+2 = 0 \\
& \Rightarrow x = 7 \text{ or } x = -2 \\
& \therefore x = 7 \text{ and } x = -8 \text{ are the two roots of the given equation.}
\end{aligned}$$

33.

**Sol:**

We have,

$$(x-5)(x-6) = \frac{25}{(24)^2}$$

$$\begin{aligned}
\Rightarrow x^2 - 5x - 6x + 30 - \frac{25}{(24)^2} &= 0 \\
\Rightarrow x^2 - 11x + \frac{30(24)^2 - 25}{(24)^2} &= 0 \\
\Rightarrow x^2 - 11x + \frac{17280 - 25}{(24)^2} &= 0 \\
\Rightarrow x^2 - 11x + \frac{17255 - 25}{(24)^2} &= 0 \\
\Rightarrow x^2 - 11x + \frac{119}{24} \times \frac{145}{24} &= 0 \quad [\because 17255 = 145 \times 119] \\
\Rightarrow x^2 - \frac{264}{24}x + \frac{119}{24} \times \frac{145}{24} &= 0 \quad [\because 11 \times 24 = 264] \\
\Rightarrow x^2 - \left(\frac{119 + 145}{24}\right)x + \frac{119}{24} \times \frac{145}{24} &= 0 \\
\Rightarrow x^2 - \frac{119}{24}x - \frac{145}{24}x + \frac{119}{24} \times \frac{145}{24} &= 0 \\
\Rightarrow x\left(x - \frac{119}{24}\right) - \frac{145}{24}\left(x - \frac{119}{24}\right) &= 0 \\
\Rightarrow \left(x - \frac{119}{24}\right)\left(x - \frac{145}{24}\right) &= 0 \\
\Rightarrow x - \frac{119}{24} = 0 \text{ and } x - \frac{145}{24} = 0 \\
\Rightarrow x = \frac{119}{24} \text{ or } x = \frac{145}{24} \\
\Rightarrow x = 4\frac{23}{24} \text{ or } x = 6\frac{1}{24} \\
\therefore x = 4\frac{23}{24} \text{ and } x = 6\frac{1}{24} \text{ are the two roots of the given equation.}
\end{aligned}$$

34.

**Sol:**

$$\text{We have, } 7x + \frac{3}{x} = 35\frac{3}{5}$$

$$\Rightarrow \frac{7x^2 + 3}{x} = 35 + \frac{3}{5}$$



$$\Rightarrow 7x^2 + 3 = \left(35 + \frac{3}{5}\right)x$$

$$\Rightarrow 7x^2 - \left(35 + \frac{3}{5}\right)x + 3 = 0$$

$$\Rightarrow 7x^2 - 35x - \frac{3}{5}x + 3 = 0$$

$$\Rightarrow 7x^2 - 35x - \frac{1}{5}(3x - 3 \times 5) = 0$$

$$\Rightarrow 7x(x-5) - \frac{3}{5}(x-5) = 0$$

$$\Rightarrow (x-5)\left(7x - \frac{3}{5}\right) = 0$$

$$\Rightarrow (x-5) = 0 \text{ or } 7x - \frac{3}{5} = 0$$

$$\Rightarrow x = 5 \text{ or } 7x = \frac{3}{5} \Rightarrow x = \frac{3}{35}$$

$\therefore x = 5$  and  $x = \frac{3}{35}$  are the two roots of the given equation.

35.

**Sol:**

We have,

$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$$

$$\Rightarrow \frac{a(x-b) + b(x-a)}{(x-a)(x-b)} = \frac{2c}{x-c}$$

$$\Rightarrow \frac{ax - ab + bx - ab}{x^2 - ax - bx + ab} = \frac{2c}{x-c}$$

$$\Rightarrow (x-c)((a+b)x - 2ab) = 2c(x^2 - (a+b)x + ab)$$

$$\Rightarrow (a+b)x^2 - 2abx - (a+b)c x + 2abc = 2cx^2 - 2c(a+b)x + 2abc$$

$$\Rightarrow (a+b-8c)x^2 - 2abx - (a+b)xc + 8c(a+b)x = 0$$

$$\Rightarrow (a+b-8c)x^2 + x(-8ab - ac - bc + 8ac + 8bc) = 0$$

$$\Rightarrow (a+b-8c)x^2 + x(-2ab + ac + bc) = 0$$

$$\Rightarrow x[x(a+b-2c) + (ac+bc-2ab)] = 0$$

$$\Rightarrow x = 0 \text{ or } x(a+b-2c) + (ac+bc-8ab) = 0$$

$$\Rightarrow x=0 \text{ or } x = -\frac{(ac+bc-8ab)}{a+b-8c}$$

$$\Rightarrow x = \frac{8ab-ac-bc}{a+b-2c}$$

$\therefore x=0$  and  $x = \frac{2ab-ac-bc}{a+b-8c}$  are the two roots of the given equation.

$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$$

$$\Rightarrow \frac{a(x-b)+b(x-a)}{(x-a)(x-b)} = \frac{2c}{x-c}$$

$$\Rightarrow \frac{ax-ab+bx-ab}{x^2-ax-bx+ab} = \frac{2c}{x-c}$$

$$\Rightarrow (x-c)((a+b)x-2ab) = 2c(x^2-(a+b)x+ab)$$

$$\Rightarrow (a+b)x^2 - 2abx - (a+b)c x + 2abc = 2cx^2 - 2c(a+b)x + 2abc$$

36.

**Sol:**

We have

$$x^2 + 2ab = (2a+b)x$$

$$\Rightarrow x^2 - (2a+b)x + 2ab = 0 \quad [\because 2ab = -8a \times -b \Rightarrow -(8a+b) = -8a-b]$$

$$\Rightarrow x^2 - 2ax - bx + 2ab = 0$$

$$\Rightarrow x - (x-8a) - b(x-2a) = 0$$

$$\Rightarrow (x-8a)(x-b) = 0$$

$$\Rightarrow x-8a=0 \text{ or } x-b=0$$

$$\Rightarrow x=8a \text{ or } x=b$$

$\therefore x=8a$  and  $x=b$  are the two roots of the given equation .

37.

**Sol:**

We have,

$$(a+b)^2 x^2 - (4ab)x - (a-b)^2 = 0$$

$$\Rightarrow (a+b)^2 x^2 - ((a+b)^2 - (a-b)^2)x - (a-b)^2 = 0 \quad [\because (a+b)^2 - (a-b)^2 = 4ab]$$

$$\Rightarrow (a+b)^2 x^2 - (a+b)^2 x + (a-b)^2 x - (a-b)^2 = 0$$

$$\Rightarrow (a+b)^2 x(x-1) + (a-b)^2 (x-1) = 0$$

$$\Rightarrow (x-1)((a+b)^2 x + (a-b)^2) = 0$$

$$\Rightarrow x-1=0 \text{ or } (a+b)^2 x + (a-b)^2 = 0$$

$$\Rightarrow x=1 \text{ or } x = -\frac{(a-b)^2}{(a+b)^2} = -\left[\frac{a-b}{a+b}\right]^2$$

$\therefore x=1$  and  $x = -\left[\frac{a-b}{a+b}\right]^2$  are the two roots of the given equation

38.

**Sol:**

We have

$$a(x^2+1) - x(a^2+1) = 0$$

$$\Rightarrow ax^2 - a^2x - x + a = 0 \quad [\because a \times a = a^2 \Rightarrow a^2 = -a^2 \times -1 - (a^2 + 1) = a^2 - 1]$$

$$\Rightarrow a \times (x-a) - 1(x-a) = 0$$

$$\Rightarrow (x-a)(ax-1) = 0$$

$$\Rightarrow x-a=0 \text{ or } ax-1=0$$

$$\Rightarrow x=a \text{ or } x = \frac{1}{a}$$

$\therefore x=a$  and  $x = \frac{1}{a}$  are the two roots of the given equation

39.

**Sol:**

We have,

$$x^2 - x - x(a+1) = 0$$

$$\Rightarrow x^2 - (a+1-a)x - a(a+1) = 0 \quad [\because -a(a+1) = -(a+1) \times a - 1 = a - (a+1)]$$

$$\Rightarrow x^2 - (a+1)x + ax + ax(-(a+1)) = 0$$

$$\Rightarrow x(x-(a+1)) + a(x-(a+1)) = 0$$

$$\Rightarrow (x-(a+1))(x+a) = 0$$

$$\Rightarrow x-(a+1) = 0 \text{ or } x+a = 0$$

$$\Rightarrow x = a+1 \text{ or } x = -a$$

$\therefore x = (a+1)$  and  $x = -a$  are the two roots of the given equation.

40.

**Sol:**

We have,

$$x^2 + \left(a + \frac{1}{a}\right)x + 1 = 0$$

$$\Rightarrow x^2 + ax + \frac{1}{a}x + a \times \frac{1}{a} = 0 \quad \left[ \because 1 = a \times \frac{1}{a} \left(a + \frac{1}{a}\right)x = ax + \frac{1}{a}x \right]$$

$$\Rightarrow x(x+a) + \frac{1}{a}(x+a) = 0$$

$$\Rightarrow (x+a) \left(x + \frac{1}{a}\right) = 0$$

$$\Rightarrow x+a=0 \text{ or } x + \frac{1}{a} = 0$$

$$\Rightarrow x = -a \text{ or } x = -\frac{1}{a}$$

$\therefore x = a$  and  $x = -\frac{1}{a}$  are the two roots of the given equation.

41.

**Sol:**

We have,

$$abx^2 + (b^2 - ac)x - bc = 0$$

$$\left[ abx - bc = -ab^2c \Rightarrow -ab^2c = b^2 \times -ac \text{ and } b^2 - ac = b^2 + (-ac) \right]$$

$$\Rightarrow abx^2 + b^2x - acx - bc = 0$$

$$\Rightarrow bx(ax+b) - c(ax+b) = 0$$

$$\Rightarrow (ax+b)(bx-c) = 0$$

$$\Rightarrow ax+b=0 \text{ or } bx-c=0$$

$$\Rightarrow x = -\frac{b}{a} \text{ or } x = \frac{c}{b}$$

$\therefore x = -\frac{b}{a}$  and  $x = \frac{c}{b}$  are the two roots of the given equation

42.

**Sol:**

We have,  $a^2b^2x^2 + b^2x - a^2x - 1 = 0$

$$\left[ -1 \times a^2b^2 = -a^2b^2 \Rightarrow -a^2b^2 = -a^2 \times b^2 \right]$$

$$\Rightarrow a^2b^2x^2 + b^2x - a^2x - 1 = 0$$

$$\Rightarrow b^2 \times (a^2x + 1) - 1(a^2x - 1) = 0$$

$$\Rightarrow (a^2x + 1)(b^2x - 1) = 0$$

$$\Rightarrow a^2x + 1 = 0 \text{ or } b^2x - 1 = 0$$

$$\Rightarrow x = -\frac{1}{a^2} \text{ and } x = \frac{1}{b^2} \text{ are the two roots of the given equation}$$

43.

**Sol:**

We have,

$$\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}, x \neq 2 \text{ and } x \neq 4$$

$$\Rightarrow \frac{(x-1)(x-4) + (x-3)(x-2)}{(x-2)(x-4)} = \frac{10}{3}$$

$$\Rightarrow \frac{x^2 - x - 4x + 4 + x^2 - 3x - 2x + 6}{x^2 - 2x - 4x + 8} = \frac{10}{3}$$

$$\Rightarrow \frac{2x^2 - 10x + 10}{x^2 - 6x + 8} = \frac{10}{3}$$

$$\Rightarrow 2(x^2 - 5x + 5) \times 3 = 5(x^2 - 6x + 8)$$

$$\Rightarrow 3x^2 - 15x + 15 = 5x^2 - 30x + 40$$

$$\Rightarrow 2x^2 - 30x + 15x + 40 - 15 = 0$$

$$\Rightarrow 2x^2 - 15x + 25 = 0$$

$$\Rightarrow 2x^2 - 10x - 5x + 25 = 0$$

$$\Rightarrow 2x(x-5) - 5(x-5) = 0$$

$$\Rightarrow (x-5)(2x-5) = 0$$

$$\Rightarrow (x-5) = 0 \text{ or } 2x-5 = 0$$

$$\Rightarrow x = 5 \text{ or } x = \frac{5}{2}$$

$$\therefore x = 5 \text{ and } x = \frac{5}{2} \text{ are the two roots of the given equation}$$

44.

**Sol:**

We have  $3x^2 - 2\sqrt{6}x + 2 = 0$  Now we solve the above quadratic equation using factorization method.

Therefore

$$\begin{aligned}3x^2 - \sqrt{6}x - \sqrt{6}x + 2 &= 0 \\ \Rightarrow \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) &= 0 \\ \Rightarrow (\sqrt{3} - \sqrt{2})(\sqrt{3}x - \sqrt{2}) &= 0 \\ \Rightarrow (\sqrt{3}x - \sqrt{2}) = 0 \text{ or } (\sqrt{3}x - \sqrt{2}) &= 0 \\ \Rightarrow \sqrt{3}x = \sqrt{2} \text{ or } \sqrt{3}x = \sqrt{2} \\ \Rightarrow x = \frac{\sqrt{2}}{\sqrt{3}} \text{ or } x = \frac{\sqrt{2}}{\sqrt{3}} \\ \Rightarrow x = \sqrt{\frac{2}{3}} \text{ or } x = \sqrt{\frac{2}{3}} \\ \text{Hence } x = \sqrt{\frac{2}{3}} \text{ or } x = \sqrt{\frac{2}{3}}\end{aligned}$$

45.

**Sol:**

We have

$$\begin{aligned}\frac{1}{x-1} - \frac{1}{x+5} &= \frac{6}{7}, x \neq 1, -5 \\ \Rightarrow \frac{x+5 - (x-1)}{(x-1)(x+5)} &= \frac{6}{7} \\ \Rightarrow \frac{x-5 - x+1}{x^2 + 5x - x - 5} &= \frac{6}{7} \\ \Rightarrow \frac{6}{x^2 + 4x - 5} &= \frac{6}{7} \\ \Rightarrow x^2 + 4x - 5 &= 7 \\ \Rightarrow x^2 + 4x - 5 - 7 &= 0 \\ \Rightarrow x^2 + 4x - 12 &= 0 \\ \Rightarrow x^2 + 6x - 2x - 12 &= 0 \\ \Rightarrow x(x+6) - 2(x+6) &= 0 \\ \Rightarrow x+6 = 0 \text{ or } x-8 &= 0 \\ \Rightarrow x = -6 \text{ or } x = 8 \\ \therefore x = -6 \text{ and } x = 8 &\text{ are the two roots of the given equation.}\end{aligned}$$

46.

**Sol:**

We have,

$$\frac{1}{x} - \frac{1}{x-2} = 3, x \neq 0, 2$$

$$\Rightarrow \frac{x-2-x}{x(x-2)} = 3$$

$$\Rightarrow -2 = 3(x^2 - 2x)$$

$$\Rightarrow 3x^2 - 6x + 2 = 0$$

$$\Rightarrow 3x^2 - (3+3)x + 2 = 0$$

$$\Rightarrow 3x^2 - (3 + \sqrt{3} + 3\sqrt{3})x + (3-1) = 0$$

$$\Rightarrow 3x^2 - \sqrt{3}(\sqrt{3}+1)x - \sqrt{3}(\sqrt{3}-1)x + [(\sqrt{3}+1)(\sqrt{3}-1)] = 0$$

$$[\because a^2 - b^2 = (a+b)(a-b)]$$

$$\Rightarrow (\sqrt{3})^2 x^2 - \sqrt{3}(\sqrt{3}-1)x + (\sqrt{3}+1)(\sqrt{3}-1) = 0$$

$$\Rightarrow \sqrt{3}(\sqrt{3}+1) - (\sqrt{3}-1)(\sqrt{3}x - (\sqrt{3}+1)) = 0$$

$$\Rightarrow [\sqrt{3}x(\sqrt{3}+1)][\sqrt{3}x - (\sqrt{3}-1)] = 0$$

$$\Rightarrow \sqrt{3}x - (\sqrt{3}+1) = 0 \text{ or } \sqrt{3}x - (\sqrt{3}-1) = 0$$

47.

**Sol:**

We have,

$$x - \frac{1}{x} = 3, x \neq 0$$

$$\Rightarrow \frac{x^2 - 1}{x} = 3$$

$$\Rightarrow x^2 - 1 = 3x$$

$$\Rightarrow x^2 - 3x - 1 = 0$$

$$\Rightarrow x^2 - \left(\frac{3}{2} + \frac{3}{2}\right)x - 1 = 0$$

$$\Rightarrow x^2 - \left(\frac{3 + \sqrt{13}}{2} + \frac{3 - \sqrt{13}}{2}\right)x + (-1) = 0$$

$$\begin{aligned}
&\Rightarrow x^2 - \left(\frac{3+\sqrt{13}}{2}\right)x - \left(\frac{3-\sqrt{13}}{2}\right)x + (-1) = 0 \\
&\Rightarrow x^2 - \left(\frac{3+\sqrt{13}}{2}\right)x - \left(\frac{3-\sqrt{13}}{2}\right)x + \left(\frac{-4}{4}\right) = 0 \\
&\Rightarrow x^2 - \left(\frac{3+\sqrt{13}}{2}\right)x - \left(\frac{3-\sqrt{13}}{2}\right)x + \left(\frac{9-13}{4}\right) = 0 \\
&\Rightarrow x^2 - \left(\frac{3+\sqrt{13}}{2}\right)x - \left(\frac{3-\sqrt{13}}{2}\right)x + \left(\frac{3^2 - (\sqrt{13})^2}{2^2}\right) = 0 \\
&\Rightarrow x^2 - \left(\frac{3+\sqrt{13}}{2}\right)x - \left(\frac{3-\sqrt{13}}{2}\right)x + \frac{(3+\sqrt{13})}{2} + \frac{(3-\sqrt{13})}{2} = 0 \\
&\Rightarrow x \left( x - \left(\frac{3+\sqrt{13}}{2}\right) \right) - \left(\frac{3-\sqrt{13}}{2}\right) \left( x - \left(\frac{3+\sqrt{13}}{2}\right) \right) = 0 \\
&\Rightarrow \left( x - \frac{3+\sqrt{13}}{2} \right) \left( x - \frac{3-\sqrt{13}}{2} \right) = 0 \\
&\Rightarrow x - \left(\frac{3+\sqrt{13}}{2}\right) = 0 \text{ or } x - \left(\frac{3-\sqrt{13}}{2}\right) = 0 \\
&\Rightarrow x = \frac{3+\sqrt{13}}{2} \text{ or } x = \frac{3-\sqrt{13}}{2} \\
&\therefore x = \frac{3+\sqrt{13}}{2} \text{ and } x = \frac{3-\sqrt{13}}{2} \text{ are the two roots of the given equation.}
\end{aligned}$$

48.

**Sol:**

We have,

$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq 4, 7$$

$$\Rightarrow \frac{x-7-(x+4)}{(x+4)(x+7)} = \frac{11}{30}$$

$$\Rightarrow \frac{x-7-x-4}{x^2-7x+4x-28} = \frac{11}{30}$$



$$\begin{aligned}
\Rightarrow \frac{-11}{x^2 - 3x - 28} &= \frac{11}{30} \\
\Rightarrow (-1) \times 30 &= 1 \times (x^2 - 3x - 28) \\
\Rightarrow -30 &= x^2 - 3x - 28 \\
\Rightarrow x^2 - 3x - 28 + 30 &= 0 \\
\Rightarrow x^2 - 3x + 2 &= 0 && [\because 2 = -2 \times -1 - 3 = -2 - 1] \\
\Rightarrow x^2 - 2x - x + 2 &= 0 \\
\Rightarrow x(x - 8) - 1(x - 8) &= 0 \\
\Rightarrow (x - 8)(x - 1) &= 0 \\
\Rightarrow x - 8 = 0 \text{ or } x - 1 &= 0 \\
\Rightarrow x = 8 \text{ or } x = 1
\end{aligned}$$

$\therefore x = 2$  and  $x = 1$  are the two roots of the given equation.

### Exercise 8.4

Find the roots of the following quadratic equations (if they exist) by the method of completing the square.

1.

**Sol:**

We have,

$$\begin{aligned}
x^2 - 4\sqrt{2}x + 6 &= 0 \\
\Rightarrow x^2 - 2 \times x \times 2\sqrt{2} + (2\sqrt{2})^2 - (2\sqrt{2})^2 + 6 &= 0 \\
\Rightarrow (x - 2\sqrt{2})^2 &= (2\sqrt{2})^2 - 6 \\
\Rightarrow (x - 2\sqrt{2})^2 &= (4 \times 2) - 6 \\
\Rightarrow (x - 2\sqrt{2})^2 &= 8 - 6 \\
\Rightarrow (x - 2\sqrt{2})^2 &= 2 \\
\Rightarrow x - 2\sqrt{2} &= \pm\sqrt{2} \\
\Rightarrow x - 2\sqrt{2} = \sqrt{2} \text{ or } x - 2\sqrt{2} &= -\sqrt{2} \\
\Rightarrow x = 3\sqrt{2} \text{ or } x = \sqrt{2}
\end{aligned}$$

$\therefore x = \sqrt{2}$  and  $x = 3\sqrt{2}$  are the roots of the given equation.

2.

**Sol:**

We have,

$$2x^2 - 7x + 3 = 0$$

$$2\left(x^2 - \frac{7}{2}x + \frac{3}{2}\right) = 0$$

$$\Rightarrow x^2 - 2 \times \frac{7}{2} \times \frac{1}{2} \times x + \frac{3}{2} = 0$$

$$\Rightarrow x^2 - 2 \times \frac{7}{4} \times x + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$$

$$\Rightarrow x^2 - 2 \times \frac{7}{4} \times x + \left(\frac{7}{4}\right)^2 - \frac{49}{16} + \frac{3}{2} = 0$$

$$\Rightarrow \left(x - \frac{7}{4}\right)^2 - \frac{49}{16} + \frac{3}{2} = 0$$

$$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{49}{16} - \frac{3}{2}$$

$$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{49 - 86}{16}$$

$$\Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$$

$$\Rightarrow x - \frac{7}{4} = \pm \left(\frac{5}{4}\right)^2$$

$$\Rightarrow x - \frac{7}{4} = \pm \frac{5}{4}$$

$$\Rightarrow x - \frac{7}{4} = \frac{5}{4} \text{ or } x - \frac{7}{4} = -\frac{5}{4}$$

$$\Rightarrow x = \frac{5}{4} + \frac{7}{4} \text{ or } x = \frac{7}{4} - \frac{5}{4}$$

$$\Rightarrow x = \frac{12}{4} \text{ or } x = \frac{2}{4}$$

$$\Rightarrow x = 3 \text{ or } x = \frac{1}{2}$$

$\therefore x = 3$  and  $x = \frac{1}{2}$  are the roots of the given quadratic equation.

3.

**Sol:**

We have,

$$3x^2 + 11x + 10 = 0$$

$$\Rightarrow x^2 + \frac{11}{3}x + \frac{10}{3} = 0$$

$$\Rightarrow x^2 + 2 \times \frac{1}{2} \times \frac{11}{3}x + \frac{10}{3} = 0$$

$$\Rightarrow x^2 + 2 \times \frac{11}{6} \times 2 + \left(\frac{11}{6}\right)^2 - \left(\frac{11}{6}\right)^2 + \frac{10}{3} = 0$$

$$\Rightarrow \left(x + \frac{11}{6}\right)^2 = \left(\frac{11}{6}\right)^2 - \frac{10}{3}$$

$$\Rightarrow \left(x + \frac{11}{6}\right)^2 = \frac{121}{36} - \frac{10}{3}$$

$$\Rightarrow \left(x + \frac{11}{6}\right)^2 = \frac{121 - 120}{36}$$

$$\Rightarrow \left(x + \frac{11}{6}\right)^2 = \frac{1}{36}$$

$$\Rightarrow \left(x + \frac{11}{6}\right)^2 = \left(\frac{1}{6}\right)^2$$

$$\Rightarrow x + \frac{11}{6} = \pm \frac{1}{6}$$

$$\Rightarrow x + \frac{11}{6} = \frac{1}{6} \text{ or } x + \frac{11}{6} = -\frac{1}{6}$$

$$\Rightarrow x = \frac{1}{6} - \frac{11}{6} \text{ or } x = -\frac{1}{6} - \frac{11}{6}$$

$$\Rightarrow x = -\frac{10}{6} \text{ or } x = -\frac{12}{6} = -2$$

$$\Rightarrow x = -\frac{5}{3} \text{ or } x = -2$$

$\therefore x = -\frac{5}{3}$  or  $x = -2$  are the two roots of the given equation.

4.

**Sol:**

We have,

$$2x^2 + x - 4 = 0$$

$$\Rightarrow 2\left(x^2 + \frac{x}{2} - \frac{4}{2}\right) = 0$$

$$\Rightarrow x^2 + 2 \times \frac{1}{2} \times \frac{1}{2} \times x - 2 = 0$$

$$\Rightarrow x^2 + 2x \times \frac{1}{4} \times x + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 2 = 0$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 + 2$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1}{16} + 2$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1 + 2 \times 16}{16}$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1 + 32}{16}$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$$

$$\Rightarrow \left(x + \frac{1}{4}\right) = \pm \sqrt{\frac{33}{16}}$$

$$\Rightarrow x + \frac{1}{4} = +\frac{\sqrt{33}}{4} \text{ or } x + \frac{1}{4} = -\frac{\sqrt{33}}{4}$$

$$\Rightarrow x = \frac{\sqrt{33}}{4} - \frac{1}{4} \text{ or } x = -\frac{\sqrt{33}}{4} - \frac{1}{4}$$

$$\Rightarrow x = \frac{\sqrt{33} - 1}{4} \text{ or } x = \frac{\sqrt{33} - 1}{4}$$

$$\therefore x = \frac{\sqrt{33} - 1}{4} \text{ or } x = -\frac{\sqrt{33} - 1}{4} \text{ are the two roots of the given equation}$$

5.

**Sol:**

We have,

$$2x^2 + x + 4 = 0$$

$$\Rightarrow x^2 + \frac{x}{2} + 2 = 0$$

$$\begin{aligned}
\Rightarrow x^2 + 2 \times \frac{1}{2} \times \frac{1}{2} \times x + 2 &= 0 \\
\Rightarrow x^2 + 2 \times \frac{1}{4} \times x + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 2 & \\
\Rightarrow \left(x + \frac{1}{4}\right)^2 &= \frac{1}{16} - 2 \\
\Rightarrow \left(x + \frac{1}{4}\right)^2 &= \frac{1-36}{16} \\
\Rightarrow \left(x + \frac{1}{4}\right)^2 &= -\frac{31}{16} \\
\Rightarrow x + \frac{1}{4} &= \pm \sqrt{\frac{-31}{16}} \\
\Rightarrow x + \frac{1}{4} &= \frac{\sqrt{-31}}{4} \text{ or } x + \frac{1}{4} = -\frac{\sqrt{31}}{4} \\
\Rightarrow x &= \frac{\sqrt{-31}-1}{4} \text{ or } x = \frac{-\sqrt{31}-1}{4}
\end{aligned}$$

Since,  $\sqrt{-31}$  is not a real number  
 $\therefore$  The roots are not real roots.

6.

**Sol:**

We have,

$$\begin{aligned}
4x^2 + 4\sqrt{3}x + 3 &= 0 \\
\Rightarrow x^2 + \frac{4\sqrt{3}}{4}x + \frac{3}{4} &= 0 \\
\Rightarrow x^2 + 2 \times \frac{1}{2} \times \sqrt{3} \times x + \frac{3}{4} &= 0 \\
\Rightarrow x^2 + 2 \times \frac{\sqrt{3}}{2} \times x + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{3}{4} &= 0 \\
\Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} + \frac{3}{4} &= 0 \\
\Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 &= 0
\end{aligned}$$

$$\Rightarrow x + \frac{\sqrt{3}}{2} = 0 \text{ and } x + \frac{\sqrt{3}}{2} = 0$$

$$\Rightarrow x = -\frac{\sqrt{3}}{2} \text{ and } x = -\frac{\sqrt{3}}{2}$$

$\therefore x = -\frac{\sqrt{3}}{2}$  and  $x = -\frac{\sqrt{3}}{2}$  are the two roots of the given equation as it is a perfect square.

7.

**Sol:**

We have,

$$\sqrt{2}x^2 - 3x - 2\sqrt{2} = 0$$

$$\Rightarrow x^2 - \frac{3x}{\sqrt{2}} - \frac{2\sqrt{2}}{\sqrt{2}} = 0$$

$$\Rightarrow x^2 - \frac{3}{\sqrt{2}}x - 2 = 0$$

$$\Rightarrow x^2 - 2 \times \frac{1}{2} \times \frac{3}{\sqrt{2}}x - 2 = 0$$

$$\Rightarrow x^2 - 2 \times \frac{3}{2\sqrt{2}} \times x + \left(\frac{3}{2\sqrt{2}}\right)^2 - \left(\frac{3}{2\sqrt{2}}\right)^2 - 2 = 0$$

$$\Rightarrow \left(x - \frac{3}{2\sqrt{2}}\right)^2 = \frac{9}{8} + 2$$

$$\Rightarrow \left(x - \frac{3}{2\sqrt{2}}\right)^2 = \frac{9+16}{8}$$

$$\Rightarrow \left(x - \frac{3}{2\sqrt{2}}\right)^2 = \frac{25}{8}$$

8.

**Sol:**

We have

$$\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$$

$$\Rightarrow x^2 + \frac{10}{\sqrt{3}}x + \frac{7\sqrt{3}}{\sqrt{3}} = 0$$

$$\Rightarrow x^2 + 2 \times \frac{1}{2} \times \frac{10}{\sqrt{3}}x + 7 = 0$$

$$\Rightarrow x^2 + 2 \times \frac{5}{\sqrt{3}} \times x + \left(\frac{5}{\sqrt{3}}\right)^2 - \left(\frac{5}{\sqrt{3}}\right)^2 + 7 = 0$$

$$\Rightarrow \left(x + \frac{5}{\sqrt{3}}\right)^2 = \frac{25}{3} - 7$$

$$\Rightarrow \left(x + \frac{5}{\sqrt{3}}\right)^2 = \frac{25 - 21}{3}$$

$$\Rightarrow \left(x + \frac{5}{\sqrt{3}}\right)^2 = \frac{4}{3}$$

$$\Rightarrow x + \frac{5}{\sqrt{3}} = \pm \sqrt{\frac{4}{3}}$$

$$\Rightarrow x + \frac{5}{\sqrt{3}} = +\frac{2}{\sqrt{3}} \text{ or } x + \frac{5}{\sqrt{3}} = -\frac{2}{\sqrt{3}}$$

$$\Rightarrow x = -\frac{3}{\sqrt{3}} \text{ or } x = -\frac{7}{\sqrt{3}}$$

$$\Rightarrow x = -\sqrt{3} \text{ or } x = -\frac{7}{\sqrt{3}}$$

$\therefore x = -\sqrt{3}$  and  $x = -\frac{7}{\sqrt{3}}$  are the roots of the given equation.

9.

**Sol:**

We have,

$$x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$$

$$\Rightarrow x^2 - 2 \times \frac{1}{2}(\sqrt{2} + 1)x + \sqrt{2} = 0$$

$$\Rightarrow x^2 - 2 \times \frac{\sqrt{2} + 1}{2}x + \left(\frac{\sqrt{2} - 1}{2}\right)^2 - \left(\frac{\sqrt{2} + 1}{2}\right)^2 + \sqrt{2} = 0$$

$$\Rightarrow \left(x - \frac{\sqrt{2} + 1}{2}\right)^2 = \left(\frac{\sqrt{2} + 1}{2}\right)^2 - \sqrt{2}$$

$$\Rightarrow \left(x - \frac{\sqrt{2} + 1}{2}\right)^2 = \frac{3 + 2\sqrt{2} - 4\sqrt{2}}{4}$$

$$\begin{aligned}
\Rightarrow \left(x - \frac{\sqrt{2}+1}{2}\right)^2 &= \frac{3-2\sqrt{2}}{4} \\
\Rightarrow \left(x - \frac{\sqrt{2}+1}{2}\right)^2 &= \frac{2+1-2\sqrt{2}}{4} \\
\Rightarrow \left(x - \frac{\sqrt{2}+1}{2}\right)^2 &= \frac{(\sqrt{2})^2 - 2\sqrt{2} + 1}{2^2} \\
\Rightarrow \left(x - \frac{\sqrt{2}+1}{2}\right)^2 &= \frac{(\sqrt{2}-1)^2}{2^2} \\
\Rightarrow \left(x - \frac{\sqrt{2}+1}{2}\right)^2 &= \left(\frac{\sqrt{2}-1}{2}\right)^2 \\
\Rightarrow x - \frac{\sqrt{2}+1}{2} &= \pm \left(\frac{\sqrt{2}-1}{2}\right) \\
\Rightarrow x - \frac{\sqrt{2}+1}{2} &= \frac{\sqrt{2}-1}{2} \text{ or } x - \frac{\sqrt{2}+1}{2} = -\frac{\sqrt{2}-1}{2} \\
\Rightarrow x &= \frac{\sqrt{2}-1}{2} + \frac{\sqrt{2}+1}{2} \text{ or } x = \frac{\sqrt{2}-1}{2} + \frac{\sqrt{2}+1}{2} \\
\Rightarrow x &= \frac{\sqrt{2}-1+\sqrt{2}+1}{2} \text{ or } x = \frac{-\sqrt{2}+1+\sqrt{2}+1}{2} \\
\Rightarrow x &= \frac{\sqrt{2}}{2} \text{ or } x = \frac{1}{2} \\
\Rightarrow x &= \sqrt{2} \text{ or } x = 1 \\
\therefore x &= \sqrt{2} \text{ and } x = 1 \text{ are the roots of the given equation}
\end{aligned}$$

10.

**Sol:**

We have,

$$\begin{aligned}
x^2 - 4ax + 4a^2 - b^2 &= 0 \\
\Rightarrow x^2 - 2 \times (2a) \times x + (2a)^2 - b^2 &= 0 \\
\Rightarrow (x - 2a)^2 &= b^2 \\
\Rightarrow x - 2a &= \pm b \\
\Rightarrow x - 2a &= b \text{ or } x - 2a = -b \\
\Rightarrow x &= 2a + b \text{ or } x = 2a - b
\end{aligned}$$



$\therefore x = 2a + b$  and  $x = 2a - b$  are the two roots of the given quadratic equation.

### Exercise 8.5

1.

**Sol:** (i)  $2x^2 - 5x + 3 = 0$

The given equation is in the form of  $ax^2 + bx + c = 0$

here  $a = 2, b = -5$  and  $c = 3$

The discriminant  $D = b^2 - 4ac$

$$\Rightarrow (-5)^2 - 4 \times 2 \times 3$$

$$\Rightarrow 25 - 24 = 1$$

$\therefore$  The discriminant of the following quadratic equation is 1.

(ii)  $x^2 + 2x + 4 = 0$

The given equation is in the form of  $ax^2 + bx + c = 0$

here  $a = 1, b = 2$  and  $c = 4$

The discriminant is  $D = b^2 - 4ac$

$$\Rightarrow (2)^2 - 4 \times 1 \times 4$$

$$\Rightarrow 4 - 16 = -12$$

$\therefore$  The discriminant of the following quadratic equation is  $-12$ .

(iii)  $(x-1)(2x-1) = 0$

The given equation is  $(x-1)(2x-1) = 0$

By solving it, we get  $2x^2 - 3x + 1 = 0$

$\therefore$  This equation is in the form of  $ax^2 + bx + c = 0$

here  $a = 2, b = -3, c = -1$

The discriminant is  $D = b^2 - 4ac$

$$\Rightarrow (-3)^2 - 4 \times 2 \times 1$$

$$\Rightarrow 9 - 8 = 1$$

$\therefore$  The discriminant D, for the following quadratic equation is 1

(iv)  $x^2 - 2x + k = 0, K \in R$

The given equation is in the form of  $ax^2 + bx + c = 0$

here  $a = 1, b = -2, c = k$  [given  $k \in R$ ]

The discriminant is  $D = b^2 - 4ac$

$$\Rightarrow (-2)^2 - 4 \times 1 \times k$$

$$\Rightarrow 4 - 4k$$

$\therefore$  The discriminant D, of the following quadratic equation is  $4 - 4k$ , where  $K \in R$

(v)  $\sqrt{3}x^2 + 2\sqrt{2}x - 2\sqrt{3} = 0$

The given equation is in the form of  $ax^2 + bx + c = 0$

here  $a = \sqrt{3}, b = 2\sqrt{2}x$  and  $c = -2\sqrt{3}$

The discriminant is  $D = b^2 - 4ac$

$$\Rightarrow (2\sqrt{2})^2 - 4 \times \sqrt{3} \times -2\sqrt{3}$$

$$\Rightarrow 8 + 24$$

$$\Rightarrow 32$$

$\therefore$  The discriminant D, of the following quadratic equation is 32.

(vi)  $x^2 - x + 1 = 0$

The given equation is in the form of  $ax^2 + bx + c = 0$

here  $a = 1, b = -1$  and  $c = 1$

The discriminant is  $D = b^2 - 4ac$

$$\Rightarrow (-1)^2 - 4 \times 1 \times 1$$

$$\Rightarrow 1 - 4 = -3$$

$\therefore$  The discriminant D, of the following quadratic equation is  $-3$ .

(vii)  $3x^2 + 2x + k = 0$

The given equation has  $3x^2 + 2x + k = 0$

here  $a = 3, b = 2, c = k$

$\Rightarrow$  given that the quadratic equation has real roots.

i.e.,  $D = b^2 - 4ac \geq 0$

$$\Rightarrow 4 - 4 \times 3 \times k \geq 0$$

$$\Rightarrow 4 - 12k \geq 0$$

$$\Rightarrow 4 \geq 12k$$

$$\Rightarrow 4 \leq \frac{4}{12}$$

$$\Rightarrow k \leq \frac{1}{3}$$

The 'k' value should not exceed  $\frac{1}{3}$  to have the real roots for the given equation.

$$(viii) 4x^2 - 3kx + 1 = 0$$

The given equation has  $4x^2 - 3kx + 1 = 0$

here  $a = 4, b = -3k, c = 1$

given that quadratic equation has real roots

$$\text{i.e., } D = b^2 - 4ac \geq 0$$

$$\Rightarrow (-3k)^2 - 4 \times 4 \times 1 \geq 0$$

$$\Rightarrow 9k^2 - 16 \geq 0$$

$$\Rightarrow 9k^2 \geq 16$$

$$\Rightarrow 3k \geq 4$$

$$\Rightarrow k \geq \frac{4}{3} \Rightarrow k \geq 1.33$$

2.

**Sol:**

$$(i) 16x^2 = 24x + 1$$

The given equation is in the form of  $16x^2 - 24x - 1 = 0$

Hence, the equation is in the form of  $ax^2 + bx + c = 0$

Here,  $a = 16, b = -24, c = -1, D = b^2 - 4ac = (-24)^2 - 4 \times 16 \times -1 = 576 + 64 = 640 > 0$

As  $D > 0$ , the given equation has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \Rightarrow \frac{-(-24) + \sqrt{640}}{2 \times 16} = \frac{24 + \sqrt{640}}{32} = \frac{3 + \sqrt{10}}{4}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} \Rightarrow \frac{-(-24) - \sqrt{640}}{2 \times 16} = \frac{24 - \sqrt{640}}{32} = \frac{3 - \sqrt{10}}{4}$$

$\therefore$  The roots of the equation are  $\frac{3 \pm \sqrt{10}}{4}$

(ii)  $x^2 + x + 2 = 0$

The given equation is in the form of  $ax^2 + bx + c = 0$

$a = 1, b = 1, c = 2.$

$$D = b^2 - 4ac = (1)^2 - 4 \times 1 \times 2 = 1 - 8 = -7 < 0$$

As  $Q < 0$ , the equation has no real roots

(iii)  $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

The given equation is in the form of  $ax^2 + bx + c = 0$

here  $a = \sqrt{3}, b = 10$  and  $c = -8\sqrt{3}$

$$D = b^2 - 4ac \Rightarrow (10)^2 - 4 \times \sqrt{3} \times -8\sqrt{3} = 100 + 96 = 196 > 0$$

As  $Q > 0$ , the given equation has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \Rightarrow \frac{10 + \sqrt{196}}{2 \times \sqrt{3}} = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}} \quad [ \because \text{Multiplying and dividing by } \sqrt{3} ]$$

$$\beta = \frac{-b - \sqrt{D}}{2a} \Rightarrow \frac{-10 - \sqrt{196}}{2 \times \sqrt{3}} = -4\sqrt{3}$$

$\therefore$  The roots of the equation are  $\frac{2}{\sqrt{3}}$  and  $-4\sqrt{3}$

(iv)  $3x^2 - 2x + 2 = 0$

The given equation is in the form of  $ax^2 + bx + c = 0$

here  $a = 3, b = -2, c = 2$

The discriminant  $Q = b^2 - 4ac$

$$\Rightarrow (-2)^2 - 4 \times 3 \times 2 = 4 - 24$$

$$\Rightarrow -20 < 0$$

Hence as  $Q < 0$ ,

The given equation has no real roots.

(v)  $2x^2 - 2\sqrt{6}x + 3 = 0$

The given equation is in the form of  $ax^2 + bx + c = 0$

here  $a = 2, b = -2\sqrt{6}, c = 3$

The discriminant  $Q = b^2 - 4ac$

$$\Rightarrow (-2\sqrt{6})^2 - 4 \times 2 \times 3 = 24 - 24$$

$$\Rightarrow 0$$

As  $Q = 0$ , the given equation has real and equal roots, They are

$$\alpha = \frac{-b + \sqrt{D}}{2a} \Rightarrow \frac{-(-2\sqrt{6}) + \sqrt{0}}{2 \times 2} = \frac{2^1 \sqrt{6}}{4_2} = \sqrt{\frac{3}{2}}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} \Rightarrow \frac{-(-2\sqrt{6}) - \sqrt{0}}{2 \times 2} = \frac{2^1 \sqrt{6}}{4_2} = \sqrt{\frac{3}{2}}$$

$\therefore$  The roots of the given equation is  $\sqrt{\frac{3}{2}}$

(vi)  $3a^2x^2 + 8abx + 4b^2 = 0, a \neq 0$

The given equation is in the form of  $ax^2 + bx + c = 0$

here  $a = 3a^2, b = 8ab, c = 4b^2$  [given  $a \neq 0$ ]

$$D = b^2 - 4ac = (8ab)^2 - 4 \times 3a^2 \times 4b^2 = 64a^2b^2 - 48a^2b^2 = 16a^2b^2 > 0$$

As  $Q = 0$ , the given equation has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \Rightarrow \frac{-(8ab) + \sqrt{16a^2b^2}}{2 \times 3a^2} = \frac{-2b}{a}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} \Rightarrow \frac{-(8ab) - \sqrt{16a^2b^2}}{2 \times 3a^2} = \frac{-2b}{a}$$

$\therefore$  The roots of the given equation are  $\frac{-2b}{a}, \frac{-2b}{a}$

(vii)  $3x^2 + 2\sqrt{b}x - b = 0$

The given equation is in the form of  $ax^2 + bx + c = 0$

here  $a = 3, b = 2\sqrt{5}, c = -5$

The discriminant  $Q = b^2 - 4ac$

$$\Rightarrow (2\sqrt{5})^2 - 4 \times 3 \times -5 = 20 + 4 \times 3 \times 5$$

$$\Rightarrow 20 + 60 - 80 > 0$$

As  $Q = 0$ , the given equation has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \Rightarrow \frac{-(2\sqrt{5}) + \sqrt{80}}{2 \times 3} = \frac{\sqrt{5}}{3}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} \Rightarrow \frac{-(2\sqrt{5}) - \sqrt{80}}{2 \times 3} = -\frac{\sqrt{5}}{3}$$

∴ The roots of the given equation is  $\sqrt{\frac{3}{2}}$

(viii)  $x^2 - 2x + 1 = 0$

The given equation is in the form of  $ax^2 + bx + c = 0$

Here  $a = 1, b = -2$  and  $c = 1$

The discriminant  $D = b^2 - 4ac$

$$\Rightarrow (-2)^2 - 4 \times 1 \times 1 = 0$$

As  $Q = 0$ , the given equation has real and equal roots

$$\Rightarrow \alpha = -\frac{b + \sqrt{D}}{2a}, \beta = -\frac{b - \sqrt{D}}{2a} \text{ i.e., } \alpha \text{ and } \beta = -\frac{b}{2a} [\because 0 = 0]$$

$$\Rightarrow \alpha \text{ and } \beta = -\frac{b}{2a} = -\frac{(-2)}{2 \times 1} = \frac{2}{2} = 1$$

∴ The roots of the given equation  $\alpha$  and  $\beta$  is 1.

(ix)  $2x^2 + 5\sqrt{3}x + 6 = 0$

The given equation is in the form of  $ax^2 + bx + c = 0$

here  $a = 2, b = 5\sqrt{3}, c = 6$

The discriminant  $D = b^2 - 4ac$

$$\Rightarrow (5\sqrt{3})^2 - 4 \times 2 \times 6 = 75 - 48$$

$$\Rightarrow 27 > 0$$

As  $Q > 0$ , the given equation has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \Rightarrow \frac{-(5\sqrt{3}) + \sqrt{27}}{2 \times 2} = \frac{\sqrt{3}(-5 + 3)}{4} = \frac{\sqrt{3} \times -2}{4} = \frac{\sqrt{3}}{2}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} \Rightarrow \frac{-(5\sqrt{3}) - \sqrt{27}}{2 \times 2} = \frac{-\sqrt{3}[5 + 3]}{4} = \frac{-8\sqrt{3}}{4} = -2\sqrt{3}$$

∴ The roots of the given equation are  $-\frac{\sqrt{3}}{2}, -2\sqrt{3}$

(x)  $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

The given equation is in the form of  $ax^2 + bx + c = 0$

here  $a = \sqrt{2}, b = 7, c = 5\sqrt{2}$

The discriminant  $\boxed{Q = b^2 - 4ac}$

$$\Rightarrow (7)^2 - 4 \times \sqrt{2} \times 5\sqrt{2} = 49 - 40$$

$$\Rightarrow 9 > 0$$

As  $D > 0$ , the given equation has real roots, given by

$$\boxed{\alpha = \frac{-b + \sqrt{D}}{2a}} \Rightarrow \frac{-7 + \sqrt{9}}{2 \times \sqrt{2}} = \frac{-7 + \sqrt{3}}{2\sqrt{2}} = \frac{-7 + \sqrt{3}}{2\sqrt{2}} = -\sqrt{2}$$

$$\boxed{\beta = \frac{-b - \sqrt{D}}{2a}} \Rightarrow \frac{-7 - \sqrt{9}}{2 \times \sqrt{2}} = \frac{-7 - \sqrt{3}}{2\sqrt{2}} = \frac{5}{2\sqrt{2}} = -\frac{5}{\sqrt{2}}$$

$\therefore$  The roots of the given equation are  $-\sqrt{2}, \frac{-5}{\sqrt{2}}$

(xi)  $2x^2 - 2\sqrt{2}x + 1 = 0$

The given equation is in the form of  $ax^2 + bx + c = 0$

here  $a = 2, b = -2\sqrt{2}, c = 1$

$$\boxed{D = b^2 - 4ac} \Rightarrow \frac{(-2\sqrt{2})^2}{2 \times 2} = \frac{2^1 \sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

as  $D > 0$ , the given equation has Real and equal roots

hence  $\boxed{\alpha \text{ and } \beta = -\frac{b}{2a}} = -\frac{(-2\sqrt{2})}{2 \times 2} = \frac{2^1 \sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\therefore$  The roots  $\frac{1}{\sqrt{2}}$  is obtained by multiplying and dividing  $\frac{\sqrt{3}}{2}$  by  $\sqrt{2}$

$\therefore$  The roots of the given equation is  $\frac{1}{\sqrt{2}}$

(xii)  $3x^2 - 5x + 2 = 0$

The given equation is in the form of  $ax^2 + bx + c = 0$

here  $a = 3, b = -5, c = 2$

$$\boxed{D = b^2 - 4ac} \Rightarrow (-5)^2 - 4 \times 3 \times 2 = 25 - 24 = 1 > 0$$

as  $D > 0$ , the given equation has Real roots, giving by

$$\boxed{\alpha = -\frac{b + \sqrt{D}}{2a}} \Rightarrow \frac{-(-5) + \sqrt{1}}{2 \times 3} = \frac{5 + \sqrt{1}}{6} = \frac{6}{6} = 1$$

$$\beta = -\frac{b - \sqrt{D}}{2a} \Rightarrow \frac{-(-5) - \sqrt{1}}{2 \times 3} = \frac{5 - \sqrt{1}}{6} = \frac{4^2}{6_3} = \frac{2}{3}$$

∴ The roots of the given equation are 1 and  $\frac{2}{3}$

3.

**Sol:** Given,

$$\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$$

$$\Rightarrow \frac{(x-1)(x-4) + (x-3)(x-2)}{(x-2)(x-4)} = \frac{10}{3}$$

[Solving improper fraction]

$$\Rightarrow \frac{(x^2 - 5x + 4) + (x^2 - 5x + 6)}{(x-2)(x-4)} = \frac{10}{3}$$

This can also be written as . .

$$3[2x^2 - 10x + 10] = 10[(x-2)(x-4)]$$

$$\Rightarrow 6x^2 - 30x + 30 = 10x^2 - 60x + 80$$

By solving them, by taking all to one side, we get

$$\Rightarrow (10x^2 - 60x + 80) - (6x^2 - 30x + 30) = 0$$

$$\Rightarrow 4x^2 - 30x + 50 = 0, \text{ here } a = 4, b = -30, c = 50$$

Hence we get  $x$  by  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow x = -\frac{(-30) + \sqrt{(-30)^2 - 4 \times 4 \times 50}}{2(4)} = 5$$

$$x = -\frac{(-30) - \sqrt{(-30)^2 - 4 \times 4 \times 50}}{2 \times 4} = \frac{5}{2}$$

∴ The value of  $x$  are 5 and  $\frac{5}{2}$

4.

**Sol:** Given  $\frac{1}{x} - \frac{1}{x-2} = 3$

$$\Rightarrow \frac{x-2}{x(x-2)} = 3$$



$$\Rightarrow \frac{-2}{x(x-2)} = 3$$

This can be written as  $-2 = 3(x^2 - 2x)$

The equation hence is  $3x^2 - 6x + 2 = 0$

This equation is in the form of  $ax^2 + bx + c = 0$

Here  $a = 3, b = -6$  and  $c = 2$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-6) + \sqrt{(-6)^2 - 4 \times 3 \times 2}}{2 \times 3} = \frac{6 + \sqrt{36 - 24}}{6} = \frac{3 + \sqrt{3}}{3}$$

$$\Rightarrow x = \frac{-(-6) - \sqrt{(-6)^2 - 4 \times 3 \times 2}}{2 \times 3} = \frac{6 - \sqrt{36 - 24}}{6} = \frac{3 - \sqrt{3}}{3}$$

$\therefore$  The value of  $x$  are  $\frac{3 \pm \sqrt{3}}{3}$

5.

**Sol:** Given  $x + \frac{1}{x} = 3, x \neq 0$

Hence this equation can be written as  $\frac{x^2 + 1}{x} = 3$

$$\Rightarrow x^2 + 1 = 3x \Rightarrow x^2 - 3x + 1 = 0$$

$\therefore$  The equation is in the form of  $ax^2 + bx + c = 0$

Here  $a = 1, b = -3, c = 1$ .

The value of ' $x$ ' can be solved by  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow x = \frac{-b + \sqrt{(-3)^2 - 4 \times 1 \times 1}}{2 \times 1} = \frac{3 + \sqrt{9 - 4}}{2} = \frac{3 + \sqrt{5}}{2}$$

$$\Rightarrow x = \frac{-b - \sqrt{(-3)^2 - 4 \times 1 \times 1}}{2 \times 1} = \frac{3 - \sqrt{9 - 4}}{2} = \frac{3 - \sqrt{5}}{2}$$

$\therefore$  The value of ' $x$ ' are  $\frac{3 \pm \sqrt{5}}{2}$

## Exercise 8.6

1.

**Sol:**

(i)  $2x^2 - 3x + 5 = 0$

The given quadratic equation is  $2x^2 - 3x + 5 = 0$

here  $a = 2, b = -3, c = 5$

$$\boxed{D = b^2 - 4ac} \Rightarrow (-3)^2 - 4 \times 2 \times 5 = 9 - 40 = -31 < 0$$

As  $D < 0$ , The discriminant of equation is negative, then the expression has no real roots

(ii)  $2x^2 - 3x + 5 = 0$

The given quadratic equation is  $2x^2 - 3x + 5 = 0$

here  $a = 2, b = -3$  and  $c = 5$

$$\therefore \boxed{D = b^2 - 4ac} \Rightarrow (-3)^2 - 4 \times 2 \times 5 = 9 - 40 = -31 < 0$$

As  $D < 0$ , the discriminant of equation is negative, the equation has no real roots

(iii)  $\frac{3}{5}x^2 - \frac{2}{3}x + 1 = 0$

The given quadratic equation is  $\frac{3}{5}x^2 - \frac{2}{3}x + 1 = 0$  can also be written as  $9x^2 - 10x + 15 = 0$

here  $a = 9, b = -10, c = 15$

$$\boxed{D = b^2 - 4ac} \Rightarrow (-10)^2 - 4 \times 9 \times 15 = 100 - 540 = -440 < 0$$

$\therefore$  as  $D < 0$ , the equation has no real roots

(iv)  $3x^2 - 4\sqrt{3}x + 4 = 0$

The given quadratic equation is  $3x^2 - 4\sqrt{3}x + 4 = 0$

here  $a = 3, b = -4\sqrt{3}, c = 4$

The discriminant  $D = b^2 - 4ac$

$$\Rightarrow (-4\sqrt{3})^2 - 4 \times 3 \times 4 = 48 - 48 = 0$$

as  $D = 0$ , the equation has real and equal roots

(v)  $3x^2 - 2\sqrt{6}x + 2 = 0$

The given quadratic equation is  $3x^2 - 2\sqrt{6}x + 2 = 0$

Here. The equation is in the form of  $ax^2 + b + c = 0$

Where  $a = 3, b = -2\sqrt{6}$  and  $c = 2$

$$D = b^2 - 4ac \Rightarrow (-2\sqrt{6})^2 - 4 \times 3 \times 2 = 24 - 24 = 0$$

as  $D = 0$ , the given quadratic equation has real and equal roots

$$(vi) (x - 2a)(x - 2b) = 4ab$$

The given equation  $(x - 2a)(x - 2b) = 4ab$  can also be written as  $x^2 - x(2a + 2b)$  and

$$c = 0 [4ab - 4ab = 0]$$

$$D = b^2 - 4ac \Rightarrow [-(2a + 2a)]^2 - 4 \times 1 \times 0 = (2a + 2b)^2 > 0$$

$\Rightarrow$  as equal root of any integers is always positive

$\Rightarrow D > 0$ , hence the discriminant of the equation is positive

$$(vii) 2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$$

The given equation is  $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$

It is in the form of the equation  $ax^2 + bx + c = 0$

Here  $a = 2(a^2 + b^2), b = 2(a + b)$  and  $c = 1$

$$\therefore \boxed{D = b^2 - 4ac}$$

$$\Rightarrow (-2abcd)^2 - 9 \times 9a^2b^2 \times 16c^2d^2$$

$$\Rightarrow b + 6a^2b^2c^2d^2 - 576a^2b^2c^2d^2 = 0$$

Hence as  $D = 0$ , the equation has Real and equal roots

$$(viii) 2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$$

The given equation is  $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$

It is in the form of the equation  $ax^2 + bx + c = 0$

Here  $a = 2(a^2 + b^2), b = 2(a + b)$  and  $c = 1$

$$\therefore \boxed{D = b^2 - 4ac} \Rightarrow [2(a + b)]^2 - 4 \times 2(a^2 + b^2) \times 1$$

$$\Rightarrow 4a^2 + ab^2 + 8ab - 8a^2 - 8b^2$$

$$\Rightarrow 8ab + 4(a^2 + b^2) < 0$$

as  $D < 0$ , The discriminant is negative and the nature of the roots are not real

$$(ix) (b+c)x^2 - (a+b+c)x + a = 0$$

The given equation is  $(b+c)x^2 - (a+b+c)x + a = 0$

Here  $a = b+c, b = -(a+b+c)$  and  $c = a$

$$\therefore \boxed{D = b^2 - 4ac} \Rightarrow [-(a+b+c)]^2 - 4 \times (b+c) \times a$$

$$\Rightarrow (a+b+c)^2 - 4abc > 0$$

$\therefore$  as  $D > 0$ , the discriminant is positive and the nature of the roots are real and unequal

2.

**Sol:**

$$(i) kx^2 + 4x + 1 = 0$$

**Sol:**

The given equation  $kx^2 + 4x + 1 = 0$  is in the form of  $ax^2 + bx + c = 0$  where  $a = k, b = 4, c = 1$

$\Rightarrow$  given that, the equation has real and equal roots

$$\text{i.e., } \boxed{D = b^2 - 4ac = 0}$$

$$\Rightarrow (4)^2 - 4 \times k \times 1 = 0$$

$$\Rightarrow 16 = 4k \Rightarrow \boxed{k = 4}$$

$\therefore$  The value of  $k = 4$

$$(ii) kx^2 - 2\sqrt{5}x + 4 = 0$$

**Sol:**

The given equation  $kx^2 - 2\sqrt{5}x + 4 = 0$  is in the form of  $ax^2 + bx + c = 0$  where  $a = k, b = -2\sqrt{5}$  and  $c = 4$

$\Rightarrow$  given that, the equation has real and equal roots

$$\text{i.e., } \boxed{D = b^2 - 4ac = 0}$$

$$\Rightarrow (-2\sqrt{5})^2 - 4 \times k \times 4 = 0$$

$$\Rightarrow 20 = 16k \Rightarrow k = \frac{20}{16} = \frac{5}{4} \quad \therefore k = \frac{5}{4}$$

$\therefore$  The value of  $k = \frac{5}{4}$

$$(iii) 3x^2 - 5x + 2k = 0$$

**Sol:**

The given equation is  $3x^2 - 5x + 2k = 0$

This equation is in the form of  $ax^2 + bx + c = 0$

Here,  $a = 3, b = -5$  and  $c = 2k$

$\Rightarrow$  given that, the equation has real and equal roots

i.e.,  $\boxed{D = b^2 - 4ac = 0}$

$$\Rightarrow (-5)^2 - 4 \times 3 \times (2k) = 0$$

$$\Rightarrow 25 = 24k$$

$$\Rightarrow \boxed{k = \frac{25}{24}}$$

$\therefore$  The value of  $k = \frac{25}{24}$

(iv)  $4x^2 + kx + 9 = 0$

**Sol:**

The given equation is  $4x^2 + kx + 9 = 0$

This equation is in the form of  $ax^2 + bx + c = 0$

Here,  $a = 4, b = k$  and  $c = 9$

$\Rightarrow$  given that, the equation has real and equal roots

i.e.,  $\boxed{D = b^2 - 4ac = 0}$

$$\Rightarrow k^2 - 4 \times 4 \times 9 = 0$$

$$\Rightarrow k^2 - 16 \times 9$$

$$\Rightarrow k = \sqrt{16 \times 9} = 4 \times 3 = 12$$

$\therefore$  The value of  $k = 12$

(v)  $2kx^2 - 40x + 25 = 0$

**Sol:**

The given equation is  $2kx^2 - 40x + 25 = 0$

This equation is in the form of  $ax^2 + bx + c = 0$

Here,  $a = 2k, b = -40$  and  $c = 25$

$\Rightarrow$  given that, the equation has real and equal roots

i.e.,  $\boxed{D = b^2 - 4ac = 0}$

$$\Rightarrow (-40)^2 - 4 \times 2k \times 25 = 0$$

$$\Rightarrow 1600 - 200k = 0$$

$$\Rightarrow k = \frac{1600}{200} = 8 \quad \boxed{\therefore k = 8}$$

∴ The value of  $k = 8$

(vi)  $9x^2 - 24x + k = 0$

**Sol:**

The given equation is  $9x^2 - 24x + k = 0$

This equation is in the form of  $ax^2 + bx + c = 0$

Here,  $a = 9, b = -24$ , and  $c = k$

⇒ given that, the nature of the roots of this equation is real and equal

i.e.,  $D = b^2 - 4ac = 0$

$$\Rightarrow (-24)^2 - 4 \times 9 \times k = 0$$

$$\Rightarrow 576 - 36k = 0$$

$$\Rightarrow k = \frac{576}{36} = 16 \quad \boxed{\therefore k = 16}$$

∴ The value of  $k = 16$

(vii)  $4x^2 - 3kx + 1 = 0$

**Sol:**

The given equation is  $4x^2 - 3kx + 1 = 0$

This equation is in the form of  $ax^2 + bx + c = 0$

Here,  $a = 4, b = -3k$ , and  $c = 1$

⇒ given that, the nature of the roots of this equation is real and equal

i.e.,  $D = b^2 - 4ac = 0$

$$\Rightarrow (-3k)^2 - 4 \times 4 \times 1 = 0$$

$$\Rightarrow k^2 = \frac{16}{9} \Rightarrow k = \sqrt{\frac{16}{9}} = \pm \frac{4}{3}$$

$$\Rightarrow \boxed{k = \pm \frac{4}{3}}$$

∴ The value of  $k$  is  $\pm \frac{4}{3}$

(viii)  $x^2 - 2(5 + 2k)x + 3(7 + 10k) = 0$

**Sol:**

The given equation is  $x^2 - 2(5 + 2k)x + 3(7 + 10k) = 0$

Here,  $a = 1, b = -2(5 + 2k)$  and  $c = 3(7 + 10k)$

⇒ given that, the nature of the roots of this equation are real and equal

$$\text{i.e., } D = b^2 - 4ac = 0$$

$$\Rightarrow \{[-2(5+2k)]\} - 4 \times 1 \times 3(7+10k) = 0$$

$$\Rightarrow 4(5+2k)^2 - 12(7+10k) = 0$$

$$\Rightarrow 25 + 4k^2 + 20k - 21 - 30k = 0$$

$$\Rightarrow 4k^2 - 10k + 4 = 0 = 2k^2 - 5k + 2 = 0$$

$$\Rightarrow k2(k-2) - 1(k-2) \Rightarrow k = 2 \text{ or } k = \frac{1}{2}$$

$\therefore$  The value of  $k$  is 2 or  $\frac{1}{2}$

$$\text{(ix) } (3k+1)x^2 + 2(k+1)x + k = 0$$

**Sol:**

The given equation is  $(3k+1)x^2 + 2(k+1)x + k = 0$

This equation is in the form of  $ax^2 + bx + c = 0$

Here  $a = 3k+1, b = 2(k+1)$  and  $c = k$

$\Rightarrow$  Given that the nature of the roots of this equation are real and equal

$$\text{i.e., } \boxed{D = b^2 - 4ac = 0}$$

$$\Rightarrow [2(k+1)]^2 - 4 \times (3k+1) \times 0$$

$$\Rightarrow 4[k+1]^2 = 4k[3k+1] = 0$$

$$\Rightarrow (k+1)^2 - k(3k+1) = 0$$

$$\Rightarrow k^2 + 1 + 2k - 3k^2 - k = 0$$

$$\Rightarrow -2k^2 + k + 1 = 0$$

This equation can also be written as  $2k^2 - k - 1 = 0$

The value of  $k$  can obtain by

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where  $a = 2, b = -1, c = 1$  from equation 2

$$k = \frac{-(-1) + \sqrt{(-1)^2 - 4 \times 2 \times (-1)}}{2 \times 2} = \frac{1 + \sqrt{9}}{4} = \frac{4}{4} = 1$$

$$k = \frac{-(-1) + \sqrt{(-1)^2 - 4 \times 2 \times (-1)}}{2 \times 2} = \frac{1 - \sqrt{9}}{4} = \frac{\cancel{2}^1}{\cancel{4}_2} = \frac{-1}{2}$$

∴ the value of k are 1 and  $\frac{-1}{2}$

(x)  $kx^2 + kx + 1 = -4x^2 - x$

**Sol:**

The given equation is  $kx^2 + kx + 1 = -4x^2 - x$  bringing all the 'x' components to one side, we get the equation as  $x^2(4+k) + x(k+1) + 1 = 0$

This equation is in the form of the general quadratic equation i.e.,  $ax^2 + bx + c = 0$

Here  $a = 4+k, b = k+1$  and  $c = 1$

⇒ Given that the nature of the roots of the given equation are real and equal

i.e.,  $D = b^2 - 4ac = 0$

$$\Rightarrow (k+1)^2 - 4(4+k) \times 1 = 0$$

$$\Rightarrow k^2 + 1 + 2k - 16 - 4k = 0$$

$$\Rightarrow k^2 - 2k - 10 = 0 \quad \dots\dots\dots(2)$$

The equation (2) is as of the form  $ax^2 + bx + c$  here  $a = 1, b = -2, c = -16 + 1 = -15$

The value of k is obtained by  $k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\Rightarrow k = \frac{-(-2) + \sqrt{(-2)^2 - 4 \times 1 \times -15}}{2 \times 1} = 5$$

$$\Rightarrow k = \frac{-(-2) - \sqrt{(-2)^2 - 4 \times 1 \times -15}}{2 \times 1} = -3$$

∴ The value of k are 5 and -3 respectively for the given quadratic equation.

(xi)  $(k+1)x^2 + 20 = (k+3)x + (k+8) = 0$

**Sol:**

The given equation is  $(k+1)x^2 + 2(k+3)x + (k+8) = 0$

Here  $a = k+1, b = 2(k+3)$  and  $c = k+8$

⇒ given that the nature of the roots of this equation are real and equal i.e.,

$$D = b^2 - 4ac = 0$$

$$\Rightarrow [2(k+3)]^2 - 4 \times (k+1) \times (k+8) = 0$$

$$\Rightarrow 4(k+3)^2 - 4(k+1)(k+8) = 0$$

$$\Rightarrow (k+3)^2 - (k+1)(k+8) = 0$$



$$\Rightarrow k^2 + 9 + 6k - [k^2 + 9k + 8] = 0$$

$$\Rightarrow k^2 + 9 + 6k - k^2 - 9k - 8 = 0$$

$$\Rightarrow -3k + 1 = 0 \Rightarrow k = \frac{1}{3}$$

$\therefore$  The value of 'k' for the given equation is  $\frac{1}{3}$

$$(xii) x^2 - 2kx + 7k - 12 = 0$$

**Sol:**

The given equation is  $x^2 - 2kx + 7k - 12 = 0$

Here  $a = 1, b = -2k$  and  $c = 7k - 12$

$\Rightarrow$  given that the nature of the roots of this equation are real and equal i.e.,

$$\boxed{D = b^2 - 4ac = 0}$$

$$\Rightarrow (-2k)^2 - 4 \times 1 \times (7k - 12) = 0$$

$$\Rightarrow 4k^2 - 28k + 48 = 0$$

$$\Rightarrow k^2 - 7k + 12 = 0$$

The value of k can be obtained by  $k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$a = 1, b = -7, c = 12 \Rightarrow k = \frac{-(-7) \pm \sqrt{49 - 48}}{2} = \frac{7 \pm \sqrt{1}}{2} = 4, 3$$

$\therefore$  The value of 'k' for the equation is 4 and 3

$$(xiii) (k+1)x^2 - 2(3k+1)x + 8k+1 = 0$$

**Sol:**

The given equation is  $(k+1)x^2 - 2(3k+1)x + 8k+1 = 0$

It is in the form of the equation  $ax^2 + bx + c = 0$

Here,  $a = k+1, b = -2(3k+1)$  and  $c = 8k+1$

$\Rightarrow$  given that the nature of the roots of the given equation are real and equal i.e.,

$$D = b^2 - 4ac = 0$$

$$\Rightarrow [-2(3k+1)]^2 - 4 \times (k+1) \times (8k+1) = 0$$

$$\Rightarrow 4(3k+1)^2 - 4(k+1)(8k+1) = 0$$

$$\Rightarrow (3k+1)^2 - (k+1)(8k+1) = 0$$

$$\Rightarrow 9k^2 + 6k + 1 - [8k^2 + 9k + 1] = 0$$

$$\Rightarrow 9k^2 + 6k + 1 - 8k^2 - 9k - 1 = 0$$

$$\Rightarrow k^2 - 3k = 0$$

$$\Rightarrow k(k - 3) = 0$$

$$\therefore k = 0 \text{ or } k = 3$$

$\therefore$  The values of 'k' for the given quadratic equation are 0 and 3

$$(xiv) 5x^2 - 4x + 2 + k(4x^2 - 2x + 1) = 0$$

**Sol:**

$$\text{The given equation is } 5x^2 - 4x + 2 + k(4x^2 - 2x + 1) = 0$$

$$\text{This can be written as } x^2[5 + 4k] - x[4 + 2k] + 2 - k = 0$$

$$\text{This equation is in the form of } ax^2 + bx + c = 0 \quad \dots\dots\dots(1)$$

$$\text{Here } a = 5 + 4k, b = -(4 + 2k) \text{ and } c = -2k$$

$\Rightarrow$  given that the nature of the roots of this equation are real and equal i.e.,

$$\boxed{D = b^2 - 4ac = 0}$$

$$\Rightarrow [-(4 + 2k)]^2 - 4(5 + 4k)(2 - k) = 0$$

$$\Rightarrow (4 + 2k)^2 - 4(5 + 4k)(2 - k) = 0$$

$$\Rightarrow 16 + 4k^2 + 16k - 4[10 - 5k + 8k - 4k^2] = 0$$

$$\Rightarrow 16 + 4k^2 + 16k - 40 + 20k - 32k + 16k^2 = 0$$

$$\Rightarrow 20k^2 - 4k - 24 = 0$$

$$5k^2 - k - 6 = 0 \quad \dots\dots\dots(2)$$

As equation (2) is of the form (1), k can be obtained

$$\text{By } \boxed{k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}} \text{ where } a = 5, b = -1, c = -6$$

$$\Rightarrow k = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) - \sqrt{1 - 4 \times 5 \times -6}}{2 \times 5} = +\frac{6}{5}$$

$$\Rightarrow k = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) - \sqrt{1 - 4 \times 5 \times -6}}{2 \times 5} = -1$$

The values of k for the given equation are  $+\frac{6}{5}$  and  $-1$

$$(xv) (4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$$

**Sol:**

The given equation is  $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$

This equation is in the form of  $ax^2 + bx + c = 0$

Here  $a = 4 - k$ ,  $b = 2k + 4$  and  $c = 8k + 1$

$\Rightarrow$  given that the nature of the roots of this equation are real and equal i.e.,

$$\boxed{D = b^2 - 4ac = 0}$$

$$\Rightarrow (2k + 4)^2 - 4(4 - k)(8k + 1) = 0$$

$$\Rightarrow 4k^2 + 16 + 16k - 4[-8k^2 + 32k + 4 - k] = 0$$

$$\Rightarrow 4k^2 + 16 + 16k + (8k^2 \times 4) - (31 \times 4)k - 16 = 0$$

$$\Rightarrow 4k^2 + \cancel{16} + 16k + 32k^2 - 124k - \cancel{16} = 0$$

$$\Rightarrow 36k^2 - 108k = 0$$

$$\Rightarrow 9k^2 - 27k = 0$$

$$\Rightarrow k^2 - 3k = 0$$

$$\Rightarrow k(k - 3) = 0$$

Hence  $k = 0$  or  $k = 3$

$\therefore$  The value of 'k' for the given quadratic equation is 0 and 3

(xvi)  $(2k + 1)x^2 + 2(k + 3)x + (k + 5) = 0$

**Sol:**

The given equation is  $(2k + 1)x^2 + 2(k + 3)x + (k + 5) = 0$

This equation is in the form of  $ax^2 + bx + c = 0$

Here  $a = 2k + 1$ ,  $b = 2(k + 3)$  and  $c = k + 5$

$\Rightarrow$  given that the nature of the roots for this equation are real and equal i.e.,

$$\boxed{D = b^2 - 4ac = 0}$$

$$\Rightarrow [2(k + 3)]^2 - 4[2k + 1][k + 5] = 0$$

$$\Rightarrow (k + 3)^2 - (2k + 1)(k + 5) = 0$$

$$\Rightarrow k^2 + 9 + 6k - [2k^2 + 11k + 5] = 0$$

$$\Rightarrow -k^2 - 5k + 4 = 0$$

$$\Rightarrow k^2 + 5k - 4 = 0 \quad \dots\dots\dots(2)$$

$$\Rightarrow k^2 + 4k + k - 4 = 0$$

The value of 'k' can be obtained by  $k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Where from (2),  $a = 1, b = 5, c = -4$

$$k = \frac{-5 \pm \sqrt{25 - 4 \times 1 \times -4}}{2 \times 1} = \frac{-5 \pm \sqrt{25 + 16}}{2} = \frac{-5 \pm \sqrt{41}}{2}$$

$$k = \frac{-5 - \sqrt{25 - 4 \times 1 \times -4}}{2 \times 1} = \frac{-5 - \sqrt{25 + 16}}{2} = \frac{-5 - \sqrt{41}}{2}$$

$\therefore$  The value of 'k' from the given equation are  $\frac{-5 \pm \sqrt{41}}{2}$

(vii)  $4x^2 - 2(k + 1)x + (k + 4) = 0$

**Sol:**

The given equation is  $4x^2 - 2(k + 1)x + (k + 4) = 0$

This equation is in the form of  $ax^2 + bx + c = 0$

Here  $a = 4, b = -2(k + 1), c = k + 4$

$\Rightarrow$  Given that the nature of the roots of this equation is real and equal i.e.  $0 = b^2 - 4ac = 0$

$$\Rightarrow [-2(k + 1)]^2 - 4 \times 4 \times (k + 4) = 0$$

$$\Rightarrow 4(k + 1)^2 - 16(k + 4) = 0$$

$$\Rightarrow (k + 1)^2 - 4(k + 4) = 0$$

$$\Rightarrow k^2 + 1 + 2k - 4k - 16 = 0$$

$$\Rightarrow k^2 - 2k - 15 = 0$$

The value of 'k' can be obtained by the formula

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ where } a = 1, b = -2, c = -15$$

$$\Rightarrow k = \frac{-(-2) \pm \sqrt{4 - 4 \times 1 \times -15}}{2 \times 1} = \frac{2 \pm \sqrt{69}}{2 \times 1} = k = -3$$

$\therefore$  The value of 'k' for the given equation are 5 and -3

(xviii)  $x^2 - 2(k + 1)x + (k + 4) = 0$

**Sol:**

The given equation is  $x^2 - 2(k + 1)x + (k + 4) = 0$

This equation is in the form of  $ax^2 + bx + c = 0$

Here  $a = 1, b = -2(k + 1)$  and  $c = k + 4$

$\Rightarrow$  The nature of the roots of this equation is given that it is real and equal

i.e.,  $0 = b^2 - 4ac = 0$

$$\Rightarrow [-2(k + 1)]^2 - 4 \times 1 \times (k + 4) = 0$$

$$\Rightarrow 4(k + 1)^2 - 4(k + 4) = 0$$

$$\Rightarrow 4(k^2 + 1 + 2k) - 4k - 16 = 0$$

$$\Rightarrow k^2 + k - 3 = 0 \quad \dots(ii)$$

The value of 'k' can be obtained by formula  $k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  where  $a = 1, b = 1, c = -3$

$$k = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-1 + \sqrt{1 - 4 \times 1 \times -3}}{2 \times 1} = \frac{1}{2}$$

$$k = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-1 - \sqrt{1 - 4 \times 1 \times -3}}{2 \times 1} = \frac{1}{2}$$

The value of 'k' for the given equation are  $\frac{1}{2}$

$$(xix) 4x^2 - 2(k + 1)x + 4 = 0$$

**Sol:**

The given equation as  $k^2x^2 - 2(2k - 1)x + 4 = 0$

It is in the form of the equation  $ax^2 + bx + c = 0$

Here  $a = k^2, b = -2(2k - 1)$  and  $c = 4$

$\Rightarrow$  Given that the nature of the roots of the equation are real and equal

i.e.,  $D = b^2 - 4ac = 0$

$$\Rightarrow [-2(2k - 1)]^2 - 4 \times k^2 \times 4 = 0$$

$$\Rightarrow 4(2k - 1)^2 - 16k^2 = 0$$

$$\Rightarrow (2k - 1)^2 - 4k^2 = 0$$

$$\Rightarrow 4k - 1 = 0$$

$$k = \frac{1}{4}$$

$\therefore$  The value of 'k' for the given equation is  $\frac{1}{4}$

$$(xx) (k + 1)x^2 - 2(k - 1)x + 1 = 0$$

**Sol:**

The given equation is  $(k + 1)x^2 - 2(k - 1)x + 1 = 0$

It is in the form of the equation  $ax^2 + bx + c = 0$

Here  $a = k + 1, b = -2(k - 1)$  and  $c = 1$

$\Rightarrow$  Given that the nature of the roots for the equation are real and equal

i.e.,  $D = b^2 - 4ac = 0$

$$\Rightarrow [-2(k - 1)]^2 - 4 \times [k + 1] \times 1 = 0$$

$$\Rightarrow 4(k - 1)^2 - 4(k + 1) = 0$$

$$\Rightarrow (k - 1)^2 - (k + 1) = 0$$

$$\Rightarrow k^2 + 1 - 2k - k - 1 = 0$$

$$\Rightarrow k^2 - 3k = 0$$

$$\Rightarrow k(k - 3) = 0$$

$\therefore$  Here  $k = 0$  or  $k = 3$

$\therefore$  The value of 'k' for the given equation is  $k = 0$  or  $k = 3$

$$(xxi) 2x^2 + kx + 3 = 0$$

**Sol:**

The given equation is  $2x^2 + kx + 3 = 0$

It is in the form of the equation  $ax^2 + bx + c = 0$

Here  $a = 2$ ,  $b = k$ , and  $c = 3$

$\Rightarrow$  Given that the roots of the equation are real and equal i.e.,  $D = b^2 - 4ac = 0$

$$\Rightarrow k^2 - 4 \times 2 \times 3 = 0$$

$$\Rightarrow k^2 = 24$$

$$\Rightarrow k = \sqrt{24} = \pm 2\sqrt{6}$$

$\therefore$  The value of  $k$  for the given equation is  $\pm 2\sqrt{6}$

$$(xxii) kx(x - 2) + 6 = 0$$

**Sol:**

The given equation is  $kx^2 - 2kx + 6 = 0$

$a = 6$ ,  $b = -2k$ ,  $c = 6$

$\Rightarrow$  Given that the roots are real and equal

i.e.,  $D = b^2 - 4ac = 0 \Rightarrow 4k^2 - 4k \times 6 = 0$

$$\Rightarrow k^2 - 6k = 0$$

$$\Rightarrow k(k - 6) = 0$$

$$\Rightarrow k = 0 \text{ or } 6$$

$\therefore$  The value of  $k$  for the given equation is 0 or 6

$$(xxiii) x^2 - 4kx + k = 0$$

**Sol:**

The given equation is  $x^2 - 4kx + k = 0$

$a = 1$ ,  $b = -4k$ ,  $c = k$

$\Rightarrow$  Given that the roots are real and equal i.e.,  $D = b^2 - 4ac = 0$

$$\Rightarrow 16k^2 - 4k = 0$$

$$\Rightarrow 4k^2 - k = 0$$

$$\Rightarrow k(4k + 1) = 0$$

$$k = 0, k = \frac{1}{4}$$

$\therefore$  The value of  $k$  for the given equation is 0 or  $\frac{1}{4}$

3.

**Sol:**

$$(i) 2x^2 + 3x + k = 0$$

**Sol:**

The given equation is  $2x^2 + 3x + k = 0$

$\Rightarrow$  given that the quadratic equation has real roots i.e.,  $D = b^2 - 4ac \geq 0$

Given here  $a = 2, b = 3, c = k$

$$\Rightarrow 9 - 4 \times 2 \times k \geq 0$$

$$\Rightarrow 9 - 8k \geq 0$$

$$\Rightarrow 9 \geq 8k \Rightarrow k \leq \frac{9}{8}$$

The value of  $k$  does not exceed  $\frac{9}{8}$  to have roots

(ii)  $2x^2 + kx + 3 = 0$

**Sol:**

The given equation is  $2x^2 + kx + 3 = 0$

$\Rightarrow$  given that the quadratic equation has real roots i.e.,  $D = b^2 - 4ac \geq 0$

here  $a = 2, b = k, c = 3$

$$\Rightarrow k^2 - 4 \times 2 \times 3 \geq 0$$

$$\Rightarrow k^2 - 24 \geq 0$$

$$\Rightarrow k^2 \geq 24$$

$$\Rightarrow k \geq \sqrt{24} \Rightarrow k \geq \pm 2\sqrt{6} \text{ or } k \leq -2\sqrt{6}$$

$\therefore$  The value of  $k$  does not exceed  $2\sqrt{6}$  and  $-2\sqrt{6}$  to have real roots

(iii)  $2x^2 - 5x - k = 0$

**Sol:**

The given equation is  $2x^2 - 5x - k = 0$

$\Rightarrow$  given that the equation has real roots i.e.,  $D = b^2 - 4ac \geq 0$

$$\Rightarrow 25 - 4 \times 2 \times -k \geq 0$$

$$\Rightarrow 25 + 8k \geq 0$$

$$\Rightarrow 8k \geq -25$$

The value of  $k$  should not exceed  $\frac{25}{8}$  to have real roots.

(iv)  $kx^2 + 6x + 1 = 0$

**Sol:**

The given equation is  $kx^2 + 6x + 1 = 0$

Here  $a = k, b = 6, c = 1$

$\Rightarrow$  given that the equation has real roots

i.e.,  $D = b^2 - 4ac \geq 0$

$$\Rightarrow 36 - 4 \times k \times 1 \geq 0$$

$$\Rightarrow 36 \geq 4k$$

$$\Rightarrow k \geq \frac{36}{7}$$

$$\Rightarrow k \leq 9$$

The value of k should not exceed the value '9' to have real roots.

$$(v) x^2 - kx + 9 = 0$$

**Sol:**

The given equation is  $x^2 - kx + 9 = 0$

Here  $a = 1, b = -k, c = 9$

$\Rightarrow$  given that the equation has real roots

$$\text{i.e., } D = b^2 - 4ac \geq 0$$

$$\Rightarrow (-k)^2 - 4 \times 1 \times 9 \geq 0$$

$$\Rightarrow k^2 - 36 \geq 0$$

$$\Rightarrow k^2 \geq 36$$

$$\Rightarrow \boxed{k \geq 6} \text{ or } \boxed{k \leq -6}$$

The 'k' value exists between  $-6$  and  $6$  to have the real roots for the given equation.

$$(vi) 2x^2 + kx + 2 = 0$$

**Sol:**

The given equation is  $2x^2 + kx + 2 = 0$

Here  $a = 2, b = k, c = 2$

$\Rightarrow$  Given that the equation has real roots

$$\text{i.e., } D = b^2 - 4ac \geq 0$$

$$\Rightarrow k^2 - 4 \times 2 \times 2 \geq 0$$

$$\Rightarrow k^2 - 16 \geq 0$$

$$\Rightarrow k \geq 16$$

$$\Rightarrow k \geq \sqrt{16}$$

$$\Rightarrow k \geq 4 \text{ or } k \leq -4$$

$\therefore$  The k value lies between  $-4$  and  $4$  to have the real roots for the given equation.

$$(vii) 3x^2 + 2x + k = 0$$

**Sol:**

The given equation has  $3x^2 + 2x + k = 0$

Here  $a = 3, b = 2, c = k$

$\Rightarrow$  Given that the quadratic equation has real roots i.e.,  $D = b^2 - 4ac \geq 0$

$$\Rightarrow 4 - 4 \times 3 \times k \geq 0$$

$$\Rightarrow 4 - 12k \geq 0$$



$$\Rightarrow k \leq \frac{4}{12}$$

$$\Rightarrow k \leq \frac{1}{3}$$

The 'k' value should not exceed  $\frac{1}{3}$  to have the real roots for the given equation

$$(viii) 4x^2 - 3k + 1 = 0$$

**Sol:**

The given equation has  $4x^2 - 3k + 1 = 0$

Here  $a = 4$ ,  $b = -3k$ ,  $c = 1$

$\Rightarrow$  Given that that quadratic equation has real roots i.e.,  $D = b^2 - 4ac \geq 0$

$$= 9k^2 - 16 \geq 0$$

$$\Rightarrow 9k^2 \geq 16 = k^2 \geq \frac{16}{9}$$

$$\Rightarrow k \geq \sqrt{\frac{16}{9}} \Rightarrow k \geq \frac{4}{3} \text{ or } k \leq -\frac{4}{3}$$

$\therefore$  The value of k should be in between  $-\frac{4}{3}$  and  $\frac{4}{3}$  to have real roots for the given equation.

$$(ix) 2x^2 + kx - 4 = 0$$

**Sol:**

The given equation is  $2x^2 + kx - 4 = 0$

Here  $a = 2$ ,  $b = k$ ,  $c = -4$

$\Rightarrow$  Given that the quadratic equation has real roots i.e.,  $D = b^2 - 4ac \geq 0$

$$\Rightarrow k^2 + 32 \geq 0$$

$$\Rightarrow k \leq \sqrt{32}$$

$$\Rightarrow k \in \mathbb{R}$$

$\therefore$  The  $k \in \mathbb{R}$  for the equation to have the real roots

4.

**Sol:**

The given equation is  $(4-k)x^2 + (2k+4)x + (8k+1) = 0$ ,

Here  $a = 4-k$ ,  $b = 2k+4$ ,  $c = 8k+1$

The discriminant  $D = b^2 - 4ac$

$$= (2k+4)^2 - 4 \times (4-k)(8k+1)$$

$$\Rightarrow 4k^2 + 16 + 4k - 4[32k + 4 - 8k^2 - k]$$

$$\Rightarrow [4k^2 + 8k^2 + 4k - 31k + 4 - 4]$$

$$\Rightarrow 4[9k^2 - 27k]$$

$$\Rightarrow D = 4[9k^2 - 27k]$$

The given equation is a perfect square

$$D = 0$$

$$\Rightarrow 4[9k^2 - 27k] = 0$$

$$\Rightarrow 9k^2 - 27k = 0$$

$$\Rightarrow 3k^2 - 9k = 0$$

$$\Rightarrow k^2 - 3k = 0$$

$$\Rightarrow k(k - 3) = 0$$

$$\therefore k = 0 \text{ or } k = 3$$

$\therefore$  The value of  $k$  is '0' or '3' for the equation to be a perfect square

5.

**Sol:**

The given equation is  $x^2 + kx + 4 = 0$

$\Rightarrow$  given that the equation has real roots

$$\text{i.e., } D = b^2 - 4ac \geq 0$$

$$\Rightarrow k^2 - 4 \times 1 \times 4 \geq 0$$

$$\Rightarrow k^2 - 16 \geq 0$$

$$\Rightarrow k \geq 4 \text{ or } k \leq -4$$

$\therefore$  The least positive value of  $k = 4$ , for the equation to have real roots

6.

**Sol:**

$$(i) \quad kx^2 + 2x + 1 = 0$$

The given equation is  $kx^2 + 2x + 1 = 0$

Here  $a = k, b = 2, c = 1$

$\Rightarrow$  given that the equation has real and distinct roots

$$\text{i.e., } D = b^2 - 4ac \geq 0$$

$$\Rightarrow 4 - 4 \times 1 \times k \geq 0$$

$$\Rightarrow 4 - 4k \geq 0 \Rightarrow 4k \leq 4$$

$$\Rightarrow 4 \leq \frac{4}{k}$$

$\therefore$  The value of  $k$  is 1 i.e.,  $k < 1$  for which the quadratic equation has real and distinct roots

$$(ii) \quad kx^2 + 6x + 1 = 0$$

The given equation is  $kx^2 + 6x + 1 = 0$

Here  $a = k, b = 6, c = 1$

$\Rightarrow$  given that the equation has real and distinct roots

Hence  $D = b^2 - 4ac \geq 0$

$$\Rightarrow 36 - 4 \times 1 \times k \geq 0$$

$$\Rightarrow 36 - 4k \geq 0$$

$$\Rightarrow 4k \leq 36$$

$$\Rightarrow k \leq \frac{36}{4} \Rightarrow k \leq 9$$

$\therefore k < 9$  for the equation to have real and distinct roots

(iii)  $x^2 - kx + 9 = 0$

The given equation is  $x^2 - kx + 9 = 0$

Here  $a = 1, b = -k, c = 9$

$\Rightarrow$  given that the equation is having real and distinct roots

Hence  $D = b^2 - 4ac \geq 0$

$$\Rightarrow k^2 - 4 \times 1 \times 9 \geq 0$$

$$\Rightarrow k^2 - 36 \geq 0$$

$$\Rightarrow k^2 \geq 36$$

$$\Rightarrow k \geq 6 \text{ or } k \leq -6$$

$\therefore$  The value of "k" lies in between  $-6$  and  $6$  to have the real roots for the given equation

7.

**Sol:**

The given equation is  $(b-c)x^2 + (c-a)x + (a-b) = 0$

This equation has the general form i.e.,  $ax^2 + bx + c = 0$

Here  $a = b-c, b = c-a$  and  $c = a-b$

$\Rightarrow$  given that the equation has real and equal roots

Hence  $b^2 - 4ac = D = 0$

$$\Rightarrow (c-a)^2 - 4 \times (b-c)(a-b) = 0$$

$$\Rightarrow c^2 + a^2 - 2ac = 4[ab - b^2 - ac + cb] = 0$$

$$\Rightarrow c^2 + a^2 - 2ac - 4ab + 4b^2 + 4ac - 4cb = 0$$

$$\Rightarrow c^2 + a^2 + 2ac - 4ab + 4b^2 - 4cb = 0$$

$$\Rightarrow (a+c)^2 - 4ab + 4b^2 + 4cb = 0$$

$$\Rightarrow (c+a-2b)^2 = 0$$

$$\Rightarrow c+a-2b = 0$$

$$\Rightarrow c + a = 2b$$

Hence, it is proved that  $c + a = 2b$

8.

**Sol:**

The given equation is  $(a^2 + b^2)x^2 - 2(x + bd)x + (c^2 + d^2) = 0$

This equation has the general form  $ax^2 + bx + c = 0$

Here  $a = a^2 + b^2, b = -2(ac + bd), c = (c^2 + d^2) = 0$

$\Rightarrow$  given here that the nature of the real and equal

i.e.,  $D = b^2 - 4ac \geq 0$

$$\Rightarrow [-2(ac + bd)]^2 - 4x(a^2 + b^2)(c^2 + d^2) = 0$$

$$\Rightarrow 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2) = 0$$

$$\Rightarrow (ac + bd)^2 - (a^2 + b^2)(c^2 + d^2) = 0$$

$$\Rightarrow a^2c^2 + b^2d^2 + 2abcd - [a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2] = 0$$

$$\Rightarrow a^2c + 2^2d^2 + 2abcd - [a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2] = 0$$

$$\Rightarrow 2abcd - a^2d^2 - b^2c^2 = 0$$

$$\Rightarrow abcd + abcd - a^2d^2 - b^2c^2 = 0$$

$$\Rightarrow ad(bc - ad) + bc(ad - bc) = 0$$

$$\Rightarrow (ad - bc)(bc - ad) = 0$$

Case i:

$$\Rightarrow ab - bc = 0$$

$$\Rightarrow ad = bc$$

$$\Rightarrow \boxed{\frac{a}{b} = \frac{c}{d}}$$

Case ii:

$$\Rightarrow (bc - ad) = 0$$

$$\Rightarrow bc = ad$$

$$\Rightarrow \boxed{\frac{a}{b} = \frac{c}{d}}$$

$\therefore$  Hence, it is proved that  $\frac{a}{b} = \frac{c}{d}$

9.

**Sol:**

Given equations are  $ax^2 + 2bx + c = 0$  and  $bx^2 - 2\sqrt{ca}x + b = 0$

Then two equations are of the form  $ax^2 + bx + c = 0$

$\Rightarrow$  given that the roots of these two equations are real. Hence  $D \geq 0$  i.e.,  $b^2 - 4ac \geq 0$

Let us assume that  $ax^2 + 2bx + c = 0$  be equations .....(1)

and  $bx^2 - 2\sqrt{ca}x + b = 0$  be equation .....(2)

from equation (1)  $\Rightarrow b^2 - 4ac \geq 0$

$\Rightarrow 4b^2 - 4ac \geq 0$  .....(3)

From equation (2)  $\Rightarrow b^2 - 4ac \geq 0$

$\Rightarrow (-2\sqrt{ac})^2 - 4b^2 \geq 0$  .....(4)

Given that the roots of (1) and (2) are simultaneously real hence equation (3) equation (4)

$\Rightarrow 4b^2 - 4ac = 4ac - 4b^2$

$\Rightarrow 8ac = 8b^2$

$\Rightarrow b^2 = ac$

$\therefore$  Hence, it is proved that  $b^2 - ac$

10.

**Sol:**

The given equation is  $(p - q)x^2 + 5(p + q) = 0$

$\Rightarrow$  given  $p, q$  are real and  $p \neq q$

The discriminant  $D = b^2 - 4ac$

$\Rightarrow [5(p + q)]^2 - 4 \times (p - q) \times (-2(p - q))$

$\Rightarrow 25(p + q)^2 + 8(p - q)^2$

We know that the square of any integer is always positive i.e., greater than zero

Hence  $D = b^2 - 4ac \geq 0$

As given that  $p, q$  are real and  $p \neq q$

$\therefore 25(p + q)^2 + 8(p - q)^2 > 0$  i.e.,  $D > 0$

$\therefore$  The roots of this equation are real and unequal

11.

**Sol:**

The given equation is  $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$

This equation is in the form of  $ax^2 + bx + c = 0$

Here  $a = c^2 - ab, b = -2(a^2 - bc)$  and  $c = b^2 - ac$

$\Rightarrow$  given that the roots of this equation are equal

Hence  $D = 0$  i.e.,  $b^2 - 4ac = 0$

$$\Rightarrow [-2(a^2 - bc)]^2 - 4(c^2 - ab)(b^2 - ac) = 0$$

$$\Rightarrow 4(a^2 - bc)^2 - 4(c^2 - ab)(b^2 - ac) = 0$$

$$\Rightarrow 4a(a^3 + b^3 + c^3 - 3abc) = 0$$

$$\Rightarrow 4a = 0 \text{ or } a^3 + b^3 + c^3 - 3abc = 0$$

$$\Rightarrow a = 0 \text{ or } a^3 + b^3 + c^3 = 3abc$$

$\therefore$  hence, it is proved

12.

**Sol:**

The given equation is  $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$

This equation is in the form of  $ax^2 + bx + c = 0$

Here  $a = 2(a^2 + b^2), b = 2(a + b)$  and  $c = 1$

$\Rightarrow$  given that  $a \neq b$

The discriminant  $D = b^2 - 4ac$

$$\Rightarrow [2(a + b)]^2 - 4 \times 2(a^2 + b^2) \times (1)$$

$$\Rightarrow 4(a + b)^2 - 8(a^2 + b^2)$$

$$\Rightarrow 4[a^2 + b^2 + 2ab] - 8a^2 - 8b^2$$

$$\Rightarrow -4a^2 - 4b^2 + 2ab$$

As given that  $a \neq b$ , as the discriminant  $D$  has negative squares,  $D$  will be less than zero

Hence  $D < 0$ , when  $a \neq b$

13.

**Sol:**

The given equation is  $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$

By solving the equation, we get it as

$$3x^2 - 2x(a + b + c) + (ab + bc + ca) = 0$$

This equation is in the form of  $ax^2 + bx + c = 0$

Here  $a = 3, b = -2(a + b + c)$  and  $c = ab + bc + ca$

The discriminant  $D = b^2 - 4ac$

$$D = (-2a + b + c)^2 - 4(3)(ab + bc + ca)$$

$$\Rightarrow 4(a + b + c)^2 - 12(ab + bc + ca)$$

$$\Rightarrow 4[(a + b + c)^2 - 3(ab + bc + ca)]$$

$$\Rightarrow 4[a^2 + b^2 + c^2 - ab - bc - ca]$$

$$\Rightarrow 2[2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca]$$

$$\Rightarrow 2[(a - b)^2 + (b - c)^2 + (c - a)^2]$$

Here clearly  $0 \geq 0$ , if  $0 = 0$  then

$$(a - b)^2 + (b - c)^2 + (c - a)^2 = 0$$

$$\Rightarrow a - b = 0, b - c = 0, c - a = 0$$

Hence  $a = b = c$

Hence, it is proved.

14.

**Sol:**

The given equations are  $ax^2 + bx + c = 0$  .....(1)

And  $-ax^2 + bx + c = 0$  .....(2)

Given equations are of the form  $ax^2 + bx + c = 0$  also given that  $a, b, c$  are real numbers and  $ac \neq 0$

The discriminant  $D = b^2 - 4ac$

For equation (1)  $\Rightarrow b^2 - 4ac$  .....(3)

For equation (2)  $\Rightarrow b^2 - 4(-a) \times (c)$

$\Rightarrow b^2 + 4ac$  .....(4)

As  $a, b, c$  are real and given that  $ac \neq 0$  hence  $b^2 - 4ac > 0$  and  $b^2 + 4ac > 0$

$\therefore 0 > 0$

### Exercise 8.7

1.

**Sol:**

Let the two consecutive natural numbers be 'x' and 'x + 1'

$\Rightarrow$  Given that the sum of their squares is 85.

Then by hypothesis, we get

$$x^2 + (x + 1)^2 = 85$$

$$\Rightarrow x^2 + x^2 + 2x + 1 = 85$$

$$\Rightarrow 2x^2 + 2x + 1 - 85 = 0$$

$$\Rightarrow 2x^2 + 2x + 84 = 0 \Rightarrow 2[x^2 + x - 42] = 0$$

$$\Rightarrow x^2 + 7x - 6x - 42 = 0 \text{ [by the method of factorisation]}$$

$$\Rightarrow x(x + 7) - 6(x + 7) = 0$$

$$\Rightarrow (x - 6)(x + 7) = 0$$

$$\Rightarrow x = 6 \text{ or } x = 7$$

Case i: if  $x = 6$   $x + 1 = 6 + 1 = 7$

Case ii: If  $x = 7$   $x + 1 = -7 + 1 = -6$

$\therefore$  The consecutive numbers that the sum of this squares be 85 are 6,7 and  $-6, -7$ .

2.

**Sol:**

Let the two parts be 'x' and  $29 - x$

$\Rightarrow$  Given that the sum of the squares of the parts is 425.

Then, by hypothesis, we have

$$\Rightarrow x^2 + (29 - x)^2 = 425$$

$$\Rightarrow 2x^2 - 58x + 841 - 425 = 0$$

$$\Rightarrow 2x^2 - 58x + 416 = 0$$

$$\Rightarrow 2[x^2 - 29x + 208] = 0$$

$$\Rightarrow x^2 - 29x + 208 = 0$$

$$\Rightarrow x^2 - 13x - 16x + 208 = 0 \text{ [By the method of factorisation]}$$

$$\Rightarrow x(x - 13) - 16(x - 13) = 0$$

$$\Rightarrow (x - 13)(x - 16) = 0$$

$$\Rightarrow x = 13 \text{ or } x = 16$$

Case i: If  $x = 13$ ;  $29 - x = 29 - 13 = 16$

Case ii:  $x = 16$ ;  $29 - x = 29 - 16 = 13$

$\therefore$  The two parts that the sum of the squares of the parts is 425 are 13, 16.

3.

**Sol:**

The given sides of two squares = x cm and  $(x + 4)$  cm

The sum of their areas =  $656 \text{ cm}^2$ .

The area of the square = side  $\times$  side.

$\therefore$  Area of the square =  $x(x + 4) \text{ cm}^2$ .

$\Rightarrow$  Given that sum of the areas is  $656 \text{ cm}^2$ .

Hence by hypothesis, we have

$$\Rightarrow x(x + 4) + x(x + 4) = 656$$



$$\Rightarrow 2x(x + 4) = 656$$

$$\Rightarrow x^2 + 4x = 328 \text{ [dividing both sides by 2]}$$

$$\Rightarrow x^2 + 4x - 328 = 0$$

$$\Rightarrow x^2 + 20x - 16x - 328 = 0 \text{ [\because By the method of factorisation]}$$

$$\Rightarrow x(x + 20) - 16(x + 20) = 0$$

$$\Rightarrow (x + 20)(x - 16) = 0 \Rightarrow x = -20 \text{ or } x = 16$$

Case i: If  $x = 16$ ;  $x + 4 = 20$

$\therefore$  The sides of the squares are 16 cm and 20 cm.

Note: No negative value is considered as the sides will never be measured negatively.

4.

**Sol:**

Given the sum of two numbers is 48

Let the two numbers be  $x$  and  $48 - x$  also given their product is 432.

$$\text{Hence } x(48 - x) = 432$$

$$\Rightarrow 48x - x^2 = 432$$

$$\Rightarrow 48x - x^2 - 432 = 0$$

$$\Rightarrow x^2 - 48x + 432 = 0$$

$$\Rightarrow x^2 - 36x - 12x + 432 = 0 \text{ [By method of factorisation]}$$

$$\Rightarrow x(x - 36) - 12(x - 36) = 0$$

$$\Rightarrow (x - 36)(x - 12) = 0$$

$$\Rightarrow x = 36 \text{ or } x = 12$$

$\therefore$  The two numbers are 12, 36.

5.

**Sol:**

Let the integer be 'x'

Given that if an integer is added to its square, the sum is 90.

$$\Rightarrow x + x^2 = 90$$

$$\Rightarrow x + x^2 - 90 = 0$$

$$\Rightarrow x^2 + 10x - 9x - 90 = 0$$

$$\Rightarrow x(x + 10) - 9(x + 10) = 0$$

$$\Rightarrow x = -10 \text{ or } x = 9$$

$\therefore$  The value of an integer are -10 or 9.

6.

**Sol:**

Let the whole number be  $x$  as it is decreased by 20  $\Rightarrow (x - 20) = 69 \cdot \left(\frac{1}{2}\right)$

$$\Rightarrow x \cdot 20 = 69 \cdot \left(\frac{1}{x}\right)$$

$$\Rightarrow x(x - 20) = 69$$

$$\Rightarrow x^2 - 20x - 69 = 0$$

$$\Rightarrow x^2 - 23 + 3x - 69 = 0$$

$$\Rightarrow x(x - 23) + 3(x - 623) = 0$$

$$\Rightarrow (x - 23)(x + 3) = 0$$

$$\Rightarrow x = 23; x = -3$$

As the whole numbers are always positive,  $x = -3$  is not considered.

$\therefore$  The whole number  $x = 23$ .

7.

**Sol:**

Let the two consecutive natural numbers be ' $x$ ' and ' $x + 2$ '

$\Rightarrow$  Given that the product of the natural numbers is 20

$$\text{Hence } \Rightarrow x(x + 1) = 20$$

$$\Rightarrow x^2 + x = 20$$

$$\Rightarrow x^2 + x - 20 = 0$$

$$\Rightarrow x^2 + 5x - 4x - 20 = 0$$

$$\Rightarrow x(x + 5) - 4(x + 5) = 0$$

$$\Rightarrow x = -5 \text{ or } x = 4$$

Considering positive value of  $x$  as  $x \in \mathbb{N}$

$$\text{For } x = 4, x + 1 = 4 + 1 = 5$$

$\therefore$  The two consecutive natural numbers are 4 and 5.

8.

**Sol:**

Let the consecutive odd positive integers are  $2x - 1$  and  $2x + 1$

Given that the sum of the squares is 394.

$$\Rightarrow (2x - 1)^2 + (2x + 1)^2 = 394$$

$$\Rightarrow 4x^2 + 1 - 4x + 4x^2 + 1 + 4x = 394$$

$$\Rightarrow 8x^2 + 2 = 394$$

$$\Rightarrow 4x^2 = 392$$

$$\Rightarrow x^2 = 98$$

$$\Rightarrow x = 7$$

$$\text{As } x = 7, 2x - 1 = 2 \times 7 - 1 = 13$$

$$2x + 1 = 2 \times 7 + 1 = 15$$

$\therefore$  The two consecutive odd positive numbers are 13 and 15.

9.

**Sol:**

Let the numbers be  $x$  and  $8 - x$

Given that the sum of these numbers is 8

And 15 times the sum of their reciprocals as 8

$$\Rightarrow 15 \left( \frac{1}{x} + \frac{1}{8-x} \right) = 8$$

$$\Rightarrow 15 \left( \frac{(8-x)+x}{x(8-x)} \right) = 8$$

$$\Rightarrow 15 ((8-x) + x) = 8(x(8-x))$$

$$\Rightarrow 15 [8 - x + x] = 8x(8-x)$$

$$\Rightarrow 120 = 64x - 8x^2$$

$$\Rightarrow 8x^2 - 64x + 120 = 0$$

$$\Rightarrow 8[x^2 - 8x + 15] = 0$$

$$\Rightarrow x^2 - 5x - 3x + 15 = 0$$

$$\Rightarrow (x-5)(x-3) = 0$$

$$\Rightarrow x = 5 \text{ or } x = 3$$

$\therefore$  The two numbers are 5 and 3.

10.

**Sol:**

Let the number be  $x$

By the hypothesis, we have

$$\Rightarrow x + \sqrt{x} = \frac{6}{25}$$

$\Rightarrow$  let us assume that  $x = y^2$ , we get

$$\Rightarrow y^2 + y = \frac{6}{25}$$

$$\Rightarrow 25y^2 + 25y - 6 = 0$$

The value of 'y' can be obtained by  $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Where  $a = 25$ ,  $b = 25$ ,  $c = -6$

$$\Rightarrow y = \frac{-25 \pm \sqrt{625 - 600}}{50}$$

$$\Rightarrow y = \frac{-25 \pm 5}{50} \Rightarrow y = \frac{1}{5} \text{ or } \frac{-11}{10}$$

$$x = y^2 = \left( \frac{1}{5} \right)^2 = \frac{1}{25}$$

$\therefore$  The number  $x = \frac{1}{25}$ .

11.

**Sol:**

Let the number be  $x$ .

Given that the sum of  $x$  and its square =  $\frac{63}{4}$

$$\Rightarrow x + x^2 = \frac{63}{4}$$

$$\Rightarrow 4x + 4x^2 - 63 = 0$$

$$\Rightarrow 4x^2 + 4x - 63 = 0$$

$$\Rightarrow 4x^2 + 4x - 63 = 0 \quad \dots(i)$$

The value of  $x$  can be found by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\Rightarrow$  here  $a = 4, b = 4$  and  $c = -63$  from (i)

$$x = \frac{-4 \pm \sqrt{16 - 4 \times 4 \times -63}}{2 \times 4}$$

$$= \frac{-4 \pm \sqrt{16 + 16 \times 63}}{2 \times 4}$$

$$x = \frac{-4 \pm \sqrt{16 + 1008}}{8} = \frac{7}{2}; x = \frac{-4 - \sqrt{16 + 1008}}{8} = \frac{-9}{2}$$

$\therefore$  The values of  $x$  i. e., the numbers is  $\frac{7}{2}, \frac{-9}{2}$ .

12.

**Sol:**

Let the three consecutive numbers  $x, x+1$  and  $x+8$ .

According to the hypothesis given

$$x^2 + (x+1)(x+2) = 154.$$

$$\Rightarrow x^2 + [x^2 + 3x + 2] = 154$$

$$\Rightarrow 2x^2 + 3x = 152$$

$$\Rightarrow 2x^2 + 3x - 152 = 0 \quad \dots(i)$$

The value of ' $x$ ' can be obtained by the formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

here  $a = 2, b = 3$  and  $c = 152$  from (i)

$$x = \frac{-3 \pm \sqrt{9 - 4 \times 2 \times -152}}{2 \times 2}$$

$$x = \frac{-3 \pm \sqrt{9 + 8 \times 152}}{4} = 8, \frac{-19}{2}$$

considering the positive value of  $x$

$$\text{If } x = 8, x + 1 = 9, x + 2 = 10$$

$\therefore$  The three consecutive integers are 8, 9, and 10

13.

**Sol:**

Given that the product of two successive integral multiples of 5 is 300.

Let the integers be  $5x$ , and  $5(x+1)$

Then, by the integers be  $5x$  and  $5(x+1)$

Then, by the hypothesis, we have

$$\begin{aligned}
5x \cdot 5(x + 1) &= 300 \\
\Rightarrow 25x(x + 1) &= 300 \\
\Rightarrow x^2 + x &= 12 \\
\Rightarrow x^2 + x - 12 &= 0 \\
\Rightarrow x^2 + 4x - 3x - 12 &= 0 \\
\Rightarrow x(x + 4) - 3(x + 4) &= 0 \\
\Rightarrow (x + 4)(x - 3) &= 0 \\
\Rightarrow x = -4 \text{ or } x = 3 \\
\text{If } x = -4, 5x = -20, 5(x + 1) &= -15 \\
x = 3, 5x = 15, 5(x + 1) &= 20 \\
\therefore \text{The two successive integral multiples are } &15, 20 \text{ or } -15, -20.
\end{aligned}$$

14.

**Sol:**

Let the number be  $x$

Then the other number =  $2x - 3$

According to the given hypothesis,

$$\Rightarrow x^2 + (2x - 3)^2 = 233$$

$$\Rightarrow x^2 + 4x^2 + 9 - 12x = 233$$

$$\Rightarrow 5x^2 - 12x - 224 = 0 \quad \dots (i)$$

The value of 'x' can be obtained by the formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here  $a = 5$ ,  $b = 12$  and  $c = -224$  from (i)

$$x = \frac{-(-12) + \sqrt{144 + 20 \times 224}}{10} = 8$$

$$x = \frac{-(-12) - \sqrt{144 + 20 \times 224}}{10} = \frac{-28}{5}$$

considering the value of  $x = 8$

$$2x - 3 = 16 - 3 = 15$$

$\therefore$  The two numbers are 8 and 15.

15.

**Sol:**

Let the consecutive even integers be  $2x$  and  $2x + 2$ .

Then according to the given hypothesis,

$$(2x)^2 + (2x + 2)^2 = 340$$

$$\Rightarrow 8x^2 + 8x - 336 = 0$$

$$\Rightarrow x^2 + x - 42 = 0$$

$$\Rightarrow x^2 + 7x - 6x - 42 = 0$$

$$\Rightarrow x(x + 7) - 6(x + 7) = 0$$

$$\Rightarrow (x + 7)(x - 6) = 0$$

$$\Rightarrow x = -7 \text{ or } x = 6$$

Considering, the positive integers of  $x$ .

$$\Rightarrow x = 6; 2x = 12 \text{ and } 2x + 2 = 14.$$

$\therefore$  The two consecutive even integers are 12 and 14.

16.

**Sol:**

Let the two numbers be  $x$  and  $x - 4$

Given that the difference of two numbers is 4.

By the given hypothesis, we have  $\frac{1}{x-4} - \frac{1}{x} = \frac{4}{21}$

$$\Rightarrow \frac{x-x+4}{x(x-4)} = \frac{4}{21}$$

$$\Rightarrow 84 = 4x(x - 4)$$

$$\Rightarrow x^2 - 4x - 21 = 0$$

$$\Rightarrow x^2 - 7x + 3x - 21 = 0$$

$$\Rightarrow x(x - 7) + 3(x - 7) = 0$$

$$\Rightarrow (x - 7)(x + 3) = 0$$

$$\Rightarrow x = 7 \text{ or } x = -3 \text{ and}$$

$$\text{If } x = -3, x - 4 = -3 - 4 = -7$$

Hence, required numbers are 3, 7 and  $-3, -7$

17.

**Sol:**

Let the numbers be  $x$  and  $x - 3$

By the given hypothesis,

$$x^2 + (x - 3)^2 = 117$$

$$\Rightarrow x^2 + x^2 + 9 - 6x - 117 = 0$$

$$\Rightarrow 2x^2 - 6x - 108 = 0$$

$$\Rightarrow x^2 - 3x - 54 = 0$$

$$\Rightarrow x(x - 9) + 6(x - 9) = 0$$

$$\Rightarrow (x - 9)(x + 6) = 0$$

$$\Rightarrow x = 9 \text{ or } x = -6$$

Considering positive value of  $x$

$$x = 9, x - 3 = 9 - 3 = 6$$

$\therefore$  The two numbers be 9 and 6.

18.

**Sol:**

Let the numbers be  $x, x + 1$  and  $x + 2$  according to the given hypothesis.

$$\begin{aligned}
x^2 + (x + 1)^2 + (x + 2)^2 &= 149 \\
\Rightarrow x^2 + x^2 + 1 + 2x + x^2 + 4 + 4x &= 149 \\
\Rightarrow 3x^2 + 6x + 5 - 149 &= 0 \\
\Rightarrow 3x^2 + x - 144 &= 0 \\
\Rightarrow x^2 + 2x - 48 &= 0 \\
\Rightarrow x(x + 8) - 6(x + 8) &= 0 \\
\Rightarrow (x + 8)(x - 6) &= 0 \\
\Rightarrow x = -8 \text{ or } x = 6
\end{aligned}$$

Considering the positive value of  $x$

$$x = 6, x + 1 = 7 \text{ and } x + 2 = 8$$

$\therefore$  The three consecutive numbers are 6, 7, 8.

19.

**Sol:**

Given that the sum of two numbers is 16.

Let the two numbers be  $x$  and  $16 - x$

By the given hypothesis, we have

$$\Rightarrow \frac{1}{x} + \frac{1}{16-x} = \frac{1}{3}$$

$$\Rightarrow \frac{16-x+x}{x(16-x)} = \frac{1}{3}$$

$$\Rightarrow 48 = 16x - x^2$$

$$\Rightarrow x^2 - 16x + 48 = 0$$

$$\Rightarrow x^2 - 12x - 4x + 48 = 0$$

$$\Rightarrow x(x - 12) - 4(x - 12) = 0$$

$$\Rightarrow (x - 12)(x - 4) = 0$$

$$\Rightarrow x = 12 \text{ or } x = 4$$

$\therefore$  The two numbers are 4 and 12.

20.

**Sol:**

Let the two consecutive multiples of 3 are  $3x$  and  $3x + 3$

Given that their product is 270

$$\Rightarrow (3x)(3x + 3) = 270$$

$$\Rightarrow x(3x + 3) = 90$$

$$\Rightarrow x^2 + x - 30 = 0$$

$$\Rightarrow x^2 + 6x - 5x - 30 = 0$$

$$\Rightarrow x(x + 6) - 5(x + 6) = 0$$

$$\Rightarrow (x + 6)(x - 5) = 0$$

$$\Rightarrow x = 5 \text{ or } x = -6$$

Considering the positive value of  $x$ .

$$\Rightarrow x = 5, 3x = 15 \text{ and } 3x + 3 = 18$$

$\therefore$  The two consecutive multiples of 3 are 15 and 18.

21.

**Sol:**

Let the number be 'x'

According to the given hypothesis

$$x + \frac{1}{x} = \frac{17}{4}$$

$$\Rightarrow \frac{x^2 + 1}{x} = \frac{17}{4}$$

$$\Rightarrow 4(x^2 + 1) = 17x$$

$$\Rightarrow 4x^2 - 17x + 4 = 0$$

$$\Rightarrow 4x^2 - 16x - x + 4 = 0$$

$$\Rightarrow 4x(x - 4) - 1(x - 4) = 0$$

$$\Rightarrow x = \frac{1}{4} \text{ or } x = 4$$

$\therefore$  The value of  $x = 4$

22.

**Sol:**

Let the two digits be  $x$  and  $x - 2$

Given that the product of their digits is 8.

$$\Rightarrow x(x - 2) = 8$$

$$\Rightarrow x^2 - 2x - 8 = 0$$

$$\Rightarrow x^2 - 4x + 2x - 8 = 0$$

$$\Rightarrow x(x - 4) + 2(x - 4) = 0$$

$$\Rightarrow (x - 4)(x + 2) = 0$$

$$\Rightarrow x = 4 \text{ or } x = -2$$

Considering the positive value  $x = 4, x - 2 = 2$ .

$\therefore$  The two digit number is 42.

23.

**Sol:**

Let the tens digit be  $x$

Then, the units digit =  $\frac{12}{x}$

$$\therefore \text{Number} = 10x + \frac{12}{x}$$

And, number obtained by interchanging the



$$\text{Digits} = 10 \times \frac{12}{x} + x = \frac{120}{x} + x.$$

$$\Rightarrow 10x + \frac{12}{x} + 36 = \frac{120}{x} + x$$

$$\Rightarrow 9x + \frac{12-120}{x} + 36 = 0$$

$$\Rightarrow 9x^2 - 108 + 36x = 0$$

$$\Rightarrow 9(x^2 + 4x - 12) = 0$$

$$\Rightarrow x^2 + 6x - 2x - 12 = 0$$

$$\Rightarrow x(x+6) - 2(x+6) = 0$$

$$\Rightarrow (x-2)(x+6) = 0 \therefore x = 2 \text{ or } -6$$

But, a digit can never be negative,  $80x = 2$

$$\text{Hence, the digit} = 10 \times 2 + \frac{12}{2} = 20 + 6 = 26$$

24.

**Sol:**

Let the two digits be:

$$\text{Tens digits be } x \text{ and units} = \frac{16}{x}$$

$$\text{Number} = 10x + \frac{16}{x}$$

$$\text{Number obtained by interchanging} = 10 \times \frac{16}{x} + x$$

$$\Rightarrow \left(10x + \frac{16}{x}\right) - \left(10 \times \frac{16}{x} + x\right) = 5u$$

$$\Rightarrow 10x + \frac{16}{x} - \frac{160}{x} + x = 54$$

$$\Rightarrow 10x^2 + 16 - 160 + x^2 = 54x$$

$$\Rightarrow 9x^2 - 54x - 144 = 0$$

$$\Rightarrow x^2 - 6x - 16 = 0$$

$$\Rightarrow x^2 - 8x + 2x - 16 = 0$$

$$\Rightarrow x(x-8)+2(x-8)=0$$

$$\Rightarrow (x=8) \text{ or } x=-2$$

But, a digit can never be negative, hence  $x=8$

$$\text{Hence the required number} = 10 \times 8 + \frac{16}{8} = 82$$

25.

**Sol:**

Let the two numbers be  $x$  and  $x-3$  given that  $x(x-3)=504$

$$\Rightarrow x^2 - 3x - 504 = 0$$

$$\Rightarrow x^2 - 24x + 21x - 504 = 0$$

$$\Rightarrow x(x-24) + 21(x-24) = 0$$

$$\Rightarrow (x-24)(x+21) = 0$$

$$\Rightarrow x = 24 \text{ or } x = 21$$

Case 1: If  $x = 24, x - 3 = 21$

Case 2: If  $x = 21, x - 3 = 18$

$\therefore$  The two numbers are 21, 18 or -21, -18

26.

**Sol:** Let the two numbers be  $x$  and  $x-4$

Given that their product is 192

$$\Rightarrow x(x-4)=192$$

$$\Rightarrow x^2 - 4x - 192 = 0$$

$$\Rightarrow x^2 - 16x + 12x - 192 = 0$$

$$\Rightarrow x(x-16) + 12(x-16) = 0$$

$$\Rightarrow (x-16)(x+12) = 0$$

$$\Rightarrow x = 16 \text{ or } x = -12$$

Considering the positive value of  $x$

$$x = 16, \Rightarrow x - 4 = 16 - 4 = 12$$

∴ The two numbers are 12,16

27.

**Sol:**

Let the digits at tens and units place of the number be  $x$  and  $y$  respectively then, it is given that  $= 10x + y$

$$\Rightarrow 10x + y = 4 \text{ (sum of digits) and } 2xy$$

$$\Rightarrow 10x + y = 4(x + y) \text{ and } 10x + y = 3xy$$

$$\Rightarrow 10x + y = 4x + 4y \text{ and } 10x + y = 3xy$$

$$\Rightarrow 6x - 3y = 0 \text{ and } 10x + y - 3xy = 0$$

$$\Rightarrow y = 2x \text{ and } 10x + 2x = 2xy(2x)$$

$$\Rightarrow 12x = 4x^2$$

$$\Rightarrow 4x^2 - 12x = 0$$

$$\Rightarrow 4x(x - 3) = 0$$

$$\Rightarrow 4x = 0 \text{ or } x = 3$$

$$\Rightarrow \text{here we have } y = 2x \Rightarrow 2 \times 3 = 6$$

$$\therefore x = 3 \text{ and } y = 6$$

$$\text{Hence } 10x + y = 10 \times 3 + 6 = 36$$

∴ The required two digit number is 36

28.

**Sol:**

Let the smaller number be  $x$ . Then square of a larger number  $= 18x$

Also, square of the smaller number  $= x^2$

It is given that the sum of the square of the integers is 208.

$$\therefore x^2 + 18x = 208$$

$$\Rightarrow x^2 + 18x - 208 = 0$$

$$\Rightarrow x^2 + 26x - 8x - 208 = 0$$

$$\Rightarrow (x + 26)(x - 8) = 0 \Rightarrow x = 8 \text{ or } x = -26$$

But, the numbers are positive. Therefore  $x = 8$

$$\therefore \text{square of the larger number} = 18x = 18 \times 8 = 144$$

$\Rightarrow$  larger number are 8 and 18.

29.

**Sol:** Let The numbers be  $x$  and  $18 - x$

$\Rightarrow$  according to the given hypothesis

$$\frac{1}{x} + \frac{1}{18 - x} = \frac{1}{4}$$

$$\Rightarrow \frac{18 - x + x}{x(18 - x)} = \frac{1}{4}$$

$$\Rightarrow 7_2 = 18x - x^2$$

$$\Rightarrow x^2 - 18x - 72 = 0$$

$$\Rightarrow x^2 - 6x - 12x - 72 = 0$$

$$\Rightarrow x(x - 6) - 12(x - 12) = 0$$

$$\Rightarrow x = 6 \text{ or } x = 12$$

$\therefore$  The two number are 6, 12

30.

**Sol:**

Let us assume a number ' $x$ '

$$\text{Such that } \frac{1}{x} + \frac{1}{15 - x} = \frac{3}{10}$$

$$\text{Hence } \Rightarrow \frac{15 - x + x}{x(15 - x)} = \frac{3}{10}$$

$$\Rightarrow 150 = 45x + 3x^2$$

$$\Rightarrow 3x^2 - 45x + 150 = 0$$

$$\Rightarrow x^2 - 15x + 50 = 0$$

$$\Rightarrow 2^2 - 10x - 5x + 50 = 0$$

$$\Rightarrow x(x-10) - 5(x-10) = 0$$

$$\Rightarrow (x-10)(x-5) = 0$$

$$\Rightarrow x = 5 \text{ or } x = 10$$

Case i: If  $x = a$ ,  $a = 5$  and

$$b = 15 - x, b = 10.$$

Case ii: if  $x = 15 + a = 15 + 10 = 5x$   $x = a = 10$

$$b = 15 - 10 = 5$$

$$\therefore a = 5, b = 10 \text{ or } a = 10 \text{ and } b = 5$$

31.

**Sol:**

Given that the sum of two numbers is 9 Let the two numbers be  $x$  and  $9 - x$

By the given hypothesis, we have

$$\frac{1}{x} + \frac{1}{9-x} = \frac{1}{2}$$

$$\Rightarrow \frac{9-x+x}{x(9-x)} = \frac{1}{2}$$

$$\Rightarrow 18 = 9x - x^2$$

$$\Rightarrow x^2 - 9x + 18 = 0$$

$$\Rightarrow x^2 - 6x - 3x + 18 = 0$$

$$\Rightarrow x(x-6) - 3(x-6) = 0$$

$$\Rightarrow (x-6)(x-3) = 0$$

$$\Rightarrow x = 6 \text{ or } x = 3$$

$\therefore$  The two numbers are 3 and 6

32.

**Sol:**

Let the three consecutive positive integers be  $x$ ,  $x+1$  and  $x+2$

According to the hypothesis, we have

$$\Rightarrow x^2 + (x+1)(x+2) = 46$$

$$\Rightarrow x^2 + x^2 + 3x + 2 = 46$$

$$\Rightarrow 2x^2 + 3x - 44 = 0$$

$$\Rightarrow 2x^2 - 8x + 11x - 44 = 0$$

$$\Rightarrow 2x(x-4) + 11(x-4) = 0$$

$$\Rightarrow (2x+11)(x-4) = 0$$

$$\Rightarrow x = 4 \text{ or } x = -\frac{11}{2}$$

Considering the positive value of  $x$

$$\Rightarrow x = 4, x+1 = 5 \text{ and } x+2 = 6$$

$\therefore$  The three consecutive numbers are 4, 5 and 6.

33.

**Sol:**

Let the smaller number be  $x$ . Then, larger number =  $2x - 5$

It is given that the difference of the square of the number is 88.

$$\Rightarrow (2x-5)^2 - x^2 = 88$$

$$\Rightarrow 4x^2 + 25 - 20x - x^2 = 88$$

$$\Rightarrow 3x^2 - 20x - 63 = 0$$

$$\Rightarrow 3x^2 - 27x + 7x - 63 = 0$$

$$\Rightarrow 3x(x-9) + 7(x-9) = 0$$

$$\Rightarrow (x-9)(3x+7) = 0$$

$$\therefore x = 9 \text{ or } -\frac{7}{3}$$

As a digit can never be negative,  $x = 9$

$$\Rightarrow \therefore \text{The numbers} = 2x - 5$$

$$= 2 \times 9 - 5 = 13$$

$\therefore$  Hence, required numbers are 9 and 13

34.

**Sol:**

Let the number be  $x$

By the given hypothesis, we have

$$x^2 - 8x = 180$$

$$\Rightarrow x^2 - 8x - 180 = 0$$

$$\Rightarrow x^2 + 10x + -18x - 180 = 0$$

$$\Rightarrow x(x+10) - 18(x+10) = 0$$

$$\Rightarrow (x+10)(x-18) = 0$$

$$\Rightarrow x = -10 \text{ or } x = 18$$

Case (i):  $x = 18$

$$8x = 8 \times 18 = 144$$

$$\therefore \text{Larger number} = \sqrt{144} = \pm 12$$

Case (ii):  $x = -10$

Square of larger number  $8x = -80$  here no perfect square exist, hence the numbers are 18,12

### Exercise 8.8

1.

**Sol:**

Let the speed of the stream be  $x$  km/hr

Given that,

Speed of the boat in still water = 8 km/hr

Now,

Speed of the boat in upstream =  $(8 - x)$  km/hr

And speed of the boat in downstream =  $(8 + x)$  km/hr

$$\text{Time taken for going 15 km upstream} = \frac{15 \text{ km}}{(8-x) \text{ km/hr}} = \frac{15}{8-x} \text{ hours}$$

$$\text{Time taken for going 22 km downstream} = \frac{22 \text{ km}}{(8+x) \text{ km/hr}} = \frac{22}{8+x} \text{ hours}$$

Given that,

Time taken for upstream + downstream = 5 hours

$$\Rightarrow \frac{15}{8-x} \text{ hours} + \frac{22}{8+x} \text{ hours} = 5 \text{ hours}$$

$$\Rightarrow \frac{15}{8-x} + \frac{22}{8+x} = 5$$

$$\Rightarrow \frac{15(8+x) + 22(8-x)}{(8-x)(8+x)} = 5$$

$$\Rightarrow \frac{120 + 15x + 176 - 22x}{8^2 - x^2} = 5$$

$$\Rightarrow 296 - 7x = 5(64 - x^2)$$

$$\Rightarrow 296 - 7x = 320 - 5x^2$$

$$\Rightarrow 5x^2 - 7x + 296 - 320 = 0$$

$$\Rightarrow 5x^2 - 7x - 24 = 0 \quad [5 \times -24 = -120 \Rightarrow -180 = 8 \times -15 - 7 = -15 + 8]$$

$$\Rightarrow 5x^2 - (15 - 8)x - 24 = 0$$

$$\Rightarrow 5x^2 - 15x + 8x - 24 = 0$$

$$\Rightarrow 5x(x - 3) + 8(x - 3) = 0$$

$$\Rightarrow (x - 3)(5x + 8) = 0$$

$$\Rightarrow x - 3 = 0 \text{ or } 5x + 8 = 0$$

$$\Rightarrow x = 3 \text{ or } x = \frac{-8}{5}$$

Since, x cannot be a negative value So, x = 3

∴ Speed of the stream is 3 km/hr

2.

**Sol:**

Let the usual speed be x km/hr.

Distance covered in the journey = 360 km

Now,

$$\text{Time taken by the train with the usual speed} = \frac{360 \text{ km}}{x \text{ km/hr}} = \frac{360}{x} \text{ hr}$$

Given that if speed is increased by 10 km/hr, the same train takes 3 hours less.

⇒ Speed of the train = (x + 10) km/hr and time taken by the train after increasing the speed

$$= \frac{360 \text{ km}}{(x+10) \text{ km/hr}} = \frac{360}{x+10} \text{ hr}$$

3.

**Sol:**

Let the speed of the slow train be x km/hr.

Given that speed of the slow train is 10 km/hr less than that of fast train



⇒ Speed of the fast train =  $(x + 10)$  km/hr

Total distance covered in the journey = 200 km

Time taken by fast train =  $\frac{200 \text{ km}}{(x+10)\text{km/hr}} = \frac{200}{x+10} \text{ hr}$  and

Time taken by slow train =  $\frac{200 \text{ km}}{x \text{ km/hr}} = \frac{200}{x} \text{ hr}$

Given that faster train takes 1 hour less than that of slow train

$$\text{i.e., } \frac{800}{x} - \frac{800}{x+10} = 1$$

$$\Rightarrow 800 \left( \frac{1}{x} - \frac{1}{x+10} \right) = 1$$

$$\Rightarrow 800 \left( \frac{x+10-x}{x(x+10)} \right) = 1$$

$$\Rightarrow 800 (10) = x (x + 10) = 1$$

$$\Rightarrow 8000 = x^2 + 10x$$

$$\Rightarrow x^2 + 10x - 2000 = 0$$

$$\Rightarrow x^2 + (50 - 40)x + (50x - 40) = 0$$

$$\Rightarrow (x^2 + 50x - 40x + (50x - 40)) = 0$$

$$\Rightarrow x(x + 50) - 40(x + 50) = 0$$

$$\Rightarrow (x + 50)(x - 40) = 0$$

$$\Rightarrow x + 50 = 0 \text{ or } x - 40 = 0$$

$$\Rightarrow x = -50 \text{ or } x = 40$$

Clearly  $x$  cannot be a negative volume since it is speed. So,  $x = 40$

∴ Speed of slow train is 40 km/hr

Now,

Speed of fast train =  $(x + 10)$  km/hr =  $(40 + 10)$  km/hr = 50 km/hr

4.

**Sol:**

Let the usual speed of the train be  $x$  km/hr

Distance covered in the journey = 150 km

⇒ Time taken by the train with usual speed =  $\frac{150 \text{ km}}{x \text{ km/hr}} = \frac{150}{x} \text{ hr}$

Given that, if the speed is increased by 5 km/hr from  $x$  to usual speed, the train takes one hour less for the same journey.

⇒ Speed of the train =  $(x + 5)$  km/hr

Now, time taken by the train after increasing the speed =  $\frac{150 \text{ km}}{(x+5)\text{km/hr}} = \frac{150}{x+5} \text{ hr}$

$$\text{We have, } \frac{150}{x} - \frac{150}{x+5} = 1$$

$$\Rightarrow 150 \left( \frac{1}{x} - \frac{1}{x+5} \right) = 1$$

$$\Rightarrow 150 \left( \frac{x+5-x}{x(x+5)} \right) = 1$$

$$\Rightarrow 150 (5) = x (x + 5)$$

$$\begin{aligned} \Rightarrow 750 &= x^2 + 5x \\ \Rightarrow x^2 + 5x - 750 &= 0 \\ \Rightarrow x^2 + 30x - 25x + (30 \times -25) &= 0 \\ \Rightarrow x(x + 30) - 25(x + 30) &= 0 \\ \Rightarrow (x + 30)(x - 25) &= 0 \\ \Rightarrow x = -30 \text{ or } (x - 25) &= 0 \\ \Rightarrow \text{Since, speed cannot be negative values, so } x &= 25. \\ \therefore \text{usual speed of the train} &= 25 \text{ km/hr} \end{aligned}$$

5.

**Sol:**

Let the going speed of the person be  $x$  km/hr

Given that, the return speed is 10 km/hr more than the going speed

$\Rightarrow$  Return speed of the person =  $(x + 10)$  km/hr

Total distance covered = 150 km.

Time taken for going =  $\frac{150 \text{ km}}{x \text{ km/hr}} = \frac{150}{x} \text{ hr}$

Time taken for returning =  $\frac{150 \text{ km}}{(x+10) \text{ km/hr}} = \frac{150}{(x+10)} \text{ hr}$

Given that, time taken for going is 2.5 hours more than the time for returning

i.e.  $\frac{150}{x} \text{ hr} - \frac{150}{x+10} \text{ hr} = 2.5 \text{ hr}$

$$\Rightarrow 150 \left( \frac{1}{x} - \frac{1}{x+10} \right) = \frac{25}{10}$$

$$\Rightarrow 150 \left( \frac{x+10-x}{x(x+10)} \right) = \frac{25}{10}$$

$$\Rightarrow 6(10) = \frac{x(x+10)}{10}$$

$$\Rightarrow 60 \times 10 = x^2 + 10x$$

$$\Rightarrow x^2 + 10x - 600 = 0$$

$$\Rightarrow x^2 + (30 - 20)x + (30 \times -20) = 0$$

$$\Rightarrow x^2 = 30x - 20x + (30 \times -20) = 0$$

$$\Rightarrow x(x + 30) - 20(x + 30) = 0$$

$$\Rightarrow (x + 30)(x - 20) = 0$$

$$\Rightarrow x + 30 = 0 \text{ or } x - 20 = 0$$

$$\Rightarrow x = -30 \text{ or } x = 20$$

Since, speed cannot be negative. So  $x = 20$

$\therefore$  speed of the person when going = 20 km/hr

Now, speed of the person when returning =  $(x + 10)$  km/hr

=  $(20 + 10)$  km/hr

= 30 km/hr

6.

**Sol:**

Let the usual speed of the plane be  $x$  km/hr

Total distance travelled = 1600 km

$$\Rightarrow \text{Time taken by the plane with usual speed} = \frac{1600 \text{ km}}{x \text{ km/hr}} = \frac{1600}{x} \text{ hr}$$

Given that, if speed is increased by 400 km/hr, the plane takes 40 minutes less than that of the usual time. Speed of the plane after increasing =  $(x + 400)$  km/hr

$$\Rightarrow \text{Time taken by the plane with increasing speed} = \frac{1600 \text{ km}}{(x+400) \frac{\text{km}}{\text{hr}}} = \frac{1600}{x+400} \text{ hr}$$

Now,

$$\frac{1600}{x} \text{ hr} - \frac{1600}{x+400} \text{ hr} = \frac{40}{60} \text{ hr} \left[ \because 40 \text{ minutes} = \frac{40}{60} \text{ hr as } 1 \text{ hr} = 60 \text{ min} \right]$$

$$\Rightarrow 1600 \left[ \frac{1}{x} - \frac{1}{x+400} \right] = \frac{40}{60}$$

$$\Rightarrow 1600 \left[ \frac{x+400-x}{x(x+400)} \right] = \frac{40}{60}$$

$$\Rightarrow 40(400 \times 60) = x(x + 400)$$

$$\Rightarrow x^2 + 400x - 960000 = 0$$

$$\Rightarrow x^2 + (1800 - 800)x + (1800 \times (-800)) = 0$$

$$\Rightarrow (x^2) + 1800x - 800x + (1800 \times -800) = 0$$

$$\Rightarrow x(x + 1800) - 800(x + 1800) = 0$$

$$\Rightarrow (x + 1800)(x - 800) = 0$$

$$\Rightarrow x = -1800 \text{ or } x - 800 = 0$$

$$\Rightarrow x = -1800 \text{ or } x = 800$$

Since, speed cannot be negative. So,  $x = 800$

$\therefore$  Usual speed of the plane is 800 km/hr.

7.

**Sol:**

Let the usual speed of the plane be  $x$  km/hr.

Distance covered in the journey = 1800 km

$$\Rightarrow \text{Time taken by the plane with usual speed} = \frac{1800 \text{ km}}{x \frac{\text{km}}{\text{hr}}} = \frac{1800}{x} \text{ hr}$$

Now, speed is increased by 100 km/hr and the time taken is 1 hour less for the same journey.

$\Rightarrow$  Speed of the plane after increased =  $(x + 100)$  km/hr and Time taken by plane with

$$\text{increased speed} = \frac{1800 \text{ km}}{(x+100) \frac{\text{km}}{\text{hr}}} = \frac{1800}{x+100} \text{ hr}$$

Now, we have

$$\frac{1800}{x} - \frac{1800}{x+100} = 1$$

$$\Rightarrow 1800 \left( \frac{1}{x} - \frac{1}{x+100} \right) = 1$$

$$\Rightarrow 1800 \left( \frac{x+100-x}{x(x+100)} \right) = 1$$

$$\Rightarrow 1200(100) = x(x + 100)$$

$$\Rightarrow 120000 = x^2 + 100x$$

$$\Rightarrow x^2 + 100x - 120000 = 0$$

$$\Rightarrow x^2 + (400 - 300)x + (400 \times -300) = 0$$

$$\Rightarrow x^2 + 400x - 300x + (400 \times -300) = 0$$

$$\Rightarrow x(x + 400) - 300(x + 400) = 0$$

$$\Rightarrow (x + 400)(x - 300) = 0$$

$$\Rightarrow x + 400 = 0 \text{ or } x - 300 = 0$$

$$\Rightarrow x = -400 \text{ or } x = 300$$

Since, speed cannot be negative so,  $x = 300 \therefore$  usual speed of the plane = 300 km/hr

8.

**Sol:**

Let the usual speed of the train be  $x$  km/hr

Distance covered in the journey = 300 km

$$\text{Time taken by the train with usual speed} = \frac{300 \text{ km}}{x \text{ km/hr}} = \frac{300}{x} \text{ hr}$$

Now,

If the speed is increased by 5 km/hr, the train takes 2 hours less for the same journey.

$\Rightarrow$  speed of the train after increasing =  $(x + 5)$  km/hr

$$\text{And time taken by the train after increasing the speed} = \frac{300 \text{ km}}{(x+5) \text{ km/hr}} = \frac{300}{x+5} \text{ hr}$$

We have,

$$\frac{300}{x} \text{ hr} - \frac{300}{x+5} \text{ hr} = 2 \text{ hrs}$$

$$\Rightarrow 300 \left( \frac{1}{x} - \frac{1}{x+5} \right) = 2$$

$$\Rightarrow 300 \left( \frac{x+5-x}{x(x+5)} \right) = 2$$

$$\Rightarrow 300(5) = 2(x(x + 5))$$

$$\Rightarrow 750 = x^2 + 5x$$

$$\Rightarrow x^2 + 5x - 750 = 0$$

$$\Rightarrow x^2 + 30x - 25x + (30 \times -25) = 0$$

$$\Rightarrow (x + 30)(x - 25) = 0$$

$$\Rightarrow x + 30 = 0 \text{ or } x - 25 = 0$$

$$\Rightarrow x = -30 \text{ or } x = 25$$

Since, speed cannot be negative. So  $x = 25$

$\therefore$  The usual speed of the train = 25 km/hr.

9.

**Sol:**

Let the original speed of the train be  $x$  km/hr

Distance covered = 90 km.

$$\Rightarrow \text{Time taken by the train with original speed} = \frac{90 \text{ km}}{x \text{ km/hr}} = \frac{90}{x} \text{ hr}$$

Now, if the speed of the train is increased by 15 km/hr, the train takes 30 minutes less for the same journey

$$\Rightarrow \text{Speed of the train after increasing} = (x + 15) \text{ km/hr and the time taken by the train after increasing the speed} = \frac{90 \text{ km}}{(x+15) \text{ km/hr}} = \frac{90}{x+15} \text{ hr}$$

Now,

$$\frac{90}{x} \text{ hr} - \frac{90}{x+15} \text{ hr} = 30 \text{ min}$$

$$\Rightarrow 90 \text{ hr} \left( \frac{1}{x} - \frac{1}{x+15} \right) = 30 \text{ min}$$

$$\Rightarrow 90 \left( \frac{1}{x} - \frac{1}{x+15} \right) = \frac{30}{60} \text{ hr} \quad [\because 1 \text{ hr} = 60 \text{ min}]$$

$$\Rightarrow 90 \left( \frac{x+15-x}{x(x+15)} \right) = \frac{1}{2}$$

$$\Rightarrow 90 \times 15 \times 2 = x(x + 15)$$

$$\Rightarrow 2700 = x^2 + 15x$$

$$\Rightarrow x^2 + 15x - 2700 = 0$$

$$\Rightarrow x^2 + (60 - 45)x + (60 \times (-45)) = 0$$

$$\Rightarrow x^2 + 60x - 45x + (60 \times -45) = 0$$

$$\Rightarrow x(x + 60) - 45(x + 60) = 0$$

$$\Rightarrow (x + 60)(x - 45) = 0$$

$$\Rightarrow x + 60 = 0 \text{ or } x - 45 = 0$$

$$\Rightarrow x = -60 \text{ or } x = 45$$

Since, speed cannot be negative. So,  $x = 45$

$\therefore$  Original speed of the train = 45 km/hr

10.

11.

**Sol:**

Let the speed of the train be  $x$  km/hr

Distance covered by the train = 360 km

$$\Rightarrow \text{Time taken by the train with initial speed} = \frac{360 \text{ km}}{x \text{ km/hr}} = \frac{360}{x} \text{ hr}$$

Now, if the speed is 5 km/hr more, the train takes 1 hour less for the same journey.

$$\Rightarrow \text{Speed of the train after increasing the speed} = (x + 5) \text{ km/hr}$$

$$\text{And time taken by the train with increased speed} = \frac{360 \text{ km}}{\frac{(x+5) \text{ km}}{\text{hr}}} = \frac{360}{x+5} \text{ hr}$$

Now,

$$\begin{aligned} \frac{360}{x} \text{ hr} - \frac{360}{x+5} \text{ hr} &= 1 \text{ hr} \\ \Rightarrow 360 \left( \frac{1}{x} - \frac{1}{x+5} \right) &= 1 \\ \Rightarrow 360 \left( \frac{x+5-x}{x(x+5)} \right) &= 1 \\ \Rightarrow 360 \times 5 &= 1 \times x(x+5) \\ \Rightarrow x^2 + 5x - 1800 &= 0 \\ \Rightarrow x^2 + 45x - 40x + (x+45) &= 0 \\ \Rightarrow x(x+45) - 40(x+45) &= 0 \\ \Rightarrow (x+45)(x-40) &= 0 \\ \Rightarrow x+45 = 0 \text{ or } x-40 &= 0 \\ \Rightarrow x = -45 \text{ or } x = 40 \end{aligned}$$

Since, speed is always a positive value i.e.  $x = 0 \Rightarrow x = 40$

$\therefore$  The speed of the train = 40 km/hr

12.

**Sol:**

Let the speed of the passenger train be  $x$  km/hr

Given that the average speed of the express train is 11 km/hr more than that of passenger train.

$\Rightarrow$  Average speed of express train =  $(x + 11)$  km/hr

Now,

Time taken by the passenger train =  $\frac{138 \text{ km}}{x \text{ km/hr}} = \frac{138}{x} \text{ hr}$

And time taken by the express train =  $\frac{132 \text{ km}}{(x+11) \text{ km/hr}} = \frac{132}{x+11} \text{ hr}$

Given that, express train takes 1 hour less than that of passenger train to reach the destiny.

$$\Rightarrow \frac{138}{x} \text{ hr} - \frac{132}{x+11} \text{ hr} = 1 \text{ hr}$$

$$\Rightarrow 138 \left( \frac{1}{x} - \frac{1}{x+11} \right) = 11$$

$$\Rightarrow 138 \left( \frac{x+11-x}{x(x+11)} \right) = 11$$

$$\Rightarrow 138 \times 11 = x(x+11) \times 1$$

$$\Rightarrow x^2 - 11x - 1452 = 0$$

$$\Rightarrow x^2 + (44x - 33x) + (44 \times -33) = 0$$

$$\Rightarrow x^2 + 44x - 33x + (44 \times -33) = 0$$

$$\Rightarrow x(x+4) - 33(x+44) = 0$$

$$\Rightarrow (x+44)(x-33) = 0$$

$$\Rightarrow x+44 = 0 \text{ or } x-33 = 0$$

$$\Rightarrow x = -44 \text{ or } x = 33$$

Since, speed cannot be in negative values. So,  $x = 33$

$\therefore$  Average speed of the slower train i.e. passenger train = 33 km/hr

And average speed of express train =  $(x + 11)$  km/hr =  $(33 + 11)$  km/hr = 44 km/hr.

13.

**Sol:**

Let the usual speed of the plane be  $x$  km/hr

Distance covered by the plane = 1250 km

$\Rightarrow$  Time taken by the plane with usual speed =  $\frac{1250 \text{ km}}{x \text{ km/hr}} = 1250 \text{ hr}$

To cover the delay of 50 minutes, the speed of the plane is increased by 250 km/hr

Now,

Speed of the plane after increasing =  $(x + 250)$  km/hr and

Time taken by the plane with increased speed =  $\frac{1250 \text{ km}}{(x+250) \text{ km/hr}} = \frac{1250}{x+250} \text{ hr}$

From the data we have,

$$\frac{1250}{x} \text{ hr} - \frac{1250}{x+250} \text{ hr} = 50 \text{ min}$$

$$\Rightarrow 1250 \text{ hr} \left( \frac{1}{x} - \frac{1}{x+250} \right) = \frac{50}{60} \text{ hr} [\because 1 \text{ hr} = 60 \text{ min}]$$

$$\Rightarrow 250 \left( \frac{x+250-x}{x(x+250)} \right) = \frac{1}{6}$$

$$\Rightarrow 250 \times 250 \times 6 = x(x + 250) \times 1$$

$$\Rightarrow 375000 = x^2 + 250x$$

$$\Rightarrow x^2 + 250x - 375000 = 0$$

$$\Rightarrow x^2 + (750 - 500)x + (750 \times -500) = 0$$

$$\Rightarrow x^2 + 750x - 500x + (750 \times -500) = 0$$

$$\Rightarrow (x + 750)(x - 500) = 0$$

$$\Rightarrow (x + 750) = 0 \text{ or } x = 500 = 0$$

$$\Rightarrow x = -750 \text{ or } x = 500$$

Since, speed cannot be a negative value. So,  $x = 500$

$\therefore$  the usual speed of the plane = 500 km/hr.

### Exercise 8.9

1.

**Sol:**

Given that, Ashu is  $x$  years old while his mother Mrs. Veena is  $x^2$  years old.

$\Rightarrow$  Ashu's present age =  $x$  years and Mrs. Veena's present age =  $x^2$  years

And also given that, after 5 years Mrs. Veena will be three times old as Ashu.

$\Rightarrow$  Ashu's age after 5 years =  $(x + 5)$  years

And Mrs. Veena's age after 5 years =  $(x^2 + 5)$  years

But given that,

$$\Rightarrow (x^2 + 5) = 3(x + 5)$$

$$\Rightarrow x^2 + 5 = 3x + 15$$

$$\Rightarrow x^2 - 3x - 10 = 0$$

$$\Rightarrow x^2 - 5x + 2x - 10 = 0$$

$$\Rightarrow x(x - 5) + 2(x - 5) = 0 \Rightarrow (x - 5)(x + 2) = 0$$

$$\Rightarrow x - 5 = 0 \text{ or } x + 2 = 0$$

$$\Rightarrow x = 5 \text{ or } x = -2$$

Since, age cannot be in negative values. So,  $x = 5$  years.

$\therefore$  Present age of Ashu is  $x = 5$  years and

Present age of Mrs. Veena is  $x^2 \Rightarrow 5^2 \text{ years} \Rightarrow 25 \text{ years}$ .

2.

**Sol:**

Let the present age of the son be  $x$  years

Given that,

sum of present ages of man and his son is 45 years.

$$\Rightarrow \text{Man's present age} = (45 - x) \text{ years}$$

And also given that,

five years ago, the product of their ages was four times the man's age at the time.

$$\Rightarrow \text{Man's age before 5 years} = (45 - x - 5) \text{ years} = (40 - x) \text{ years}$$

$$\text{And son's age before 5 years} = (x - 5) \text{ years}$$

$$\text{But, given that } (40 - x)(x - 5) = 4(40 - x)$$

$$\Rightarrow x - 5 = 4$$

$$\Rightarrow x = 9 \text{ years}$$

$$\Rightarrow \text{Son's present age} \Rightarrow x = 9 \text{ years}$$

$$\text{Now, Man's present age} \Rightarrow (45 - x) \text{ years} = (45 - 9) \text{ years} = 36 \text{ years}$$

$\therefore$  The present ages of man and son are 36 years and 9 years respectively.

3.

**Sol:**

Let the present age of shika be  $x$  years.

Given that,

The product of her age five years ago and her age 8 years later is 30

Now,

$$\text{Shika's age five years ago} = (x - 5) \text{ years}$$

$$\text{And Shika's age 8 years later} = (x + 8) \text{ years}$$

Given that.

$$(x - 5)(x + 8) = 30$$



$$\begin{aligned} \Rightarrow x^2 + 8x - 5x - 40 &= 30 \\ \Rightarrow x^2 + 3x - 70 &= 0 \\ \Rightarrow x^2 + 10x - 7x - 70 &= 0 \\ \Rightarrow x(x + 10) - 7(x + 10) &= 0 \\ \Rightarrow (x + 10)(x - 7) &= 0 \\ \Rightarrow x + 10 = 0 \text{ or } x - 7 &= 0 \\ \Rightarrow x = -10 \text{ or } x = 7 \end{aligned}$$

Since, age cannot be in negative values, So  $x = 7$  years

$\therefore$  The present age of shika is 7 years.

4.

**Sol:**

Let the present age of Ramu be a  $x$  years

Given that,

The product of his age five years ago and his age  $y$  nine years later is 15.

Now, Ramu's age five years ago =  $(x - 5)$  years

And Ramu's age nine years later =  $(x + 9)$  years

Given that,

$$\begin{aligned} (x - 5)(x + 9) &= 15 \\ \Rightarrow x^2 + 9x - 5x - 45 &= 15 \\ \Rightarrow x^2 + 4x - 60 &= 0 \\ \Rightarrow x^2 + 10x - 6x - 60 &= 0 \\ \Rightarrow x(x + 10) - 6(x + 10) &= 0 \\ \Rightarrow (x + 10)(x - 6) &= 0 \\ \Rightarrow x + 10 = 0 \text{ or } x - 6 &= 0 \\ \Rightarrow x = -10 \text{ or } x = 6 \end{aligned}$$

Since, age cannot be in negative values, So  $x = 6$  years

$\therefore$  The present age of shika is 6 years.

5.

**Sol:**

Let the present age of friend 1 be a  $x$  years

Given that,

Sum of the ages of two friends = 20 years

$\Rightarrow$  Present age of friend 2 =  $(20 - x)$  years

And also given that, four years ago, the product of their age was 48.

$\Rightarrow$  Age of friend 1 before 4 years =  $(x - 4)$  years

And age of friend 2 before 4 years =  $(20 - x - 4)$  years =  $(16 - x)$  years

Given that,

$$(x - 4)(16 - x) = 48$$

$$\Rightarrow 16x - x^2 - 64 + 4x = 48$$

$$\Rightarrow x^2 - 20x + 112 = 0$$

Let D be the discriminant of this quadratic equation.

$$\text{Then, } D = (-20)^2 - 4 \times 112 \times 1 = 400 - 448 = -48 < 0$$

We know that, to have real roots for a quadratic equation that discriminant D must be greater than or equal to 0 i.e.  $D \geq 0$

But  $D < 0$  in the above. So, above equation does not have real roots

Hence, the given situation is not possible.

6.

**Sol:**

Let the age of girls sister be a x years

Given that,

Girl is twice as old as her sister

$$\Rightarrow \text{Girls age} = 2 \times x \text{ years} = 2x \text{ years}$$

Given that, after 4 years, the product of their ages will be 160.

$$\Rightarrow \text{Girls age after 4 years} = (2x + 4) \text{ years}$$

$$\text{And sisters age after 4 years} = (x + 4) \text{ years}$$

Given that,

$$(2x + 4)(x + 4) = 160$$

$$\Rightarrow 2x^2 + 8x + 4x + 16 = 160$$

$$\Rightarrow 2x^2 + 12x - 144 = 0$$

$$\Rightarrow 2(x^2 + 6x - 72) = 0$$

$$\Rightarrow x^2 + 6x - 72 = 0$$

$$\Rightarrow x^2 + 12x - 6x - 72 = 0$$

$$\Rightarrow x(x + 12) - 6(x + 12) = 0$$

$$\Rightarrow (x + 12)(x - 6) = 0$$

$$\Rightarrow x + 12 = 0 \text{ or } x - 6 = 0$$

$$\Rightarrow x = -12 \text{ or } x = 6$$

Since, age cannot be in a negative value.

So,  $x = 6$ .

$\therefore$  Age of girls sister is  $x = 6$  years.

And age of girl is  $2x = 2 \times 6$  years = 12 years

Hence, the present ages of girl and her sister are 12 years and 6 years respectively.

7.

**Sol:**

Let the present age of Rehman be x years.

Now,

Rehman's age 3 years ago =  $(x - 3)$  years

And Rehman's age 5 years later =  $(x + 5)$  years

Given that,

The sum of reciprocals of Rehman's ages 3 years ago and 5 years later is  $\frac{1}{3}$

$$\Rightarrow \frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$$

$$\Rightarrow (2x + 2) \times 3 = 1(x - 3)(x + 5)$$

$$\Rightarrow 6x + 6 = x^2 + 5x - 3x - 15$$

$$\Rightarrow x^2 + 2x - 6x - 15 - 6 = 0$$

$$\Rightarrow x^2 - 4x - 21 = 0$$

$$\Rightarrow x^2 - 7x + 3x - 21 = 0$$

$$\Rightarrow x(x - 7) + 3(x - 7) = 0$$

$$\Rightarrow (x - 7)(x + 3) = 0 \Rightarrow x - 7 = 0 \text{ or } x + 3 = 0$$

$$\Rightarrow x = 7 \text{ or } x = -3$$

Since, age cannot be in negative values. So,  $x = 7$  years Hence, the present age of Rehman is 7 years.

### Exercise 8.10

1.

**Sol:**

Let the length of the shortest side be  $x$  cm

Given that the length of the largest side is 5cm more than that of smaller side

$\Rightarrow$  longest side =  $(x + 5)$  cm

And also, given that

Hypotenuse = 25cm

So, let us consider a right angled triangle ABC right angled at B

We have, hypotenuse (AC) = 25 cm

BC =  $x$  cm and AB =  $(x + 5)$  cm

Since, ABC is a right angled triangle

We have,  $(BC)^2 + (AB)^2 = (AC)^2$

$$\Rightarrow x^2 \text{ cm}^2 + (x + 5)^2 \text{ cm}^2 = (25)^2 \text{ cm}^2$$

$$\Rightarrow x^2 + x^2 + 10x + 25 = 625$$

$$\Rightarrow 2x^2 + 10x - 600 = 0$$

$$\Rightarrow 2(x^2 + 5x - 300) = 0$$

$$\Rightarrow x^2 + 5x - 300 = 0$$

$$\Rightarrow x^2 + 20x - 15x + (20x - 15) = 0$$

$$\Rightarrow x(x + 20) - 15(x + 20) = 0$$

$$\Rightarrow (x + 20)(x - 15) = 0$$

$$\Rightarrow (x + 20) = 0 \text{ or } (x - 15) = 20$$

2.

**Sol:**

Using Pythagoras theorem,

$$(AB)^2 + (BC)^2 = (AC)^2$$

$$\Rightarrow (2y)^2 cm^2 + (3x)^2 cm^2 = (9\sqrt{5})^2 cm^2$$

$$\Rightarrow 4y^2 + 9x^2 = 81 \times 5$$

$$\Rightarrow 4y^2 + 9x^2 = 405$$

$$\Rightarrow 4(90 - x^2) + 9x^2 = 405 \quad [\because x^2 + y^2 = 90]$$

$$\Rightarrow 4 \times 90 - 4x^2 + 9x^2 = 405$$

$$\Rightarrow 5x^2 = 405 - 360$$

$$\Rightarrow 5x^2 = 405 - 360$$

$$\Rightarrow 5x^2 = 45$$

$$\Rightarrow x^2 = 9$$

$$\Rightarrow x = \sqrt{3^2} \Rightarrow x = \pm 3$$

Since,  $x$  cannot be a negative value. So  $x = 3\text{cm}$

We have,

$$x^2 + y^2 = 90$$

$$\Rightarrow y^2 = 90 - (3)^2$$

$$\Rightarrow y^2 = 90 - 9$$

$$\Rightarrow y^2 = 81 \Rightarrow y = \sqrt{81} \Rightarrow y = \pm 9$$

Since,  $y$  cannot be a negative value. So,  $y = 9\text{cm}$

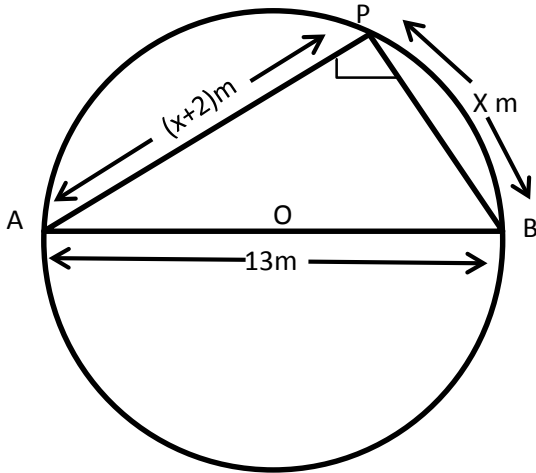
$\therefore$  hence, the length of the smaller side is 3 cm and the length of the longer side is 9cm.

3.

**Sol:**

Yes, it is possible to do so as in the given condition

This can be proved as below,



Let P be the required location of the pole such that its distance from gate B is x meter i.e.  $BP = x$  meters and also  $AP - BP = 7\text{m}$

$$\Rightarrow AP = BP + 7\text{m} = (x + 7)\text{m}$$

Since, AB is a diameter and P is a point on the boundary of the semi-circle,  $\Delta APB$  is right angled triangle, right angled at P.

Using Pythagoras theorem,

$$(AB)^2 = (AP)^2 + (BP)^2$$

$$\Rightarrow (13)^2 m^2 = (x + 7)^2 m^2 + (x)^2 m^2$$

$$\Rightarrow 169 = x^2 + 14x + 49 + x^2$$

$$\Rightarrow 2x^2 + 14x + 49 - 169 = 0$$

$$\Rightarrow 2x^2 + 14x - 120 = 0$$

$$\Rightarrow 2(x^2 + 7x - 60) = 0$$

$$\Rightarrow x^2 + 7x - 60 = 0$$

$$\Rightarrow x^2 + 12x - 5x - (12 \times -5) = 0$$

$$\Rightarrow x(x + 12) - 5(x + 12) = 0$$

$$\Rightarrow (x + 12)(x - 5) = 0$$

$$\Rightarrow x + 12 = 0 \text{ or } x - 5 = 0$$

$$\Rightarrow x = -12 \text{ or } x = 5$$

Since, x cannot be a negative value, So  $x = 5$

$$\Rightarrow BP = 5\text{m}$$

$$\text{Now, } AP = (BP + 7)\text{m} = (5 + 7)\text{m} = 12\text{ m}$$

$\therefore$  The pole has to be erected at a distance 5 mtrs from the gate B and 12 m from the gate A.

4.

**Sol:**

120 m, 90 m

### Exercise 8.11

1.

**Sol:**

Let the breadth of the rectangle be  $x$  meters

Given that,

Perimeter = 82 m and Area =  $400\text{m}^2$

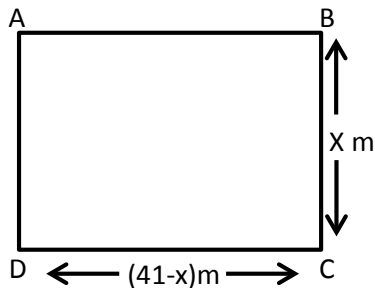
We know that

Perimeter of a rectangle =  $2(\text{length} + \text{breadth})$

$$\Rightarrow 82 = 2(\text{length} + x)$$

$$\Rightarrow 41 = \text{length} + x$$

$$\Rightarrow \text{length} = (41 - x)\text{m}$$



We have

Area of rectangle = length  $\times$  breadth

$$\Rightarrow 400\text{ m}^2 = (41 - x)\text{ m} \times x\text{ m}$$

$$\Rightarrow 400 = 41x - x^2$$

$$\Rightarrow 400x^2 - 41x + 400 = 0$$

$$\Rightarrow x^2 - 25x - 16x + (-25 \times -16) = 0$$

$$\Rightarrow x(x - 25) - 16(x - 25) = 0$$

$$\Rightarrow (x - 25)(x - 16) = 0$$

$$\Rightarrow (x - 25)(x - 16) = 0$$

$$\Rightarrow x - 25 = 0 \text{ or } x - 16 = 0$$

$$\Rightarrow x = 25 \text{ or } x = 16$$

Hence, breadth of the rectangle is 25m or 16m

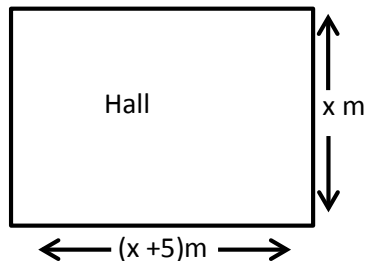
2.

**Sol:**

Let the breadth of the rectangle (hall) be  $x$  meter.

Given that,

Length of the hall is 5m more than its breadth i.e. length =  $(x + 5)\text{m}$



And also given that,

$$\text{Area of the hall} = 84m^2$$

Since, hall is in the shape of a rectangle,

Area of the rectangular hall = length  $\times$  breadth

$$\Rightarrow 84m^2 = xm \times (x + 5)m$$

$$\Rightarrow 84 = x(x + 5)$$

$$\Rightarrow 84 = x^2 + 5x$$

$$\Rightarrow x^2 + 5x - 84 = 0$$

$$\Rightarrow x^2 + 12x - 7x - 84 = 0$$

$$\Rightarrow x(x + 12) - 7(x + 12) = 0$$

$$\Rightarrow (x - 7)(x + 12) = 0$$

$$\Rightarrow x = 7m \text{ or } x = -12m$$

Since,  $x$  cannot be negative. So, breadth of the hall =  $7m$

Hence, length of the hall =  $(x + 5)m = (7 + 5)m = 12m$ .

3.

**Sol:**

Let  $S_1$  and  $S_2$  be two squares.

Let  $x$  cm be the side of square  $S_1$  and  $(x + 4)$  cm be the side of square  $S_2$ .

We know that,

$$\text{Area of a square} = (\text{Side})^2$$

$$\Rightarrow \text{Area of square } S_1 = (x)^2 = x^2 cm^2$$

$$\Rightarrow \text{Area of square } S_2 = (x + 4)^2 cm^2$$

Given that,

$$\text{Area of square } S_1 + \text{Area of square } S_2 = 656 cm^2$$

$$\Rightarrow x^2 cm^2 + (x + 4)cm^2 = 656 cm^2$$

$$\Rightarrow x^2 + x^2 + 8x + 16 = 656$$

$$\Rightarrow 2x^2 + 8x + 16 - 656 = 0$$

$$\Rightarrow 2x^2 + 8x - 640 = 0$$

$$\Rightarrow 2(x^2 + 4x - 320) = 0$$

$$\Rightarrow x^2 + 4x - 320 = 0$$

$$\Rightarrow x^2 + 20x - 16x + (20x - 16) = 0$$

$$\Rightarrow x + 20 = 0 \text{ or } x - 16 = 0$$

$$\Rightarrow x = -20 \text{ cm or } x = 16 \text{ cm}$$

Since,  $x$  cannot be negative. So,  $x = 16 \text{ cm}$

$\therefore$  Side of square  $S_1 \Rightarrow x = 16 \text{ cm}$  and

Side of square  $S_2 \Rightarrow (x + 4) = (16 + 4) \text{ cm} = 20 \text{ cm}$

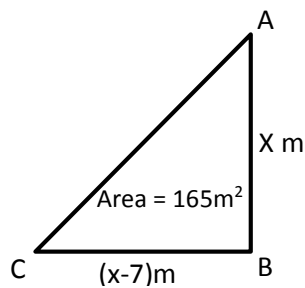
4.

**Sol:**

Let the altitude of the right angled triangle be denoted by  $x$  meter

Given that altitude exceeds the base of the triangle by  $7 \text{ m}$ .

$$\Rightarrow \text{Base} = (x - 7) \text{ m}$$



We know that,

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\Rightarrow 165 \text{ m}^2 = \frac{1}{2} \times (x - 7) \text{ m} \times x \text{ m} \quad [\because \text{Area} = 165 \text{ m}^2 \text{ given}]$$

$$\Rightarrow 2 \times 165 = x(x - 7)$$

$$\Rightarrow x^2 - 7x = 330$$

$$\Rightarrow x^2 - 7x - 330 = 0$$

$$\Rightarrow (x - 22) + 15(x - 22) = 0$$

$$\Rightarrow (x - 22)(x + 15) = 0$$

$$\Rightarrow x = 22 \text{ or } x = -15$$

Since,  $x$  cannot be negative. So,  $x = 22 \text{ m}$

$\therefore$  Altitude of the triangle  $\Rightarrow x = 22 \text{ m}$

And base of the triangle  $\Rightarrow (x - 7) \text{ m} = (22 - 7) \text{ m} = 15 \text{ m}$

5.

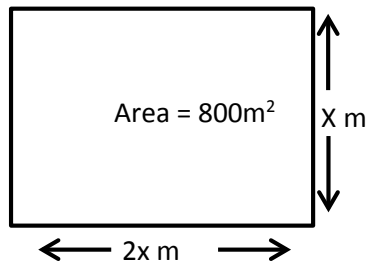
**Sol:**

Let the breadth of the rectangular mango grove be  $x$  meter.

Given that length is twice that of breadth  $\Rightarrow \text{length} = 2 \times x \text{ m} = 2x \text{ m}$

Given that area of the grove is  $800 \text{ m}^2$ .





But we know that

Area of a rectangle = length  $\times$  breadth

$$\Rightarrow 800\text{m}^2 = 2x \text{ m} \times x \text{ m}$$

$$\Rightarrow 2x^2 = 800$$

$$\Rightarrow x^2 = 400$$

$$\Rightarrow x = \sqrt{400} = \sqrt{(20)^2} = \pm 20$$

$$\Rightarrow x = 20 \text{ or } x = -20$$

Since,  $x$  cannot be a negative value.

So,  $x = 20 \text{ m}$

$\therefore$  Breadth of the mango grove = 20m and length of the mango grove

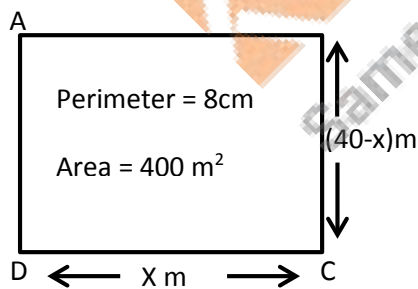
$$= 2x \text{ m} = 2 \times 20 \text{ m} = 40\text{m}$$

Yes. It is possible to design a rectangular mango grove whose length is twice its breadth and the area is 800m<sup>2</sup>.

6.

**Sol:**

To prove the given condition, let us assume that the length of the rectangular park be denoted by  $x \text{ m}$ .



Given that

$$\text{Perimeter} = 80\text{m} \text{ and } \text{Area} = 400\text{m}^2$$

We know that,

$$\text{Perimeter of a rectangle} = 2(\text{length} + \text{breadth})$$

$$\Rightarrow 80\text{m} = 2(x + \text{breadth})$$

$$\Rightarrow \text{breadth} = \left(\frac{80}{2} - x\right) \text{ m}$$

$$\Rightarrow \text{breadth} = (40 - x)m$$

And also,

Area of a rectangle = length  $\times$  breadth

$$\Rightarrow 400m^2 = x m \times (40 - x)m$$

$$\Rightarrow 400 = x(40 - x)$$

$$\Rightarrow 400 = 40x - x^2$$

$$\Rightarrow x^2 - 40x + 400 = 0$$

$$\Rightarrow x^2 - 2 \times 20 \times x + (20)^2 = 0$$

$$\Rightarrow (x - 20)^2 = 0$$

$$\Rightarrow (x - 20 = 0) \Rightarrow x = 20$$

$\therefore$  length of the rectangular park  $\Rightarrow x = 20$  m and breadth of the rectangular park  $\Rightarrow (40 - x)m = (40 - 20)m = 20m$

Yes. It is possible to design a rectangular park of perimeter 80m and area 400m<sup>2</sup>.

7.

**Sol:**

Let the two squares be denoted as  $S_1$  and  $S_2$  and let side of squares  $S_1$  be denoted as  $x$  meter and that of square  $S_2$  be  $y$  m.

Given that,

Difference of their perimeter is 64m.

We know that

Perimeter of a square =  $4 \times$  side

$$\Rightarrow \text{Perimeter of square } S_1 = 4 \times x m = 4x m$$

$$\Rightarrow \text{Perimeter of square } S_2 = 4 \times y m = 4y m$$

Now, difference of perimeter = perimeter of square  $S_1$  - Perimeter of square  $S_2$

$$\Rightarrow 64 m = (4x - 4y)m$$

$$\Rightarrow 64 = 4(x - y)$$

$$\Rightarrow x - y = 16$$

$$\Rightarrow x = y + 16$$

And also,

Given that sum of areas of two squares = 640 m<sup>2</sup>.

We know that,

Area of a square = (Side)<sup>2</sup>

$$\Rightarrow \text{Area of square } S_1 = x^2 m^2$$

$$\Rightarrow \text{Area of square } S_2 = y^2 m^2$$

Now,

Sum of areas of two squares = Area of square  $S_1$  + Area of square  $S_2$

$$\Rightarrow 640m^2 = x^2 m^2 + y^2 m^2$$

$$\Rightarrow 640 = (y + 16)^2 + y^2 \quad [\because x = y + 16]$$

$$\Rightarrow y^2 + 32y + 256 + y^2 = 640$$

$$\begin{aligned}
&\Rightarrow 2y^2 + 32y + 256 - 640 = 0 \\
&\Rightarrow 2y^2 + 32y - 384 = 0 \\
&\Rightarrow 2(y^2 + 16y - 192) = 0 \\
&\Rightarrow y^2 + 16y - 192 = 0 \\
&\Rightarrow y^2 + 24y - 8y + (24 \times -8) = 0 \\
&\Rightarrow y(y + 24) - 8(y + 24) = 0 \\
&\Rightarrow (y + 24)(y - 8) = 0 \\
&\Rightarrow y + 24 = 0 \text{ or } y - 8 = 0 \\
&\Rightarrow y = -24 \text{ or } y = 8
\end{aligned}$$

Since,  $y$  cannot be a negative value. So,  $y = 8\text{m}$

$\therefore$  Side of the square  $S_2$  is  $y = 8\text{m}$

And side of the square  $S_1$  is  $x = (y + 16)\text{m} = (8 + 16)\text{m} = 24\text{m}$

Hence, sides of the two squares is  $24\text{m}$  and  $8\text{m}$ .

### Exercise 8.12

1.

**Sol:**

Let B takes  $x$  days to complete the piece of work.

$$\Rightarrow \text{B's one days work} = \frac{1}{x}$$

Now, A takes 10 days less than that of B to finish the same piece of work i.e.  $(x - 10)$  days

$$\Rightarrow \text{A's one days work} = \frac{1}{x-10}$$

Given that, both A and B together can finish the same work in 12 days.

$$\Rightarrow \text{(A and B)'s one days work} = \frac{1}{12}$$

Now,

$$\text{(A's one days work)} + \text{(B's one days work)} = \frac{1}{x} + \frac{1}{x-10} \text{ and } \text{(A + B)'s one days work} = \frac{1}{12}$$

$$\Rightarrow \frac{1}{x} + \frac{1}{x-10} = \frac{1}{12}$$

$$\Rightarrow \frac{x-10+x}{x(x-10)} = \frac{1}{12}$$

$$\Rightarrow (2x - 10) \times 12 = x(x - 10)$$

$$\Rightarrow 24x - 120 = x^2 - 10x$$

$$\Rightarrow x^2 - 10x - 24x + 120 = 0$$

$$\Rightarrow x^2 - 34x + 120 = 0$$

$$= x^2 - 30x - 4x + (-30 \times -4) = 0$$

$$\Rightarrow x(x - 30) - 4(x - 30) = 0$$

$$\Rightarrow (x - 30)(x - 4) = 0$$

$$\Rightarrow (x - 30) = 0 \text{ or } (x - 4) = 0$$

$$\Rightarrow x = 30 \text{ or } x = 4$$

We can observe that, the value of  $x$  cannot be less than 10.

∴ The time taken by B to finish the work is 30 days.

2.

**Sol:**

Let  $x$  be no. of students planned for a picnic

Given that budget for food was Rs 480

$$\Rightarrow \text{Share of each student} = \frac{\text{Total budget}}{\text{No. of students}} = \text{Rs } \frac{480}{x}$$

Given that 8 students failed to go

⇒ No. of students will be  $(x - 8)$

Now,

Share of each student will be equal to

$$= \frac{\text{total budget}}{\text{No. of students}} = \text{Rs. } \frac{480}{x-8}$$

Given that if 8 students failed to go, then cost of food for each member increased by Rs. 10.

3.

**Sol:**

Let the time taken by the top of smaller diameter to fill the tank be  $x$  hours

$$\Rightarrow \text{Portion of tank filled by smaller pipe in one hour} = \frac{1}{x}$$

Now, larger diameter pipe takes 10 hours less than that of smaller diameter pipe  
i.e.  $(x - 10)$  hours

$$\Rightarrow \text{Portion of tank filled by larger diameter pipe in one hour} = \frac{1}{x-10}$$

Given that,

$$\text{Two tops together can fill the tank in } 9\frac{3}{8} \text{ hours} \Rightarrow \frac{75}{8} \text{ hours}$$

Now,

$$\text{Portion of tank filled by both the tops together in one hour} = \frac{1}{75/8} = \frac{8}{75}$$

We have,

Portion of tank filled by smaller pipe in 1 hr + Portion of tank filled by larger pipe in 1 hr.

$$= \frac{1}{x} + \frac{1}{x-10} \Rightarrow \frac{8}{75} = \frac{1}{x} + \frac{1}{x-10}$$

4.

**Sol:**

Let us take the time taken by the faster pipe to fill the tank as  $x$  minutes.

$$\Rightarrow \text{Portion of tank filled by faster pipe in one minute} = \frac{1}{x}$$

Now, time taken by the slower pipe to fill the same tank is 5 minutes more than that of faster pipe i.e.  $(x + 5)$  minutes.

$$\Rightarrow \text{Portion of tank filled by slower pipe in one minute} = \frac{1}{x+5}$$

Given that,

$$\text{The two pipes together can fill the tank in } 11\frac{1}{9} \text{ minutes} \Rightarrow \frac{100}{9} \text{ minutes}$$

$\Rightarrow$  portion of tank filled by faster pipe in 1min + Portions of tank filled by slower pipe in

$$1\text{min i.e. } \frac{9}{100} = \frac{1}{x} + \frac{1}{x+5}$$

$$\Rightarrow \frac{9}{100} = \frac{x+5+x}{x(x+5)}$$

### Exercise 8.13

1.

**Sol:**

Let initial length of the cloth be  $x$  m, and cost per each meter of cloth be Rs  $y$

$\Rightarrow$  Total cost of piece of cloth will be length of cloth  $\times$  cost per each meter

$\Rightarrow xy$

$$\text{But given that } xy = \text{Rs. } 35 \Rightarrow y = \text{Rs. } \frac{35}{x}$$

And also,

Given that if the piece were 4m longer and each meter costs Rs. 1 less the cost would remain unchanged.

$\Rightarrow$  Length of the cloth will be  $(x + 4)$ m and cost per each meter of cloth will be Rs  $(y - 1)$

$\Rightarrow$  Total cost of piece of cloth will be Rs.  $(x + 4)(y - 1)$

But,

$$\text{Rs } (x + 4)(y - 1) = \text{Rs } 35$$

$$\Rightarrow xy + 4y - x - 4 = 35$$

$$\Rightarrow 35 + 4\left(\frac{35}{x}\right) - x - 4 = 35 \quad \left[ \because xy = 35 \text{ \& } y = \frac{35}{x} \right]$$

$$\Rightarrow \frac{140 - x^2 - 4x}{x} = 0$$

$$\Rightarrow x^2 + 4x - 140 = 0$$

$$\Rightarrow x^2 + 14x - 10x + (14x - 10) = 0 \quad [\because 140 = 14x - 10 = 4x = 14x - 10x]$$

$$\Rightarrow x(x + 14) - 10(x + 14) = 0$$

$$\Rightarrow (x + 14)(x - 10) = 0$$

$$\Rightarrow (x + 14) = 0 \text{ or } (x - 10) = 0$$

$$\Rightarrow x = -14 \text{ or } x = 10$$

Since length of the cloth cannot be in negative integers, the required length of cloth is 10m.

2.

**Sol:**

Let  $x$  be no. of students planned for a picnic

Given that budget for food was Rs 480

$$\Rightarrow \text{Share of each student} = \frac{\text{Total budget}}{\text{No. of students}} = \text{Rs } \frac{480}{x}$$

Given that 8 students failed to go

$$\Rightarrow \text{No. of students will be } (x - 8)$$

Now,

Share of each student will be equal to

$$= \frac{\text{total budget}}{\text{No. of students}} = \text{Rs. } \frac{480}{x-8}$$

Given that if 8 students failed to go, then cost of food for each member increased by Rs. 10.

3.

**Sol:**

Let the cost price of the article be Rs  $x$

Given that gain percentage of the article is as much as cost price i.e.  $x$

$\Rightarrow$  Selling price = cost price + gain

$$= \text{Rs } x + \text{cost price} \times \text{gain percentage}$$

$$= \text{Rs } x + \text{Rs } x \times \frac{x}{100}$$

$$= \text{Rs } \left( x + \frac{x^2}{100} \right)$$

Given that selling price = Rs 24

$$\Rightarrow \text{Rs } 24 = \text{Rs } \left( x + \frac{x^2}{100} \right)$$

$$\Rightarrow 24 = x + \frac{x^2}{100}$$

$$\Rightarrow \frac{x^2}{100} + x - 24 = 0$$

$$\Rightarrow x^2 + 100x - 2400 = 0$$

$$\Rightarrow x^2 + 120x - 20x + (120 \times -80) = 0$$

$$\Rightarrow x^2(x + 180) - 80(x + 180) = 0$$

$$\Rightarrow (x + 180)(x - 20) = 0$$

$$\Rightarrow x + 120 = 0 \text{ or } x - 20 = 0$$

$$\Rightarrow x = -120 \text{ or } x = 20$$

Since, cost price of the article cannot be negative, the required cost price of the article is

Rs 20

$$\Rightarrow \text{Rs } \frac{480}{x-8} - \text{Rs } \frac{480}{x} = \text{Rs } 10$$

$$\Rightarrow \frac{480}{x-8} - \frac{480}{x} = 10$$

$$\Rightarrow 480 \left( \frac{1}{x-8} - \frac{1}{x} \right) = 10$$

$$\Rightarrow 48 \left( \frac{x-(x-8)}{x(x-8)} \right) = 1$$

$$\begin{aligned}
\Rightarrow 48 \left( \frac{x-x+8}{x^2-8x} \right) &= 1 \\
\Rightarrow 48(8) &= x^2 - 8x \\
\Rightarrow x^2 - 8x - 384 &= 0 \\
\Rightarrow x^2 - 24x + 16x + (-24 \times 16) &= 0 \\
\Rightarrow x(x - 24) + 16(x - 24) &= 0 \\
\Rightarrow (x - 24)(x + 16) &= 0 \\
\Rightarrow (x - 24) = 0 \text{ or } (x + 16) &= 0 \\
\Rightarrow x - 24 \text{ or } x = -16 &
\end{aligned}$$

Since the value of number of students cannot be negative, the required number of students attended the picnic is 24.

4.

**Sol:**

Let total number of swans be  $x$

Given that  $\frac{7}{2}$  times the square root of the total number of swans are playing on the shore of a pond i.e.  $\frac{7}{2}\sqrt{x}$  and the two remaining ones are swinging in water

$$\Rightarrow \text{Total number of swans } x = \frac{7}{2}\sqrt{x} + 2$$

$$\Rightarrow x = \frac{7}{2}\sqrt{x} + 2 \quad [\text{Let } \sqrt{x} = y \Rightarrow x = y^2]$$

$$\Rightarrow y^2 = \frac{7}{2}y + 2$$

$$\Rightarrow y^2 - \frac{7}{2}y - 2 = 0$$

$$\Rightarrow 2y^2 - 7y - 4 = 0$$

$$\Rightarrow 2y^2 - 8y + y - 4 = 0$$

$$\Rightarrow 2y(y - 4) + 1(y - 4) = 0$$

$$\Rightarrow (y - 4)(2y + 1) = 0$$

$$\Rightarrow (y - 4) = 0 \text{ or } (2y + 1) = 0$$

$$\Rightarrow y = 4 \text{ or } y = \frac{-1}{2}$$

$$\Rightarrow y^2 = 4^2 = 16 \text{ or } y^2 = \left(\frac{-1}{2}\right)^2 = \frac{1}{4}$$

Since, the value of number of swans cannot be a fraction, the required number of swans  $x = 16$

5.

**Sol:**

Let initial list price of the toy be Rs  $x$

Given that total cost of toys = Rs 360

$$\Rightarrow \text{Initially number of toys a person can buy} = \frac{\text{Total cost}}{\text{list price of each toy}} = \frac{\text{Rs } 360}{\text{Rs } x} \Rightarrow \frac{360}{x}$$

Now, if the list price is reduced by Rs 2 i.e. Rs.  $(x - 2)$

Number of toys a person can buy is 2 more for Rs 360

$$\Rightarrow \text{Number of toys a person can buy when price is reduced} = \frac{\text{Total cost}}{\text{list price}} = \frac{\text{Rs } 360}{\text{Rs } x-2} = \frac{360}{x-2}$$

Now,

$$\frac{360}{x-2} - \frac{360}{x} = 2$$

$$\Rightarrow 360 \left( \frac{1}{x-2} - \frac{1}{x} \right) = 2$$

$$\Rightarrow 360 \left( \frac{x-(x-2)}{x(x-2)} \right) = 2$$

$$\Rightarrow 360 \left( \frac{x-x+2}{x^2-2x} \right) = 2$$

$$\Rightarrow 360 \left( \frac{2}{x^2-2x} \right) = 2$$

$$\Rightarrow 360 = x^2 - 2x$$

6.

**Sol:**

Let the original number of persons be  $x$ ,

Total amount to be divided equally is Rs. 9000

$$\Rightarrow \text{Share of each person will be equal to} = \frac{\text{Total amount}}{\text{No. of persons}} = \text{Rs } \frac{9000}{x}$$

Given that if there had been 20 more persons

$\Rightarrow$  Final number of persons will be  $x + 20$ , then each would have got Rs 160 less

Now,

$$\text{Final share of each person will be equal to} = \frac{\text{Total amount}}{\text{No. of persons}} = \text{Rs } \frac{9000}{x+20}$$

We have,

$$\text{Rs } \frac{9000}{x} - \text{Rs } \frac{9000}{x+20} = \text{Rs } 160$$

$$\Rightarrow 9000 \left( \frac{1}{x} - \frac{1}{x+20} \right) = 160$$

$$\Rightarrow 9000 \left( \frac{7+80-x}{x(x+20)} \right) = 160$$

$$\Rightarrow 9000 \left( \frac{20}{x^2+20x} \right) = 160$$

$$\Rightarrow 1125 = x^2 + 20x$$

$$\Rightarrow x^2 + 20x - 1125 = 0$$

$$\Rightarrow x^2 + 45x - 25x + (45 \times -25) = 0$$

$$\Rightarrow x(x + 45) - 25(x + 45) = 0$$

$$\Rightarrow (x + 45)(x - 25) = 0$$

$$\Rightarrow x + 45 = 0 \text{ or } x - 25 = 0$$

$$\Rightarrow x = -45 \text{ or } x = 25$$



Since, share of each person cannot be negative value, the required share of each person is Rs 25.

7.

**Sol:**

Let the number of students planned for the picnic be  $x$

Given budget for food = Rs. 500

$$\Rightarrow \text{Initially share of food for each student} = \frac{\text{total budget}}{\text{no. of students}} = \text{Rs } \frac{500}{x}$$

Given that 5 students failed to go for the picnic

$$\Rightarrow \text{No. of students attended the picnic will be } (x - 5)$$

$$\text{Now, share of food for each student will be equal to} = \frac{\text{total budget}}{\text{no. of students attended}} = \text{Rs } \frac{500}{x-5}$$

Given that, share of food for each student is increased

$$\Rightarrow \text{Rs } \frac{500}{x-5} - \text{Rs } \frac{500}{x} = \text{Rs } 5$$

$$\Rightarrow \frac{500}{x-5} - \frac{500}{x} = 5$$

$$\Rightarrow 500 \left( \frac{1}{x-5} - \frac{1}{x} \right) = 5$$

$$\Rightarrow 500 \left( \frac{x - (x-5)}{x(x-5)} \right) = 5$$

$$\Rightarrow 500 \left( \frac{x - x + 5}{x^2 - 5x} \right) = 5$$

$$\Rightarrow 500 \left( \frac{5}{x^2 - 5x} \right) = 5$$

$$\Rightarrow 500 = x^2 - 5x$$

$$\Rightarrow x^2 - 5x - 500 = 0$$

$$\Rightarrow x^2 - 25x + 20x - 500 = 0$$

$$\Rightarrow x(x - 25) + 20(x - 25) = 0$$

$$\Rightarrow (x - 25)(x + 20) = 0$$

$$\Rightarrow (x - 25) = 0 \text{ or } (x + 20) = 0$$

$$\Rightarrow x = 25 \text{ or } x = -20$$

Since, the value of  $x$  cannot be negative,

$$\Rightarrow x = 25$$

Here,  $x$  is the no. of students planned,

Given that 5 students failed to go

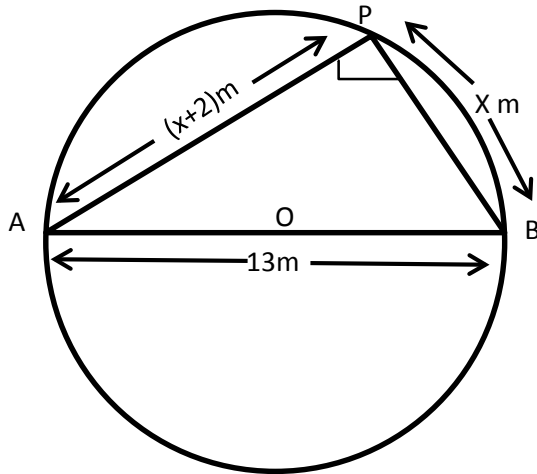
$$\Rightarrow \text{No. of students attended the picnic} = x - 5 = 25 - 5 = 20$$

$$\therefore \text{No. of students attended the picnic} = 20$$

8.

**Sol:**

Let P be the required location of the pole such that its distance from gate B is x meters i.e.  $BP = x$  meters and also  $AP - BP = 7\text{m} \Rightarrow AP = (x + 7)\text{m}$



Since, AB is a diameter and P is a point in the semi-circle  $\triangle APB$  is right angled at P.

Now,  $(x + 7)^2 + (x)^2 = (13)^2$  [ $\because AP^2 + BP^2 = AB^2$  and  $AB = 13\text{m}$ ]

$$\Rightarrow x^2 + 14x + 49 + x^2 = 169$$

$$\Rightarrow 2x^2 + 14x + (49 - 169) = 0$$

$$\Rightarrow 2x^2 + 14x - 120 = 0$$

$$\Rightarrow 2(x^2 + 7x - 60) = 0$$

$$\Rightarrow x^2 + 7x - 60 = 0$$

$$\Rightarrow x^2 + 12x - 5x + (12x - 5) = 0$$

$$\Rightarrow x(x + 12) - 5(x + 12) = 0$$

$$\Rightarrow (x + 12)(x - 5) = 0$$

$$\Rightarrow x + 12 = 0 \text{ or } x - 5 = 0$$

$$\Rightarrow x = -12\text{m or } x = 5\text{m}$$

Since, BP cannot be in negative value (or) distances cannot be negative values,

The required values of BP and AP are 5m and 12m respectively.

$\therefore$  The pole has to be erected at a distance 5meters from the gate B.

9.

**Sol:**

Let number of marks obtained by P in mathematics and science be x and y respectively.

Given that sum of these two is 28

$$\Rightarrow x + y = 28 \Rightarrow x = 28 - y$$

Given that if x becomes  $(x + 3)$  i.e. marks in mathematics is increased by 3 and y

becomes  $(y - 4)$  i.e. marks in science is decreased by 4, The product of these two

becomes by 4,

$$\Rightarrow (x + 3)(y - 4) = 180$$

$$\Rightarrow (28 - y + 3)(y - 4) = 180 \quad [\because x = 28 - y]$$

$$\begin{aligned}
&\Rightarrow (31 - y)(y - 4) = 180 \\
&\Rightarrow 31y - 31 \times 4 - y^2 + 4y = 180 \\
&\Rightarrow 35y - y^2 - 124 = 180 \\
&\Rightarrow y^2 - 35y + 180 + 124 = 0 \\
&\Rightarrow y^2 - 35y + 304 = 0 \\
&\Rightarrow y^2 - 19y - 16y + (-19 \times -16) = 0 \\
&\Rightarrow y(y - 19) - 16(y - 19) = 0 \\
&\Rightarrow (y - 19)(y - 16) = 0 \\
&\Rightarrow y - 19 = 0 \text{ or } y - 16 = 0 \\
&\Rightarrow y = 19 \text{ or } y = 16
\end{aligned}$$

We have,

$$x + y = 28$$

$$\text{if } y = 19 \Rightarrow x = 28 - y = 28 - 19 = 9 \text{ and}$$

$$\text{if } y = 16 \Rightarrow x = 28 - y = 28 - 16 = 12$$

$\therefore$  Marks in mathematics = 9 and Marks in Science = 19 or  
 Marks in mathematics = 12 and Marks in Science = 16

10.

**Sol:**

Let marks of shefali in Mathematics and English be  $x$  and  $y$  respectively.

Given that sum of these two is 30  $\Rightarrow x + y = 30 \Rightarrow x = 30 - y$

Given that if  $x$  becomes  $(x + 2)$  i.e. marks in mathematics is increased by 2 and  $y$  becomes  $(y - 3)$  i.e. marks in English is decreased by 3,

the product at these two becomes 210

$$\text{i.e. } (x + 2)(y - 3) = 210$$

$$\Rightarrow (30 - y + 2)(y - 3) = 210 \quad [\because x = 30 - y]$$

$$\Rightarrow (32 - y)(y - 3) = 210$$

$$\Rightarrow (32 - y)(y - 3) = 210$$

$$\Rightarrow 32y - 32 \times 3 - y \times 3y = 210$$

$$\Rightarrow 35y - 96 - y^2 = 210$$

$$\Rightarrow y^2 - 35y + 210 + 96 = 0$$

$$\Rightarrow y^2 - 35y + 306 = 0$$

$$\Rightarrow y^2 - 17y - 18y + (-17 \times -18) = 0 \quad [\because 306 = 17 \times 18 = -17 \times -18]$$

$$\Rightarrow y(y - 17) - 18(y - 17) = 0$$

$$\Rightarrow (y - 17)(y - 18) = 0$$

$$\Rightarrow y - 17 = 0 \text{ or } y - 18 = 0$$

$$\Rightarrow y = 17 \text{ or } y = 18$$

We have,

$$x + y = 30$$

$$\text{if } y = 17 \Rightarrow x = 30 - y = 30 - 17 = 13 \text{ and}$$

$$\text{if } y = 18 \Rightarrow x = 30 - y = 30 - 18 = 18$$

$\therefore$  Marks in Mathematics = 13 and marks in English = 17 or

Marks in Mathematics = 12 and marks in English = 18.

11.

**Sol:**

Let the number of articles produced on a particular day be  $x$ .

Total cost of production on that particular day = Rs 90

$$\text{Given } \Rightarrow \text{Cost of production of each article} = \frac{\text{Total cost of production}}{\text{no. of articles produced}} = \text{Rs } \frac{90}{x}$$

But given that, the cost of production of each article was 3 more than twice the no. of articles produced on that day i.e. Rs  $(2x + 3)$

$$\Rightarrow \text{Rs } (2x + 3) = \text{Rs } \frac{90}{x}$$

$$\Rightarrow 2x + 3 = \frac{90}{x}$$

$$\Rightarrow x(2x + 3) = 90 \Rightarrow 2x^2 + 3x - 90 = 0$$

$$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$$

$$\Rightarrow x(2x + 15) - 6(2x + 15) = 0$$

$$\Rightarrow (2x + 15)(x - 6) = 0$$

$$\Rightarrow 2x + 15 = 0 \text{ or } x - 6 = 0$$

$$\Rightarrow x = \frac{-15}{2} \text{ or } x = 6$$

Since, number of articles  $x$  cannot be a negative value, the required value of number of articles produced on a particular day  $x = 6$ .

