Answer each of the following questions either in one word or one sentence or as per the requirement of the questions :

Question 1.
Define an identity.
Solution:
An identity is an equation that is true for all values of the variable (s) involved.

Question 2.
What is the value of $(1 - \cos^2 \theta) \operatorname{cosec}^2 \theta$?
Solution:

$$(1 - \cos^2 \theta) \operatorname{cosec}^2 \theta = \sin^2 \theta \times \operatorname{cosec}^2 \theta$$

$$= (\sin \theta \operatorname{cosec} \theta)^2 \qquad \left\{ \because 1 - \cos^2 \theta = \sin^2 \theta \text{ and } \sin \theta \operatorname{cosec} \theta = 1 \right\}$$

$$= (1)^2 = 1$$

Question 3.
What is the value of $(1 + \cot^2 \theta) \sin^2 \theta$?
Solution:
$(1 + \cot^2 \theta) \sin^2 \theta = \operatorname{cosec}^2 \sin^2 \theta$ $\{1 + \cot^2 \theta = \operatorname{cosec}^2 \theta\}$
$= (\operatorname{cosec} \theta \sin \theta)^2 = (1)^2 = 1$ ($\because \sin \theta \operatorname{cosec} \theta = 1$)

Question 4.
What is the value of $\sin^2 \theta + \frac{1}{1 + \tan^2 \theta}$?

Solution:

$$\sin^2 \theta + \frac{1}{1 + \tan^2 \theta} = \sin^2 \theta + \frac{1}{\sec^2 \theta}$$

$$= \sin^2 \theta + \cos^2 \theta \qquad \left\{ \because 1 + \tan^2 \theta = \sec^2 \theta,\ \frac{1}{\sec \theta} = \cos \theta \text{ and } \sin^2 \theta + \cos^2 \theta = 1 \right.$$

$$= 1$$

Question 5.
If $\sec \theta + \tan \theta = x$, write the value of $\sec \theta - \tan \theta$ in terms of x.
Solution:
$\sec \theta + \tan \theta = x$
We know that
$\sec^2 \theta - \tan^2 \theta = 1$.
$\Rightarrow (\sec \theta + \tan \theta) (\sec \theta - \tan \theta) = 1$ $\{a^2 - b^2 = (a + b) (a - b)\}$
$\Rightarrow x (\sec \theta - \tan \theta) = 1$
$\Rightarrow \sec \theta - \tan \theta = 1x$

Question 6.
If cosec θ – cot θ = α, write the value of cosec θ + cot α.
Solution:
cosec θ – cot θ = α
We know that,
$\text{cosec}^2\ \theta - \cot^2\ \theta = 1$
⇒ (cosec θ – cot θ) (cosec θ + cot θ) = 1
⇒ a (cosec θ + cot θ) = 1 $\{a^2 - b^2 = (a + b)(a - 6)\}$
⇒ cosec θ + cot θ = 1/α

Question 7.
Write the value of $\text{cosec}^2 (90° - \theta) - \tan^2\theta$
Solution:

$$\text{cosec}^2 (90° - \theta) - \tan^2 \theta$$
$$= \sec^2 \theta - \tan^2 \theta = 1$$

$$\left\{\begin{array}{l}\because \text{cosec}(90° - \theta) = \sec\theta \\ \text{and } \sec^2\theta - \tan^2\theta = 1\end{array}\right.$$

Question 8.
Write the value of sin A cos (90° – A) + cos A sin (90° – A)
Solution:

$$\sin A \cos (90° - A) + \cos A \sin (90° - A)$$
$$= \sin A \sin A + \cos A \cos A$$

$$\left\{\begin{array}{l}\because \cos(90° - A) = \sin A \\ \sin(90° - A) = \cos A \\ \sin^2 A + \cos^2 A = 1\end{array}\right.$$

$$= \sin^2 A + \cos^2 A = 1$$

Question 9.

Write the value of $\cot^2 \theta - \dfrac{1}{\sin^2 \theta}$

Solution:

$$\cot^2 \theta - \frac{1}{\sin^2 \theta} = \cot^2 \theta - \text{cosec}^2 \theta$$

$$\left\{\because \frac{1}{\sin\theta} = \text{cosec}\,\theta\right.$$

$$= -(\text{cosec}^2 \theta - \cot^2 \theta) = -1$$

Question 10.
If x = a sin θ and y = b cos θ, what is the value of $b^2x^2 + a^2y^2$?
Solution:
x = a sin θ, y = b cos θ

$$\frac{x}{a} = \sin\theta, \ \frac{y}{b} = \cos\theta$$

Squaring and adding we get,

$$\therefore \ \frac{x^2}{a^2} = \sin^2\theta, \ \frac{y^2}{b^2} = \cos^2\theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \sin^2\theta + \cos^2\theta$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{b^2x^2 + a^2y^2}{a^2b^2} = 1 \Rightarrow b^2x^2 + a^2y^2 = a^2b^2$$

$$\therefore \ b^2x^2 + a^2y^2 = a^2b^2$$

Question 11.
If sin θ = 45, What is the value of cot θ + cosec θ?

Solution:

$$\sin\theta = \frac{1}{\text{cosec }\theta} = \frac{4}{5} \Rightarrow \text{cosec }\theta = \frac{5}{4}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\sqrt{1-\sin^2\theta}}{\sin^2\theta} = \frac{\sqrt{1-\left(\frac{4}{5}\right)^2}}{\frac{4}{5}}$$

$$= \frac{\sqrt{1-\frac{16}{25}}}{\frac{4}{5}} = \frac{\sqrt{\frac{25-16}{25}}}{\frac{4}{5}} = \frac{\sqrt{\frac{9}{25}}}{\frac{4}{5}}$$

$$= \frac{3}{5} \times \frac{5}{4} = \frac{3}{4}$$

$$\text{Now } \cot\theta + \text{cosec }\theta = \frac{3}{4} + \frac{5}{4} = \frac{3+5}{4}$$

$$= \frac{8}{4} = 2$$

Question 12.
What is the value of $9\cot^2\theta - 9\text{cosec}^2\theta$?
Solution:
$9\cot^2\theta - 9\text{cosec}^2\theta$
$= -(9\text{cosec}^2\theta - 9\cot^2\theta)$
$= -9(\text{cosec}^2\theta - \cot^2\theta) = -9 \times 1$
$= -9 \{\because \text{cosec}^2\theta - \cot^2\theta = 1\}$

Question 13.

What is the value of $6\tan^2\theta - \dfrac{6}{\cos^2\theta}$?

Solution:

$$6\tan^2\theta - \frac{6}{\cos^2\theta} = 6\tan^2\theta - 6\sec^2\theta$$

$$\left\{\because \frac{1}{\cos\theta} = \sec\theta\right.$$

$$= -6(\sec^2\theta - \tan^2\theta)\ \{\sec^2\theta - \tan^2\theta = 1\}$$

$$= -6 \times 1 = -6$$

Question 14.

What is the value of $\dfrac{\tan^2\theta - \sec^2\theta}{\cot^2\theta - \operatorname{cosec}^2\theta}$?

Solution:

$$\frac{\tan^2\theta - \sec^2\theta}{\cot^2\theta - \operatorname{cosec}^2\theta} = \frac{-(\sec^2\theta - \tan^2\theta)}{-(\operatorname{cosec}^2\theta - \cot^2\theta)}$$

$$= \frac{-1}{-1} = 1 \qquad \left\{\begin{array}{l}\because \sec^2\theta - \tan^2\theta = 1 \\ \operatorname{cosec}^2\theta - \cot^2\theta = 1\end{array}\right.$$

Question 15.
What is the value of $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$?
Solution:

$$(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$$
$$= (1 + \tan^2\theta)(1 - \sin^2\theta)$$

$$\left\{\begin{array}{l}\because (a-b)(a+b) = a^2 - b^2 \\ 1 - \sin^2\theta = \cos^2\theta \\ \text{and } 1 + \tan^2\theta = \sec^2\theta\end{array}\right.$$

$$= (1 + \tan^2\theta)\cos^2\theta = \sec^2\theta\cos^2\theta$$
$$\{\cos\theta \times \sec\theta = 1\}$$
$$= 1$$

Question 16.
If cos A = 725, find the value of tan A +*cot A. (C.B.S.E. 2008)*

Solution:

$$\cos A = \frac{7}{25}$$

$$\therefore \sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\left(\frac{7}{25}\right)^2}$$

$$= \sqrt{1-\frac{49}{625}} = \sqrt{\frac{625-49}{625}}$$

$$= \sqrt{\frac{576}{625}} = \frac{24}{25}$$

$$\text{Now } \tan A + \cot A = \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= \frac{\frac{24}{25}}{\frac{7}{25}} + \frac{\frac{7}{25}}{\frac{24}{25}} = \frac{24}{25} \times \frac{25}{7} + \frac{7}{25} \times \frac{25}{24}$$

$$= \frac{24}{7} + \frac{7}{24}$$

$$= \frac{576+49}{168} = \frac{625}{168}$$

Question 17.
If sin θ = 13, then find the value of 2 cot² θ + 2. (C.B.S.E. 2009)
Solution:

$$\sin \theta = \frac{1}{3}$$

$$\therefore \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{3}{1}$$

Now $2 \cot^2 \theta + 2 = 2 (\cot^2 \theta + 1)$
$= 2 \operatorname{cosec}^2 \theta = 2 \times (3)^2$
$= 2 \times 9 = 18$

Question 18.
If cos θ = 34, then find the value of 9 tan² θ + 9.

Solution:

$$\cos\theta = \frac{3}{4}$$

$$\therefore\ \sec\theta = \frac{1}{\cos\theta} = \frac{4}{3}$$

Now $9\tan^2\theta + 9 = 9(\tan^2\theta + 1)$

$$= 9\sec^2\theta = 9 \times \left(\frac{4}{3}\right)^2$$

$$= 9 \times \frac{16}{9} = 16$$

Question 19.
If $\sec^2\theta\,(1 + \sin\theta)(1 - \sin\theta) = k$, then find the value of k. (C.B.S.E. 2009)
Solution:

$$\sec^2\theta\,(1 + \sin\theta)(1 - \sin\theta) = k$$

$$\Rightarrow \sec^2\theta\,(1 - \sin^2\theta) = k$$

$$\left\{\begin{array}{l} \because (a+b)(a-b) \\ = a^2 - b^2 \\ 1 - \sin^2\theta = \cos^2\theta \\ \cos\theta\ \sec\theta = 1 \end{array}\right\}$$

$$\Rightarrow \sec^2\theta\cos^2\theta = k$$

$$1 = k$$

$$\therefore\ k = 1$$

Question 20.
If $\text{cosec}^2\theta\,(1 + \cos\theta)(1 - \cos\theta) = \lambda$, then find the value of λ.
Solution:
$\text{cosec}^2\theta\,(1 + \cos\theta)(1 - \cos\theta) = \lambda$
$\Rightarrow \text{cosec}^2\theta\,(1 - \cos^2\theta) = \lambda$ $\{(a + b)(a - b) = a^1 - b^2)\}$
$\Rightarrow \text{cosec}^2\theta \times \sin\theta = \lambda$ $(1 - \cos^2\theta = \sin^2\theta)$
$\Rightarrow 1 = \lambda$ $(\sin\theta\ \text{cosec}\,\theta = 1)$
$\therefore \lambda = 1$

Question 21.
If $\sin^2\theta\cos^2\theta\,(1 + \tan^2\theta)(1 + \cot^2\theta) = \lambda$, then find the value of λ.
Solution:
$\sin^2\theta\cos^2\theta\,(1 + \tan^2\theta)(1 + \cot^2\theta) = \lambda$

$\sin^2\theta \cos^2\theta (\sec^2\theta)(\mathrm{cosec}^2\theta) = \lambda$

$$\left\{\begin{array}{l} \because 1+\tan^2\theta = \sec^2\theta \\ \quad 1+\cot^2\theta = \mathrm{cosec}^2\theta \end{array}\right\}$$

$\Rightarrow \sin^2\theta\, \mathrm{cosec}^2\theta \cos^2\theta \sec^2\theta = \lambda$

$= 1 \times 1 = \lambda$ $\quad \left\{\begin{array}{l} \because \sin\theta\, \mathrm{cosec}\,\theta = 1 \\ \quad \cos\theta \sec\theta = 1 \end{array}\right\}$

$\therefore \lambda = 1$

Question 22.

If $5x = \sec\theta$ and $\frac{5}{x} = \tan\theta$, find the value of $5\left(x^2 - \frac{1}{x^2}\right)$. [CBSE 2010]

Solution:

If $5x = \sec\theta$, $\frac{5}{x} = \tan\theta$

$$5x + \frac{5}{x} = \sec\theta + \tan\theta$$

$$5\left(x + \frac{1}{x}\right) = \sec\theta + \tan\theta$$

$$x + \frac{1}{x} = \frac{1}{5}(\sec\theta + \tan\theta)$$

and $x - \frac{1}{x} = \frac{1}{5}(\sec\theta - \tan\theta)$

Now, $5\left(x^2 - \frac{1}{x^2}\right) = 5\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

$$= 5\left[\frac{1}{5}(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)\right]$$

$$= 5 \times \frac{1}{25}(\sec^2\theta - \tan^2\theta)$$

$$= \frac{1}{5}[1] \qquad (\because \sec^2\theta - \tan^2\theta = 1)$$

$$= \frac{1}{5}$$

Question 23.
If cosec θ = 2x and cot θ = 2x, find the value of 2 (x² – 1x2 [CBSE 2010]

Solution:

$$\text{cosec } \theta = 2x, \cot \theta = \frac{2}{x}$$

$$2x + \frac{2}{x} = \text{cosec } \theta + \cot \theta$$

$$2\left(x + \frac{1}{x}\right) = \text{cosec } \theta + \cot \theta$$

$$\Rightarrow x + \frac{1}{x} = \frac{1}{2} [\text{cosec } \theta + \cot \theta]$$

$$\text{and } x - \frac{1}{x} = \frac{1}{2} [\text{cosec } \theta - \cot \theta]$$

$$\text{Now, } 2\left(x^2 - \frac{1}{x^2}\right) = 2\left[\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)\right]$$

$$= 2\left[\frac{1}{2}(\text{cosec}\theta + \cot\theta) \times \frac{1}{2}(\text{cosec}\theta - \cot\theta)\right]$$

$$= 2 \times \frac{1}{2} \times \frac{1}{2} [\text{cosec}^2 \theta - \cot^2 \theta]$$

$$= \frac{1}{2} \times 1 \qquad \{\because \text{cosec}^2 \theta - \cot^2 \theta = 1\}$$

$$= \frac{1}{2}$$

Question 24.
Write 'True' or 'False' and justify your answer in each of the following:
(i) The value of sin θ is x + 1x, where 'x' is a positive real number.
(ii) cos θ = a2+b22ab , where a and b are two lab distinct numbers such that ab > 0.
(iii) The value of $\cos^2 23 - \sin^2 67$ is positive.
(iv) The value of the expression sin 80° – cos 80° is negative.
(v) The value of sin θ + cos θ is always greater than 1.
Solution:

(*i*) False.

We know that $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 \geq 0$ or

$\left(x+\frac{1}{x}\right) \geq 2$, but $\sin\theta$ is not greater than 1.

Alternatively, there exists the following three posibilities:

Case 1: If $x < 1$, then $\left(x+\frac{1}{x}\right) < 1$

Case 2: If $x = 1$, then $\left(x+\frac{1}{x}\right) = 1$

Case 3: If $x > 1$, then $\left(x+\frac{1}{x}\right) > 1$

However, $\sin\theta$ cannot be greater than 1.

(*ii*) False.

Given, a and b are two distinct numbers such that $ab > 0$.

Using, AM > GM

[since, AM and GM of two number a and b

are $\frac{a+b}{2}$ and $\sqrt{ab}$, respectively]

$$\Rightarrow \frac{a^2+b^2}{2} > \sqrt{a^2 \cdot b^2}$$

$$\Rightarrow a^2 + b^2 > 2ab$$

$\Rightarrow \dfrac{a^2+b^2}{2ab} > 1$ $\left[\because \cos\theta = \dfrac{a^2+b^2}{2ab}\right]$

$\Rightarrow \cos\theta > 1$ $[\because -1 \le \cos\theta \le 1]$

Which is not possible.

Hence, $\cos\theta \neq \dfrac{a^2+b^2}{2ab}$

(*iii*) False.

$\cos^2 23 - \sin^2 67$

$= \sin^2 (90 - 23) - \sin^2 67$

$= \sin^2 67 - \sin^2 67 = 0$

(*iv*) False.

$\sin 80° - \cos 80°$

$= \sin 80° - \sin (90° - 80°)$

$= \sin 80° - \sin 10°$ is +ve

$\because$ $\sin\theta$ increases as θ increases.

(*v*) False.

The value of $(\sin\theta + \cos\theta)$ for $\theta = 0°$ is 1.