Exercise 3.10

Q1

Points A and B are 70 km. a part on a highway. A car starts from A and another car starts from B simulataneously. If they travel in the same direction, they meet in 7 hours, but if they travel towards each other, they meet in one hour. Find the speed of the two cars.

Solution

Let X and Y be two cars starting from points A and B respectively.
Let the speed of car X be x km/hr and that of car Y be y km/hr.

CaseI, When two cars move in the same directions:
Suppose two cars meet at point Q. Then,
Distance travelled by car $X = AQ$
Distance travelled by car $Y = BQ$
It is given that two cars meet in 7 hours.
$\therefore$ Distance travelled by car X in 7 hours = $7x$ km
$\Rightarrow AQ = 7x$
Distance travelled by car Y in 7 hours = $7y$ km
$\Rightarrow BQ = 7y$
Clearly, $AQ - BQ = AB$
$\Rightarrow 7x - 7y = 70$ $[\because AB = 70 \text{ km}]$
$\Rightarrow 7(x - y) = 70$
$\Rightarrow x - y = 10$ ---(i)

CaseII, When two cars move in opposite directions:
Suppose two cars meet at point P. Then,
Distance travelled by car $X = AP$,
Distance travelled by car $Y = BP$.
In this case, two cars meet in 1 hour.
$\therefore$ Distance travelled by car X in 1 hour = x km
$\Rightarrow AP = x$
Distance travelled by car Y in 1 hour = y km
$\Rightarrow BP = y$
Clearly, $AP + BP = AB$
$\Rightarrow x + y = 70$ ---(ii)
Adding equation (i) and equation (ii), we get
$2x = 10 + 70$
$\Rightarrow 2x = 80$
$\Rightarrow x = \frac{80}{2} = 40$
Putting $x = 40$ in equation (ii), we get
$40 + y = 70$
$\Rightarrow y = 70 - 40 = 30$

Hence, Speed of car X is 40 km/hr and speed of car Y is 30 km/hr.

Q2

A sailor goes 8 km downstream in 40 minutes and returns in 1 hours. Determine the speed of the sailor in still water and the speed of the current.

Solution

Let the speed of the sailor in still water be x km/hr and the speed of the current be y km/hr.
Then,
Speed downstream = $(x+y)$ km/hr
Speed in return journey = $(x-y)$ km/hr

Now, Time taken to cover 8 km downstream = $\frac{8}{x+y}$ hrs

But, Time taken to cover 8 km downstream is 40 minutes

$\Rightarrow \frac{8}{x+y} = \frac{40}{60}$ $\left[\because 40 \text{ minutes} = \frac{40}{60} \text{ hrs}\right]$

$\Rightarrow \frac{8}{x+y} = \frac{2}{3}$

$\Rightarrow \frac{8 \times 3}{2} = x+y$

$\Rightarrow 4 \times 3 = x+y$

$\Rightarrow x+y = 12$ ---(i)

and, Time taken in return journey = $\frac{8}{x-y}$ km/hr

But, Time taken in return journey is 1 hour

$\Rightarrow \frac{8}{x-y} = 1$

$\Rightarrow x-y = 8$ ---(ii)

Adding equation (i) and equation (ii), we get

$2x = 12+8$

$\Rightarrow 2x = 20$

$\Rightarrow x = \frac{20}{2} = 10$

Putting $x = 10$ in equation (i), we get

$10 + y = 12$

$\Rightarrow y = 12-10$

$\Rightarrow y = 2$

Hence, Speed of the sailor in still water = 10 km/hr
Speed of the current = 2 km/hr

Q3

The boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of stream and that of the boat in still water.

Solution

Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/h

Then,
Speed upstream = $(x-y)$ km/hr
Speed downstream = $(x+y)$ km/hr

Now, Time taken to cover 30 km upstream = $\frac{30}{x-y}$ hrs

Time taken to cover 44 km downstream = $\frac{44}{x+y}$ hrs

But, Total time of journey is 10 hours

$\therefore \quad \frac{30}{x-y}+\frac{44}{x+y}=10 \quad$ ---(i)

Time taken to cover 40 km upstream = $\frac{40}{x-y}$ hrs

Time taken to cover 55 km downstream = $\frac{55}{x+y}$ hrs

But, Total time of journey is 13 hours.

$\therefore \quad \frac{40}{x-y}+\frac{55}{x+y}=13 \quad$ ---(ii)

Putting $\frac{1}{x-y}=u$ and $\frac{1}{x+y}=v$, in equation (i) and (ii), we get

$30u+44v=10 \quad$ ---(iii)

$40u+55v=13 \quad$ ---(iv)

By cross-multiplication, we get

$$\frac{u}{44\times(-13)-(-10)\times 55}=\frac{-v}{30\times(-13)-(-10)\times 40}=\frac{1}{30\times 55-44\times 40}$$

$$\Rightarrow \frac{u}{-572+550}=\frac{-v}{-390+400}=\frac{1}{1650-1760}$$

$$\Rightarrow \frac{u}{-22}=\frac{-v}{10}=\frac{1}{-110}$$

$$\Rightarrow u=\frac{-22}{-110} \text{ and } -v=\frac{10}{-110}$$

$$\Rightarrow u=\frac{1}{5} \text{ and } v=\frac{1}{11}$$

Now, $u=\frac{1}{5}$

$\Rightarrow \frac{1}{x-y}=\frac{1}{5}$

$\Rightarrow x-y=5 \quad$ ---(v)

And, $v=\frac{1}{11}$

$\Rightarrow \frac{1}{x+y}=\frac{1}{11}$

$\Rightarrow x+y=11 \quad$ ---(vi)

Adding equation (v) and equation (vi), we get

$2x=5+11$

$\Rightarrow x=\frac{16}{2}=8$

Putting $x=8$ in equation (vi), we get

$8+y=11$

$\Rightarrow y=11-8$

$\Rightarrow y=3$

Hence, Speed of the stream = 3 km/hr
Speed of the boat in still water = 8 km/hr

Q4

A boat goes 24 km upstream and 28 km downstream in 6 hrs. It goes 30 km upstream and 21 km downstream in $6\frac{1}{2}$ hrs. Find the speed of the boat in still water and also speed of the stream.

Solution

Let

Speed of the boat be x and speed of the stream be y

From the given data

we get

$\frac{24}{x-y}+\frac{28}{x+y}=6$

$\frac{30}{x-y}+\frac{21}{x+y}=6.5$

Let

$\frac{1}{x-y}=X$

$\frac{1}{x+y}=Y$

Then the equation becomes

$24X+28Y=6----(i)$

$30X+21Y=6.5---(ii)$

Solving (i) and (ii) we get

$X=\frac{1}{6}$ and $Y=\frac{1}{14}$

So x-y=6 and x+y=14

Hence

x=10kmph and y=4kmph

Speed of the boat is 10kmph

Speed of the stream be 4kmph

Q5

A man walks a certain distance with certain speed. If he walks 1/2 km an hour faster, he takes 1 hour less. But, if he walks 1 km an hour slower, he takes 3 more hours. Find the distance covered by the man and his original rate of walking.

Solution

Let the original speed of man be x km/hr and the actual time taken by y hours. Then,

Distance covered = (xy) km ---(i) $[\because D = S \times T]$

If the speed is increased by $\frac{1}{2}$ km/hr then time of journey is reduced by 1 hour i.e., when speed is $\left(x + \frac{1}{2}\right)$ km/hr, time of journey is $(y - 1)$ hours.

$\therefore$ Distance covered = $\left(x + \frac{1}{2}\right)(y - 1)$

$\Rightarrow xy = xy - x + \frac{1}{2}y - \frac{1}{2}$ [using(i)]

$\Rightarrow x - \frac{1}{2}y + \frac{1}{2} = 0$

$\Rightarrow 2x - y + 1 = 0$ ---(ii)

When the speed is reduced by 1 km/hr, then the time of journey is increased by 3 hours i.e., when speed is $(x - 1)$ km/hr time of journey is $(y + 3)$ hours.

$\therefore$ Distance covered = $(x - 1)(y + 3)$

$\Rightarrow xy = (x - 1)(y + 3)$ [using(i)]

$\Rightarrow xy = xy + 3x - y - 3$

$\Rightarrow 0 = 3x - y - 3$

$\Rightarrow 3x - y - 3 = 0$ ---(iii)

Thus, we obtain the following system of equations:

$2x - y + 1 = 0$

$3x - y - 3 = 0$

By using cross-multiplication, we have

$$\frac{x}{(-1)\times(-3) - (1)\times(-1)} = \frac{-y}{(2)\times(-3) - (1)\times(3)} = \frac{1}{(2)\times(-1) - (-1)\times(3)}$$

$\Rightarrow \frac{x}{3+1} = \frac{-y}{-6-3} = \frac{1}{-2+3}$

$\Rightarrow \frac{x}{4} = \frac{-y}{-9} = \frac{1}{1}$

$\Rightarrow \frac{x}{4} = \frac{y}{9} = 1$

$\Rightarrow \frac{x}{4} = 1$ and $\frac{y}{9} = 1$

$\Rightarrow x = 4$ and $y = 9$

Putting the value of x and y in equation(i), we obtain

Distance = (4×9) km = 36 km

Hence, distance covered by man = 36 km

Original rate of walking = 4 km/hr

Q6

A person rowing at the rate of 5 km/h in still water, takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of stream.

Solution

Given, speed of boat in still water = 5 km / hr

Let the speed of the stream be x km/hr.

∴ Speed of the boat upstream = (5 − x) km / hr

Speed of the boat downstream = (5 + x) km / hr

It is given that

Time to cover 40 km upstream = 3 x time to cover 40 km downstream

$\Rightarrow \frac{40}{5-x} = 3 \times \frac{40}{5+x}$

$\Rightarrow \frac{40}{5-x} = \frac{120}{5+x}$

$\Rightarrow \frac{1}{5-x} = \frac{3}{5+x}$

$\Rightarrow 5 + x = 15 - 3x$

$\Rightarrow 4x = 10$

$\Rightarrow x = \frac{10}{4}$

$\Rightarrow x = 2.5$

Thus, the speed of the stream is 2.5 km/hr.

Q7

Ramesh travels 760 km to his home partly by train and partly by car. He takes 8 hours if he travels 160 km. by train and the rest by car. He takes 12 minutes more if the travels 240 km by train and the rest by car. Find the speed of the train and car respectively.

Solution

Let the speed of the train be x km/hr and that of the car be y km/hr. We have following cases:
CaseI When Ramesh travels 160 km by train and the rest by car:

In this case, we have,

Time taken by Ramesh to travel 160 km by train = $\frac{160}{x}$ hrs

Time taken by Ramesh to travel $(760 - 160) = 600$ km by car = $\frac{600}{y}$ hrs

$\therefore$ Total time taken by Ramesh to cover 760 km = $\frac{160}{x} + \frac{600}{y}$

It is given that the total time taken is 8 hours.

$$\therefore \quad \frac{160}{x} + \frac{600}{y} = 8$$

$$\Rightarrow \quad 8\left[\frac{20}{x} + \frac{75}{y}\right] = 8$$

$$\Rightarrow \quad \frac{20}{x} + \frac{75}{y} = 1 \qquad \text{---(i)}$$

CaseII When Ramesh travels 240 km by train and the rest by car:
In this case, we have

Time taken by Ramesh to travel 240 km by train = $\frac{240}{x}$ hrs

Time taken by Ramesh to travel $(760 - 240) = 520$ km by car = $\frac{520}{y}$

In this case, total time of the journey is 8 hrs 12 minutes

$$\therefore \quad \frac{240}{x} + \frac{520}{y} = 8 \text{ hrs } 12 \text{ minutes}$$

$$\Rightarrow \quad \frac{240}{x} + \frac{520}{y} = 8\frac{12}{60}$$

$$\Rightarrow \quad \frac{240}{x} + \frac{520}{y} = \frac{41}{5} \qquad \text{---(ii)}$$

Thus, we obtain the following system of equations:

$$\frac{20}{x} + \frac{75}{y} = 1$$

$$\frac{240}{x} + \frac{520}{y} = \frac{41}{5}$$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, the above system reduces to

$$20u + 75v = 1 \qquad \text{---(iii)}$$

$$240u + 520v = \frac{41}{5} \qquad \text{---(iv)}$$

Multiplying equation(iii) by 12, we get

$$240u + 900v = 12 \qquad \text{---(v)}$$

Subtracting equation(iv) by equation (v), we get

$$900v - 520v = 12 - \frac{41}{5}$$

$$\Rightarrow \quad 380v = \frac{60 - 41}{5}$$

$$\Rightarrow \quad 380v = \frac{19}{5}$$

$$\Rightarrow \quad v = \frac{19}{5} \times \frac{1}{380}$$

$\Rightarrow \quad v = \frac{1}{5} \times \frac{1}{20}$

$\Rightarrow \quad v = \frac{1}{100}$

Putting $v = \frac{1}{100}$ in equation (v), we get

$240u + 900 \times \frac{1}{100} = 12$

$\Rightarrow \quad 240u + 9 = 12$

$\Rightarrow \quad 240u = 12 - 9 = 3$

$\Rightarrow \quad u = \frac{3}{240} = \frac{1}{80}$

Now, $u = \frac{1}{80}$

$\Rightarrow \quad \frac{1}{x} = \frac{1}{80}$

$\Rightarrow \quad x = 80$

And, $v = \frac{1}{100}$

$\Rightarrow \quad \frac{1}{y} = \frac{1}{100}$

$\Rightarrow \quad y = 100$

Hence, speed of the train = 80 km/hr

Speed of the car = 100 km/hr.

Q8

A man travels 600 km partly by train and partly by car. If the dcovers 400 km by train and the rest by car, it takes him 6 hours and 30 minutes. But, if the travels 200 km by train and the rest by car, he takes half an hour longer. Find the speed of the train and that of the car.

Solution

Let the speed of the train be x km/hr and that of the car be y km/hr. We have following cases:

CaseI When Ramesh travels 400 km by train and the rest by car:

In this case, we have

Time taken by the man to travel 400 km by train $= \frac{400}{x}$

Time taken by the man to travel $(600-400) = 200$ km by car $= \frac{200}{y}$

In this case, total time of the journey is 6 hrs 30 minutes.

$\therefore \quad \frac{400}{x} + \frac{200}{y} = 6$ hrs 30 minutes

$\Rightarrow \quad \frac{400}{x} + \frac{200}{y} = 6\frac{1}{2}$

$\Rightarrow \quad \frac{400}{x} + \frac{200}{y} = \frac{13}{2} \qquad ---(i)$

CaseII When he travels 200 km by train and the rest by car:
In this case, we have

Time taken by the man to travel 200 km by train $= \frac{200}{x}$ hrs

Time taken by the man to travel $(600-200) = 400$ km by car $= \frac{400}{y}$ hrs

In this case, total time of journey is $\left(\frac{13}{2} + \frac{1}{2}\right) = 7$ hrs.

$\therefore \quad \frac{200}{x} + \frac{400}{y} = 7 \qquad ---(ii)$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, in equation (i) and (ii), we get

$400u + 200v = \frac{13}{2} \qquad ---(iii)$

$200u + 400v = 7 \qquad ---(iv)$

Multiplying equation (iii) by 2, we get

$800u + 400v = 13 \qquad ---(v)$

Subtracting equation (iv) by equation (v), we get

$800u - 200u = 13 - 7$

$\Rightarrow \quad 600u = 6$

$\Rightarrow \quad u = \frac{6}{600} = \frac{1}{100}$

Putting $u = \frac{1}{100}$ in equation (iv), we get

$200 \times \frac{1}{100} + 400v = 7$

$\Rightarrow \quad 2 + 400v = 7$

$\Rightarrow \quad 400v = 7 - 2$

$\Rightarrow \quad 400v = 5$

$\Rightarrow \quad v = \frac{5}{400} = \frac{1}{80}$

Now, $u = \frac{1}{100}$

$\Rightarrow \quad \frac{1}{x} = \frac{1}{100}$

$\Rightarrow \quad x = 100$

And, $v = \frac{1}{80}$

$\Rightarrow \quad \frac{1}{y} = \frac{1}{80}$

$\Rightarrow \quad y = 80$

Hence, speed of the train = 100 km/hr

Speed of the car = 80 km/hr.

Q9

Places A and B are 80 km apart from each other on a highway. A car starts from A and other from B at the same time. If they move in the same direction, they meet in 8 hours and if they move in opposite directions, they meet in 1 hour and 20 minutes. Find the speeds of the cars.

Solution

Let X and Y be two cars starting from points A and B respectively.
Let the speed of car X be x km/hr and that of car Y be y km/hr.

CaseI, When two cars move in the same directions:
Suppose two cars meet at point Q. Then,
Distance travelled by car $X = AQ$
Distance travelled by car $Y = BQ$

It is given that two cars meet in 8 hours.
$\therefore$ Distance travelled by car X in 8 hours $= 8x$ km
$\Rightarrow AQ = 8x$

Distance travelled by car Y in 8 hours $= 8y$ km
$\Rightarrow BQ = 8y$

Clearly, $AQ - BQ = AB$
$\Rightarrow 8x - 8y = 80 \quad [\because AB = 80 \text{ km}]$
$\Rightarrow 8(x - y) = 80$
$\Rightarrow x - y = 10 \quad$ ---(i)

CaseII, When two cars move in opposite directions:
Suppose two cars meet at point P. Then,
Distance travelled by car $X = AP$,
Distance travelled by car $Y = BP$.

In this case, two cars meet in 1 hour 20 minutes $= 1\frac{1}{3} = \frac{4}{3}$ hrs

$\therefore$ Distance travelled by car X in $\frac{4}{3}$ hours $= \frac{4}{3}x$ km

$\Rightarrow AP = \frac{4}{3}x$

Distance travelled by car Y in $\frac{4}{3}$ hours $= \frac{4}{3}y$ km

$\Rightarrow BP = \frac{4}{3}y$

Clearly, $AP + BP = AB$
$\Rightarrow \frac{4}{3}x + \frac{4}{3}y = 80$
$\Rightarrow \frac{4}{3}(x + y) = 80$
$\Rightarrow x + y = \frac{80 \times 3}{4}$
$\Rightarrow x + y = 60 \quad$ ---(ii)

Adding equation (i) and equation (ii), we get
$2x = 10 + 60$
$\Rightarrow 2x = 70$
$\Rightarrow x = \frac{70}{2} = 35$
Putting $x = 35$ in equation (ii), we get,
$35 + y = 60$
$\Rightarrow y = 60 - 35 = 25$

Hence, speed of car X is 35 km/hr and speed of car Y is 25 km/hr.

Q10

A boat goes 12 km upstream and 40 km downstream in 8 hours. It can go 16 km upstream and 32 km downstream in the same time. Find the speed of the boat in still water and the speed of the stream.

Solution

Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr.

Then,
Speed upstream = $(x-y)$ km/hr
Speed downstream = $(x+y)$ km/hr

Now, Time taken to cover 12 km upstream = $\frac{12}{x-y}$ hrs

Time taken to cover 40 km downstream = $\frac{40}{x+y}$ hrs

But, Total time of journey is 8 hours

$\therefore \quad \frac{12}{x-y}+\frac{40}{x+y}=8 \qquad$ ---(i)

Time taken to cover 16 km upstream = $\frac{16}{x-y}$ hrs

Time taken to cover 32 km downstream = $\frac{32}{x+y}$ hrs

But, Total time of journey is 8 hours.

$\therefore \quad \frac{16}{x-y}+\frac{32}{x+y}=8 \qquad$ ---(ii)

Putting $\frac{1}{x-y}=u$ and $\frac{1}{x+y}=v$, in equation (i) and (ii), we get

$12u+40v=8$

$\Rightarrow \quad 4(3u+10v)=8$

$\Rightarrow \quad 3u+10v=2$

$\Rightarrow \quad 3u+10v-2=0 \qquad$ ---(iii)

And, $16u+32v=8$

$\Rightarrow \quad 8(2u+4v)=8$

$\Rightarrow \quad 2u+4v=1$

$\Rightarrow \quad 2u+4v-1=0 \qquad$ ---(iv)

By cross-multiplication, we get

$$\frac{u}{10\times(-1)-(-2)\times 4}=\frac{-v}{3\times(-1)-(-2)\times 2}=\frac{1}{3\times 4-2\times 10}$$

$\Rightarrow \quad \frac{u}{-10+8}=\frac{-v}{-3+4}=\frac{1}{12-20}$

$\Rightarrow \quad \frac{u}{-2}=\frac{-v}{1}=\frac{1}{-8}$

$\Rightarrow \quad \frac{u}{-2}=\frac{1}{-8}$ and $\frac{-v}{1}=\frac{1}{-8}$

$\Rightarrow \quad u=\frac{2}{8}$ and $v=\frac{1}{8}$

$\Rightarrow \quad u=\frac{1}{4}$ and $v=\frac{1}{8}$

Now, $u=\frac{1}{4}$

$\Rightarrow \quad \frac{1}{x-y}=\frac{1}{4}$

$\Rightarrow \quad x-y=4 \qquad$ ---(v)

And, $v = \frac{1}{8}$

$\Rightarrow \quad \frac{1}{x+y} = \frac{1}{8}$

$\Rightarrow \quad x + y = 8 \qquad$ ---(vi)

Adding equation (v) and equation (vi), we get

$2x = 4 + 8$

$\Rightarrow \quad x = \frac{12}{2} = 6$

Putting $x = 16$ in equation (vi), we get

$6 + y = 8$

$\Rightarrow \quad y = 8 - 6 = 2$

Hence, speed of the boat in still water = 6 km/hr

Speed of the stream = 2 km/hr

Q11

Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Solution

Let the speed of the train be x km/hr and that of the bus be y km/hr. We have the following cases:

CaseI When Roohi travels 60 km by train and the rest by bus:
In this case, we have

Time taken by Roohi to travel 60 km by train = $\frac{60}{x}$ hrs

Time taken by Roohi to travel $(300-60) = 240$ km by bus = $\frac{240}{y}$ hrs

$\therefore$ Total time taken by Roohi to cover 300 km = $\frac{60}{x} + \frac{240}{y}$

It is given that the total time taken is 4 hours.

$$\therefore \quad \frac{60}{x} + \frac{240}{y} = 4$$

$$\Rightarrow \quad 4\left[\frac{15}{x} + \frac{60}{y}\right] = 4$$

$$\Rightarrow \quad \frac{15}{x} + \frac{60}{y} = 1 \quad \text{---(i)}$$

CaseII When Roohi travels 100 km by train and the rest by bus:
In this case, we have

Time taken by Roohi to travel 100 km by train = $\frac{100}{x}$ hrs

Time taken by Roohi to travel $(300-100) = 200$ km by bus = $\frac{200}{y}$ hrs

In this case, total time of the journey is 4 hrs 10 minutes

$$\therefore \quad \frac{100}{x} + \frac{200}{y} = 4 \text{ hrs } 10 \text{ minutes}$$

$$\Rightarrow \quad \frac{100}{x} + \frac{200}{y} = 4\frac{1}{6}$$

$$\Rightarrow \quad \frac{100}{x} + \frac{200}{y} = \frac{25}{6}$$

$$\Rightarrow \quad 25\left(\frac{4}{x} + \frac{8}{y}\right) = \frac{25}{6}$$

$$\Rightarrow \quad \frac{4}{x} + \frac{8}{y} = \frac{1}{6}$$

$$\Rightarrow \quad 6\left(\frac{4}{x} + \frac{8}{y}\right) = 1$$

$$\Rightarrow \quad \frac{24}{x} + \frac{48}{y} = 1 \quad \text{---(ii)}$$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, in equation(i) and(ii), we get

$$15u + 60v = 1 \quad \text{---(iii)}$$

$$24u + 48v = 1 \quad \text{---(iv)}$$

By cross-multiplication, we have

$$\frac{u}{60 \times (-1) - 48 \times (-1)} = \frac{-v}{15 \times (-1) - 24 \times (-1)} = \frac{1}{15 \times 48 - 60 \times 24}$$

$$\Rightarrow \quad \frac{u}{-60+48} = \frac{-v}{-15+24} = \frac{1}{720-1440}$$

$\Rightarrow \quad \frac{u}{-12} = \frac{-v}{9} = \frac{1}{-720}$

$\Rightarrow \quad \frac{u}{-12} = \frac{1}{-720}$ and $\frac{-v}{9} = \frac{1}{-720}$

$\Rightarrow \quad u = \frac{-12}{-720}$ and $v = \frac{-9}{-720}$

$\Rightarrow \quad u = \frac{1}{60}$ and $v = \frac{1}{80}$

Now, $u = \frac{1}{60}$

$\Rightarrow \quad \frac{1}{x} = \frac{1}{60}$

$\Rightarrow \quad x = 60$ km/hr

And, $v = \frac{1}{80}$

$\Rightarrow \quad \frac{1}{y} = \frac{1}{80}$

$\Rightarrow \quad y = 80$ km/hr

Hence, speed of the train = 60 km/hr

Speed of the car = 80 km/hr.

Q12

Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

Solution

Let the speed of Ritu in still water and the speed of stream be x km/h and y km/h respectively.

Speed of Ritu while rowing upstream = (x - y) km/h
Speed of Ritu while rowing downstream = (x + y) km/h

According to the question,

$2(x+y) = 20$

$\Rightarrow x + y = 10 \quad \ldots (1)$

$2(x-y) = 4$

$\Rightarrow x - y = 2 \quad \ldots (2)$

Adding equations (1) and (2), we obtain:

$2x = 12$

$\Rightarrow x = 6$

Putting the value of x in equation (1), we obtain:

y = 4

Thus, Ritu's speed in still water is 6 km/h and the speed of the current is 4 km/h.

Q13

A motor boat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of the stream.

Solution

Let the speed of boat in still water be x km / hr and
the speed of the stream be y km/hr.

∴ Speed of the boat upstream = $(x - y)$ km / hr

Speed of the boat downstream = $(x - y)$ km / hr

Now, time taken by boat to travel 30 km upstream $= \frac{30}{x-y}$

Time taken by boat to travel 28 km downstream $= \frac{28}{x+y}$

Then, we have $\frac{30}{x-y} + \frac{28}{x+y} = 7$ ....(i)

Also, time taken by boat to travel 21 km upstream $= \frac{21}{x-y}$

Time taken by boat to travel 21 km downstream $= \frac{21}{x+y}$

Then, we have $\frac{21}{x-y} + \frac{21}{x+y} = 5$ ....(ii)

Putting $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = v$ in equations (i) and (ii), we get

$30u + 28v = 7$

$\Rightarrow 30u + 28v - 7 = 0$ ....(iii)

$21u + 21v = 5$

$\Rightarrow 21u + 21v - 5 = 0$ ....(iv)

By cross multiplication, we have

$$\frac{u}{28 \times (-5) - 21 \times (-7)} = \frac{-v}{30 \times (-5) - 21 \times (-7)} = \frac{1}{30 \times 21 - 21 \times 28}$$

$$\Rightarrow \frac{u}{-140 + 147} = \frac{-v}{-150 + 147} = \frac{1}{630 - 588}$$

$$\Rightarrow \frac{u}{7} = \frac{v}{3} = \frac{1}{42}$$

Now, $\Rightarrow \frac{u}{7} = \frac{1}{42} \Rightarrow 42u = 7 \Rightarrow u = \frac{7}{42} = \frac{1}{6} \Rightarrow \frac{1}{x-y} = \frac{1}{6}$

$\Rightarrow x - y = 6$ ....(iii)

And, $\frac{v}{3} = \frac{1}{42} \Rightarrow 42v = 3 \Rightarrow v = \frac{3}{42} = \frac{1}{14} \Rightarrow \frac{1}{x+y} = \frac{1}{14}$

$\Rightarrow x + y = 14$ ....(iv)

Adding (i) and (ii), we get $2x = 20 \Rightarrow x = 10$

$\Rightarrow 10 + y = 14 \Rightarrow y = 4$

Thus, the speed of the boat in still water is 10 km/hr
and the speed of the stream is 4 km/hr.

Q14

Abdul travelled 300 km by train and 200 km by taxi, it took him 5 hours 30 minutes. But if he travels 260 km by train and 240 km by taxi he takes 6 minutes longer. Find the speed of the train and that of the taxi.

Solution

Let the speed of the train be x km/hr and that of the taxi be y km/hr. We have the followin cases:

CaseI When Abdul travels 300 km by train and 200 km by taxi:
In this case, we have

Time taken by Abdul to travel 300 km by train = $\frac{300}{x}$ hrs

Time taken by Abdul to travel 200 km by taxi = $\frac{200}{y}$ hrs

$\therefore$ Total time taken by Abdul = $\frac{300}{x}+\frac{200}{y}$

It is given that the total time taken is 5 hours 30 minutes.

$$\therefore \quad \frac{300}{x}+\frac{200}{y} = 5 \text{ hours } 30 \text{ minutes}$$

$$\Rightarrow \quad \frac{300}{x}+\frac{200}{y} = 5\frac{1}{2}$$

$$\Rightarrow \quad \frac{300}{x}+\frac{200}{y} = \frac{11}{2}$$

$$\Rightarrow \quad \frac{600}{x}+\frac{400}{y} = 11 \qquad \text{---(i)}$$

CaseII When Abdul travels 260 km by train and 240 km by taxi:
In this case, we have

Time taken by Abdul to travel 200 km by train = $\frac{260}{x}$ hrs

Time taken by Abdul to travel 240 km by taxi = $\frac{240}{y}$ hrs

In this case, total time of the journey is $(5 \text{ hours } 30 \text{ minutes} + 6 \text{ minutes})$

$$= 5\frac{1}{2}+\frac{1}{10}$$

$$= \frac{11}{2}+\frac{1}{10}$$

$$= \frac{55+1}{10}$$

$$= \frac{56}{10}$$

$$= \frac{28}{5} \text{ hrs}$$

$$\therefore \quad \frac{260}{x}+\frac{240}{y} = \frac{28}{5}$$

$$\Rightarrow \quad 4\left(\frac{65}{x}+\frac{60}{y}\right) = \frac{28}{5}$$

$$\Rightarrow \quad \frac{65}{x}+\frac{60}{y} = \frac{7}{5}$$

$$\Rightarrow \quad \frac{65\times 5}{x}+\frac{60\times 5}{y} = 7$$

$$\Rightarrow \quad \frac{325}{x}+\frac{300}{y} = 7 \qquad \text{---(ii)}$$

Putting $\frac{1}{x}=u$ and $\frac{1}{y}=v$, in equation (i) and (ii), we get

$$600u + 400v = 11$$

$$\Rightarrow \quad 600u + 400v - 11 = 0 \qquad \text{---(iii)}$$

And, $325u + 300v = 7$

$\Rightarrow 325u + 300v - 7 = 0$ ---(iv)

By cross-multiplication, we have

$$\frac{u}{400\times(-7)-(-11)\times 300} = \frac{-v}{600\times(-7)-(-11)\times 325} = \frac{1}{600\times 300 - 400\times 325}$$

$$\Rightarrow \frac{u}{-2800+3300} = \frac{-v}{-4200+3575} = \frac{1}{180000-130000}$$

$$\Rightarrow \frac{u}{500} = \frac{-v}{-625} = \frac{1}{50000}$$

$$\Rightarrow \frac{u}{500} = \frac{v}{625} = \frac{1}{50000}$$

$$\Rightarrow \frac{u}{500} = \frac{1}{50000} \text{ and } \frac{v}{625} = \frac{1}{50000}$$

$$\Rightarrow u = \frac{500}{50000} \text{ and } v = \frac{625}{50000}$$

$$\Rightarrow u = \frac{1}{100} \text{ and } v = \frac{1}{80}$$

Now, $u = \frac{1}{100}$

$\Rightarrow \frac{1}{x} = \frac{1}{100}$

$\Rightarrow x = 100$

And, $v = \frac{1}{80}$

$\Rightarrow \frac{1}{y} = \frac{1}{80}$

$\Rightarrow y = 80$

Hence, speed of the train = 100 km/hr

Speed of the taxi = 80 km/hr.

Q15

A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Solution

Let the speed of the train be x km/h and the time taken by train to travel the given distance be t hours and the distance to travel be d km.

Now, $\text{Speed} = \dfrac{\text{Distance travelled}}{\text{Time taken to travel that distance}}$

$x = \dfrac{d}{t}$

Or, d = xt ... (1)

According to the question,

$(x+10) = \dfrac{d}{(t-2)}$

$(x+10)(t-2) = d$

$xt + 10t - 2x - 20 = d$

By using equation (1), we obtain:

$-2x + 10t = 20$ ... (2)

$(x-10) = \dfrac{d}{(t+3)}$

$(x-10)(t+3) = d$

$xt - 10t + 3x - 30 = d$

By using equation (1), we obtain:

3x - 10t = 30 ... (3)

Adding equations (2) and (3), we obtain:

x = 50

Substituting the value of x in equation (2), we obtain:

(-2) x (50) + 10t = 20

-100 + 10t = 20

10t = 120

t = 12

From equation (1), we obtain:

d = xt = 50 x 12 = 600

Thus, the distance covered by the train is 600 km.

Concept insight: To solve this problem, it is very important to remember the relation $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$. Now, all these three quantities are unknown. So, we will represent these

by three different variables. By using the given conditions, a pair of equations will be obtained. Mind one thing that the equations obtained will not be linear. But they can

be reduced to linear form by using the fact that $\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$. Then two linear equations can be formed which can

be solved easily by elimination method.

Q16

Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hours. What are the speeds of two cars?

Solution

Let the speed of first car and second car be u km/h and v km/h respectively.

According to the question,

$5(u-v) = 100$

$\Rightarrow u - v = 20$ ...(1)

$1(u+v) = 100$

$\Rightarrow u + v = 100$ ...(2)

Adding equations (1) and (2), we obtain:

$2u = 120$

$u = 60$

Substituting the value of u in equation (2), we obtain:

v = 40

Hence, speed of the first car is 60 km/h and speed of the second car is 40 km/h.

Q17

While covering a distance of 30 km. Ajeet takes 2 hours more than Amit. If Ajeet doubles his speed, he would take 1 hour less than Amit. Find their speeds of walking.

Solution

Let the speed of Ajeet and Amit be x km/hr and y km/hr respectively. Then,

Time taken by Ajeet to cover 30 km = $\frac{30}{x}$ hrs

And, Time taken by Amit to cover 30 km = $\frac{30}{y}$ hrs

By the given conditions, we have

$$\frac{30}{x}-\frac{30}{y}=2$$

$$\Rightarrow \frac{15}{x}-\frac{15}{y}=1 \quad ---(i)$$

If Ajeet doubles his place, then speed of Ajeet is $2x$ km/hr

$\therefore$ Times taken by Ajeet to cover 30 km = $\frac{30}{2x}$ hrs

Times taken by Amit to cover 30 km = $\frac{30}{y}$ hrs

According to the given conditions, we have

$$\frac{30}{y}-\frac{30}{2x}=1$$

$$\Rightarrow \frac{30}{y}-\frac{15}{x}=1 \quad ---(ii)$$

Putting $\frac{1}{x}=u$ and $\frac{1}{y}=v$, in equations(i) and(ii), we get

$$15u-15v=1 \quad ---(iii)$$

$$30v-15u=1 \quad ---(iv)$$

Adding equation(iii) and equation(iv), we get

$$30v-15v=1+1$$

$$\Rightarrow 15v=2$$

$$\Rightarrow v=\frac{2}{15}$$

Putting $v=\frac{2}{15}$ in equation(iii), we get

$$15u-15\times\frac{2}{15}=1$$

$$\Rightarrow 15u-2=1$$

$$\Rightarrow 15u=1+2$$

$$\Rightarrow 15u=3$$

$$\Rightarrow u=\frac{3}{15}=\frac{1}{5}$$

Now, $u=\frac{1}{5}$

$$\Rightarrow \frac{1}{x}=\frac{1}{5}$$

$$\Rightarrow x=5$$

And, $v=\frac{2}{15}$

$$\Rightarrow \frac{1}{y}=\frac{2}{15}$$

$$\Rightarrow y=\frac{15}{2}=7.5$$

Hence, Ajeet's speed = 5 km/hr and, Amit's speed = 7.5 km/hr.

Q18

A takes 3 hours more than B to walk a distance of 30 km. But, if A doubles his pace (speed) he is ahead of B by $1\frac{1}{2}$ hours. Find the speeds of A and B.

Solution

Let the speed A and B be x km/hr and y km/hr respectively. Then,

Time taken by A to cove 30 km $= \frac{30}{x}$ hrs

And, Time taken by B to cover 30 km $= \frac{30}{y}$ hrs

By the given conditions, we have

$$\frac{30}{x} - \frac{30}{y} = 3$$

$$\Rightarrow \frac{10}{x} - \frac{10}{y} = 1 \quad \text{---(i)}$$

If A doubles his pace, then speed of A is $2x$ km/hr

$\therefore$ Time taken by A to cover 30 km $= \frac{30}{2x}$ hrs

Time taken by B to cover 30 km $= \frac{30}{y}$ hrs

According to the given conditions, we have

$$\frac{30}{y} - \frac{30}{2x} = 1\frac{1}{2}$$

$$\Rightarrow \frac{30}{y} - \frac{30}{2x} = \frac{3}{2}$$

$$\Rightarrow \frac{10}{y} - \frac{10}{2x} = \frac{1}{2}$$

$$\Rightarrow \frac{10}{y} - \frac{5}{x} = \frac{1}{2}$$

$$\Rightarrow \frac{-5}{x} + \frac{10}{y} = \frac{1}{2}$$

$$\Rightarrow \frac{-10}{x} + \frac{20}{y} = 1 \quad \text{---(ii)}$$

Putting $\frac{1}{x} = u$ and $\frac{1}{y} = v$, in equatin (i) and (ii), we get

$$\Rightarrow 10u - 10v = 1 \quad \text{---(iii)}$$

$$-10u + 20v = 1 \quad \text{---(iv)}$$

Adding equations (iii) and (iv), we get

$$-10v + 20v = 1 + 1$$

$$\Rightarrow 10v = 2$$

$$\Rightarrow v = \frac{2}{10} = \frac{1}{5}$$

Putting $v = \frac{1}{5}$ in equation (iii), we get

$$10u - 10 \times \frac{1}{5} = 1$$

$$\Rightarrow 10u - 2 = 1$$

$$\Rightarrow 10u = 1 + 2$$

$$\Rightarrow 10u = 3$$

$$\Rightarrow u = \frac{3}{10}$$

Now, $u = \frac{3}{10}$

$$\Rightarrow \frac{1}{x} = \frac{3}{10}$$

$$\Rightarrow x = \frac{10}{3}$$

And, $v = \frac{1}{5}$

$$\Rightarrow \frac{1}{y} = \frac{1}{5}$$

$$\Rightarrow y = 5$$

Hence, A's speed $= \frac{10}{3}$ km/hr and B's speed $= 5$ km/hr.