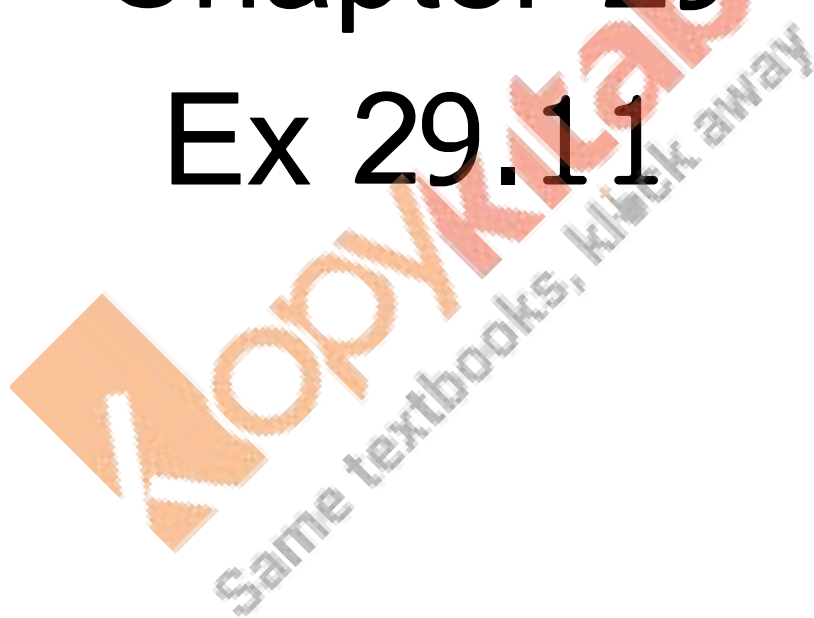


RD Sharma
Solutions
Class 11 Maths
Chapter 29
Ex 29.11



Limits Ex 29.11 Q1

$$\begin{aligned}\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n &= e^{\lim_{n \rightarrow \infty} \left(\frac{x}{n}\right)^n} \\ &= e^x\end{aligned}$$

Limits Ex 29.11 Q2

$$\begin{aligned}\lim_{x \rightarrow 0^+} \left\{1 + \tan^2 \sqrt{x}\right\}^{1/2x} \\ &= e^{\lim_{x \rightarrow 0^+} \left\{\frac{\tan^2 \sqrt{x}}{2x}\right\}} \\ &= e^{\lim_{x \rightarrow 0^+} \left\{\frac{\tan^2 \sqrt{x}}{2x}\right\}} \\ &= e^{\lim_{x \rightarrow 0^+} \left\{\frac{\sin^2 \sqrt{x}}{2x \cos^2 \sqrt{x}}\right\}} \\ &= e^{\frac{1}{2} \lim_{x \rightarrow 0^+} \left\{\left(\frac{\sin \sqrt{x}}{\sqrt{x}}\right)^2\right\} \lim_{x \rightarrow 0^+} \left\{\frac{1}{\cos^2 \sqrt{x}}\right\}} \\ &= e^{\frac{1}{2}} \\ &= \sqrt{e}\end{aligned}$$

Limits Ex 29.11 Q3

$$\begin{aligned}\lim_{x \rightarrow 0} (\cos x)^{1/\sin x} &= \lim_{x \rightarrow 0} (1 + \cos x - 1)^{1/\sin x} \\ &= \lim_{x \rightarrow 0} (1 - (1 - \cos x))^{1/\sin x} \\ &= \lim_{x \rightarrow 0} \left(1 - 2 \sin^2 \left(\frac{x}{2}\right)\right)^{1/\sin x} \\ &= e^{\lim_{x \rightarrow 0} \left(-2 \sin^2 \left(\frac{x}{2}\right)\right) \times (1/\sin x)}\end{aligned}$$

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$$\begin{aligned}
 & \lim_{x \rightarrow 0} \left(\frac{-2 \sin^2\left(\frac{x}{2}\right)}{\sin x} \right) \\
 &= e \\
 & \lim_{x \rightarrow 0} \left(\frac{-2 \sin^2\left(\frac{x}{2}\right)}{2 \sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right)} \right) \\
 &= e \\
 &= e^{\lim_{x \rightarrow 0} -\tan x} \\
 &= e^0 \\
 &= 1
 \end{aligned}$$

Limits Ex 29.11 Q4

$$\begin{aligned}
 & \lim_{x \rightarrow 0} (\cos x + \sin x)^{1/x} \\
 &= \lim_{x \rightarrow 0} (1 + (\cos x + \sin x - 1))^{1/x} \\
 &= e^{\lim_{x \rightarrow 0} \frac{(\cos x + \sin x - 1)}{x}} \\
 &= e^{\lim_{x \rightarrow 0} \frac{(\sin x - (1 - \cos x))}{x}} \\
 &= e^{\lim_{x \rightarrow 0} \frac{(\sin x - 2 \sin^2(x/2))}{x}} \\
 &= e^{\lim_{x \rightarrow 0} \frac{\sin x}{x} - \lim_{x \rightarrow 0} \frac{2 \sin(x/2) \sin(x/2)}{2 \left(\frac{x}{2}\right)}} \\
 &= e^{\lim_{x \rightarrow 0} \frac{\sin x}{x} - \lim_{x \rightarrow 0} \frac{\sin(x/2) \sin(x/2)}{\left(\frac{x}{2}\right)}} \\
 &= e \\
 &= e^{1-0} \\
 &= e
 \end{aligned}$$

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Limits Ex 29.11 Q5

$$\begin{aligned} & \lim_{x \rightarrow 0} (\cos x + a \sin bx)^{1/x} \\ &= \lim_{x \rightarrow 0} (1 + (\cos x + a \sin bx - 1))^{1/x} \\ &= e^{\lim_{x \rightarrow 0} \frac{(\cos x + a \sin bx - 1)}{x}} \\ &= e^{\lim_{x \rightarrow 0} \frac{(a \sin bx - (1 - \cos x))}{x}} \\ &= e^{\lim_{x \rightarrow 0} \frac{(a \sin bx - 2 \sin^2(x/2))}{x}} \\ &= \lim_{x \rightarrow 0} \frac{ab \sin bx}{bx} \lim_{x \rightarrow 0} \frac{2 \sin(x/2) \sin(x/2)}{2 \left(\frac{x}{2}\right)} \\ &= e \\ &= \lim_{x \rightarrow 0} \frac{ab \sin bx}{bx} \lim_{x \rightarrow 0} \frac{\sin(x/2) \sin(x/2)}{\left(\frac{x}{2}\right)} \\ &= e^{ab-0} \\ &= e^{ab} \end{aligned}$$

Limits Ex 29.11 Q6

$$\begin{aligned} & \lim_{x \rightarrow \infty} \left\{ \frac{x^2 + 2x + 3}{2x^2 + x + 5} \right\}^{\frac{3x-2}{3x+2}} \\ &= e^{\lim_{x \rightarrow \infty} \left\{ \left(\frac{3x-2}{3x+2} \right) \ln \left(\frac{x^2 + 2x + 3}{2x^2 + x + 5} \right) \right\}} \\ &= e^{\lim_{x \rightarrow \infty} \left\{ \left(\frac{3-\frac{2}{x}}{3+\frac{2}{x}} \right) \left(\ln \left(\frac{x^2 + 2x + 3}{2x^2 + x + 5} \right) \right) \right\}} \end{aligned}$$

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$$\begin{aligned}
 & \lim_{x \rightarrow \infty} \left\{ \left(\frac{3-\frac{2}{x}}{3+\frac{2}{x}} \right)^{\left(\ln \left(\frac{1+\frac{2}{x}+\frac{3}{x^2}}{2+\frac{1}{x}+\frac{5}{x^2}} \right) \right)} \right\} \\
 &= e \\
 &= e^{1 \ln\left(\frac{1}{2}\right)} \\
 &= \frac{1}{2}
 \end{aligned}$$

Limits Ex 29.11 Q7

$$\begin{aligned}
 & \lim_{x \rightarrow 1} \left\{ \frac{x^3 + 2x^2 + x + 1}{x^2 + 2x + 3} \right\}^{\frac{1 - \cos(x-1)}{(x-1)^2}} \\
 &= e^{\lim_{x \rightarrow 1} \left\{ \frac{1 - \cos(x-1)}{(x-1)^2} \ln \left(\frac{x^3 + 2x^2 + x + 1}{x^2 + 2x + 3} \right) \right\}} \\
 &= e^{\lim_{x \rightarrow 1} \left\{ \frac{2 \sin^2(x-1)}{4 \left(\frac{x-1}{2} \right)^2} \ln \left(\frac{x^3 + 2x^2 + x + 1}{x^2 + 2x + 3} \right) \right\}} \\
 &= e^{\frac{2}{4} \ln\left(\frac{5}{6}\right)} \\
 &= e^{\ln\left(\frac{5}{6}\right)^{\frac{1}{2}}} \\
 &= \left(\frac{5}{6}\right)^{\frac{1}{2}} = \sqrt{\frac{5}{6}}
 \end{aligned}$$

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Limits Ex 29.11 Q8

$$\lim_{x \rightarrow 0} \left\{ \frac{e^x + e^{-x} - 2}{x^2} \right\}^{1/x^2}$$

$$= e^{\lim_{x \rightarrow 0} \left\{ \frac{e^x + e^{-x} - 2}{x^2} \right\}}$$

Applying L'Hospital's Rule

$$= e^{\lim_{x \rightarrow 0} \left\{ \frac{2+e^x(-2+x)+x}{2(-1+e^x)x^2} \right\}}$$

Applying L'Hospital's Rule again

$$= e^{\frac{1}{2} \lim_{x \rightarrow 0} \left\{ \frac{1+e^x(-1+x)}{x(-2+e^x(2+x))} \right\}}$$

Applying L'Hospital's Rule again

$$= e^{\frac{1}{2} \lim_{x \rightarrow 0} \left\{ \frac{e^x x}{-2+e^x(2+4x+x^2)} \right\}}$$

Applying L'Hospital's Rule again

$$= e^{\frac{1}{2} \lim_{x \rightarrow 0} \left\{ \frac{1+x}{6+6x+x^2} \right\}}$$

$$= e^{\frac{1}{2} \left\{ \frac{\lim_{x \rightarrow 0} (1+x)}{\lim_{x \rightarrow 0} (6+6x+x^2)} \right\}}$$

$$= e^{\frac{1}{12}}$$

$$= \sqrt[12]{e}$$

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Limits Ex 29.11 Q9

$$\begin{aligned}
 & \lim_{x \rightarrow a} \left\{ \frac{\sin x}{\sin a} \right\}^{\frac{1}{x-a}} \\
 &= \lim_{x \rightarrow a} \left\{ 1 + \left(\frac{\sin x}{\sin a} - 1 \right) \right\}^{\frac{1}{x-a}} \\
 &= e^{\lim_{x \rightarrow a} \left\{ \frac{\left(\frac{\sin x}{\sin a} - 1 \right)}{x-a} \right\}} \\
 &= e^{\lim_{x \rightarrow a} \left\{ \frac{\left(\frac{\sin x - \sin a}{\sin a} \right)}{x-a} \right\}} \\
 &= e^{\lim_{x \rightarrow a} \left\{ \frac{\sin x - \sin a}{\sin a(x-a)} \right\}} \\
 &= e^{\lim_{x \rightarrow a} \left\{ \frac{2 \cos \left(\frac{x+a}{2} \right) \sin \left(\frac{x-a}{2} \right)}{\sin a(x-a)} \right\}} \\
 &= e^{\lim_{x \rightarrow a} \left\{ \frac{2 \cos \left(\frac{x+a}{2} \right)}{\sin a} \right\} \lim_{x \rightarrow a} \left\{ \frac{\sin \left(\frac{x-a}{2} \right)}{2 \left(\frac{x-a}{2} \right)} \right\}} \\
 &= e^{\frac{2 \cos a}{2 \sin a}} \\
 &= e^{\cot a}
 \end{aligned}$$

Limits Ex 29.11 Q10

$$\begin{aligned}
 & \lim_{x \rightarrow \infty} \left\{ \frac{3x^2 + 1}{4x^2 - 1} \right\}^{\frac{x^3}{1+x}} \\
 &= \lim_{x \rightarrow \infty} \left\{ 1 + \frac{-x^2 + 2}{4x^2 - 1} \right\}^{\frac{x^3}{1+x}} \\
 &= e^{\lim_{x \rightarrow \infty} \left\{ \frac{(-x^2 + 2)(x^3)}{4x^2 - 1} \right\}} \\
 &= e^{\lim_{x \rightarrow \infty} \left\{ \frac{-x^5 + 2x^3}{4x^2 - 1 + 4x^3 - x} \right\}} \\
 &= e^{-\infty} \\
 &= 0
 \end{aligned}$$

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