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Solutions

Class 10 Maths

Chapter 6

Ex 6.2

Q1) If cosθ=45 $\cos\theta = \frac{4}{5}$, find all other trigonometric ratios of angle ΘΘ.

Solution:

We have:

sinΘ=√1−cos²Θ=√1−(45)² $\sin\Theta = \sqrt{1-\cos^2\Theta} = \sqrt{1-(\frac{4}{5})^2}$

= √1−1625 $\sqrt{1-\frac{16}{25}}$

= √25−1625 $\sqrt{\frac{25-16}{25}}$

= √925=35 $\sqrt{\frac{9}{25}} = \frac{3}{5}$

Therefore, sinΘ=35 $\sin\Theta = \frac{3}{5}$

tanΘ=sinΘcosΘ=3545=34 secΘ=1cosΘ=145=54 $\tan\Theta = \frac{\sin\Theta}{\cos\Theta} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$ $\sec\Theta = \frac{1}{\cos\Theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$

i.e. cosecΘ=1secΘ=135=53 cotΘ=1tanΘ=134=43
$\mathrm{cosec}\Theta = \frac{1}{\sec\Theta} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$ $\cot\Theta = \frac{1}{\tan\Theta} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$

Q2) If sinΘ=1√2 $\sin\Theta = \frac{1}{\sqrt{2}}$, find all other trigonometric ratios of angle ΘΘ.

Solution:

We have,

cosΘ=√1−sin²Θ=√1−(1√2)² $\cos\Theta = \sqrt{1-\sin^2\Theta} = \sqrt{1-(\frac{1}{\sqrt{2}})^2}$

= √1−12 $\sqrt{1-\frac{1}{2}}$

= √2−12 $\sqrt{\frac{2-1}{2}}$

= cosΘ=1√2 $\cos\Theta = \frac{1}{\sqrt{2}}$

= tanΘ=sinΘcosΘ=1√21√2=1 $\tan\Theta = \frac{\sin\Theta}{\cos\Theta} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

= cosecΘ=1sinΘ=11√2=√2 $\mathrm{cosec}\Theta = \frac{1}{\sin\Theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

= secΘ=1cosΘ=11√2=√2 $\sec\Theta = \frac{1}{\cos\Theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

= cotΘ=1tanΘ=11=1 $\cot\Theta = \frac{1}{\tan\Theta} = \frac{1}{1} = 1$

Q3) If tanΘ=1√2 $\tan\Theta = \frac{1}{\sqrt{2}}$ **, find the value of cosec²Θ−sec²Θcosec²Θ+cot²Θ** $\frac{\mathrm{cosec}^2\Theta - \sec^2\Theta}{\mathrm{cosec}^2\Theta + \cot^2\Theta}$**.**

Solution:

We know that secΘ=√1+tan²Θ $\sec\Theta = \sqrt{1+\tan^2\Theta}$

=√1+(1√2)² $\sqrt{1+\left(\frac{1}{\sqrt{2}}\right)^2}$

= √1+12=√32 $\sqrt{1+\frac{1}{2}} = \sqrt{\frac{3}{2}}$

= cotΘ=1tanΘ=11√2=√2 $\cot\Theta = \frac{1}{\tan\Theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

= cosecΘ=√1+cot²Θ=√1+2=√3 $\mathrm{cosec}\Theta = \sqrt{1+\cot^2\Theta} = \sqrt{1+2} = \sqrt{3}$

Substituting it in equation (1) we get

= (√3)²−(√32)²(√3)²+(√2)²=3−323+2=325=310 $\frac{(\sqrt{3})^2-(\sqrt{\frac{3}{2}})^2}{(\sqrt{3})^2+(\sqrt{2})^2} = \frac{3-\frac{3}{2}}{3+2} = \frac{\frac{3}{2}}{5} = \frac{3}{10}$

Q4) If tanΘ=34 $\tan\Theta = \frac{3}{4}$ **, find the value of 1−cosΘ1+cosΘ** $\frac{1-\cos\Theta}{1+\cos\Theta}$

Solution:

We know that

secΘ=√1+tan²Θ $\sec\Theta = \sqrt{1+\tan^2\Theta}$

= √1+(34)² $\sqrt{1+\left(\frac{3}{4}\right)^2}$

= √1+916 $\sqrt{1+\frac{9}{16}}$

=√16+916 $\sqrt{\frac{16+9}{16}}$

= √2516 $\sqrt{\frac{25}{16}}$

= secΘ=54 $\sec\Theta = \frac{5}{4}$

= secΘ=1cosΘ=154=45=cosΘ $\sec\Theta = \frac{1}{\cos\Theta} = \frac{1}{\frac{5}{4}} = \frac{4}{5} = \cos\Theta$

Therefore, Weget 1−451+45=1595=19 We get $\frac{1-\frac{4}{5}}{1+\frac{4}{5}} = \frac{\frac{1}{5}}{\frac{9}{5}} = \frac{1}{9}$

Q5) If tanΘ=125 $\tan\Theta = \frac{12}{5}$, find the value of 1+sinΘ1−sinΘ $\frac{1+\sin\Theta}{1-\sin\Theta}$.

Solution:

cotΘ=1tanΘ=1125=512 $\cot\Theta = \frac{1}{\tan\Theta} = \frac{1}{\frac{12}{5}} = \frac{5}{12}$

= cosecΘ=√1+cot²Θ=√1+[512]²=√144+25144=√169144=1312

$\text{cosec}\Theta = \sqrt{1+\cot^2\Theta} = \sqrt{1+[\frac{5}{12}]^2} = \sqrt{\frac{144+25}{144}} = \sqrt{\frac{169}{144}} = \frac{13}{12}$

= sinΘ=1cosecΘ=11312=1213 $\sin\Theta = \frac{1}{\text{cosec}\Theta} = \frac{1}{\frac{13}{12}} = \frac{12}{13}$

i.e. Weget 1+12131−1213=13+121813−1218=251=25 We get $\frac{1+\frac{12}{13}}{1-\frac{12}{13}} = \frac{\frac{13+12}{18}}{\frac{13-12}{18}} = \frac{25}{1} = 25$.

Q6) If cotΘ=1√3 $\cot\Theta = \frac{1}{\sqrt{3}}$, find the value of 1−cos²Θ2−sin²Θ $\frac{1-\cos^2\Theta}{2-\sin^2\Theta}$.

Solution:

cosecΘ=√1+cot²Θ=√1+13=√43 $\text{cosec}\Theta = \sqrt{1+\cot^2\Theta} = \sqrt{1+\frac{1}{3}} = \sqrt{\frac{4}{3}}$

= cosecΘ=2√3 $\text{cosec}\Theta = \frac{2}{\sqrt{3}}$

= sinΘ=1cosecΘ=12√3=√32 $\sin\Theta = \frac{1}{\text{cosec}\Theta} = \frac{1}{\frac{2}{\sqrt{3}}} = \frac{\sqrt{3}}{2}$

= and 1cotΘ=sinΘcosΘ=cosΘ=sinΘ×cotΘ=√32×1√3=12

and $\frac{1}{\cot\Theta} = \frac{\sin\Theta}{\cos\Theta} = \cos\Theta = \sin\Theta \times \cot\Theta = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{1}{2}$

Therefore, on substituting we get

= 1−(12)²2−(√32)²=1−142−34=34 54=35 $\frac{1-(\frac{1}{2})^2}{2-(\frac{\sqrt{3}}{2})^2} = \frac{1-\frac{1}{4}}{2-\frac{3}{4}} = \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{5}$.

Q7) If cosecA=√2$\text{cosec}A = \sqrt{2}$ **, find the value of 2sin²A+3cot²A4(tan²A−cos²A)** $\frac{2\sin^2 A+3\cot^2 A}{4(\tan^2 A-\cos^2 A)}$**.**

Solution:

We know that cotA=√cosec²A−1 $\cot A = \sqrt{\text{cosec}^2 A - 1}$

= √(2)²−1=√2−1 $\sqrt{(2)^2 - 1} = \sqrt{2-1}$ =1.

= tanA=1cotA=11=1 $\tan A = \frac{1}{\cot A} = \frac{1}{1} = 1$

= sinA=1cosecA=1√2 $\sin A = \frac{1}{\text{cosec}A} = \frac{1}{\sqrt{2}}$

= sinA=1√2 $\sin A = \frac{1}{\sqrt{2}}$

cosA=√1−sin²A=√1−(1√2)²=√1√2=1√2 $\cos A = \sqrt{1-\sin^2 A} = \sqrt{1-(\frac{1}{\sqrt{2}})^2} = \sqrt{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}}$

On substituting we get:

= 2[1√2]²+3[1]²4[[1]−[1√2]²]=2×12+34[1−12] $\frac{2[\frac{1}{\sqrt{2}}]^2+3[1]^2}{4[[1]-[\frac{1}{\sqrt{2}}]^2]} = \frac{2\times\frac{1}{2}+3}{4[1-\frac{1}{2}]}$

⇒1+34.12=42=2⇒ $\frac{1+3}{4.\frac{1}{2}} = \frac{4}{2} = 2$

Q8) If cotΘ=√3$\cot\Theta = \sqrt{3}$ **, find the value of cosec²Θ+cot²Θcosec²Θ−sec²Θ** $\frac{\text{cosec}^2\Theta+\cot^2\Theta}{\text{cosec}^2\Theta-\sec^2\Theta}$**.**

Solution:

cosecΘ=√1+cot²Θ=√1+(√3)²=√1+3=2 $\text{cosec}\Theta = \sqrt{1+\cot^2\Theta} = \sqrt{1+(\sqrt{3})^2} = \sqrt{1+3} = 2$

sinΘ=1cosecΘ=12cotΘ=cosΘsinΘcosΘ=cotΘ.sinΘ $\sin\Theta = \frac{1}{\text{cosec}\Theta} = \frac{1}{2}\cot\Theta = \frac{\cos\Theta}{\sin\Theta}$ $\cos\Theta = \cot\Theta.\sin\Theta$ ⇒cosΘ=√32⇒ $\cos\Theta = \frac{\sqrt{3}}{2}$

= secΘ=1cosΘ=2√3 $\sec\Theta = \frac{1}{\cos\Theta} = \frac{2}{\sqrt{3}}$

On substituting we get:

(2)²+(√3)²(2)²−(2√3)²=4+312−43=783 $\frac{(2)^2+(\sqrt{3})^2}{(2)^2-(\frac{2}{\sqrt{3}})^2} = \frac{4+3}{\frac{12-4}{3}} = \frac{7}{\frac{8}{3}}$

= 218 $\frac{21}{8}$

Q9) If 3cosΘ=1 $3\cos\Theta = 1$ **, find the value of 6sin²Θ+tan²Θ4cosΘ** $\frac{6\sin^2\Theta+\tan^2\Theta}{4\cos\Theta}$**.**

Solution:

cosΘ=13,sinΘ=√1−cos²Θ $\cos\Theta = \frac{1}{3}$, $\sin\Theta = \sqrt{1-\cos^2\Theta}$

= √1−19=√89=2√23 $\sqrt{1-\frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

tanΘ=sinΘcosΘ=2√23.13=2√2 $\tan\Theta = \frac{\sin\Theta}{\cos\Theta} = \frac{2\sqrt{2}}{3.\frac{1}{3}} = 2\sqrt{2}$

On substituting we get

6[2√23]²+(2√2)²4.13=163+843=16+24343 $\frac{6[\frac{2\sqrt{2}}{3}]^2+(2\sqrt{2})^2}{4.\frac{1}{3}} = \frac{\frac{16}{3}+8}{\frac{4}{3}} = \frac{\frac{16+24}{3}}{\frac{4}{3}}$

= 404=10 $\frac{40}{4} = 10$

Q10) If √3tanΘ=sinΘ $\sqrt{3}\tan\Theta = \sin\Theta$ **, find the value of sin²Θ−cos²Θ** $\sin^2\Theta - \cos^2\Theta$ **.**

Solution:

√3sinΘcosΘ=sinΘ $\sqrt{3}\frac{\sin\Theta}{\cos\Theta} = \sin\Theta$

$= \cos\Theta = \frac{\sqrt{3}}{3} \Rightarrow \frac{1}{\sqrt{3}}$

$= \sin\Theta = \sqrt{1-\cos^2\Theta} = \sqrt{1-\left(\frac{1}{\sqrt{3}}\right)^2}$

$= \sin^2\Theta - \cos^2\Theta = \left(\sqrt{\frac{2}{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2$

$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

Q11) If $\mathrm{cosec}\Theta = \frac{13}{12}$, find the value of $\frac{2\sin\Theta - 3\cos\Theta}{4\sin\Theta - 9\cos\Theta}$.

Solution:

$\sin\Theta = \frac{1}{\mathrm{cosec}\Theta} = \frac{1}{\frac{13}{12}} = \frac{12}{13}$

$= \cos\Theta = \sqrt{1-\sin^2\Theta} = \sqrt{1-\left[\frac{12}{13}\right]^2} = \sqrt{1-\frac{144}{169}}$

$= \sqrt{\frac{25}{169}} = \frac{5}{13}$

$\Rightarrow \frac{2.\frac{12}{13} - 3.\frac{5}{13}}{4.\frac{12}{13} - 9.\frac{5}{13}} = \frac{\frac{24-15}{13}}{\frac{48-15}{13}} = \frac{9}{3} = 3$

Q12) If $\sin\Theta + \cos\Theta = \sqrt{2}\cos(90^\circ - \Theta)$, find $\cot\Theta$.

Solution:

$\sin\Theta + \cos\Theta = \sqrt{2}\sin\Theta \quad [\cos(90-\Theta) = \sin\Theta]$

$\Rightarrow \cos\Theta = \sqrt{2}\sin\Theta - \sin\Theta$

$\Rightarrow \cos\Theta = \sin\Theta(\sqrt{2}-1)$

Divide both sides with $\sin\Theta$ we get

$= \frac{\cos\Theta}{\sin\Theta} = \frac{\sin\Theta}{\sin\Theta}(\sqrt{2}-1)$

$= \cot\Theta = \sqrt{2}-1$.

Q-13. If $2\sin^2\Theta - \cos^2\Theta = 2$, then find the value of Θ.

Solution.

$2\sin^2\Theta - \cos^2\Theta = 2$

$\Rightarrow 2\sin^2\Theta - (1 - \sin^2\Theta) = 2$ $\Rightarrow 2\sin^2\Theta - 1 + \sin^2\Theta = 2$ $\Rightarrow 3\sin^2\Theta = 3$ $\Rightarrow \sin^2\Theta = 1$ $\Rightarrow \sin\Theta = 1$ $\Rightarrow \sin\Theta = \sin 90^\circ$ $\Rightarrow \Theta = 90^\circ$

Q-14. If $\sqrt{3}\tan\Theta - 1 = 0$, find the value of $\sin^2\Theta - \cos^2\Theta$.

Solution.

$\sqrt{3}\tan\Theta - 1 = 0$ $\Rightarrow \sqrt{3}\tan\Theta = 1$ $\Rightarrow \sqrt{3}\tan\Theta = \frac{1}{\sqrt{3}}$ $\sqrt{3}\tan\Theta = \tan 30^\circ$ $\Theta = 30^\circ$

Now,

$\sin^2\Theta - \cos^2\Theta$

$= \sin^2(30^\circ) - \cos^2(30^\circ)$

$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{4} - \frac{3}{4} = \frac{-2}{4} = \frac{-1}{2}$