RD SHARMA

Solutions

Class 10 Maths

Chapter 6

Ex 6.1

Prove the following trigonometric identities

Q1: $(1-\text{Cos}^2 A)\ \text{Cosec}^2 A = 1$

Ans: $(1-\text{Cos}^2 A)\ \text{Cosec}^2 A = \text{Sin}^2 A\ \text{Cosec}^2 A$

$= (\text{Sin } A\ \text{Cosec } A)^2$

$= (\text{Sin } A \times (1/\text{Sin } A))^2$

$= (1)^2 = 1$

Q2: $(1 + \text{Cot}^2 A)\ \text{Sin}^2 A = 1$

Ans: We know, $\text{Cosec}^2 A - \text{Cot}^2 A = 1$

So,

$(1 + \text{Cot}^2 A)\ \text{Sin}^2 A = \text{Cosec}^2 A\ \text{Sin}^2 A$

$= (\text{Cosec } A\ \text{Sin } A)^2$

$= ((1/\text{Sin } A) \times \text{Sin } A)^2$

$= (1)^2 = 1$

Q3: **$\tan^2\theta\cos^2\theta$**$\tan^2\theta\ \cos^2\theta$ **$= 1-\cos^2\theta$**$1-\cos^2\theta$

A3: We know ,

$\sin^2\theta+\cos^2\theta=1$ $\sin^2\theta+\cos^2\theta = 1$

So,

$\tan^2\theta\cos^2\theta$ $\tan^2\theta\ \cos^2\theta$ $= (\tan\theta\times\cos\theta)^2$ $(\tan\theta \times \cos\theta)^2$

$=(\sin\theta\cos\theta\times\cos\theta)^2 = \left(\frac{\sin\theta}{\cos\theta} \times \cos\theta\right)^2$ $=(\sin\theta)^2 = (\sin\theta)^2$ $=\sin^2\theta = \sin^2\theta$ $1-\cos^2\theta$ $1-\cos^2\theta$

Q4: **$\text{cosec}\theta\sqrt{1-\cos^2\theta}=1$**$\text{cosec}\theta\sqrt{1-\cos^2\theta} = 1$

A4: We know ,

$\sin^2\theta+\cos^2\theta=1$ $\sin^2\theta+\cos^2\theta = 1$

So,

$\text{cosec}\theta\sqrt{1-\cos^2\theta}=\text{cosec}\theta\sqrt{\sin^2\theta}$ $\text{cosec}\theta\sqrt{1-\cos^2\theta} = \text{cosec}\theta\sqrt{\sin^2\theta}$

$= =\text{cosec}\theta\sin\theta = \text{cosec}\theta\ \sin\theta$

$= =1\sin\theta\ \sin\theta = \frac{1}{\sin\theta}\ \sin\theta$

$= 1$

Q5 : **$(\sec^2\theta-1)(\text{cosec}^2\theta-1)=1$**$(\sec^2\theta - 1)(\text{cosec}^2\theta - 1) = 1$

A5: We know that,

$(\sec^2\theta-\tan^2\theta)=1(\sec^2\theta-\tan^2\theta)=1$ $(\text{cosec}^2\theta-\cot^2\theta)=1(\text{cosec}^2\theta-\cot^2\theta)=1$

So,

$(\sec^2\theta-1)(\text{cosec}^2\theta-1)=\tan^2\theta\times\cot^2\theta(\sec^2\theta-1)(\text{cosec}^2\theta-1)=\tan^2\theta\times\cot^2\theta$ $=(\tan\theta\times\cot\theta)^2=(\tan\theta\times\cot\theta)^2$ $=(\tan\theta\times 1\tan\theta)^2=(\tan\theta\times\frac{1}{\tan\theta})^2$

$= 1^2 = 1$

Q6: $\tan\theta+1\tan\theta=\sec\theta\,\text{cosec}\theta\tan\theta+\frac{1}{\tan\theta}=\sec\theta\ \text{cosec}\theta$

A6: We know that,

$(\sec^2\theta-\tan^2\theta)=1(\sec^2\theta-\tan^2\theta)=1$

So,

$\tan\theta+1\tan\theta=\tan^2\theta+1\tan\theta\,\tan\theta+\frac{1}{\tan\theta}=\frac{\tan^2\theta+1}{\tan\theta}$ $=\sec^2\theta\tan\theta=\frac{\sec^2\theta}{\tan\theta}$ $=\sec\theta\sec\theta\tan\theta=\sec\theta\frac{\sec\theta}{\tan\theta}$ $=\sec\theta\,1\cos\theta\,\sin\theta\cos\theta=\sec\theta\frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$ $=\sec\theta\,1\sin\theta$ $=\sec\theta\frac{1}{\sin\theta}$ Undefined control sequence \thetacosec

Q7: $\cos\theta 1-\sin\theta=1+\sin\theta\cos\theta\,\frac{\cos\theta}{1-\sin\theta}=\frac{1+\sin\theta}{\cos\theta}$

A7: We know ,

$\sin^2\theta+\cos^2\theta=1\sin^2\theta+\cos^2\theta=1$

So, Multiplying both numerator and denominator by $(1+\sin\theta)(1+\sin\theta)$, we have

$\cos\theta 1-\sin\theta=\cos\theta(1+\sin\theta)(1-\sin\theta)(1+\sin\theta)\,\frac{\cos\theta}{1-\sin\theta}=\frac{\cos\theta(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$ $=\cos\theta(1+\sin\theta)(1-\sin^2\theta)=\frac{\cos\theta(1+\sin\theta)}{(1-\sin^2\theta)}$ $=\cos\theta(1+\sin\theta)\cos^2\theta=\frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$ $=(1+\sin\theta)\cos\theta=\frac{(1+\sin\theta)}{\cos\theta}$

Q8: $\cos\theta 1+\sin\theta=1-\sin\theta\cos\theta\,\frac{\cos\theta}{1+\sin\theta}=\frac{1-\sin\theta}{\cos\theta}$

A8: We know ,

$\sin^2\theta+\cos^2\theta=1\sin^2\theta+\cos^2\theta=1$

Multiplying both numerator and denominator by $(1-\sin\theta)(1-\sin\theta)$, we have

$\cos\theta 1+\sin\theta=\cos\theta(1-\sin\theta)(1+\sin\theta)(1-\sin\theta)\,\frac{\cos\theta}{1+\sin\theta}=\frac{\cos\theta(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$ $=\cos\theta(1-\sin\theta)(1-\sin^2\theta)=\frac{\cos\theta(1-\sin\theta)}{(1-\sin^2\theta)}$ $=\cos\theta(1-\sin\theta)(\cos^2\theta)=\frac{\cos\theta(1-\sin\theta)}{(\cos^2\theta)}$ $=(1-\sin\theta)\cos\theta=\frac{(1-\sin\theta)}{\cos\theta}$ $=(1-\sin\theta)\cos\theta=\frac{(1-\sin\theta)}{\cos\theta}$

Q 9: $\cos^2A + 11+\cot^2A\,\frac{1}{1+\cot^2A} = 1$

A9: We know that,

$\text{Sin}^2A + \cos^2A = 1$

$\text{cosec}^2A - \cot^2A = 1$

So, $\cos^2A+\frac{1}{1+\cot^2A}=\cos^2A+\frac{1}{cosec^2A}$ $\cos^2A+\frac{1}{1+\cot^2A}=\cos^2A+\frac{1}{cosec^2A}$

$=\cos^2A+(\frac{1}{cosecA})^2$ $=\cos^2A+(\frac{1}{cosecA})^2$ $=\cos^2A+\sin A^2$ $=\cos^2A+\sin A^2$

= 1

Q10: $\sin A^2+\frac{1}{1+\tan^2A}=1$ $\sin A^2+\frac{1}{1+\tan^2A}=1$

A10: We know,

$\sin^2A+\cos^2A=1$

$\sec^2A-\tan^2A=1$

So,

$\sin A^2+\frac{1}{1+\tan^2A}=\sin A^2+\frac{1}{\sec^2A}$ $\sin A^2+\frac{1}{1+\tan^2A}=\sin A^2+\frac{1}{\sec^2A}$ $=\sin A^2+(\frac{1}{\sec A})^2$ $=\sin A^2+(\frac{1}{\sec A})^2$ $=\sin A^2+\cos^2A$ $=\sin A^2+\cos^2A$

= 1

Q11: $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=cosec\theta-\cot\theta$ $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=cosec\theta-\cot\theta$

A11: We know ,

$\sin^2\theta+\cos^2\theta=1$ $\sin^2\theta+\cos^2\theta=1$

Multiplying both numerator and denominator by $(1-\cos\theta)$ $(1-\cos\theta)$, we have

$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}}$ $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}}$ $=\sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$ $=\sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$ $=\sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$

$=\sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$ $=\frac{(1-\cos\theta)}{\sin\theta}$ $=\frac{(1-\cos\theta)}{\sin\theta}$ $=\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}$ $=\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}$

\(=\frac{1}{sin\theta}-\frac{cos\theta}{sin\theta}[/latex\]

Q12: [latex]\frac{1-cos\theta }{sin\theta}=\frac{sin\theta}{1+cos\theta}\)

A12: We know ,

$\sin^2\theta+\cos^2\theta=1$ $\sin^2\theta+\cos^2\theta=1$

Multiplying both numerator and denominator by $(1+\cos\theta)$ $(1+\cos\theta)$, we have

$=\frac{(1-\cos^2\theta)}{(1+\cos\theta)(\sin\theta)}$ $=\frac{(1-\cos^2\theta)}{(1+\cos\theta)(\sin\theta)}$ $=\frac{(\sin^2\theta)}{(1+\cos\theta)(\sin\theta)}$ $=\frac{(\sin^2\theta)}{(1+\cos\theta)(\sin\theta)}$ $=\frac{(\sin\theta)}{(1+\cos\theta)}$ $=\frac{(\sin\theta)}{(1+\cos\theta)}$

Q13. $\frac{\sin\theta}{1-\cos\theta}$ $\frac{\sin\theta}{1-\cos\theta}$ **= cosec**$\theta\theta$ **+ cot**$\theta\theta$

Ans:

Given, L.H.S = $\frac{\sin\theta}{1-\cos\theta}$ $\frac{\sin\theta}{1-\cos\theta}$

Rationalize both nr and dr with 1+cos $\theta\theta$

$= \frac{\sin\theta}{1-\cos\theta} * \frac{1+\cos\theta}{1+\cos\theta}$

We know that, (a-b)(a+b) = $a^2 - b^2$

$=> \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$

Here, $(1-\cos^2\theta) = \sin^2\theta$

$=> \frac{\sin\theta + (\sin\theta * \cos\theta)}{\sin^2\theta}$

$=> \frac{\sin\theta}{\sin^2\theta} + \frac{\sin\theta * \cos\theta}{\sin^2\theta}$

$=> \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$

=> $\text{cosec}\,\theta + \cot\theta$

Hence, L.H.S = R.H.S

Q14. $\frac{1-\sin\theta}{1+\sin\theta}$ **=** $(\sec\theta - \tan\theta)^2$

Ans:

Given, L.H.S = $\frac{1-\sin\theta}{1+\sin\theta}$

Rationalize with nr and dr with $1 - \sin\theta$

$=> \frac{1-\sin\theta}{1+\sin\theta} * \frac{1-\sin\theta}{1-\sin\theta}$

Here, $(1-\sin\theta)(1+\sin\theta) = \cos^2\theta$

$=> \frac{(1-\sin\theta)^2}{\cos^2\theta}$

$=> \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$

$=> \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$

$=> (\sec\theta - \tan\theta)^2$

Hence, L.H.S = R.H.S

Q15. $\frac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta}$ **= cot\theta**

Ans:

Given, L.H.S = $\frac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta}$

Here, $1 + \cot^2\theta = \text{cosec}^2\theta$

$=> \frac{\text{cosec}^2\theta * \tan\theta}{\sec^2\theta}$

$=> \frac{1}{\sin^2\theta} * \frac{\cos^2\theta}{1} * \frac{\sin\theta}{\cos\theta}$

$=> \frac{\cos\theta}{\sin\theta}$

=> $\cot\theta$

Hence, L.H.S = R.H.S

Q16. $\tan^2\theta - \sin^2\theta = \tan^2\theta * \sin^2\theta$

Ans:

Given, L.H.S = $\tan^2\theta - \sin^2\theta$

Here, $\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$

=> $\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$

=> $\sin^2\theta\left[\frac{1}{\cos^2\theta} - 1\right]$

=> $\sin^2\theta\left[\frac{1-\cos^2\theta}{\cos^2\theta}\right]$

=> $\frac{\sin^2\theta}{\cos^2\theta} * \sin^2\theta$

=> $\tan^2\theta * \sin^2\theta$

Hence, L.H.S = R.H.S

Q17. **(cosec θ + sin θ)(cosec θ – sin θ) = $\cot^2\theta + \cos^2\theta$**

Ans:

Given, L.H.S = (cosec θ + sin θ)(cosec θ – sin θ)

Here, $(a + b)(a - b) = a^2 - b^2$

$\text{cosec}^2\theta$ can be written as $1 + \cot^2\theta$ and $\sin^2\theta$ can be written as $1 - \cos^2\theta$

=> $1 + \cot^2\theta - (1 - \cos^2\theta)$

=> $1 + \cot^2\theta - 1 + \cos^2\theta$

=> $\cot^2\theta + \cos^2\theta$

Hence, L.H.S = R.H.S

Q18. **(sec θ + cos θ)(sec θ – cos θ) = $\tan^2\theta + \sin^2\theta$**

Ans:

Given, L.H.S = (sec θ + cos θ)(sec θ – cos θ)

Here, $(a + b)(a - b) = a^2 - b^2$

$\sec^2\theta$ can be written as $1 + \tan^2\theta$ and $\cos^2\theta$ can be written as $1 - \sin^2\theta$

=> $1 + \tan^2\theta - (1 - \sin^2\theta)$

=> $1 + \tan^2\theta - 1 + \sin^2\theta$

=> $\tan^2\theta + \sin^2\theta$

Hence, L.H.S = R.H.S

Q19. secA(1 – sinA)(secA + tanA) = 1

Ans:

Given, L.H.S = secA(1 – sinA)(secA + tanA)

Here, secA = 1cosA $\frac{1}{\cos A}$ and tanA = sinAcosA $\frac{\sin A}{\cos A}$

=> 1cosA $\frac{1}{\cos A}$ * (1 – sinA) * 1+sinAcosA $\frac{1+\sin A}{\cos A}$

=> \(\frac{cos^{2}A}{cos^{2}A}\)

=> 1

Hence, L.H.S = R.H.S

Q20. (cosecA – sinA)(secA – cosA)(tanA + cotA) = 1

Ans:

Given, L.H.S = (cosecA – sinA)(secA – cosA)(tanA + cotA)

Here, cosecA = 1sinA $\frac{1}{\sin A}$, secA = 1cosA $\frac{1}{\cos A}$, tanA = sinAcosA $\frac{\sin A}{\cos A}$, cotA = cosAsinA $\frac{\cos A}{\sin A}$

Substitute the above values in L.H.S

=> (1sinA $\frac{1}{\sin A}$ – sinA)(1cosA $\frac{1}{\cos A}$ – cosA)(sinAcosA $\frac{\sin A}{\cos A}$ + cosAsinA $\frac{\cos A}{\sin A}$)

=> (1−sin^{2}AsinA $\frac{1-\sin^2 A}{\sin A}$) * (1−cos^{2}AcosA $\frac{1-\cos^2 A}{\cos A}$) * (

sin^{2}A+cos^{2}AsinA∗cosA $\frac{\sin^2 A + \cos^2 A}{\sin A * \cos A}$

Here, (\(\frac{1\;-sin^{2}A}) = cos^{2}A, ([latex]\frac{1\;-cos^{2}A}) = sin^{2}A, sin^{2}A + cos^{2}A = 1

=> [latex]\frac{sin^{2}A\;*\;cos^{2}A\;*\;1}{sin^{2}A\;*\;cos^{2}A}\)

=> 1

Hence, L.H.S = R.H.S

Q21. (1 + tan^{2}θ$\tan^2\theta$)(1 – sin θθ)(1 + sin θθ) = 1

Ans:

Given, L.H.S = (1 + tan^{2}θ$\tan^2\theta$)(1 – sin\theta)(1 + sin\theta)

We know that,

Sin^2 θθ + cos^2 θθ = 1

And sec^2 θθ – tan^2 θθ = 1

So,

(1 + tan^{2}θ$\tan^2\theta$)(1 – sin θθ)(1 + sin θθ) = (1 + tan^{2}θ$\tan^2\theta$){(1 – sin θθ)(1 + sin θθ)}

= (1 + tan^{2}θ$\tan^2\theta$)((1 – sin^{2}θ$\sin^2\theta$)

= sec^{2}θ∗tan^{2}θ$\sec^2\theta * \tan^2\theta$

= (1cos^{2}θ)∗cos^{2}θ$(\frac{1}{\cos^2\theta}) * \cos^2\theta$

= 1

hence, L.H.S = R.H.S

Q22. $(\sin^2 A * \cot^2 A)+(\cos^2 A; * \tan^2 A)$ $(\sin^2 A \; * \; \cot^2 A) \; + \; (\cos^2 A; * \tan^2 A)$ **= 1**

Ans:

Given, L.H.S = Undefined control sequence \A

Here, \((sin^{2}A\;+\;cos^{2}A) = 1

So,

[latex](sin^{2}\A\;*\;cot^{2}A)\;+\;(cos^{2}A\;*\;tan^{2}A)\) = $\sin^2 A$($\cos^2 A \sin^2 A \frac{\cos^2 A}{\sin^2 A}$) + $\cos^2 A$($\sin^2 A \cos^2 A \frac{\sin^2 A}{\cos^2 A}$

= $\cos^2 A + \sin^2 A$

= 1

Hence , L.H.S = R.H.S

Q23:

1. **cot θ**θ **– tan θ**θ **=** $2\cos^2\theta - 1 \sin\theta * \cos\theta \frac{2\cos^2\theta - 1}{\sin\theta * \cos\theta}$

Ans:

Give, L.H.S = cot θθ – tan θθ

Here, $\sin^2$ θθ + $\cos^2$ θθ = 1

So,

=> cot θθ – tan θθ = $\cos\theta\sin\theta \frac{\cos\theta}{\sin\theta}$ – $\sin\theta\cos\theta \frac{\sin\theta}{\cos\theta}$

= $\cos^2\theta - \sin^2\theta \sin\theta * \cos\theta \frac{\cos^2\theta - \sin^2\theta}{\sin\theta * \cos\theta}$

= $\cos^2\theta - (1 - \cos^2\theta) \sin\theta * \cos\theta \frac{\cos^2\theta - (1 - \cos^2\theta)}{\sin\theta * \cos\theta}$

= $\cos^2\theta - 1 - \cos^2\theta \sin\theta * \cos\theta \frac{\cos^2\theta - 1 - \cos^2\theta}{\sin\theta * \cos\theta}$

= ($2\cos^2\theta - 1 \sin\theta * \cos\theta$)$\left(\frac{2\cos^2\theta - 1}{\sin\theta * \cos\theta}\right)$

Hence, L.H.S = R.H.S

1. **tanθ−cotθ=**$\tan\theta \; - \; \cot\theta \; =$ **(** $2\sin^2\theta - 1 \sin\theta * \cos\theta$ **)**$\left(\frac{2\sin^2\theta - 1}{\sin\theta * \cos\theta}\right)$

Sol:

Given, L.H.S = \(tan\theta\;-\;cot\theta

We know that,

$\sin^2$ [latex]\theta\) + $\cos^2$ θθ = 1

tanθ−cotθ=[latex] $\sin\theta\cos\theta$ $\tan\theta \; - \; \cot\theta \; = \;$ [latex]$\frac{\sin\theta}{\cos\theta}$ – $\cos\theta\sin\theta \frac{\cos\theta}{\sin\theta}$

= $\sin^2\theta - \cos^2\theta \sin\theta\cos\theta \frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}$

= sin²θ−(1−sin²θ)sinθcosθ $\frac{\sin^2\theta - (1 - \sin^2\theta)}{\sin\theta\cos\theta}$

= sin²θ−1+sin²θsinθcosθ $\frac{\sin^2\theta - 1 + \sin^2\theta}{\sin\theta\cos\theta}$

= (2sin²θ−1sinθ∗cosθ)$(\frac{2\sin^2\theta - 1}{\sin\theta * \cos\theta})$

Hence, L.H.S = R.H.S

Q24. cos²θsinθ−cosecθ+sinθ $\frac{\cos^2\theta}{\sin\theta} - \operatorname{cosec}\theta + \sin\theta$ **= 0**

Ans:

Given, L.H.S cos²θsinθ−cosecθ+sinθ $\frac{\cos^2\theta}{\sin\theta} - \operatorname{cosec}\theta + \sin\theta$

We know that,

$\sin^2\theta\theta + \cos^2\theta\theta = 1$

So,

cos²θsinθ−cosecθ+sinθ $\frac{\cos^2\theta}{\sin\theta} - \operatorname{cosec}\theta + \sin\theta$ = (cos²θsinθ−cosecθ)+sinθ $\frac{\cos^2\theta}{\sin\theta} - \operatorname{cosec}\theta) + \sin\theta$

= (cos²θsinθ−1sinθ)+sinθ$(\frac{\cos^2\theta}{\sin\theta} - \frac{1}{\sin\theta}) + \sin\theta$

= (cos²θ−1sinθ)+sinθ$(\frac{\cos^2\theta - 1}{\sin\theta}) + \sin\theta$

= (−sin²θsinθ)+sinθ$(\frac{-\sin^2\theta}{\sin\theta}) + \sin\theta$

= −sinθ+sinθ$-\sin\theta + \sin\theta$

= 0

Hence, L.H.S = R.H.S

Q 25 . 11+sinA $\frac{1}{1+\sin A}$ **+ 11−sinA** $\frac{1}{1-\sin A}$ **= 2 sec² A**

Ans:

LHS = 11+sinA $\frac{1}{1+\sin A}$ + 11−sinA $\frac{1}{1-\sin A}$

(1−sinA)+(1+sinA)(1+sinA)(1−sinA) $\frac{(1-\sin A)+(1+\sin A)}{(1+\sin A)(1-\sin A)}$ 1−sinA+1+sinA1−sin²A $\frac{1-\sin A+1+\sin A}{1-\sin^2 A}$

⇒ 21−sin²A ⇒ $\frac{2}{1-\sin^2 A}$ [Since , (1 + sin A)(1 – sin A) = 1−sin²A $1 - \sin^2 A$]

⇒ 2cos²A ⇒ $\frac{2}{\cos^2 A}$ [Since , 1-sin² A = cos2 A]

⇒2sec²A⇒ $2\sec^2 A$

LHS = RHS Hence proved

Q 26 . 1+sinθcosθ+cosθ1+sinθ=2secθ $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$

Ans:

LHS = 1+sinθcosθ+cosθ1+sinθ $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}$

$= \frac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)}$

$= \frac{1+\sin^2\theta+2\sin\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$

$\Rightarrow \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)} = 2\sec\theta$

LHS = RHS Hence proved

Q 27 . $\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta} = \frac{1+\sin^2\theta}{1-\sin^2\theta}$

Ans:

We know that $\sin^2\theta + \cos^2\theta = 1$

So,

LHS =

$$\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta}$$
$$= \frac{(1+2\sin\theta+\sin^2\theta)+(1-2\sin\theta+\sin^2\theta)}{2\cos^2\theta}$$
$$= \frac{1+2\sin\theta+\sin^2\theta+1-2\sin\theta+\sin^2\theta}{2\cos^2\theta}$$
$$= \frac{2+2\sin^2\theta}{2\cos^2\theta}$$
$$= \frac{2(1+\sin^2\theta)}{2(1-\sin^2\theta)}$$
$$= \frac{(1+\sin^2\theta)}{(1-\sin^2\theta)}$$

LHS = RHS Hence proved

Q 28 . $\frac{1+\tan^2\theta}{1+\cot^2\theta} = \left[\frac{1-\tan\theta}{\cot\theta}\right]^2 - \tan^2\theta$

Ans :

LHS = $\frac{1+\tan^2\theta}{1+\cot^2\theta}$

$= \frac{\sec^2\theta}{\csc^2\theta}$ [Since , $\tan^2\theta + 1 = \sec^2\theta$, $1 + \cot^2\theta = \mathrm{cosec}^2\theta$]

$= \frac{1}{\cos^2\theta \cdot 1}\sin^2\theta$

$= \tan^2\theta$

LHS = RHS Hence proved

Q 29 . $\frac{1+\sec\theta}{\sec\theta} = \frac{\sin^2\theta}{1-\cos\theta}$

Ans :

LHS = $\frac{1+\sec\theta}{\sec\theta}$

= $\frac{1+\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}}$

= $\frac{\cos\theta+1}{\cos\theta} \cdot \cos\theta$

= $1+\cos\theta$

RHS = $\frac{\sin^2\theta}{1-\cos\theta}$

= $\frac{1-\cos^2\theta}{1-\cos\theta}$

= $\frac{(1-\cos\theta)(1+\cos\theta)}{1-\cos\theta}$

= $1+\cos\theta$

LHS = RHS Hence proved

Q 30 . $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$ **=** $1+\tan\theta+\cot\theta$

Ans:

LHS = $\frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\cot\theta}{1-\tan\theta}$

= $\frac{\tan^2\theta}{\tan\theta-1} + \frac{\cot\theta}{1-\tan\theta}$

= $\frac{1}{1-\tan\theta}\left[\frac{1}{\tan\theta} - \tan^2\theta\right]$

= $\frac{1}{1-\tan\theta}\left[\frac{1-\tan^3\theta}{\tan\theta}\right]$

= $\frac{1}{1-\tan\theta}\frac{(1-\tan\theta)(1+\tan\theta+\tan^2\theta)}{\tan\theta}$ [Since , $a^3-b^3=(a-b)(a^2+ab+b^2)$]

= $\frac{1+\tan\theta+\tan^2\theta}{\tan\theta}$

= $\frac{1}{\tan\theta} + \frac{\tan\theta}{\tan\theta} + \frac{\tan^2\theta}{\tan\theta}$

= $1+\tan\theta+\cot\theta$

LHS = RHS Hence proved

Q 31 . $\sec^6\theta = \tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$

Ans :

We know that $\sec^2\theta - \tan^2\theta = 1$

Cubing both sides

$(\sec^2\theta-\tan^2\theta)^3=1$ $(\sec^2\theta-\tan^2\theta)^3=1$

$\sec^6\theta-\tan^6\theta-3\sec^2\theta\tan^2\theta(\sec^2\theta-\tan^2\theta)=1$ $\sec^6\theta-\tan^6\theta-3\sec^2\theta\tan^2\theta(\sec^2\theta-\tan^2\theta)=1$ [Since , $a^3-b^3=(a-b)(a^2+ab+b^2)$ $a^3-b^3=(a-b)(a^2+ab+b^2)$]

$\sec^6\theta-\tan^6\theta-3\sec^2\theta\tan^2\theta=1$ $\sec^6\theta-\tan^6\theta-3\sec^2\theta\tan^2\theta=1$ $\Rightarrow\sec^6\theta=\tan^6\theta+3\sec^2\theta\tan^2\theta+1$
$\Rightarrow\sec^6\theta=\tan^6\theta+3\sec^2\theta\tan^2\theta+1$

Hence proved.

Q 32 . $\mathrm{cosec}^6\theta=\cot^6\theta+3\cot^2\theta\,\mathrm{cosec}^2\theta+1$ $\mathrm{cosec}^6\theta=\cot^6\theta+3\cot^2\theta\,\mathrm{cosec}^2\theta+1$

Ans :

We know that $\mathrm{cosec}^2\theta-\cot^2\theta=1$ $\mathrm{cosec}^2\theta-\cot^2\theta=1$

Cubing both sides

$(\mathrm{cosec}^2\theta-\cot^2\theta)^3=1$ $(\mathrm{cosec}^2\theta-\cot^2\theta)^3=1$

$\mathrm{cosec}^6\theta-\cot^6\theta-3\mathrm{cosec}^2\theta\cot^2\theta(\mathrm{cosec}^2\theta-\cot^2\theta)=1$
$\mathrm{cosec}^6\theta-\cot^6\theta-3\mathrm{cosec}^2\theta\cot^2\theta(\mathrm{cosec}^2\theta-\cot^2\theta)=1$ [Since , $a^3-b^3=(a-b)(a^2+ab+b^2)$
$a^3-b^3=(a-b)(a^2+ab+b^2)$]

$\mathrm{cosec}^6\theta-\cot^6\theta-3\mathrm{cosec}^2\theta\cot^2\theta=1$ $\mathrm{cosec}^6\theta-\cot^6\theta-3\mathrm{cosec}^2\theta\cot^2\theta=1$ $\Rightarrow\mathrm{cosec}^6\theta=\cot^6\theta+3\mathrm{cosec}^2\theta\cot^2\theta+1$
$\Rightarrow\mathrm{cosec}^6\theta=\cot^6\theta+3\mathrm{cosec}^2\theta\cot^2\theta+1$

Hence proved.

Q 33 . $(1+\tan^2\theta)\cot\theta\,\mathrm{cosec}^2\theta=\tan\theta$ $\frac{(1+\tan^2\theta)\cot\theta}{\mathrm{cosec}^2\theta}=\tan\theta$

Ans :

We know that $\sec^2\theta-\tan^2\theta=1$ $\sec^2\theta-\tan^2\theta=1$

Therefore , $\sec^2\theta=1+\tan^2\theta$ $\sec^2\theta=1+\tan^2\theta$

LHS = $\sec^2\theta\cdot\cot\theta\,\mathrm{cosec}^2\theta$ $\frac{\sec^2\theta\cdot\cot\theta}{\mathrm{cosec}^2\theta}$

= $1\cdot\sin^2\theta\cos^2\theta\cdot\cos\theta\sin\theta$ $\frac{1\cdot\sin^2\theta}{\cos^2\theta}\cdot\frac{\cos\theta}{\sin\theta}$ [$\because\sec\theta=1\cos\theta,\mathrm{cosec}\theta=1\sin\theta,\cot\theta=\cos\theta\sin\theta$]

$\Rightarrow\sin\theta\cos\theta=\tan\theta\Rightarrow\frac{\sin\theta}{\cos\theta}=\tan\theta$

LHS = RHS Hence proved

Q 34 . \(\frac{ 1+cosA}{sin ^{2} A}\) = 11−cosA $\frac{1}{1-\cos A}$

Ans:

We know that $\sin^2A+\cos^2A$ $\sin^2A+\cos^2A=1$

$\sin^2A=1-\cos^2A$ $\sin^2A=1-\cos^2A$ $\Rightarrow\sin^2A=(1-\cos A)(1+\cos A)\Rightarrow\sin^2A=(1-\cos A)(1+\cos A)$ $\Rightarrow$LHS= $(1+\cos A)(1-\cos A)(1+\cos A)\Rightarrow$ LHS $=\frac{(1+\cos A)}{(1-\cos A)(1+\cos A)}$

$= \Rightarrow \text{LHS} = \frac{1}{(1-\cos A)}$

$\therefore$ LHS = RHS Hence proved

Q 35 . $\frac{\sec A-\tan A}{\sec A+\tan A} = \frac{\cos^2 A}{(1+\sin A)^2}$

Ans:

LHS = $\frac{\sec A-\tan A}{\sec A+\tan A}$

Rationalizing the denominator by multiplying and dividing with sec A + tan A , we get

$\frac{\sec A-\tan A}{\sec A+\tan A} \times \frac{\sec A+\tan A}{\sec A+\tan A}$

$= \frac{\sec^2 A-\tan^2 A}{(\sec A+\tan A)^2}$

$= \frac{1}{(\sec A+\tan A)^2}$

$= \frac{1}{(\sec^2 A+\tan^2 A+2\sec A\tan A)}$

$= \frac{1}{\left(\frac{1}{\cos^2 A}+\frac{\sin^2 A}{\cos^2 A}+\frac{2\sin A}{\cos A}\right)}$

$\Rightarrow \frac{\cos^2 A}{1+\sin^2 A+2\sin A}$

$= \frac{\cos^2 A}{(1+\sin A)^2}$

$\therefore$ LHS = RHS Hence proved

Q 36 . $\frac{1+\cos A}{\sin A} = \frac{\sin A}{1-\cos A}$

Ans:

LHS = $\frac{1+\cos A}{\sin A}$

Multiply both numerator and denominator with (1 – cos A) we get ,

$\frac{(1+\cos A)(1-\cos A)}{\sin A(1-\cos A)}$

$= \frac{1-\cos^2 A}{\sin A(1-\cos A)}$

$= \frac{\sin^2 A}{\sin A(1-\cos A)}$

$= \frac{\sin A}{1-\cos A}$

$\therefore$ LHS = RHS Hence proved

37.

(i) $\sqrt{\frac{1+\sin A}{1-\sin A}}$ **= sec A + tan A**

Ans:

To prove,

$$\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

Considering left hand side (LHS),

Rationalize the numerator and denominator with $\sqrt{1+\sin A}$

$$\sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}} = \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

$$= \frac{(1+\sin A)}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

= sec A + tan A

Therefore, LHS = RHS

Hence proved

(ii) $\sqrt{\frac{(1-\cos A)}{(1+\cos A)}} + \sqrt{\frac{(1+\cos A)}{(1-\cos A)}}$ **= 2cosec A**

Ans:

To prove,

$$\sqrt{\frac{(1-\cos A)}{(1+\cos A)}} + \sqrt{\frac{(1+\cos A)}{(1-\cos A)}} = 2\operatorname{cosec} A$$

Considering left hand side (LHS),

Rationalize the numerator and denominator.

$$= \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}} + \sqrt{\frac{(1+\cos A)(1+\cos A)}{(1-\cos A)(1+\cos A)}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{(1-\cos^2 A)}} + \sqrt{\frac{(1+\cos A)^2}{(1-\cos^2 A)}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{(\sin^2 A)}} + \sqrt{\frac{(1+\cos A)^2}{(\sin^2 A)}}$$

$$= \frac{(1-\cos A)}{(\sin A)} + \frac{(1+\cos A)}{(\sin A)}$$

$$= \frac{(1-\cos A+1+\cos A)}{(\sin A)}$$

$$= \frac{(2)}{(\sin A)}$$

= 2cosec A

Therefore, LHS = RHS

Hence proved

38. **Prove that:**

(i) $\sqrt{(\sec\Theta-1)(\sec\Theta+1)}+\sqrt{(\sec\Theta+1)(\sec\Theta-1)}\sqrt{\frac{(\sec\Theta-1)}{(\sec\Theta+1)}}+\sqrt{\frac{(\sec\Theta+1)}{(\sec\Theta-1)}}$ **= 2cosec ΘΘ**

Ans:

To prove,

$= \sqrt{(\sec\Theta-1)(\sec\Theta+1)}+\sqrt{(\sec\Theta+1)(\sec\Theta-1)}\sqrt{\frac{(\sec\Theta-1)}{(\sec\Theta+1)}}+\sqrt{\frac{(\sec\Theta+1)}{(\sec\Theta-1)}}$ = 2cosec ΘΘ

Considering left hand side (LHS),

Rationalize the numerator and denominator.

$= \sqrt{(\sec\Theta-1)(\sec\Theta-1)(\sec\Theta+1)(\sec\Theta-1)}+\sqrt{(\sec\Theta+1)(\sec\Theta+1)(\sec\Theta-1)(\sec\Theta+1)}\sqrt{\frac{(\sec\Theta-1)(\sec\Theta-1)}{(\sec\Theta+1)(\sec\Theta-1)}}+\sqrt{\frac{(\sec\Theta+1)(\sec\Theta+1)}{(\sec\Theta-1)(\sec\Theta+1)}}$

$= \sqrt{(\sec\Theta-1)^2(\sec^2\Theta-1)}+\sqrt{(\sec\Theta+1)^2(\sec^2\Theta-1)}\sqrt{\frac{(\sec\Theta-1)^2}{(\sec^2\Theta-1)}}+\sqrt{\frac{(\sec\Theta+1)^2}{(\sec^2\Theta-1)}}$

$= \sqrt{(\sec\Theta-1)^2\tan^2\Theta}+\sqrt{(\sec\Theta+1)^2\tan^2\Theta}\sqrt{\frac{(\sec\Theta-1)^2}{\tan^2\Theta}}+\sqrt{\frac{(\sec\Theta+1)^2}{\tan^2\Theta}}$

$= (\sec\Theta-1)\tan\Theta+(\sec\Theta+1)\tan\Theta\frac{(\sec\Theta-1)}{\tan\Theta}+\frac{(\sec\Theta+1)}{\tan\Theta}$

$= (\sec\Theta-1+\sec\Theta+1)\tan\Theta\frac{(\sec\Theta-1+\sec\Theta+1)}{\tan\Theta}$

$= (2\cos\Theta)\cos\Theta\sin\Theta\frac{(2\cos\Theta)}{\cos\Theta\sin\Theta}$

$= 2\sin\Theta\frac{2}{\sin\Theta}$

= 2cosec ΘΘ

Therefore, LHS = RHS

Hence proved

(ii) $\sqrt{(1+\sin\Theta)(1-\sin\Theta)}+\sqrt{(1-\sin\Theta)(1+\sin\Theta)}\sqrt{\frac{(1+\sin\Theta)}{(1-\sin\Theta)}}+\sqrt{\frac{(1-\sin\Theta)}{(1+\sin\Theta)}}$ **= 2sec ΘΘ**

Ans:

To prove,

$= \sqrt{(1+\sin\Theta)(1-\sin\Theta)}+\sqrt{(1-\sin\Theta)(1+\sin\Theta)}\sqrt{\frac{(1+\sin\Theta)}{(1-\sin\Theta)}}+\sqrt{\frac{(1-\sin\Theta)}{(1+\sin\Theta)}}$ = 2sec ΘΘ

Considering left hand side (LHS),

Rationalize the numerator and denominator.

$= \sqrt{(1+\sin\Theta)(1+\sin\Theta)(1-\sin\Theta)(1+\sin\Theta)}+\sqrt{(1-\sin\Theta)(1-\sin\Theta)(1+\sin\Theta)(1-\sin\Theta)}\sqrt{\frac{(1+\sin\Theta)(1+\sin\Theta)}{(1-\sin\Theta)(1+\sin\Theta)}}+\sqrt{\frac{(1-\sin\Theta)(1-\sin\Theta)}{(1+\sin\Theta)(1-\sin\Theta)}}$

$= \sqrt{(1+\sin\Theta)^2(1-\sin^2\Theta)}+\sqrt{(1-\sin\Theta)^2(1-\sin^2\Theta)}\sqrt{\frac{(1+\sin\Theta)^2}{(1-\sin^2\Theta)}}+\sqrt{\frac{(1-\sin\Theta)^2}{(1-\sin^2\Theta)}}$

$= \sqrt{(1+\sin\Theta)^2(\cos^2\Theta)}+\sqrt{(1-\sin\Theta)^2(\cos^2\Theta)}\sqrt{\frac{(1+\sin\Theta)^2}{(\cos^2\Theta)}}+\sqrt{\frac{(1-\sin\Theta)^2}{(\cos^2\Theta)}}$

$= (1+\sin\Theta)(\cos\Theta)+(1-\sin\Theta)(\cos\Theta)\frac{(1+\sin\Theta)}{(\cos\Theta)}+\frac{(1-\sin\Theta)}{(\cos\Theta)}$

$= (1+\sin\Theta+1-\sin\Theta)(\cos\Theta)\ \frac{(1+\sin\Theta+1-\sin\Theta)}{(\cos\Theta)}$

$= (2)(\cos\Theta)\ \frac{(2)}{(\cos\Theta)}$

$= 2\sec\Theta\, 2\sec\Theta$

Therefore, LHS = RHS

Hence proved

(iii) $\sqrt{(1+\cos\Theta)(1-\cos\Theta)}\ \sqrt{\frac{(1+\cos\Theta)}{(1-\cos\Theta)}}$ **\sqrt{\frac{(1-cos \Theta)}{(1+cos \Theta)}} =** $\mathbf{2cosec\Theta}\, 2\operatorname{cosec}\Theta$

Ans:

To prove,

$\sqrt{(1-\cos\Theta)(1+\cos\Theta)} + \sqrt{(1+\cos\Theta)(1-\cos\Theta)}\ \sqrt{\frac{(1-\cos\Theta)}{(1+\cos\Theta)}} + \sqrt{\frac{(1+\cos\Theta)}{(1-\cos\Theta)}}$ = 2cosec \Theta

Considering left hand side (LHS),

Rationalize the numerator and denominator.

$= \sqrt{(1-\cos\Theta)(1-\cos\Theta)(1+\cos\Theta)(1-\cos\Theta)} + \sqrt{(1+\cos\Theta)(1+\cos\Theta)(1-\cos\Theta)(1+\cos\Theta)}\ \sqrt{\frac{(1-\cos\Theta)(1-\cos\Theta)}{(1+\cos\Theta)(1-\cos\Theta)}} + \sqrt{\frac{(1+\cos\Theta)(1+\cos\Theta)}{(1-\cos\Theta)(1+\cos\Theta)}}$

$= \sqrt{(1-\cos\Theta)^2(1-\cos^2\Theta)} + \sqrt{(1+\cos\Theta)^2(1-\cos^2\Theta)}\ \sqrt{\frac{(1-\cos\Theta)^2}{(1-\cos^2\Theta)}} + \sqrt{\frac{(1+\cos\Theta)^2}{(1-\cos^2\Theta)}}$

$= \sqrt{(1-\cos\Theta)^2(\sin^2\Theta)} + \sqrt{(1+\cos\Theta)^2(\sin^2\Theta)}\ \sqrt{\frac{(1-\cos\Theta)^2}{(\sin^2\Theta)}} + \sqrt{\frac{(1+\cos\Theta)^2}{(\sin^2\Theta)}}$

$= (1-\cos\Theta)(\sin\Theta) + (1+\cos\Theta)(\sin\Theta)\ \frac{(1-\cos\Theta)}{(\sin\Theta)} + \frac{(1+\cos\Theta)}{(\sin\Theta)}$

$= (1-\cos\Theta+1+\cos\Theta)(\sin\Theta)\ \frac{(1-\cos\Theta+1+\cos\Theta)}{(\sin\Theta)}$

$= (2)(\sin\Theta)\ \frac{(2)}{(\sin\Theta)}$

= 2cosec \Theta

Therefore, LHS = RHS

Hence proved

(iv) $\sec\Theta-1\sec\Theta+1\ \frac{\sec\Theta-1}{\sec\Theta+1}$ **=** $(\sin\Theta 1+\cos\Theta)^2\left(\frac{\sin\Theta}{1+\cos\Theta}\right)^2$

Ans:

To prove,

$\sec\Theta-1\sec\Theta+1\ \frac{\sec\Theta-1}{\sec\Theta+1} = (\sin\Theta 1+\cos\Theta)^2\left(\frac{\sin\Theta}{1+\cos\Theta}\right)^2$

Considering left hand side (LHS),

$= \sec\Theta-1\sec\Theta+1\ \frac{\sec\Theta-1}{\sec\Theta+1}$

$= 1-\cos\Theta 1+\cos\Theta\ \frac{1-\cos\Theta}{1+\cos\Theta}$

Multiply and divide with $(1+\cos\Theta\Theta)$

$= (1-\cos\Theta)(1+\cos\Theta)(1+\cos\Theta)(1+\cos\Theta)\ \frac{(1-\cos\Theta)(1+\cos\Theta)}{(1+\cos\Theta)(1+\cos\Theta)}$

$= (1-\cos^2\Theta)(1+\cos\Theta)^2 \frac{(1-\cos^2\Theta)}{(1+\cos\Theta)^2}$

$= (\sin^2\Theta)(1+\cos\Theta)^2 \frac{(\sin^2\Theta)}{(1+\cos\Theta)^2}$

$= (\sin\Theta 1+\cos\Theta)^2 (\frac{\sin\Theta}{1+\cos\Theta})^2$

Therefore, LHS = RHS

Hence proved

39. **$(\sec A - \tan A)^2 = 1-\sin A 1+\sin A \frac{1-\sin A}{1+\sin A}$**

Ans:

To prove,

$(\sec A - \tan A)^2 = 1-\sin A 1+\sin A \frac{1-\sin A}{1+\sin A}$

Considering left hand side (LHS),

$= (\sec A - \tan A)^2$

$= [1\cos A - \sin A\cos A]^2 [\frac{1}{\cos A} - \frac{\sin A}{\cos A}]^2$

$= (1-\sin A)^2\cos^2 A \frac{(1-\sin A)^2}{\cos^2 A}$

$= (1-\sin A)^2 1-\sin^2 A \frac{(1-\sin A)^2}{1-\sin^2 A}$

$= (1-\sin A)^2(1+\sin A)(1-\sin A) \frac{(1-\sin A)^2}{(1+\sin A)(1-\sin A)}$

$= (1-\sin A)(1+\sin A) \frac{(1-\sin A)}{(1+\sin A)}$

Therefore, LHS = RHS

Hence proved

40. **$1-\cos A 1+\cos A \frac{1-\cos A}{1+\cos A} = (\cot A - \operatorname{cosec} A)^2$**

Ans:

To prove,

$1-\cos A 1+\cos A \frac{1-\cos A}{1+\cos A} = (\cot A - \operatorname{cosec} A)$

Considering left hand side (LHS),

Rationalize the numerator and denominator with (1 – cos A)

$= (1-\cos A)(1-\cos A)(1+\cos A)(1-\cos A) \frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}$

$= (1-\cos A)^2(1-\cos^2 A) \frac{(1-\cos A)^2}{(1-\cos^2 A)}$

$= (1-\cos A)^2(\sin^2 A) \frac{(1-\cos A)^2}{(\sin^2 A)}$

$= (1\sin A - \cos A\sin A)^2 (\frac{1}{\sin A} - \frac{\cos A}{\sin A})^2$

$= (\text{cosec } A - \cot A)^2$

$= (\cot A - \text{cosec})^2$

Therefore, LHS = RHS

Hence proved

41. **1secA–1 + 1secA+1 =2cosecAcotA** $\frac{1}{\sec A-1} + \frac{1}{\sec A+1} = 2\text{cosec}A\cot A$

Ans:

To prove,

1secA–1 + 1secA+1 =2cosecAcotA $\frac{1}{\sec A-1} + \frac{1}{\sec A+1} = 2\text{cosec}A\cot A$

Considering left hand side (LHS),

= secA+1+secA−1(secA+1)(secA−1) $\frac{\sec A+1+\sec A-1}{(\sec A+1)(\sec A-1)}$

= 2secA(sec²A−1) $\frac{2\sec A}{(\sec^2 A-1)}$

= 2secA(tan²A) $\frac{2\sec A}{(\tan^2 A)}$

= 2cos²A(cosAsin²A) $\frac{2\cos^2 A}{(\cos A\sin^2 A)}$

= 2cosA(sin²A) $\frac{2\cos A}{(\sin^2 A)}$

= 2cosA(sinA)(sinA)) $\frac{2\cos A}{(\sin A)(\sin A))}$

= 2cosec A cot A

Therefore, LHS = RHS

Hence proved

42. **cosA1−tanA + sinA1−cotA** $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A}$ **= sin A + cos A**

Ans:

To prove,

cosA1−tanA + sinA1−cotA $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A}$ = sin A + cos A

Considering left hand side (LHS),

= cosA1−tanA + sinA1−cotA $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A}$

= cosA1−sinAcosA + sinA1−cosAsinA $\frac{\cos A}{1-\frac{\sin A}{\cos A}} + \frac{\sin A}{1-\frac{\cos A}{\sin A}}$

= cos²AcosA−sinA − sin²AcosA−sinA $\frac{\cos^2 A}{\cos A-\sin A} - \frac{\sin^2 A}{\cos A-\sin A}$

= cos²A−sin²AcosA−sinA $\frac{\cos^2 A-\sin^2 A}{\cos A-\sin A}$

= (cosA+sinA)(cosA−sinA)cosA−sinA $\frac{(\cos A+\sin A)(\cos A-\sin A)}{\cos A-\sin A}$

= cos A + sin A

Therefore, LHS = RHS

Hence proved

43. **(cosecA)(cosecA−1) + (cosecA)(cosecA+1)** $\frac{(\text{cosecA})}{(\text{cosecA}-1)} + \frac{(\text{cosecA})}{(\text{cosecA}+1)}$ **= 2sec^2 A**

Ans:

To prove,

(cosecA)(cosecA−1) + (cosecA)(cosecA+1) $\frac{(\text{cosecA})}{(\text{cosecA}-1)} + \frac{(\text{cosecA})}{(\text{cosecA}+1)}$ = 2sec^2 A

Considering left hand side (LHS),

= (cosecA)(cosecA+1+cosecA−1)(cosec^{2}A−1)) $\frac{(\text{cosecA})(\text{cosecA}+1+\text{cosecA}-1)}{(\text{cosec}^2\text{A}-1)}$)

= (2cosec^{2}A)cot^{2}A $\frac{(2\text{cosec}^2\text{A})}{\cot^2\text{A}}$

= (2sin^{2}A)sin^{2}A.cos^{2}A $\frac{(2\sin^2\text{A})}{\sin^2\text{A}.\cos^2\text{A}}$

= 2cos^{2}A $\frac{2}{\cos^2\text{A}}$

= 2sec^{2}A$2\sec^2\text{A}$

Therefore, LHS = RHS

Hence proved

44. tan^{2}A1+tan^{2}A + cot^{2}A1+cot^{2}A $\frac{\tan^2\text{A}}{1+\tan^2\text{A}} + \frac{\cot^2\text{A}}{1+\cot^2\text{A}}$ **= 1**

Ans:

To prove,

tan^{2}A1+tan^{2}A + cot^{2}A1+cot^{2}A $\frac{\tan^2\text{A}}{1+\tan^2\text{A}} + \frac{\cot^2\text{A}}{1+\cot^2\text{A}}$ = 1

Considering left hand side (LHS),

= sin^{2}Acos^{2}A cos^{2}A+sin^{2}Acos^{2}A + cos^{2}Asin^{2}A cos^{2}A+sin^{2}Asin^{2}A $\frac{\frac{\sin^2\text{A}}{\cos^2\text{A}}}{\frac{\cos^2\text{A}+\sin^2\text{A}}{\cos^2\text{A}}} + \frac{\frac{\cos^2\text{A}}{\sin^2\text{A}}}{\frac{\cos^2\text{A}+\sin^2\text{A}}{\sin^2\text{A}}}$

= sin^{2}Acos^{2}A+sin^{2}A + cos^{2}Acos^{2}A+sin^{2}A $\frac{\sin^2\text{A}}{\cos^2\text{A}+\sin^2\text{A}} + \frac{\cos^2\text{A}}{\cos^2\text{A}+\sin^2\text{A}}$

= sin^{2}A+cos^{2}Acos^{2}A+sin^{2}A $\frac{\sin^2\text{A}+\cos^2\text{A}}{\cos^2\text{A}+\sin^2\text{A}}$

= 1

Therefore, LHS = RHS

Hence proved

45. **cotA−cosAcotA+cosA** $\frac{\text{cotA}-\text{cosA}}{\text{cotA}+\text{cosA}}$ **= cosecA−1cosecA+1** $\frac{\text{cosecA}-1}{\text{cosecA}+1}$

Ans:

To prove,

$$\frac{\cot A-\cos A}{\cot A+\cos A}=\frac{\operatorname{cosec}A-1}{\operatorname{cosec}A+1}$$

Considering left hand side (LHS),

$$=\frac{\frac{\cos A}{\sin A}-\cos A}{\frac{\cos A}{\sin A}+\cos A}$$

$$=\frac{\cos A\operatorname{cosec}A-\cos A}{\cos A\operatorname{cosec}A+\cos A}$$

$$=\frac{\cos A(\operatorname{cosec}A-1)}{\cos A(\operatorname{cosec}A+1)}$$

$$=\frac{(\operatorname{cosec}A-1)}{(\operatorname{cosec}A+1)}$$

Therefore, LHS = RHS

Hence proved

46. $\mathbf{\frac{1+\cos\Theta-\sin^2\Theta}{\sin\Theta(1+\cos\Theta)}=\cot\Theta}$

Ans:

To prove,

$$\frac{1+\cos\Theta-\sin^2\Theta}{\sin\Theta(1+\cos\Theta)}=\cot\Theta$$

Considering left hand side (LHS),

$$=\frac{1+\cos\Theta-(1-\cos^2\Theta)}{\sin\Theta(1+\cos\Theta)}$$

$$=\frac{1+\cos\Theta-1+\cos^2\Theta}{\sin\Theta(1+\cos\Theta)}$$

$$=\frac{\cos\Theta+\cos^2\Theta}{\sin\Theta(1+\cos\Theta)}$$

$$=\frac{\cos\Theta(1+\cos\Theta)}{\sin\Theta(1+\cos\Theta)}$$

$$=\frac{(\cos\Theta)}{(\sin\Theta)}$$

$= \cot\Theta$

Therefore, LHS = RHS

Hence, proved.

(i) $\mathbf{\frac{1+\cos\Theta+\sin\Theta}{1+\cos\Theta-\sin\Theta}=\frac{1+\sin\Theta}{\cos\Theta}}$

Ans:

To prove,

$$\frac{1+\cos\Theta+\sin\Theta}{1+\cos\Theta-\sin\Theta}=\frac{1+\sin\Theta}{\cos\Theta}$$

Dividing the numerator and denominator with $\cos\Theta$

Considering LHS, we get,

$$=\frac{\frac{1+\cos\Theta+\sin\Theta}{\cos\Theta}}{\frac{1+\cos\Theta-\sin\Theta}{\cos\Theta}}$$

$= \frac{\sec\Theta+1+\tan\Theta}{\sec\Theta+1-\tan\Theta}$

$= \frac{1+\sec\Theta+\tan\Theta}{1+\sec\Theta-\tan\Theta}$

[As we know,

$(\sec^2\Theta)-(\tan^2\Theta)=1$]

$(\sec\Theta+\tan\Theta)(\sec\Theta-\tan\Theta)=1$

$(\sec\Theta+\tan\Theta)=\frac{1}{(\sec\Theta-\tan\Theta)}$

$= \frac{\frac{1}{(\sec\Theta-\tan\Theta)}+1}{1+\sec\Theta-\tan\Theta}$

$= \frac{1+\sec\Theta-\tan\Theta}{1+\sec\Theta-\tan\Theta} \times \frac{1}{\sec\Theta-\tan\Theta}$

$= \sec\Theta+\tan\Theta$

$= \frac{1+\sin\Theta}{\cos\Theta}$

Therefore, LHS = RHS

Hence proved

(ii) $\frac{\sin\Theta-\cos\Theta+1}{\sin\Theta+\cos\Theta-1} = \frac{1}{\sec\Theta-\tan\Theta}$

Ans:

To prove,

$\frac{\sin\Theta-\cos\Theta+1}{\sin\Theta+\cos\Theta-1} = \frac{1}{\sec\Theta-\tan\Theta}$

Considering LHS, we get,

$\frac{\sin\Theta-\cos\Theta+1}{\sin\Theta+\cos\Theta-1}$

Dividing the numerator and denominator with $\cos\Theta$, we get,

$= \frac{\tan\Theta+\sec\Theta-1}{\tan\Theta-\sec\Theta+1}$

[As we know, $(\sec\Theta+\tan\Theta)=\frac{1}{(\sec\Theta-\tan\Theta)}$]

$= \frac{\frac{1}{(\sec\Theta-\tan\Theta)}-1}{\tan\Theta-\sec\Theta+1}$

$= \frac{\tan\Theta-\sec\Theta+1}{\tan\Theta-\sec\Theta+1} \times \frac{1}{(\sec\Theta-\tan\Theta)}$

$= \frac{1}{(\sec\Theta-\tan\Theta)}$

Therefore, LHS = RHS

Hence proved

(iii) $\frac{\cos\Theta-\sin\Theta+1}{\cos\Theta+\sin\Theta-1}$ **=** $\text{cosec}\,\Theta+\cot\Theta$

Ans:

To prove,

$\frac{\cos\Theta-\sin\Theta+1}{\cos\Theta+\sin\Theta-1} = \text{cosec}\,\Theta+\cot\Theta$

Considering LHS, we get,

Dividing the numerator and denominator with $\sin\Theta$, we get,

= $\frac{\frac{\cos\Theta-\sin\Theta+1}{\sin\Theta}}{\frac{\cos\Theta+\sin\Theta-1}{\sin\Theta}}$

= $\frac{\cot\Theta+\text{cosec}\Theta-1}{\cot\Theta-\text{cosec}\Theta+1}$

[As we know,

$$(\text{cosec}^2\Theta)-(\cot^2\Theta)=1$$
$$(\text{cosec}\Theta+\cot\Theta)(\text{cosec}\Theta-\cot\Theta)=1$$
$$(\text{cosec}\Theta+\cot\Theta)=\frac{1}{(\text{cosec}\Theta-\cot\Theta)}$$
]

= $\frac{\frac{1}{(\text{cosec}\Theta-\cot\Theta)}-1}{\cot\Theta-\text{cosec}\Theta+1}$

= $\frac{\cot\Theta-\text{cosec}\Theta+1}{\cot\Theta-\text{cosec}\Theta+1} \times \frac{1}{(\text{cosec}\Theta-\cot\Theta)}$

= $\frac{1}{(\text{cosec}\Theta-\cot\Theta)}$

= $\text{cosec}\Theta+\cot\Theta$

Therefore, LHS = RHS

Hence proved

(iv) $(\sin\Theta+\cos\Theta)(\tan\Theta+\cot\Theta)$ = $\sec\Theta+\text{cosec}\Theta$

Ans:

To prove,

$(\sin\Theta+\cos\Theta)(\tan\Theta+\cot\Theta)$ = $\text{cosec}\Theta+\text{cosec}\Theta$

Considering LHS, we get,

= $(\sin\Theta+\cos\Theta)(\frac{\sin\Theta}{\cos\Theta}+\frac{\cos\Theta}{\sin\Theta})$

= $(\frac{\sin^2\Theta}{\cos\Theta}+\cos\Theta+\sin\Theta+\frac{\cos^2\Theta}{\sin\Theta})$

= $\sin\Theta(\tan\Theta+1)+\cos\Theta(\frac{1}{\tan\Theta}+1)$

= $\sin\Theta(\tan\Theta+1)+\frac{\cos\Theta}{\tan\Theta}(\tan\Theta+1)$

= $(\sin\Theta+\frac{\cos\Theta}{\tan\Theta})(\tan\Theta+1)$

= $(\frac{\sin^2\Theta+\cos^2\Theta}{\sin\Theta})(\tan\Theta+1)$

= $(\frac{1}{\sin\Theta})(\tan\Theta+1)$

= Undefined control sequence \Thetasin

= $\sec\Theta+\text{cosec}\Theta$

Therefore, LHS = RHS

Hence proved

50. **\frac{tanA}{1+secA}-\frac{tanA}{1-secA}= 2 cosec A**

Ans:

To prove,

\frac{tanA}{1+secA}-\frac{tanA}{1-secA}= 2 cosec A

Considering LHS, we get,

= sinAcosA cosA+1cosA − sinAcosA cosA−1cosA $\frac{\frac{\sin A}{\cos A}}{\frac{\cos A+1}{\cos A}} - \frac{\frac{\sin A}{\cos A}}{\frac{\cos A-1}{\cos A}}$

= sinAcosA+1 − sinAcosA−1 $\frac{\sin A}{\cos A+1} - \frac{\sin A}{\cos A-1}$

= sinA(1cosA+1 − 1cosA−1) $\sin A\left(\frac{1}{\cos A+1} - \frac{1}{\cos A-1}\right)$

= sinA(cosA−1−cosA−1cos²A−1) $\sin A\left(\frac{\cos A-1-\cos A-1}{\cos^2 A-1}\right)$

= sinA(cosA−1−cosA−1cos²A−1) $\sin A\left(\frac{\cos A-1-\cos A-1}{\cos^2 A-1}\right)$

= sinA(−2−sin²A) $\sin A\left(\frac{-2}{-\sin^2 A}\right)$

= 2sinA) $\frac{2}{\sin A}$)

= 2 cosec A

Therefore, LHS = RHS

Hence proved

Q51: 1+ cot²Θ1+cosecΘ =cosecΘ $1 + \frac{\cot^2\Theta}{1+\operatorname{cosec}\Theta} = \operatorname{cosec}\Theta$

Ans:

1+ cosec²Θ−11+cosecΘ [∵cot²Θ=cosec²Θ−1] $1 + \frac{(\operatorname{cosec}\Theta-1)(\operatorname{cosec}\Theta+1)}{1+\operatorname{cosec}\Theta}$ 1+ (cosecΘ−1)(cosecΘ+1)1+cosecΘ =1+cosecΘ−1[∵(a+b)(a−b)=a^2-b^2)] =cosecΘ= $\operatorname{cosec}\Theta$

Therefore, LHS = RHS

Hence, proved.

Q52: cosΘcosecΘ+1 + cosΘcosecΘ−1 =2tanΘ $\frac{\cos\Theta}{\operatorname{cosec}\Theta+1} + \frac{\cos\Theta}{\operatorname{cosec}\Theta-1} = 2\tan\Theta$

Ans:

cosΘ 1sinΘ+1 + cosΘ 1sinΘ−1 $\frac{\cos\Theta}{\frac{1}{\sin\Theta}+1} + \frac{\cos\Theta}{\frac{1}{\sin\Theta}-1}$ cosΘ 1+sinΘsinΘ + cosΘ 1−sinΘsinΘ $\frac{\cos\Theta}{\frac{1+\sin\Theta}{\sin\Theta}} + \frac{\cos\Theta}{\frac{1-\sin\Theta}{\sin\Theta}}$ (cosΘ)(sinΘ)1+sinΘ + (cosΘ)(sinΘ)1−sinΘ $\frac{(\cos\Theta)(\sin\Theta)}{1+\sin\Theta} + \frac{(\cos\Theta)(\sin\Theta)}{1-\sin\Theta}$ (1−sinΘ)(sinΘcosΘ)+(sinΘcosΘ)(1+sinΘ)(1−sinΘ) $\frac{(1-\sin\Theta)(\sin\Theta\cos\Theta)+(\sin\Theta\cos\Theta)}{(1+\sin\Theta)(1-\sin\Theta)}$ sinΘcosΘ−sinΘcosΘ+sinΘcosΘ+sin²Θcos²Θ1−sin²Θ $\frac{\sin\Theta\cos\Theta-\sin\Theta\cos\Theta+\sin\Theta\cos\Theta+\sin^2\Theta\cos^2\Theta}{1-\sin^2\Theta}$ =sinΘcosΘcos²Θ $= \frac{\sin\Theta\cos\Theta}{\cos^2\Theta}$ =2sinΘcosΘ $= \frac{2\sin\Theta}{\cos\Theta}$ =2tanΘ= $2\tan\Theta$

Therefore, LHS = RHS

Hence, proved

Q53) (1+tan²A)+(1+ 1tan²A)= 1sin²A−sin⁴A $(1+\tan^2 A) + \left(1 + \frac{1}{\tan^2 A}\right) = \frac{1}{\sin^2 A - \sin^4 A}$

Ans:

LHS = (1+ sin^{2}Acos^{2}A)+(1+ cos^{2}Asin^{2}A) $(1+\frac{\sin^2 A}{\cos^2 A})+(1+\frac{\cos^2 A}{\sin^2 A})$

=> cos^{2}A+sin^{2}Acos^{2}A + sin^{2}A+cos^{2}Asin^{2}A $\frac{\cos^2 A+\sin^2 A}{\cos^2 A}+\frac{\sin^2 A+\cos^2 A}{\sin^2 A}$

=> 1cos^{2}A + 1sin^{2}A [∵sin^{2}A+cos^{2}A=1

=> sin^{2}A+cos^{2}Asin^{2}Acos^{2}A = 1sin^{2}A(1−sin^{2}A) [∵cos^{2}A=1−sin^{2}A]

=> 1sin^{2}A−sin^{4}A $\frac{1}{\sin^2 A-\sin^4 A}$

Therefore, LHS = RHS.

Hence Proved.

Q54) sin^{2}Acos^{2}B – cos^{2}Asin^{2}B = sin^{2}A – sin^{2}B

Ans:

LHS = sin^{2}Acos^{2}B – cos^{2}Asin^{2}B

= sin^{2}A(1−sin^{2}B)−(1−sin^{2}A)(sin^{2}A)[∵cos^{2}A=1−sin^{2}A]

= sin^{2}A−sin^{2}Asin^{2}B−sin^{2}B+sin^{2}Asin^{2}B $\sin^2 A-\sin^2 A\sin^2 B-\sin^2 B+\sin^2 A\sin^2 B$

= sin^{2}A−sin^{2}B $\sin^2 A-\sin^2 B$

= RHS

Hence Proved.

Q55: (i) cotA+tanBcotB+tanA =cotAtanB $\frac{\cot A+\tan B}{\cot B+\tan A}=\cot A\tan B$

Ans:

LHS = cotA+tanBcotB+tanA $\frac{\cot A+\tan B}{\cot B+\tan A}$

= cosAsinA + sinBcosB cosBsinB + sinAcosA $\frac{\frac{\cos A}{\sin A}+\frac{\sin B}{\cos B}}{\frac{\cos B}{\sin B}+\frac{\sin A}{\cos A}}$

= cosAcosB+sinAsinBsinAcosB cosAcosB+sinAsinBcosAsinB $\frac{\frac{\cos A\cos B+\sin A\sin B}{\sin A\cos B}}{\frac{\cos A\cos B+\sin A\sin B}{\cos A\sin B}}$

= cosAcosB+sinAsinBsinAcosB × cosAsinBcosAcosB+sinAsinB $\frac{\cos A\cos B+\sin A\sin B}{\sin A\cos B}\times\frac{\cos A\sin B}{\cos A\cos B+\sin A\sin B}$

= cosAsinBsinAcosB $\frac{\cos A\sin B}{\sin A\cos B}$

= cotAtanB

= RHS

Hence Proved.

(ii) tanA+tanBcotA+cotB =tanAtanB $\frac{\tan A+\tan B}{\cot A+\cot B}=\tan A\tan B$

Ans:

LHS = tanA+tanBcotA+cotB $\frac{\tan A+\tan B}{\cot A+\cot B}$

$= \text{sinAcosA} + \text{sinBcosB cosAsinA} + \text{cosBsinB} \ \dfrac{\frac{\sin A}{\cos A}+\frac{\sin B}{\cos B}}{\frac{\cos A}{\sin A}+\frac{\cos B}{\sin B}}$

$= \text{sinAcosB+cosAsinBcosAcosB cosAsinB+cosBsinAsinAsinB} \ \dfrac{\frac{\sin A\cos B+\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\sin B+\cos B\sin A}{\sin A\sin B}}$

$= \text{sinAcosB+cosAsinBcosAcosB} \times \text{sinAsinBcosAsinB+cosBsinA} \ \dfrac{\sin A\cos B+\cos A\sin B}{\cos A\cos B} \times \dfrac{\sin A\sin B}{\cos A\sin B+\cos B\sin A}$

$= \text{sinAsinBcosAcosB} \ \dfrac{\sin A\sin B}{\cos A\cos B}$

= tanAtanB

= RHS

Hence Proved.

Q56) $\mathbf{cot^2Acosec^2B-cot^2Bcosec^2A=cot^2A-cot^2B}$ $\cot^2 A\operatorname{cosec}^2 B-\cot^2 B\operatorname{cosec}^2 A=\cot^2 A-\cot^2 B$

Ans:

LHS = $cot^2Acosec^2B-cot^2Bcosec^2A$ $\cot^2 A\operatorname{cosec}^2 B-\cot^2 B\operatorname{cosec}^2 A$

= $cot^2A(1+cot^2B)-cot^2B(1+cot^2A)$[∵$cosec^2θ=1+cot^2θ$]

= $cot^2A+cot^2Acot^2B-cot^2B-cot^2Bcot^2A$ $\cot^2 A+\cot^2 A\cot^2 B-\cot^2 B-\cot^2 B\cot^2 A$

= $cot^2A – cot^2B$

= RHS

Hence Proved.

Q57) $\mathbf{tan^2Asec^2B-sec^2Atan^2B=tan^2A-tan^2B}$ $\tan^2 A\sec^2 B-\sec^2 A\tan^2 B=\tan^2 A-\tan^2 B$

Ans:

LHS = $tan^2Asec^2B-sec^2Atan^2B$ $\tan^2 A\sec^2 B-\sec^2 A\tan^2 B$

= $tan^2A(1+tan^2B)-sec^2A(tan^2A)$ $\tan^2 A(1+\tan^2 B)-\sec^2 A(\tan^2 A)$

= $tan^2A+tan^2Atan^2B-tan^2B(1+tan^2A)$[∵$sec^2A=1+tan^2A$]

= $tan^2A+tan^2Atan^2B-tan^2B-tan^2Atan^2B$ $\tan^2 A+\tan^2 A\tan^2 B-\tan^2 B-\tan^2 A\tan^2 B$

= tan^2A-tan^2B $\tan^2 A-\tan^2 B$

= RHS

Hence Proved.

Q58) If x = $\mathbf{asecθ+btanθ}$ $a\sec\theta+b\tan\theta$ **and y = $\mathbf{atanθ+bsecθ}$** $a\tan\theta+b\sec\theta$ **, prove that $x^2 – y^2 = a^2 – b^2$.**

Ans:

LHS = $x^2 – y^2$

= $(asecθ+btanθ)^2-(atanθ+bsecθ)^2$ $(a\sec\theta+b\tan\theta)^2-(a\tan\theta+b\sec\theta)^2$

$= a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta\tan\theta - a^2\tan^2\theta - b^2\sec^2\theta - 2ab\sec\theta\tan\theta$

$= a^2\sec^2\theta + b^2\tan^2\theta - a^2\tan^2\theta - b^2\sec^2\theta$

$= a^2\sec^2\theta - b^2\sec^2\theta + b^2\tan^2\theta - a^2\tan^2\theta$

$= \sec^2\theta(a^2 - b^2) + \tan^2\theta(b^2 - a^2)$

$= \sec^2\theta(a^2 - b^2) - \tan^2\theta(a^2 - b^2)$

$= (\sec^2\theta - \tan^2\theta)(a^2 - b^2)$

$= a^2 - b^2$

= RHS

Hence Proved.

Q59) If $3\sin\theta + 5\cos\theta = 5$, prove that $5\sin\theta - 3\cos\theta = \pm 3$.

Ans:

Given $3\sin\theta + 5\cos\theta = 5$

$3\sin\theta = 5 - 5\cos\theta$ $\quad 3\sin\theta = 5(1 - \cos\theta)$ $\quad 3\sin\theta = \frac{5(1-\cos\theta)(1-\cos\theta)}{1+\cos\theta}$ $\quad 3\sin\theta = \frac{5(1-\cos^2\theta)}{1+\cos\theta}$ $\quad 3\sin\theta = \frac{5\sin^2\theta}{1+\cos\theta}$ $\quad 3 + 3\cos\theta = 5\sin\theta$ $\quad 3 = 5\sin\theta - 3\cos\theta$

= RHS

Hence Proved.

Q60) If $\text{cosec}\theta + \cot\theta = m$ and $\text{cosec}\theta - \cot\theta = n$, prove that mn = 1.

Ans:

LHS = mn

$= (\text{cosec}\theta + \cot\theta)(\text{cosec}\theta - \cot\theta)$

$= \text{cosec}^2\theta - \cot^2\theta$

= 1

= RHS

Hence Proved.

Q 62 . If $T_n = \sin^n\theta + \cos_n\theta$, prove that $\frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}$.

Ans:

LHS = $\frac{(\sin^3\theta + \cos^3\theta) - (\sin^5\theta + \cos^5\theta)}{\sin\theta + \cos\theta}$

$= \frac{\sin^3\theta(1 - \sin^2\theta) + \cos^3\theta(1 - \cos^2\theta)}{\sin\theta + \cos\theta}$

$= \frac{\sin^3\theta \times \cos^2\theta + \cos^3\theta \times \sin^2\theta}{\sin\theta + \cos\theta}$

$= \frac{\sin^2\theta\cos^2\theta(\sin\theta+\cos\theta)}{\sin\theta+\cos\theta}$

$= \sin^2\theta\cos^2\theta$

RHS = Missing close brace

= Missing close brace

$= \frac{\sin^5\theta\times\cos^2\theta+\cos^5\theta\times\sin^2\theta}{\sin^3\theta+\cos^3\theta}$

$= \frac{\sin^2\theta\cos^2\theta(\sin^3\theta+\cos^3\theta)}{\sin\theta+\cos\theta}$

$= \sin^2\theta\cos^2\theta$

LHS = RHS Hence proved .

Q 63 . $(\tan\theta + \frac{1}{\cos\theta})^2 + (\tan\theta - \frac{1}{\cos\theta})^2$ **=** $2(\frac{1+\sin^2\theta}{1-\sin^2\theta})$

Ans:

$(\tan\theta + \sec\theta)^2 + (\tan\theta - \sec\theta)^2$

$= \tan^2\theta + \sec^2\theta + 2\tan\theta\sec\theta + \tan^2\theta + \sec^2\theta - 2\tan\theta\sec\theta$

$= 2\tan^2\theta + 2\sec^2\theta$

$= 2[\tan^2\theta + \sec^2\theta]$

$= 2[\frac{\sin^2\theta}{\cos^2\theta} + \frac{1}{\cos^2\theta}]$

$= 2(\frac{1+\sin^2\theta}{\cos^2\theta})$

$= 2(\frac{1+\sin^2\theta}{1-\sin^2\theta})$

= RHS

LHS = RHS Hence proved .

Q 64 . $(\frac{1}{\sec^2\theta-\cos^2\theta} + \frac{1}{\mathrm{cosec}^2\theta-\sin^2})\sin^2\theta\cos^2\theta$ **=** $\frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$

Ans:

$[\frac{1}{\frac{1}{\cos^2\theta}-\cos^2\theta} + \frac{1}{\frac{1}{\sin^2\theta}-\sin^2\theta}]\sin^2\theta\cos^2$

$= [\frac{1}{\frac{1-\cos^4\theta}{\cos^2\theta}} + \frac{1}{\frac{1-\sin^4\theta}{\sin^2\theta}}]\sin^2\theta\cos^2\theta$

$= [\frac{\cos^2\theta}{1-\cos^4\theta} + \frac{\sin^2\theta}{1-\sin^4\theta}]\sin^2\theta\cos^2\theta$

$= [\frac{\cos^2\theta}{\cos^2\theta+\sin^2\theta-\cos^4\theta} + \frac{\sin^2\theta}{\cos^2\theta+\sin^2\theta-\sin^4\theta}]\sin^2\theta\cos^2\theta$

$= [\frac{\cos^2\theta}{\cos^2\theta(1-\cos^2\theta)+\sin^2\theta} + \frac{\sin^2\theta}{\cos^2\theta+\sin^2\theta(1-\sin^2\theta)}]\sin^2\theta\cos^2\theta$

$= [\cos^2\theta\cos^2\theta\sin^2\theta+\sin^2\theta + \sin^2\theta\cos^2\theta+\sin^2\theta\cos^2\theta]\sin^2\theta\cos^2\theta[\frac{\cos^2\theta}{\cos^2\theta\sin^2\theta+\sin^2\theta} + \frac{\sin^2\theta}{\cos^2\theta+\sin^2\theta\cos^2\theta}]\sin^2\theta\cos^2\theta$

$= [\cos^2\theta\sin^2\theta(\cos^2\theta+1) + \sin^2\theta\cos^2\theta(\sin^2\theta+1)]\sin^2\theta\cos^2\theta[\frac{\cos^2\theta}{\sin^2\theta(\cos^2\theta+1)} + \frac{\sin^2\theta}{\cos^2\theta(\sin^2\theta+1)}]\sin^2\theta\cos^2\theta$

$= \cos^4\theta(\sin^2\theta+1)+\sin^4\theta(\cos^2\theta+1)\sin^2\theta\cos^2\theta(\cos^2\theta+1)(\sin^2\theta+1)\sin^2\theta\cos^2\theta\frac{\cos^4\theta(\sin^2\theta+1)+\sin^4\theta(\cos^2\theta+1)}{\sin^2\theta\cos^2\theta(\cos^2\theta+1)(\sin^2\theta+1)}\sin^2\theta\cos^2\theta$

$= \cos^4\theta(\sin^2\theta+1)+\sin^4\theta(\cos^2\theta+1)(\cos^2\theta+1)(\sin^2\theta+1)\frac{\cos^4\theta(\sin^2\theta+1)+\sin^4\theta(\cos^2\theta+1)}{(\cos^2\theta+1)(\sin^2\theta+1)}$

$= \cos^4\theta+\cos^4\theta\sin^2\theta+\sin^4\theta+\sin^4\theta\cos^2\theta 1+\sin^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta\frac{\cos^4\theta+\cos^4\theta\sin^2\theta+\sin^4\theta+\sin^4\theta\cos^2\theta}{1+\sin^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta}$

$= 1-2\sin^2\theta\cos^2\theta+\sin^2\theta\cos^2\theta(\cos^2\theta+\sin^2\theta)1+1+\cos^2\theta\sin^2\theta\frac{1-2\sin^2\theta\cos^2\theta+\sin^2\theta\cos^2\theta(\cos^2\theta+\sin^2\theta)}{1+1+\cos^2\theta\sin^2\theta}$

$= 1-\sin^2\theta\cos^2\theta 2+\sin^2\theta\cos^2\theta\frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$

LHS = RHS   Hence proved .

Q 65 . (i) . $[1+\sin\theta-\cos\theta 1+\sin\theta+\cos\theta]^2[\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}]^2 = 1-\cos\theta 1+\cos\theta\frac{1-\cos\theta}{1+\cos\theta}$

Ans:

$= (1+\sin\theta-\cos\theta 1+\sin\theta+\cos\theta \times 1+\sin\theta-\cos\theta 1+\sin\theta-\cos\theta)^2(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta} \times \frac{1+\sin\theta-\cos\theta}{1+\sin\theta-\cos\theta})^2$

$= [(1+\sin\theta-\cos\theta)^2(1+\sin\theta)^2-\cos^2\theta]^2[\frac{(1+\sin\theta-\cos\theta)^2}{(1+\sin\theta)^2-\cos^2\theta}]^2$

$= [(1)^2+\sin^2\theta+\cos^2\theta+2\times1\times\sin\theta+2\times\sin\theta(-\cos\theta)-2\cos\theta 1-\cos^2\theta+\sin^2\theta+2\sin\theta]^2[\frac{(1)^2+\sin^2\theta+\cos^2\theta+2\times1\times\sin\theta+2\times\sin\theta(-\cos\theta)-2\cos\theta}{1-\cos^2\theta+\sin^2\theta+2\sin\theta}]^2$

$= [1+1+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta\sin^2\theta+\sin^2\theta+2\sin\theta]^2[\frac{1+1+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{\sin^2\theta+\sin^2\theta+2\sin\theta}]^2$

$= [2+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta 2\sin^2\theta+2\sin\theta]^2[\frac{2+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{2\sin^2\theta+2\sin\theta}]^2$

$= [2(1+\sin\theta)-2\cos\theta(\sin\theta+1)2\sin\theta(\sin\theta+1)]^2[\frac{2(1+\sin\theta)-2\cos\theta(\sin\theta+1)}{2\sin\theta(\sin\theta+1)}]^2$

$= [(1+\sin\theta)(2-2\cos\theta)2\sin\theta(\sin\theta+1)]^2[\frac{(1+\sin\theta)(2-2\cos\theta)}{2\sin\theta(\sin\theta+1)}]^2$

$= [(2-2\cos\theta)2\sin\theta]^2[\frac{(2-2\cos\theta)}{2\sin\theta}]^2$

$= [(1-\cos\theta)\sin\theta]^2[\frac{(1-\cos\theta)}{\sin\theta}]^2$

$= (1-\cos\theta)^2 1-\cos^2\theta\frac{(1-\cos\theta)^2}{1-\cos^2\theta}$

$= (1-\cos\theta)\times(1-\cos\theta)(1-\cos\theta)(1+\cos\theta)\frac{(1-\cos\theta)\times(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$= 1-\cos\theta 1+\cos\theta\frac{1-\cos\theta}{1+\cos\theta}$

LHS = RHS   Hence proved .

Q 65 (ii) . $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta} = \frac{1-\sin\theta}{\cos\theta}$

Ans:

= LHS = $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}$

= $\frac{(\sec^2\theta-\tan^2\theta)+(\sec\theta-\tan\theta)}{1+\sec\theta+\tan\theta}$ [since , $\sec^2\theta-\tan^2\theta=1$]

= $\frac{(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)+(\sec\theta-\tan\theta)}{1+\sec\theta+\tan\theta}$

= $\frac{(\sec\theta-\tan\theta)(1+\sec\theta+\tan\theta)}{1+\sec\theta+\tan\theta}$

= $(\sec\theta-\tan\theta)$

= $\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

= $\frac{1-\sin\theta}{\cos\theta}$

LHS = RHS Hence proved .

Q 66 . (sec A + tan A – 1)(sec A – tan A + 1) = 2 tan A

Ans:

= $(\sec A+\tan A-\{\sec^2A-\tan^2A\})[\sec A-\tan A+(\sec^2A-\tan^2A)]$

= $(\sec A+\tan A-(\sec A+\tan A)(\sec A-\tan A))[\sec A-\tan A+(\sec A-\tan A)(\sec A+\tan A)]$

= $(\sec A+\tan A)(1-(\sec A-\tan A))(\sec A-\tan A)(1+(\sec A+\tan A))$

= $(\sec^2A-\tan^2A)(1-\sec A+\tan A)(1+\sec A+\tan A)$

= $(1-\frac{1}{\cos A}+\frac{\sin A}{\cos A})(1+\frac{1}{\cos A}+\frac{\sin A}{\cos A})$

= $(\frac{\cos A-1+\sin A}{\cos A})(\frac{\cos A+1+\sin A}{\cos A})$

= $(\frac{(\cos A+\sin A)^2-1}{\cos^2A})$

= $(\frac{\cos^2A+\sin^2A+2\sin A\cos B-1}{\cos^2A})$

= $(\frac{1+2\sin A\cos B-1}{\cos^2A})$

= $(\frac{2\sin A\cos B}{\cos^2A})$

= 2 tan A

LHS = RHS Hence proved .

Q 67 . (1 + cot A – cosec A)(1 + tan A + sec A) = 2

Ans:

LHS = (1 + cot A – cosec A)(1 + tan A + sec A)

= (1+ cosAsinA − 1sinA)(1+ sinAcosA + 1cosA) $(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A})(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A})$

= (sinA+cosA−1sinA)(cosA+sinA+1cosA) $(\frac{\sin A+\cos A-1}{\sin A})(\frac{\cos A+\sin A+1}{\cos A})$

= ((sinA−cosA)^2−1sinAcosA) $(\frac{(\sin A-\cos A)^2-1}{\sin A\cos A})$

= sin^{2}A+2sinAcosA+cos^{2}A−1sinAcosA $\frac{\sin^2 A+2\sin A\cos A+\cos^2 A-1}{\sin A\cos A}$

= (1+2sinAcosA−1sinAcosA) $(\frac{1+2\sin A\cos A-1}{\sin A\cos A})$

= 2

LHS = RHS Hence proved .

Q 68 . (cosecθ−secθ)(cotθ−tanθ) $(\text{cosec}\theta-\sec\theta)(\cot\theta-\tan\theta)$ **= (cosecθ+secθ)(secθcosecθ–2)** $(\text{cosec}\theta+\sec\theta)(\sec\theta\text{cosec}\theta-2)$

Ans:

LHS = (cosecθ−secθ)(cotθ−tanθ) $(\text{cosec}\theta-\sec\theta)(\cot\theta-\tan\theta)$

[1sinθ − 1cosθ][cosθsinθ − sinθcosθ] $[\frac{1}{\sin\theta}-\frac{1}{\cos\theta}][\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}]$ [cosθ−sinθsinθcosθ][cos^{2}θ−sin^{2}θsinθcosθ]

$[\frac{\cos\theta-\sin\theta}{\sin\theta\cos\theta}][\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}]$ [(cosθ−sinθ)^2(cosθ+sinθ)sin^{2}θcos^{2}θ] $[\frac{(\cos\theta-\sin\theta)^2(\cos\theta+\sin\theta)}{\sin^2\theta\cos^2\theta}]$

RHS = (cosecθ+secθ)(secθcosecθ–2) $(\text{cosec}\theta+\sec\theta)(\sec\theta\text{cosec}\theta-2)$

[1sinθ + 1cosθ][1cosθ − 1sinθ −2] $[\frac{1}{\sin\theta}+\frac{1}{\cos\theta}][\frac{1}{\cos\theta}-\frac{1}{\sin\theta}-2]$

= [sinθ+cosθsinθcosθ][1−2sinθcosθsinθcosθ] $[\frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}][\frac{1-2\sin\theta\cos\theta}{\sin\theta\cos\theta}]$

= [sinθ+cosθsinθcosθ][cos^{2}θ+sin^{2}θ−2sinθcosθsinθcosθ] $[\frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}][\frac{\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta}{\sin\theta\cos\theta}]$

= [(cosθ−sinθ)^2(cosθ+sinθ)sin^{2}θcos^{2}θ] $[\frac{(\cos\theta-\sin\theta)^2(\cos\theta+\sin\theta)}{\sin^2\theta\cos^2\theta}]$ [∵cos^{2}θ+sin^{2}θ=1]

LHS = RHS Hence proved .

Q 70 . cosAcosecA−sinAsecAcosA+sinA $\frac{\cos A\,\text{cosec}A-\sin A\sec A}{\cos A+\sin A}$ **= cosec A – sec A**

Ans:

LHS = cosAcosecA−sinAsecAcosA+sinA $\frac{\cos A\,\text{cosec}A-\sin A\sec A}{\cos A+\sin A}$

= cosA× 1sinA −sinA× 1cosA cosA+sinA $\frac{\cos A\times\frac{1}{\sin A}-\sin A\times\frac{1}{\cos A}}{\cos A+\sin A}$

= cosAsinA − sinAcosA cosA+sinA $\frac{\frac{\cos A}{\sin A}-\frac{\sin A}{\cos A}}{\cos A+\sin A}$

= cos^{2}A−sin^{2}AcosAsinA cosA+sinA $\frac{\frac{\cos^2 A-\sin^2 A}{\cos A\sin A}}{\cos A+\sin A}$

= cos²A−sin²AcosAsinA × 1cosA+sinA $\frac{\cos^2 A-\sin^2 A}{\cos A\sin A} \times \frac{1}{\cos A+\sin A}$

= (cosA−sinA)(cosA+sinA)cosAsinA×(cosA+sinA) $\frac{(\cos A-\sin A)(\cos A+\sin A)}{\cos A\sin A\times(\cos A+\sin A)}$

= (cosA−sinA)cosAsinA $\frac{(\cos A-\sin A)}{\cos A\sin A}$

= cosAcosAsinA − sinAcosAsinA $\frac{\cos A}{\cos A\sin A} - \frac{\sin A}{\cos A\sin A}$

= 1sinA − 1cosA $\frac{1}{\sin A} - \frac{1}{\cos A}$

= cosecA−secA $\operatorname{cosec} A - \sec A$

= RHS

∴ LHS = RHS Hence proved .

Q 71 . sinAsecA+tanA−1 + cosAcosecA+cotA−1 $\frac{\sin A}{\sec A+\tan A-1} + \frac{\cos A}{\operatorname{cosec} A+\cot A-1}$ **= 1**

Ans:

LHS : sinAsecA+tanA−1 + cosAcosecA+cotA−1 $\frac{\sin A}{\sec A+\tan A-1} + \frac{\cos A}{\operatorname{cosec} A+\cot A-1}$

= sinA1cosA+sinAcosA−1 + cosA1sinA+cosAsinA−1 $\frac{\sin A}{\frac{1}{\cos A}+\frac{\sin A}{\cos A}-1} + \frac{\cos A}{\frac{1}{\sin A}+\frac{\cos A}{\sin A}-1}$

= sinA1+sinA−cosAcosA + cosA1+cosA−sinAsinA $\frac{\sin A}{\frac{1+\sin A-\cos A}{\cos A}} + \frac{\cos A}{\frac{1+\cos A-\sin A}{\sin A}}$

= sinAcosA1+sinA−cosA + cosAsinA1+cosA−sinA $\frac{\sin A\cos A}{1+\sin A-\cos A} + \frac{\cos A\sin A}{1+\cos A-\sin A}$

= (sinAcosA)[11+sinA−cosA + 11+cosA−sinA] $(\sin A\cos A)\left[\frac{1}{1+\sin A-\cos A} + \frac{1}{1+\cos A-\sin A}\right]$

= (sinAcosA)[2cosA−sinA+sinA+sinAcosA−sin²A−cosA−cos²A+cosAsinA] $(\sin A\cos A)\left[\frac{2}{\cos A-\sin A+\sin A+\sin A\cos A-\sin^2 A-\cos A-\cos^2 A+\cos A\sin A}\right]$

= (sinAcosA)[21−sin²A−cos²A+2sinAcosA] $(\sin A\cos A)\left[\frac{2}{1-\sin^2 A-\cos^2 A+2\sin A\cos A}\right]$

= (sinAcosA)[21−(sin²A−cos²A)+2sinAcosA] $(\sin A\cos A)\left[\frac{2}{1-(\sin^2 A-\cos^2 A)+2\sin A\cos A}\right]$

= (sinAcosA)[21−1+2sinAcosA] $(\sin A\cos A)\left[\frac{2}{1-1+2\sin A\cos A}\right]$

= (sinAcosA)× 22sinAcosA $(\sin A\cos A) \times \frac{2}{2\sin A\cos A}$

= 1

= RHS

∴ LHS = RHS Hence proved .

Q 72 . tanA(1+tan²A)² + cotA(1+cot²A)² $\frac{\tan A}{(1+\tan^2 A)^2} + \frac{\cot A}{(1+\cot^2 A)^2}$ **= sin A cos A**

Ans:

tanA(sec²A)² + cotA(cosec²A)² $\frac{\tan A}{(\sec^2 A)^2} + \frac{\cot A}{(\operatorname{cosec}^2 A)^2}$ [1 +tan² A = sec² A , 1 +cot² A = cosec² A]

= sinAcosAsec⁴A + cosAsinAcosec⁴A $\frac{\frac{\sin A}{\cos A}}{\sec^4 A} + \frac{\frac{\cos A}{\sin A}}{\operatorname{cosec}^4 A}$

= sinAcosA 1cos^{4}A + cosAsinA 1sin^{4}A $\frac{\frac{\sin A}{\cos A}}{\frac{1}{\cos^4 A}} + \frac{\frac{\cos A}{\sin A}}{\frac{1}{\sin^4 A}}$

= sinAcosA × cos^{4}A1 + cosAsinA × sin^{4}A1 $\frac{\sin A}{\cos A} \times \frac{\cos^4 A}{1} + \frac{\cos A}{\sin A} \times \frac{\sin^4 A}{1}$

= sinA×cos^{3}A+cosA×sin^{3}A$\sin A \times \cos^3 A + \cos A \times \sin^3 A$

= sinAcosA(cos^{2}A+sin^{2}A)$\sin A\cos A\,(\cos^2 A + \sin^2 A)$

= sinAcosA$\sin A\cos A$

LHS = RHS   Hence proved .

Q73. sec^{4}A(1−sin^{4}A)−2tan^{2}A=1$\sec^4 A(1 - \sin^4 A) - 2\tan^2 A = 1$

Ans:

Given, L.H.S = \(sec^{4}A(1\;-\;sin^{4}A)\;-\;2tan^{2}A

= [latex]sec^{4}A\;-\;sec^{4}A\;\times \;sin^{4}A\;-\;2tan^{4}A\)

= sec^{4}A−(1cos^{4}A ×sin^{4}A)−2tan^{4}A$\sec^4 A - (\frac{1}{\cos^4 A} \times \sin^4 A) - 2\tan^4 A$

= sec^{4}A−tan^{4}A−2tan^{4}A$\sec^4 A - \tan^4 A - 2\tan^4 A$

= (sec^{2}A)^2−tan^{4}A−2tan^{4}A$(\sec^2 A)^2 - \tan^4 A - 2\tan^4 A$

= (1+tan^{2}A)^2−tan^{4}A−2tan^{4}A$(1 + \tan^2 A)^2 - \tan^4 A - 2\tan^4 A$

= 1+tan^{4}A+2tan^{2}A−tan^{4}A−2tan^{4}A$1 + \tan^4 A + 2\tan^2 A - \tan^4 A - 2\tan^4 A$

= 1

Hence, L.H.S = R.H.S

Q74. cot^{2}A(secA−1)1+sinA $\frac{\cot^2 A(\sec A - 1)}{1 + \sin A}$ **= sec^{2}A[1−sinA1+sinA]**$\sec^2 A[\frac{1 - \sin A}{1 + \sin A}]$

Ans:

Given, L.H.S = cot^{2}A(secA−1)1+sinA $\frac{\cot^2 A(\sec A - 1)}{1 + \sin A}$

Here, sin^{2}A+cos^{2}A$\sin^2 A + \cos^2 A$ = 1

= cos^{2}Asin^{2}A(1cosA −1)1+sinA $\frac{\frac{\cos^2 A}{\sin^2 A}(\frac{1}{\cos A} - 1)}{1+\sin A}$

= cos^{2}Asin^{2}A(1−cosAcosA)1+sinA $\frac{\frac{\cos^2 A}{\sin^2 A}(\frac{1-\cos A}{\cos A})}{1+\sin A}$

= cosA×cosA(1−cos^{2}A)(1−cosAcosA)1+sinA $\frac{\frac{\cos A \times \cos A}{(1-\cos^2 A)}(\frac{1-\cos A}{\cos A})}{1+\sin A}$

= (cosA)(1+cosA) 11+sinA $\frac{(\cos A)}{(1+\cos A)}\ \frac{1}{1+\sin A}$

Solving,

RHS => sec^{2}a[1−sinA1+secA]$\sec^2 a[\frac{1-\sin A}{1+\sec A}]$

= 1cos^{2}A [1−sinA1+secA]$\frac{1}{\cos^2 A}[\frac{1-\sin A}{1+\sec A}]$

= $1\cos^2 A[1-\sin A 1+\sec A]$ $\frac{1}{\cos^2 A}\left[\frac{1-\sin A}{1+\sec A}\right]$

= $1\cos^2 A[1-\sin A\cos A+1]\cos A$ $\frac{1}{\cos^2 A}\left[\frac{1-\sin A}{\cos A+1}\right]\cos A$

= $(1-\sin A)(\cos A+1)(\cos A)$ $\frac{(1-\sin A)}{(\cos A+1)(\cos A)}$

Multiplying Nr. And Dr. with (1+SinA)

= $(1-\sin A)(\cos A+1)(\cos A) \times 1+\sin A 1+\sin A$ $\frac{(1-\sin A)}{(\cos A+1)(\cos A)} \times \frac{1+\sin A}{1+\sin A}$

= $(1^2-\sin^2 A)(\cos A+1)(\cos A)(1+\sin A)$ $\frac{(1^2-\sin^2 A)}{(\cos A+1)(\cos A)(1+\sin A)}$

= $\cos^2 A(\cos A+1)(\cos A)(1+\sin A)$ $\frac{\cos^2 A}{(\cos A+1)(\cos A)(1+\sin A)}$

= $\cos A(\cos A+1)(1+\sin A)$ $\frac{\cos A}{(\cos A+1)(1+\sin A)}$

Hence, LHS= RHS

Q75. (1+cotA+tanA)(sinA−cosA)$(1 + \cot A + \tan A)(\sin A - \cos A)$ **= secAcosec²A** $\frac{\sec A}{\mathrm{cosec}^2 A}$ **− cosecAsec²A** $\frac{\mathrm{cosec} A}{\sec^2 A}$ **= sinAtanA – cotAcosA**

Ans:

Given, L.H.S = (1+cotA+tanA)(sinA−cosA)$(1 + \cot A + \tan A)(\sin A - \cos A)$

=> sinA – cosA + cotAsinA – cotAcosA + sinAtanA – tanAcosA

=> sinA – cosA + cosAsinA ×sinA $\frac{\cos A}{\sin A} \times \sin A$ – cotAcosA + sinAtanA – sinAcosA ×cosA $\frac{\sin A}{\cos A} \times \cos A$

=> sinA – cosA + cosA – cotAcosA + sinAtanA – sinA

=> sinAtanA – cosAcotA

=> secAcosec²A $\frac{\sec A}{\mathrm{cosec}^2 A}$ – cosecAsec²A $\frac{\mathrm{cosec} A}{\sec^2 A}$

Here, secA = 1cosA $\frac{1}{\cos A}$ and cosecA = 1sinA $\frac{1}{\sin A}$

=> sin²AcosA $\frac{\sin^2 A}{\cos A}$ – cos²AsinA $\frac{\cos^2 A}{\sin A}$

=> sin²A−cos²AcosAsinA $\frac{\sin^2 A - \cos^2 A}{\cos A \sin A}$

=> (sinA× sinAcosA)$(\sin A \times \frac{\sin A}{\cos A})$ – (cosA× cosAcotA)$(\cos A \times \frac{\cos A}{\cot A})$

=> sinAtanA – cosAcotA

Hence, L.H.S = R.H.S

Q76. If xa cosθ+yb sinθ$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta$ **= 1 and xa cosθ−yb sinθ**$\frac{x}{a}\cos\theta - \frac{y}{b}\sin\theta$ **= 1, prove that x²a²+y²b²** $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ **= 2**

Ans:

Given,

=> (xa cosθ+ yb sinθ$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta$)² + (xa cosθ− yb sinθ$\frac{x}{a}\cos\theta - \frac{y}{b}\sin\theta$)² = 1² + 1²

=> x²a² cos²θ+ y²b² sin²θ+ 2xyab cosθsinθ+ x²a² sin²θ+ y²b² − 2xyab sinθcosθ
$\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + \frac{2xy}{ab}\cos\theta\sin\theta + \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2} - \frac{2xy}{ab}\sin\theta\cos\theta$ = 1 + 1

=> $x^2a^2\cos^2\theta + y^2b^2\sin^2\theta + x^2a^2\sin^2\theta + y^2b^2\sin^2\theta \frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\sin^2\theta$ = 2

=> $\cos^2\theta[x^2a^2 + y^2b^2]\cos^2\theta[\frac{x^2}{a^2} + \frac{y^2}{b^2}] + \sin^2\theta[x^2a^2 + y^2b^2]\sin^2\theta[\frac{x^2}{a^2} + \frac{y^2}{b^2}]$ = 2

=> $(\cos^2\theta + \sin^2\theta)[x^2a^2 + y^2b^2](\cos^2\theta + \sin^2\theta)[\frac{x^2}{a^2} + \frac{y^2}{b^2}]$ = 2

Here $\cos^2A + \sin^2A = 1$

=> (1) [\frac{x^{2}}{a^{2}}\;+\;\frac{y^{2}}{b^{2}}][/latex] = 2

=> [\frac{x^{2}}{a^{2}}\;+\;\frac{y^{2}}{b^{2}}][/latex] = 2

Q77. If $\mathrm{cosec}\theta - \sin\theta = a^3 \mathrm{cosec}\theta - \sin\theta = a^3$, $\sec\theta - \cos\theta = b^3 \sec\theta - \cos\theta = b^3$, prove that $a^2b^2(a^2+b^2)a^2b^2(a^2 + b^2)$ = 1

Ans:

Given, $\mathrm{cosec}\theta - \sin\theta = a^3 \mathrm{cosec}\theta - \sin\theta = a^3$

Here, $\mathrm{cosec}\theta = 1\sin\theta\, \mathrm{cosec}\theta = \frac{1}{\sin\theta}$

=> $1\sin\theta \frac{1}{\sin\theta} - \sin\theta\sin\theta = a^3a^3$

=> $1 - \sin^2\theta\sin\theta \frac{1 - \sin^2\theta}{\sin\theta} = a^3a^3$

Here $\cos^2A + \sin^2A = 1$

=> $\cos^2\theta\sin\theta \frac{\cos^2\theta}{\sin\theta} = a^3a^3$

=> $\cos^{23}\theta\sin^{13}\theta \frac{\cos^{\frac{2}{3}}\theta}{\sin^{\frac{1}{3}}\theta} = a$

Squaring on both sides

=> $a^2 = \cos^{43}\theta\sin^{23}\theta \frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta}$

$\sec\theta - \cos\theta = b^3 \sec\theta - \cos\theta = b^3$

=> $1\cos\theta \frac{1}{\cos\theta} - \cos\theta\cos\theta = b^3b^3$

=> $1 - \cos^2\theta\cos\theta \frac{1 - \cos^2\theta}{\cos\theta} = b^3b^3$

=> $\sin^2\theta\cos\theta \frac{\sin^2\theta}{\cos\theta} = b^3b^3$

=> $\sin^{23}\theta\cos^{13}\theta \frac{\sin^{\frac{2}{3}}\theta}{\cos^{\frac{1}{3}}\theta} = b$

Squaring on both sides

=> $b^2 = \sin^{43}\theta\cos^{23}\theta \frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta}$

Now, $a^2b^2(a^2+b^2)a^2b^2(a^2 + b^2)$

=> $\cos^{43}\theta\sin^{23}\theta \frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta} \times\times \sin^{43}\theta\cos^{23}\theta \frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta}(\cos^{43}\theta\sin^{23}\theta \frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta} + \sin^{43}\theta\cos^{23}\theta \frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta})$

=> $\cos^{23}\theta\cos^{\frac{2}{3}}\theta \sin^{23}\theta\sin^{\frac{2}{3}}\theta$ $\sin^{23}\theta\sin^{\frac{2}{3}}\theta$}[/latex]

= 1

Hence, L.H.S = R.H.S

Q78. If $acos^3$ θθ + 3acosθθ $sin^2θsin^2θ$ = m, $asin^3θsin^3θ$ + $3acos^2θcos^2θ$ sinθsinθ = n, prove that $(m+n)^{23}(m + n)^{\frac{2}{3}}$ + $(m−n)^{23}(m − n)^{\frac{2}{3}}$ = $2(a)^{23}(a)^{\frac{2}{3}}$

Ans:

Given, $(m+n)^{23}(m + n)^{\frac{2}{3}}$ + $(m−n)^{23}(m − n)^{\frac{2}{3}}$

Substitute the values of m and n in the above equation

=> (\\(($acos^3$ [latex]\theta\\) + 3acosθθ $sin^2θsin^2θ$ \;+\; $asin^3θsin^3θ$ + $3acos^2θcos^2θ$ sinθsinθ)^{\frac{2}{3}}[/latex] + (\\(($acos^3$[latex]\theta\\) + 3acosθθ $sin^2θsin^2θ$ \;-\; $asin^3θsin^3θ$ – $3acos^2θcos^2θ$ sinθsinθ)^{\frac{2}{3}}[/latex]

=> $(a)^{23}(a)^{\frac{2}{3}}$ ((\\((cos^3 [latex]\theta\\) + 3cosθθ $sin^2θsin^2θ$ \;+\; $sin^3θsin^3θ$ + $3cos^2θcos^2θ$ sinθsinθ)^{\frac{2}{3}}[/latex] + $(a)^{23}(a)^{\frac{2}{3}}$ ((\\((cos^3 [latex]\theta\\) + 3cosθθ $sin^2θsin^2θ$ \;-\; $sin^3θsin^3θ$ – $3cos^2θcos^2θ$ sinθsinθ)^{\frac{2}{3}}[/latex]

=> $(a)^{23}(a)^{\frac{2}{3}}$ $((cosθ+sinθ)^3)^{23}((cosθ + sinθ)^3)^{\frac{2}{3}}$ + $(a)^{23}(a)^{\frac{2}{3}}$ $((cosθ−sinθ)^3)^{23}((cosθ − sinθ)^3)^{\frac{2}{3}}$

=> $(a)^{23}(a)^{\frac{2}{3}}$ [$(cosθ+sinθ)^2(cosθ + sinθ)^2$] + $(a)^{23}(a)^{\frac{2}{3}}$ [$(cosθ−sinθ)^2(cosθ − sinθ)^2$]

=> $(a)^{23}(a)^{\frac{2}{3}}$ ($(cos^2θ+sin^2θ+2sinθcosθ)(cos^2θ + sin^2θ + 2sinθcosθ)$) + $(a)^{23}(a)^{\frac{2}{3}}$ ($(cos^2θ+sin^2θ−2sinθcosθ)(cos^2θ + sin^2θ − 2sinθcosθ)$)

=> $(a)^{23}(a)^{\frac{2}{3}}$ [1 + 2sinθcosθ2sinθcosθ] + $(a)^{23}(a)^{\frac{2}{3}}$ [1 – 2sinθcosθ2sinθcosθ]

=> $(a)^{23}(a)^{\frac{2}{3}}$ [1 + 2sinθcosθ2sinθcosθ] + 1 – 2sinθcosθ2sinθcosθ]

=> $(a)^{23}(a)^{\frac{2}{3}}$ (1 + 1)

=> $2(a)^{23}(a)^{\frac{2}{3}}$

Hence, L.H.S = R.H.S

Q79) Ifx=acos^{3}Θ,y=bsin^{3}Θ,provethat$(xa)^{23}+(yb)^{23}=1$ If $x = acos^3Θ,\ y = bsin^3Θ$, prove that $(\frac{x}{a})^{\frac{2}{3}} + (\frac{y}{b})^{\frac{2}{3}} = 1$

Ans:

x=acos^{3}Θ:y=bsin^{3}Θ $x = acos^3Θ :\ y = bsin^3Θ$ xa=cos^{3}Θ:yb=sin^{3}Θ $\frac{x}{a} = cos^3Θ :\ \frac{y}{b} = sin^3Θ$

L.H.S = $[xa]^{23}+[yb]^{23}[\frac{x}{a}]^{\frac{2}{3}} + [\frac{y}{b}]^{\frac{2}{3}}$

=$[cos^3Θ]^{23}+[sin^3Θ]^{23}$= $[cos^3Θ]^{\frac{2}{3}} + [sin^3Θ]^{\frac{2}{3}}$ =$cos^2Θ+sin^2Θ$(∵$cos^2Θ+sin^2Θ=1$)

=1

Hence proved.

Q80) IfacosΘ+bsinΘ=mandasinΘ−bcosΘ=n,provethat$a^2+b^2=m^2+n^2$
If $acosΘ + bsinΘ = m$ and $asinΘ − bcosΘ = n$, prove that $a^2 + b^2 = m^2 + n^2$

Ans:

R.H.S=m^2+n^2R. H. S = $m^2 + n^2$

$=(a\cos\Theta+b\sin\Theta)^2+$

$(a\sin\Theta-b\cos\Theta)^2=a^2\cos^2\Theta+b^2\sin^2\Theta+2ab\sin\Theta\cos\Theta+a^2\sin^2\Theta+b^2\cos^2\Theta-2ab\sin\Theta\cos\Theta=a^2\cos^2\Theta+b^2\cos^2\Theta+b^2\sin^2\Theta+a^2\sin^2\Theta$

$=a^2(\sin^2\Theta+\cos^2\Theta)+b^2(\sin^2\Theta+\cos^2\Theta)=a^2+b^2[\because\sin^2\Theta+\cos^2\Theta=1]$

$= m^2 + n^2$

Q81: If $\cos A+\cos^2 A=1$, prove that $\sin^2 A+\sin^4 A=1$ If $\cos A + \cos^2 A = 1$, prove that $\sin^2 A + \sin^4 A = 1$

Ans:

Given- $\cos A + \cos^2 A = 1$

We have to prove $\sin^2 A + \sin^4 A = 1$

Now, $\cos A + \cos^2 A = 1$

$\cos A = 1- \cos^2 A$

$\cos A = \sin^2 A$

$\sin^2 A = \cos A$

Therefore, we have $\sin^2 A + \sin^4 A = \cos A + (\cos A)^2 = \cos A + \cos^2 A = 1$

Hence proved.

Q82:

If $\cos\Theta+\cos^2\Theta=1$, prove that $\sin^{12}\Theta+3\sin^{10}\Theta+3\sin^8\Theta+\sin^6\Theta+2\sin^4\Theta+2\sin^2\Theta-2=1$

If $\cos\Theta + \cos^2\Theta = 1$, prove that $\sin^{12}\Theta + 3\sin^{10}\Theta + 3\sin^8\Theta + \sin^6\Theta + 2\sin^4\Theta + 2\sin^2\Theta - 2 = 1$

Ans:

$\cos\Theta+\cos^2\Theta=1$ $\cos\Theta + \cos^2\Theta = 1$ $\cos\Theta=1-\cos^2\Theta$ $\cos\Theta = 1 - \cos^2\Theta$

$\cos\Theta=\sin^2\Theta$ $\cos\Theta = \sin^2\Theta$ ……(i)

Now, $\sin^{12}\Theta+3\sin^{10}\Theta+3\sin^8\Theta+\sin^6\Theta+2\sin^4\Theta+2\sin^2\Theta-2$

Now, $\sin^{12}\Theta + 3\sin^{10}\Theta + 3\sin^8\Theta + \sin^6\Theta + 2\sin^4\Theta + 2\sin^2\Theta - 2$ $=(\sin^4\Theta)^3+3\sin^4\Theta.\sin^2\Theta(\sin^4\Theta+\sin^2\Theta)+(\sin^2\Theta)^3+2(\sin^2\Theta)^2+2\sin^2\Theta-2$

$= (\sin^4\Theta)^3 + 3\sin^4\Theta.\sin^2\Theta(\sin^4\Theta + \sin^2\Theta) + (\sin^2\Theta)^3 + 2(\sin^2\Theta)^2 + 2\sin^2\Theta - 2$ Using $(a+b)^3=a^3+b^3+3ab(a+b)$ and also from (i) $\cos\Theta=\sin^2\Theta$

Using $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ and also from (i) $\cos\Theta = \sin^2\Theta$ $(\sin^4\Theta+\sin^2\Theta)^3+2\cos^2\Theta+2\cos\Theta-2$

$(\sin^4\Theta + \sin^2\Theta)^3 + 2\cos^2\Theta + 2\cos\Theta - 2$ $((\sin^2\Theta)^2+\sin^2\Theta)^3+2\cos^2\Theta+2\cos\Theta-2$

$((\sin^2\Theta)^2 + \sin^2\Theta)^3 + 2\cos^2\Theta + 2\cos\Theta - 2$ $(\cos^2\Theta+\sin^2\Theta)^3+2\cos^2\Theta+2\cos\Theta-2$

$(\cos^2\Theta+\sin^2\Theta)^3+2\cos^2\Theta+2\cos\Theta-2$ 1+2cos²Θ+2sin²Θ−2[∵sin²Θ+cos²Θ=1]

1+2(cos²Θ+sin²Θ)−2 $1+2(\cos^2\Theta+\sin^2\Theta)-2$ 1+2(1)−2 $1+2(1)-2$ =1 $=1$

L.H.S = R.H.S

Hence proved.

Q83: Given that: (1+cosα)(1+cosβ)(1+cosγ)=(1−cosα)(1−cosβ)(1−cosγ) $(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma)=(1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)$ **. Show that one of the values of each member of this equality is sinαsinβsinγ** $\sin\alpha\ \sin\beta\ \sin\gamma$ **.**

Ans:

We know that 1+cosΘ=1+cos²Θ2−sin²Θ2=2cos²Θ2 $1+\cos\Theta=1+\cos^2\frac{\Theta}{2}-\sin^2\frac{\Theta}{2}=2\cos^2\frac{\Theta}{2}$

⇒2cos²α2.2cos²β2.2cos²γ2…..(i) $\Rightarrow 2\cos^2\frac{\alpha}{2}.2\cos^2\frac{\beta}{2}.2\cos^2\frac{\gamma}{2}\ .....(i)$ Multiply(i)withsinαsinβsinγanddivideitwithsameweget

Multiply (i) with sinα sinβ sinγ and divide it with same we get 8cos²α2.cos²β2.cos²γ2sinα.sinβ.sinγ ×sinα.sinβ.sinγ

$\frac{8\cos^2\frac{\alpha}{2}.\cos^2\frac{\beta}{2}.\cos^2\frac{\gamma}{2}}{\sin\alpha.\sin\beta.\sin\gamma}\times\sin\alpha.\sin\beta.\sin\gamma$ ⇒2cos²α2.cos²β2.cos²γ2×sinα.sinβ.sinγsinα2.sinβ2.sinγ2

$\Rightarrow\frac{2\cos^2\frac{\alpha}{2}.\ \cos^2\frac{\beta}{2}.\ \cos^2\frac{\gamma}{2}\times\sin\alpha.\ \sin\beta.\ \sin\gamma}{\sin\frac{\alpha}{2}.\ \sin\frac{\beta}{2}.\ \sin\frac{\gamma}{2}}$ sinα.sinβ.sinγ×cotα2.cotβ2.cotγ2 $\sin\alpha.\ \sin\beta.\ \sin\gamma\times\cot\frac{\alpha}{2}.\ \cot\frac{\beta}{2}.\ \cot\frac{\gamma}{2}$ RHS(1−cosα)(1−cosβ)(1−cosγ)RHS $(1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)$

We know that 1−cosΘ=1−cos²Θ2+sin²Θ2=2sin²Θ2 $1-\cos\Theta=1-\cos^2\frac{\Theta}{2}+\sin^2\frac{\Theta}{2}=2\sin^2\frac{\Theta}{2}$

⇒2.sin²α2 2.sin²β2 2.sin²γ2 $\Rightarrow 2.\sin^2\frac{\alpha}{2}\ 2.\sin^2\frac{\beta}{2}\ 2.\sin^2\frac{\gamma}{2}$ Multiply(i)withsinαsinβsinγanddivideitwithsameweget

Multiply (i) with sinα sinβ sinγ and divide it with same we get 8sin²α2.sin²β2.sin²γ2sinα.sinβ.sinγ ×sinα.sinβ.sinγ

$\frac{8\sin^2\frac{\alpha}{2}.\sin^2\frac{\beta}{2}.\sin^2\frac{\gamma}{2}}{\sin\alpha.\sin\beta.\sin\gamma}\times\sin\alpha.\sin\beta.\sin\gamma$ ⇒8sin²α2.sin²β2.sin²γ2×sinα.sinβ.sinγ2sinα2cosα2.2sinβ2cosβ2.2sinγ2cosγ2

$\Rightarrow\frac{8\sin^2\frac{\alpha}{2}.\ \sin^2\frac{\beta}{2}.\ \sin^2\frac{\gamma}{2}\times\sin\alpha.\ \sin\beta.\ \sin\gamma}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}.\ 2\sin\frac{\beta}{2}\cos\frac{\beta}{2}.\ 2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2}}$ ⇒sinα.sinβ.sinγ×tanα2.tanβ2.tanγ2 $\Rightarrow\sin\alpha.\ \sin\beta.\ \sin\gamma\times\tan\frac{\alpha}{2}.\ \tan\frac{\beta}{2}.\ \tan\frac{\gamma}{2}$

Hence sinαsinβsinγ $\sin\alpha\ \sin\beta\ \sin\gamma$ is the member of equality.

Q84: If sinΘ+cosΘ=x,provethatsin⁶Θ+cos⁶Θ=4−3(x²−1)²4 $\sin\Theta+\cos\Theta=x,\ \text{prove that}\ \sin^6\Theta+\cos^6\Theta=\frac{4-3(x^2-1)^2}{4}$ **.**

Ans:

sinΘ+cosΘ=x $\sin\Theta+\cos\Theta=x$

Squaring on both sides

(sinΘ+cosΘ)²=x² $(\sin\Theta+\cos\Theta)^2=x^2$ ⇒sinΘ²+cosΘ²+2sinΘcosΘ=x²

$\Rightarrow\sin\Theta^2+\cos\Theta^2+2\sin\Theta\cos\Theta=x^2$ ∴sinΘcosΘ=x²−12……(i) Weknowthatsin²Θ+cos²Θ=1

We know that $\sin^2\Theta+\cos^2\Theta=1$

Cubing on both sides

(sin²Θ+cos²Θ)³=1³ $(\sin^2\Theta+\cos^2\Theta)^3=1^3$ sin⁶Θ+cos⁶Θ+3sin²Θcos²Θ(sin²Θ+cos²Θ)=1

$\sin^6\Theta+\cos^6\Theta+3\sin^2\Theta\cos^2\Theta(\sin^2\Theta+\cos^2\Theta)=1$ ⇒sin⁶Θ+cos⁶Θ=1−3sin²Θcos²Θ

$\Rightarrow\sin^6\Theta+\cos^6\Theta=1-3\sin^2\Theta\cos^2\Theta$ ⇒sin⁶Θ+cos⁶Θ=1−3(x²−1)²4

$\Rightarrow\sin^6\Theta+\cos^6\Theta=1-\frac{3(x^2-1)^2}{4}$ ∴sin⁶Θ+cos⁶Θ=4−3(x²−1)²4

Q85. If x = asecθcosϕ $a\sec\theta\cos\phi$ **, y =bsecθsinϕ** $b\sec\theta\sin\phi$ **and z= ctanϕ** $c\ \tan\phi$ **, show that x²a²+y²b²−z²c²** $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}$ **= 1**

Ans:

Given, $x = a\sec\theta\cos\phi$

$y = b\sec\theta\sin\phi$

$z = c\tan\phi$

squaring x,y,z on the sides

$x^2 = a^2\sec^2\theta\cos^2\phi$

$\frac{x^2}{a^2} = \sec^2\theta\cos^2\phi$ — 1

$y^2 = b^2\sec^2\theta\sin^2\phi$

$\frac{y^2}{b^2} = \sec^2\theta\sin^2\phi$ — 2

$z^2 = c^2\tan^2\phi$

$\frac{z^2}{c^2} = \tan^2\phi$ —— 3

Substitute eq 1,2,3 in $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$

=> $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$

=> $\sec^2\theta\cos^2\phi + \sec^2\theta\sin^2\phi - \tan^2\phi$

=> $\sec^2\theta(\cos^2\phi + \sin^2\phi) - \tan^2\phi$

We know that, $\cos^2\phi + \sin^2\phi = 1$

=> $\sec^2\theta(1) - \tan^2\phi$

And, $\sec^2\theta - \tan^2\theta = 1$

=> 1

Hence, L.H.S= R.H.S

Q86. If $\sin\theta + 2\cos\theta$ prove that $2\sin\theta - \cos\theta = 2$

Ans:

Given, $\sin\theta + 2\cos\theta = 1$

Squaring on both sides

=> $(\sin\theta + 2\cos\theta)^2 = 1^2$

=> $\sin^2\theta + 4\cos^2\theta + 4\sin\theta\cos\theta = 1$

=> $4\cos^2\theta + 4\sin\theta\cos\theta = 1 - \sin^2\theta$

Here, $1 - \sin^2\theta = \cos^2\theta$

=> $4\cos^2\theta + 4\sin\theta\cos\theta - \cos^2\theta = 0$

=> $3\cos^2\theta + 4\sin\theta\cos\theta = 0$ — 1

We have, $2\sin\theta - \cos\theta = 2$

Squaring L.H.S

$(2\sin\theta - \cos\theta)^2$ = $4\sin^2\theta + \cos^2\theta - 4\sin\theta\cos\theta$

Here, $4\sin\theta\cos\theta$ = $3\cos^2\theta$

= $4\sin^2\theta + \cos^2\theta + 3\cos^2\theta$

= $4\sin^2\theta + 4\cos^2\theta$

= $4(\sin^2\theta + \cos^2\theta)$

= 4(1)

= 4

$(2\sin\theta - \cos\theta)^2$ = 4

=> $2\sin\theta - \cos\theta$ = 2

Hence proved