

**Q.1) Evaluate the following : (i)  $\sin 20 \cos 70 \frac{\sin 20}{\cos 70}$**

**Sol (i) :** Given that,  $\sin 20 \cos 70 \frac{\sin 20}{\cos 70}$

Since  $\sin (90 - \Theta) = \cos \Theta$

$$\Rightarrow \Rightarrow \sin 20 \cos 70 \frac{\sin 20}{\cos 70} = \sin(90-70) \cos 70 \frac{\sin(90-70)}{\cos 70}$$

$$\Rightarrow \Rightarrow \sin 20 \cos 70 \frac{\sin 20}{\cos 70} = \cos 70 \cos 70 \frac{\cos 70}{\cos 70}$$

$$\Rightarrow \Rightarrow \sin 20 \cos 70 \frac{\sin 20}{\cos 70} = 1$$

Therefore  $\sin 20 \cos 70 \frac{\sin 20}{\cos 70} = 1$

**(ii)  $\cos 19 \sin 71 \frac{\cos 19}{\sin 71}$**

**Soln.(ii):** Given that,  $\cos 19 \sin 71 \frac{\cos 19}{\sin 71}$

$$\Rightarrow \Rightarrow \cos 19 \sin 71 \frac{\cos 19}{\sin 71} = \cos(90-71) \sin 71 \frac{\cos(90-71)}{\sin 71}$$

$$\Rightarrow \Rightarrow \cos 19 \sin 71 \frac{\cos 19}{\sin 71} = \sin 71 \sin 71 \frac{\sin 71}{\sin 71}$$

$$\Rightarrow \Rightarrow \cos 19 \sin 71 \frac{\cos 19}{\sin 71} = 1$$

Since  $\cos(90-\Theta) = \sin \Theta$

Therefore  $\cos 19 \sin 71 \frac{\cos 19}{\sin 71} = 1$

**(iii)  $\sin 21 \cos 69 \frac{\sin 21}{\cos 69}$**

**Soln.(iii):** Given that,  $\sin 21 \cos 69 \frac{\sin 21}{\cos 69}$

Since  $\sin(90-\Theta) = \cos \Theta$

$$\Rightarrow \Rightarrow \sin 21 \cos 69 \frac{\sin 21}{\cos 69} = \sin(90-69) \cos 69 \frac{\sin(90-69)}{\cos 69}$$

$$\Rightarrow \Rightarrow \sin 21 \cos 69 \frac{\sin 21}{\cos 69} = \cos 69 \cos 69 \frac{\cos 69}{\cos 69}$$

$$\Rightarrow \Rightarrow \sin 21 \cos 69 \frac{\sin 21}{\cos 69} = 1$$

(iv)  $\tan 10 \cot 80 \frac{\tan 10}{\cot 80}$

**Soln.(iv):** We are given that,  $\tan 10 \cot 80 \frac{\tan 10}{\cot 80}$

Since  $\tan(90-\Theta) = \cot \Theta$

$$\Rightarrow \Rightarrow \tan 10 \cot 80 \frac{\tan 10}{\cot 80} = \tan(90-80) \cot 80 \frac{\tan(90-80)}{\cot 80}$$

$$\Rightarrow \Rightarrow \tan 10 \cot 80 \frac{\tan 10}{\cot 80} =$$

$$\cot 80 \cot 80 \frac{\cot 80}{\cot 80}$$

$$\Rightarrow \Rightarrow \tan 10 \cot 80 \frac{\tan 10}{\cot 80} = 1$$

Therefore  $\tan 10 \cot 80 \frac{\tan 10}{\cot 80} = 1$

(v)  $\sec 11 \operatorname{cosec} 79 \frac{\sec 11}{\operatorname{cosec} 79}$

**Soln.(v):**

Given that,  $\sec 11 \operatorname{cosec} 79 \frac{\sec 11}{\operatorname{cosec} 79}$

Since  $\sec(90-\Theta) = \operatorname{cosec} \Theta$

$$\Rightarrow \Rightarrow \sec 11 \operatorname{cosec} 79 \frac{\sec 11}{\operatorname{cosec} 79} = \sec(90-79) \operatorname{cosec} 79 \frac{\sec(90-79)}{\operatorname{cosec} 79}$$

$$\Rightarrow \Rightarrow \sec 11 \operatorname{cosec} 79 \frac{\sec 11}{\operatorname{cosec} 79} = \operatorname{cosec} 79 \operatorname{cosec} 79 \frac{\operatorname{cosec} 79}{\operatorname{cosec} 79}$$

$$\Rightarrow \Rightarrow \sec 11 \operatorname{cosec} 79 \frac{\sec 11}{\operatorname{cosec} 79} = 1$$

Therefore  $\sec 11 \operatorname{cosec} 79 \frac{\sec 11}{\operatorname{cosec} 79} = 1$

**Q.2: EVALUATE THE FOLLOWING :**

$$(i) (\sin 49^\circ \cos 41^\circ)^2 \left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + (\cos 41^\circ \sin 49^\circ)^2 \left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2$$

**Soln.(i):**

We have to find:  $(\sin 49^\circ \cos 41^\circ)^2 \left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + (\cos 41^\circ \sin 49^\circ)^2 \left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2$

Since  $\sec 70^\circ \operatorname{cosec} 20^\circ = \frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ} + \sin 59^\circ \cos 31^\circ = \frac{\sin 59^\circ}{\cos 31^\circ}$   $\sin(90^\circ - 90^\circ - \Theta) = \cos \Theta$  and  $\cos(90^\circ - 90^\circ - \Theta) = \sin \Theta$

So

$$\begin{aligned} & (\sin(90^\circ - 41^\circ) \cos 41^\circ)^2 \left(\frac{\sin(90^\circ - 41^\circ)}{\cos 41^\circ}\right)^2 + (\cos(90^\circ - 49^\circ) \sin 49^\circ)^2 \left(\frac{\cos(90^\circ - 49^\circ)}{\sin 49^\circ}\right)^2 = (\cos 41^\circ \cos 41^\circ)^2 \\ & \left(\frac{\cos 41^\circ}{\cos 41^\circ}\right)^2 + (\sin 49^\circ \sin 49^\circ)^2 \left(\frac{\sin 49^\circ}{\sin 49^\circ}\right)^2 \end{aligned}$$

$$= 1 + 1 = 2$$

So value of  $(\sin 49^\circ \cos 41^\circ)^2 \left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + (\cos 41^\circ \sin 49^\circ)^2 \left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2$  is 2

**(ii)  $\cos 48^\circ \cos 48^\circ - \sin 42^\circ \sin 42^\circ$**

**Soln.(ii)**

We have to find:  $\cos 48^\circ \cos 48^\circ - \sin 42^\circ \sin 42^\circ$

Since  $\cos(90^\circ - \Theta) = \sin \Theta$  and  $\sin(90^\circ - \Theta) = \cos \Theta$ . So

$$\cos 48^\circ \cos 48^\circ - \sin 42^\circ \sin 42^\circ = \cos(90^\circ - 42^\circ) \cos(90^\circ - 42^\circ) - \sin 42^\circ \sin 42^\circ$$

$$= \cos 42^\circ \cos 42^\circ - \sin 42^\circ \sin 42^\circ = 0$$

So value of  $\cos 48^\circ \cos 48^\circ - \sin 42^\circ \sin 42^\circ$  is 0

**(iii)  $\cot 40^\circ \tan 50^\circ - 12 \left(\cos 35^\circ \sin 55^\circ\right) \frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2} \left(\frac{\cos 35^\circ}{\sin 55^\circ}\right)$**

**Soln.(iii)**

We have to find:

$$\cot 40^\circ \tan 50^\circ - 12 \left( \cos 35^\circ \sin 55^\circ \right) \frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2} \left( \frac{\cos 35^\circ}{\sin 55^\circ} \right)$$

Since  $\cot(90^\circ - \Theta) = \tan \Theta$  and  $\cos(90^\circ - \Theta) = \sin \Theta$

$$\cot 40^\circ \tan 50^\circ - 12 \left( \cos 35^\circ \sin 55^\circ \right) \frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2} \left( \frac{\cos 35^\circ}{\sin 55^\circ} \right) = \cot(90^\circ - 50^\circ) \tan 50^\circ - 12 \left( \cos(90^\circ - 55^\circ) \sin 55^\circ \right)$$

$$\frac{\cot(90^\circ - 50^\circ)}{\tan 50^\circ} - \frac{1}{2} \left( \frac{\cos(90^\circ - 55^\circ)}{\sin 55^\circ} \right)$$

$$= \tan 50^\circ \tan 50^\circ - 12 \left( \sin 55^\circ \sin 55^\circ \right) \frac{\tan 50^\circ}{\tan 50^\circ} - \frac{1}{2} \left( \frac{\sin 55^\circ}{\sin 55^\circ} \right)$$

$$= 1 - 12 - \frac{1}{2} = -11 \frac{1}{2}$$

So value of  $\cot 40^\circ \tan 50^\circ - 12 \left( \cos 35^\circ \sin 55^\circ \right) \frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2} \left( \frac{\cos 35^\circ}{\sin 55^\circ} \right)$  is  $-11 \frac{1}{2}$

$$(iv) \left( \sin 27^\circ \cos 63^\circ \right)^2 \left( \frac{\sin 27^\circ}{\cos 63^\circ} \right)^2 - \left( \cos 63^\circ \sin 27^\circ \right)^2 \left( \frac{\cos 63^\circ}{\sin 27^\circ} \right)^2$$

**Soln(iv)**

$$\text{We have to find: } \left( \sin 27^\circ \cos 63^\circ \right)^2 \left( \frac{\sin 27^\circ}{\cos 63^\circ} \right)^2 - \left( \cos 63^\circ \sin 27^\circ \right)^2 \left( \frac{\cos 63^\circ}{\sin 27^\circ} \right)^2$$

Since  $\sin(90^\circ - \Theta) = \cos \Theta$  and  $\cos(90^\circ - \Theta) = \sin \Theta$

$$\left( \sin 27^\circ \cos 63^\circ \right)^2 \left( \frac{\sin 27^\circ}{\cos 63^\circ} \right)^2 - \left( \cos 63^\circ \sin 27^\circ \right)^2 \left( \frac{\cos 63^\circ}{\sin 27^\circ} \right)^2 = \left( \sin(90^\circ - 63^\circ) \cos 63^\circ \right)^2 \left( \frac{\sin(90^\circ - 63^\circ)}{\cos 63^\circ} \right)^2 -$$

$$\left( \cos(90^\circ - 27^\circ) \sin 27^\circ \right)^2 \left( \frac{\cos(90^\circ - 27^\circ)}{\sin 27^\circ} \right)^2$$

$$= \left( \cos 63^\circ \cos 63^\circ \right)^2 \left( \frac{\cos 63^\circ}{\cos 63^\circ} \right)^2 - \left( \sin 27^\circ \sin 27^\circ \right)^2 \left( \frac{\sin 27^\circ}{\sin 27^\circ} \right)^2 = 1 - 1 = 0$$

So value of  $\left( \sin 27^\circ \cos 63^\circ \right)^2 \left( \frac{\sin 27^\circ}{\cos 63^\circ} \right)^2 - \left( \cos 63^\circ \sin 27^\circ \right)^2 \left( \frac{\cos 63^\circ}{\sin 27^\circ} \right)^2$  is 0

$$(v) \tan 35^\circ \cot 55^\circ + \cot 78^\circ \tan 12^\circ - 1 \frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ} - 1$$

**Soln.(v)**

We have to find:

$$\tan 35^\circ \cot 55^\circ + \cot 78^\circ \tan 12^\circ = 1 \frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ} = 1$$

Since  $\tan(90^\circ - \theta) = \cot \theta$  and  $\cot(90^\circ - \theta) = \tan \theta = 1$

So value of  $\tan 35^\circ \cot 55^\circ + \cot 78^\circ \tan 12^\circ$  is  $1 \frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ}$  is 1

**(vi)  $\sec 70^\circ \operatorname{cosec} 20^\circ + \sin 59^\circ \cos 31^\circ = \frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ} + \frac{\sin 59^\circ}{\cos 31^\circ}$**

**Soln.(vi)**

We have to find:  $\sec 70^\circ \operatorname{cosec} 20^\circ + \sin 59^\circ \cos 31^\circ = \frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ} + \frac{\sin 59^\circ}{\cos 31^\circ}$

Since  $\sec 70^\circ \operatorname{cosec} 20^\circ + \sin 59^\circ \cos 31^\circ = \frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ} + \frac{\sin 59^\circ}{\cos 31^\circ}$  and  $\sec(90^\circ - \theta) = \operatorname{cosec} \theta$

So

$$\sec 70^\circ \operatorname{cosec} 20^\circ + \sin 59^\circ \cos 31^\circ = \frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ} + \frac{\sin 59^\circ}{\cos 31^\circ} = (\sec(90^\circ - 20^\circ) \operatorname{cosec} 20^\circ) \left( \frac{\sec(90^\circ - 20^\circ)}{\operatorname{cosec} 20^\circ} \right)^2 - (\sin(90^\circ - 31^\circ) \cos 31^\circ) \left( \frac{\sin(90^\circ - 31^\circ)}{\cos 31^\circ} \right)^2$$

$$= \operatorname{cosec} 20^\circ \operatorname{cosec} 20^\circ + \cos 31^\circ \cos 31^\circ \frac{\operatorname{cosec} 20^\circ}{\operatorname{cosec} 20^\circ} + \frac{\cos 31^\circ}{\cos 31^\circ} = 1 + 1 = 2$$

So value of  $\sec 70^\circ \operatorname{cosec} 20^\circ + \sin 59^\circ \cos 31^\circ = \frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ} + \frac{\sin 59^\circ}{\cos 31^\circ}$  is 2

**(vii)  $\operatorname{cosec} 31^\circ - \sec 59^\circ \operatorname{cosec} 31^\circ - \sec 59^\circ$**

**Soln(vii)**

We have to find:  $\operatorname{cosec} 31^\circ - \sec 59^\circ \operatorname{cosec} 31^\circ - \sec 59^\circ$

Since  $\operatorname{cosec}(90^\circ - \theta) = \sec \theta$ . So

$$= \operatorname{cosec} 31^\circ - \sec 59^\circ \operatorname{cosec} 31^\circ - \sec 59^\circ$$

$$= \operatorname{cosec}(90^\circ - 59^\circ) - \sec 59^\circ \operatorname{cosec}(90^\circ - 59^\circ) - \sec 59^\circ = \sec 59^\circ - \sec 59^\circ \sec 59^\circ - \sec 59^\circ = 0$$

So value of  $\operatorname{cosec} 31^\circ - \sec 59^\circ \operatorname{cosec} 31^\circ - \sec 59^\circ$  is 0

$$(viii)(\sin 72^\circ + \cos 18^\circ)(\sin 72^\circ + \cos 18^\circ) (\sin 72^\circ - \cos 18^\circ)(\sin 72^\circ - \cos 18^\circ)$$

**Soln.(viii)**

We have to find:  $(\sin 72^\circ + \cos 18^\circ)(\sin 72^\circ + \cos 18^\circ) (\sin 72^\circ - \cos 18^\circ)(\sin 72^\circ - \cos 18^\circ)$

Since  $\sin(90^\circ - \Theta)\sin(90^\circ - \Theta) = \cos\Theta\Theta$ , So

$$(\sin 72^\circ + \cos 18^\circ)(\sin 72^\circ + \cos 18^\circ) (\sin 72^\circ - \cos 18^\circ)(\sin 72^\circ - \cos 18^\circ) = (\sin 72^\circ)^2 (\sin 72^\circ)^2 - (\cos 18^\circ)^2 (\cos 18^\circ)^2$$

$$= [\sin(90^\circ - 18^\circ)]^2 - (\cos 18^\circ)^2 [\sin(90^\circ - 18^\circ)]^2 - (\cos 18^\circ)^2$$

$$= (\cos 18^\circ)^2 (\cos 18^\circ)^2 - (\cos 18^\circ)^2 (\cos 18^\circ)^2$$

$$= \cos^2 18^\circ - \cos^2 18^\circ \cos^2 18^\circ - \cos^2 18^\circ = 0$$

So value of  $(\sin 72^\circ + \cos 18^\circ)(\sin 72^\circ + \cos 18^\circ) (\sin 72^\circ - \cos 18^\circ)(\sin 72^\circ - \cos 18^\circ)$  is 0.

$$(ix) \sin 35^\circ \sin 55^\circ \sin 35^\circ \sin 55^\circ - \cos 35^\circ \cos 55^\circ \cos 35^\circ \cos 55^\circ$$

**Soln(ix)**

We find :

$$\sin 35^\circ \sin 55^\circ \sin 35^\circ \sin 55^\circ - \cos 35^\circ \cos 55^\circ \cos 35^\circ \cos 55^\circ$$

Since  $\sin(90^\circ - \Theta)\sin(90^\circ - \Theta) = \cos\Theta\Theta$  and  $\cos(90^\circ - \Theta)\cos(90^\circ - \Theta) = \sin\Theta\Theta$

$$\sin 35^\circ \sin 55^\circ \sin 35^\circ \sin 55^\circ - \cos 35^\circ \cos 55^\circ \cos 35^\circ \cos 55^\circ = \sin(90^\circ - 55^\circ)\sin 55^\circ \sin(90^\circ - 55^\circ)\sin 55^\circ - \cos(90^\circ - 55^\circ)\cos 55^\circ \cos(90^\circ - 55^\circ)\cos 55^\circ = 1-1 = 0$$

So value of  $\sin 35^\circ \sin 55^\circ \sin 35^\circ \sin 55^\circ - \cos 35^\circ \cos 55^\circ \cos 35^\circ \cos 55^\circ$  is 0

$$(x) \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$$

**Soln.(x)**

We have to find  $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$

Since  $\tan(90^\circ - \Theta) \tan(90^\circ - \Theta) = \cot \Theta \Theta$ . So

$\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$

$$\frac{\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ}{\tan(90^\circ - 42^\circ) \tan(90^\circ - 67^\circ) \tan 42^\circ \tan 67^\circ}$$

$$= \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ$$

$$= (\tan 67^\circ \cot 67^\circ) (\tan 42^\circ \cot 42^\circ) (\tan 67^\circ \cot 67^\circ) (\tan 42^\circ \cot 42^\circ) = 1 \times 1 = 1$$

So value of  $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$  is 1

**(xi)  $\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ \sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ$**

**Soln.(xi)**

We have to find  $\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ \sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ$

Since  $\cos(90^\circ - \Theta) \cos(90^\circ - \Theta) = \sin \Theta \Theta$ ,  $\sec(90^\circ - \Theta) \sec(90^\circ - \Theta) = \operatorname{cosec} \Theta \Theta$  and  $\sin \Theta \Theta \cdot \operatorname{cosec} \Theta \Theta = 1$ . So

$$\frac{\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ \sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ}{\sec(90^\circ - 40^\circ) \sin 40^\circ + \cos(90^\circ - 50^\circ) \operatorname{cosec} 50^\circ \cos(90^\circ - 50^\circ) \operatorname{cosec} 50^\circ} = 1 + 1 = 2$$

So value of  $\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ \sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ$  is 2.

**Q.3) Express  $\cos 75^\circ 75^\circ + \cot 75^\circ 75^\circ$  in terms of angle between  $0^\circ$  and  $30^\circ$ .**

**Soln. 3 :**

Given that:  $\cos 75^\circ 75^\circ + \cot 75^\circ 75^\circ$

$$= \cos 75^\circ 75^\circ + \cot 75^\circ 75^\circ$$

$$= \cos(90^\circ - 15^\circ) + \cot(90^\circ - 15^\circ) \cos(90^\circ - 15^\circ) + \cot(90^\circ - 15^\circ)$$

$$= \sin 15^\circ 15^\circ + \tan 15^\circ 15^\circ$$

Hence the correct answer is  $\sin 15^\circ \cos 15^\circ + \tan 15^\circ \sin 15^\circ$

**Q.4) If  $\sin 3A = \cos(A - 26^\circ)$ , where  $3A$  is an acute angle, find the value of  $A$ .**

**Soln.4:**

We are given  $3A$  is an acute angle

We have:  $\sin 3A = \cos(A - 26^\circ)$

$$\Rightarrow \sin 3A = \sin(90^\circ - (A - 26^\circ))$$

$$\Rightarrow \sin 3A = \sin(116^\circ - A)$$

$$\Rightarrow 3A = 116^\circ - A$$

$$\Rightarrow 4A = 116^\circ$$

$$\Rightarrow A = 29^\circ$$

Hence the correct answer is  $29^\circ$

**Q.5) If  $A, B, C$  are the interior angles of a triangle  $ABC$ , prove that,**

(i)  $\tan\left(\frac{C+A}{2}\right) = \cot\frac{B}{2}$

(ii)  $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

**Soln.5:**

(i) We have to prove:  $\tan\left(\frac{C+A}{2}\right) = \cot\frac{B}{2}$

Since we know that in triangle  $ABC$

$$A+B+C=180$$

$$\Rightarrow C+A=180^\circ - B$$

$$\Rightarrow \frac{C+A}{2} = 90^\circ - \frac{B}{2}$$



$$\Rightarrow \Rightarrow \tan C + A = \tan(90^\circ - B) \tan \frac{C+A}{2} = \tan(90^\circ - \frac{B}{2})$$

$$\Rightarrow \Rightarrow \tan(C+A) = \cot B \tan(\frac{C+A}{2}) = \cot \frac{B}{2}$$

Hence proved

$$(ii) \text{ We have to prove : } \sin(B+C) = \cos A \sin(\frac{B+C}{2}) = \cos \frac{A}{2}$$

Since we know that in triangle ABC

$$A+B+C=180$$

$$\Rightarrow \Rightarrow B+C=180^\circ - A$$

$$\Rightarrow \Rightarrow B+C = 90^\circ - A \Rightarrow \frac{B+C}{2} = 90^\circ - \frac{A}{2}$$

$$\Rightarrow \Rightarrow \sin(B+C) = \sin(90^\circ - A) = \sin(\frac{B+C}{2}) = \sin(90^\circ - \frac{A}{2})$$

$$\sin(B+C) = \cos A \sin(\frac{B+C}{2}) = \cos \frac{A}{2}$$

Hence proved

**Q.6) Prove that :**

$$(i) \tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ = 1$$

$$(ii) \sin 48^\circ \sec 48^\circ + \cos 48^\circ \csc 42^\circ = 2$$

$$(iii) \sin 70^\circ \cos 20^\circ + \csc 20^\circ \sec 70^\circ - 2 \cos 70^\circ \csc 20^\circ = 0$$

$$\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\csc 20^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \csc 20^\circ = 0$$

$$(iv) \cos 80^\circ \sin 10^\circ + \cos 59^\circ \csc 31^\circ = 2 \frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \csc 31^\circ = 2$$

**Soln.6:**

(i) Therefore

$$\tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ$$

$$= \tan(90^\circ - 70^\circ) \tan(90^\circ - 70^\circ) \tan(90^\circ - 55^\circ) \tan(90^\circ - 55^\circ) \tan 45^\circ \tan 55^\circ \tan 70^\circ$$

$$= \cot 70^\circ \cot 70^\circ \cot 55^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ$$

$$= (\tan 70^\circ \cot 70^\circ)(\tan 55^\circ \cot 55^\circ) \tan 45^\circ (\tan 70^\circ \cot 70^\circ) (\tan 55^\circ \cot 55^\circ) \tan 45^\circ = 1 \times 1 \times 1 = 1$$

Hence proved

(ii) We will simplify the left hand side

$$\sin 48^\circ 48' \cdot \sec 48^\circ 48' + \cos 48^\circ 48' \cdot \operatorname{cosec} 42^\circ 42' = \sin 48^\circ 48' \cdot \sec(90^\circ - 48^\circ)$$

$$\sec(90^\circ - 48^\circ) \cos 48^\circ 48' \cdot \operatorname{cosec}(90^\circ - 48^\circ) \operatorname{cosec}(90^\circ - 48^\circ)$$

$$= \sin 48^\circ 48' \cdot \cos 48^\circ 48' + \cos 48^\circ 48' \cdot \sin 48^\circ 48' = 1 + 1 = 2$$

Hence proved

(iii) We have,  $\sin 70^\circ \cos 20^\circ + \operatorname{cosec} 20^\circ \sec 70^\circ - 2 \cos 70^\circ \cdot \operatorname{cosec} 20^\circ = 0$

$$\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \cdot \operatorname{cosec} 20^\circ = 0$$

So we will calculate left hand side

$$\sin 70^\circ \cos 20^\circ + \operatorname{cosec} 20^\circ \sec 70^\circ - 2 \cos 70^\circ \cdot \operatorname{cosec} 20^\circ = 0$$

$$\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \cdot \operatorname{cosec} 20^\circ = 0 = \sin 70^\circ \cos 20^\circ + \cos 70^\circ \sin 20^\circ - 2 \cos 70^\circ \cdot \operatorname{cosec}(90^\circ - 70^\circ)$$

$$\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\cos 70^\circ}{\sin 20^\circ} - 2 \cos 70^\circ \cdot \operatorname{cosec}(90^\circ - 70^\circ)$$

$$= \sin(90^\circ - 20^\circ) \cos 20^\circ \frac{\sin(90^\circ - 20^\circ)}{\cos 20^\circ} + \cos(90^\circ - 20^\circ) \sin 20^\circ \frac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} - 2 \cos 70^\circ \cdot \operatorname{cosec}(90^\circ - 70^\circ)$$

$$2 \cos 70^\circ \cdot \operatorname{cosec}(90^\circ - 70^\circ)$$

$$= \cos 20^\circ \cos 20^\circ + \sin 20^\circ \sin 20^\circ - 2 \times 1 \frac{\cos 20^\circ}{\cos 20^\circ} + \frac{\sin 20^\circ}{\sin 20^\circ} - 2 \times 1 = 1 + 1 - 2 = 2 - 2 = 0$$

Hence proved

(iv) We have  $\cos 80^\circ \sin 10^\circ + \cos 59^\circ \cdot \operatorname{cosec} 31^\circ = 2 \frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \cdot \operatorname{cosec} 31^\circ = 2$

We will simplify the left hand side

$$\cos 80^\circ \sin 10^\circ + \cos 59^\circ \cdot \operatorname{cosec} 31^\circ$$

$$\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \cdot \operatorname{cosec} 31^\circ = \cos(90^\circ - 10^\circ) \sin 10^\circ + \cos 59^\circ \cdot \operatorname{cosec}(90^\circ - 59^\circ)$$

$$\frac{\cos(90^\circ - 10^\circ)}{\sin 10^\circ} + \cos 59^\circ \cdot \operatorname{cosec}(90^\circ - 59^\circ)$$

$$= \sin 10^\circ \sin 10^\circ + \cos 59^\circ \cdot \sec 59^\circ \frac{\sin 10^\circ}{\sin 10^\circ} + \cos 59^\circ \cdot \sec 59^\circ = 1 + 1 = 2$$

Hence proved.

### Question 7

If A,B,C are the interior of triangle ABC , show that

$$(i) \sin(B+C) = \cos A \sin\left(\frac{B+C}{2}\right) = \cos \frac{A}{2}$$

#### Solution

$$A+B+C=180^\circ$$

$$B + C = 180^\circ - A \Rightarrow \frac{B+C}{2} = \frac{180^\circ - A}{2}$$

LHS=RHS

$$(ii) \cos(90^\circ - A) = \sin A \cos\left(90^\circ - \frac{A}{2}\right) = \sin \frac{A}{2}$$

LHS=RHS

### Question 8

If  $2\theta + 45^\circ$  and  $30^\circ - \theta$  are acute angles , find the degree measure of

$$\theta \text{ satisfying } \sin(2\theta + 45^\circ) = \cos(30^\circ + \theta) \Rightarrow \sin(2\theta + 45^\circ) = \cos(30^\circ + \theta)$$

#### Solution

$$\text{Here } 2\theta + 45^\circ = 60^\circ + \theta \Rightarrow \theta = 15^\circ$$

$$\text{We know that } \sin(90^\circ - \theta) = \cos(\theta)$$

$$= \sin(2\theta + 45^\circ) = \sin(90^\circ - (30^\circ - \theta)) = \sin(2\theta + 45^\circ) = \sin(90^\circ - (30^\circ - \theta))$$

$$= \sin(2\theta + 45^\circ) = \sin(90^\circ - 30^\circ + \theta) = \sin(2\theta + 45^\circ) = \sin(90^\circ - 30^\circ + \theta)$$

$$= \sin(2\theta + 45^\circ) = \sin(60^\circ + \theta) = \sin(60^\circ + \theta)$$

On equating sin of angle of we get,

$$= 2\Theta + 45^\circ = 60^\circ + \Theta + 45^\circ = 60^\circ + \Theta \Rightarrow \Theta = 15^\circ$$

### Question 9

If  $\Theta$  is a positive acute angle such that  $\sec \Theta = \csc 60^\circ$ , find

$$2\cos^2 \Theta - 1$$

### Solution

$$\text{We know that, } \sec(90^\circ - \Theta) = \csc \Theta \Rightarrow \sec(90^\circ - \Theta) = \csc \Theta$$

$$= \sec(\Theta) = \sec(90^\circ - 60^\circ) \Rightarrow \sec(\Theta) = \sec(90^\circ - 60^\circ)$$

$$\Rightarrow \Theta = 30^\circ \Rightarrow 2\cos^2 \Theta - 1$$

$$= 2\cos^2 30^\circ - 1$$

$$= 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1$$

$$= 2\left(\frac{3}{4}\right) - 1 = \left(\frac{3}{2}\right) - 1 = \left(\frac{1}{2}\right)$$

**Q10.** If  $\sin 3\Theta = \cos(\Theta - 6^\circ)$  where  $3\Theta$  and  $\Theta - 6^\circ$  are acute angles, find the value of  $\Theta$ .

**Soln:**

$$\text{We have, } \sin 3\Theta = \cos(\Theta - 6^\circ) \Rightarrow \sin 3\Theta = \cos(\Theta - 6^\circ)$$

$$\cos(90^\circ - 3\Theta) = \cos(\Theta - 6^\circ) \Rightarrow 90^\circ - 3\Theta = \Theta - 6^\circ$$

$$90^\circ - 3\Theta = \Theta - 6^\circ \Rightarrow -3\Theta - \Theta = 6^\circ - 90^\circ \Rightarrow -4\Theta = 96^\circ - 90^\circ \Rightarrow -4\Theta = 6^\circ \Rightarrow \Theta = \frac{-6^\circ}{-4} = 1.5^\circ$$

$$\Theta = \frac{-6^\circ}{-4} = 1.5^\circ$$

**Q11.** If  $\sec 2A = \csc(A - 42^\circ)$  where  $2A$  is acute angle, find the value of  $A$ .

**Soln:** we know that  $\sec(90^\circ - 3\Theta) = \csc \Theta \Rightarrow \sec(90^\circ - 3\Theta) = \csc \Theta$

$$\sec 2A = \sec(90 - (A - 42)) \quad \sec 2A = \sec(90 - (A - 42)) \quad \sec 2A = \sec(90 - A + 42)$$
$$\sec 2A = \sec(90 - A + 42) \quad \sec 2A = \sec(132 - A) \quad \sec 2A = \sec(132 - A)$$

Now equating both the angles we get

$$2A = 132 - A$$

$$A = \frac{132}{3}$$

$$A = 44$$

