Evaluate each of the following:

Q 1 . sin 45°45° sin 30°30° + cos 45°45° cos 30°30°

Solution:

Sin 45°45° sin 30°30° + cos 45°45° cos 30°30°
[1]

We know that by trigonometric ratios we have ,

sin45°=1√2 $\sin 45^\circ = \frac{1}{\sqrt{2}}$ sin30°=12 $\sin 30^\circ = \frac{1}{2}$

cos45°=1√2 $\cos 45^\circ = \frac{1}{\sqrt{2}}$ cos30°=√32 $\cos 30^\circ = \frac{\sqrt{3}}{2}$

Substituting the values in equation 1 , we get

1√2 · 12 + 1√2 · √32 $\frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}$

= 1√2 · √32√2 $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2\sqrt{2}}$

= √3+12√2 $\frac{\sqrt{3}+1}{2\sqrt{2}}$

Q 2 . sin 60°60° cos 30°30° + cos 60°60° sin 30°30°

Solution:

sin 60°60° cos 30°30° + cos 60°60° sin 30°30°
[1]

By trigonometric ratios we have ,

sin60°=√32 $\sin 60^\circ = \frac{\sqrt{3}}{2}$ sin30°=12 $\sin 30^\circ = \frac{1}{2}$

cos30°=√32 $\cos 30^\circ = \frac{\sqrt{3}}{2}$ cos60°=12 $\cos 60^\circ = \frac{1}{2}$

Substituting the values in equation 1 , we get

= √32 · √32 + 12 · 12 $\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2}$

= 34 + 14 $\frac{3}{4} + \frac{1}{4}$ = 44 $\frac{4}{4}$ = 1

Q 3 . cos 60°60° **cos 45°**45° **– sin 60°**60° **sin 45°**45°

Solution:

cos 60°60° cos 45°45° – sin 60°60° sin 45°45° [1]

We know that by trigonometric ratios we have ,

cos60°=12$\cos 60^\circ = \frac{1}{2}$ cos45°=1√2$\cos 45^\circ = \frac{1}{\sqrt{2}}$

sin60°=√32$\sin 60^\circ = \frac{\sqrt{3}}{2}$ sin45°=1√2$\sin 45^\circ = \frac{1}{\sqrt{2}}$

Substituting the values in equation 1 , we get

12 · 1√2 − √32 · 1√2 $\frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$

= 1−√32√2 $\frac{1-\sqrt{3}}{2\sqrt{2}}$

Q.4: sin²30°+sin²45°+sin²60°+sin²90°$\sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 90^\circ$

Solution:

sin²30°+sin²45°+sin²60°+sin²90°$\sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 90^\circ$ [1]

We know that by trigonometric ratios we have ,

sin30°=12$\sin 30^\circ = \frac{1}{2}$ sin45°=1√2$\sin 45^\circ = \frac{1}{\sqrt{2}}$

sin60°=√32$\sin 60^\circ = \frac{\sqrt{3}}{2}$ sin90°$\sin 90^\circ$ = 1

Substituting the values in equation 1 , we get

= [12]²+[1√2]²+[√32]²+1$\left[\frac{1}{2}\right]^2 + \left[\frac{1}{\sqrt{2}}\right]^2 + \left[\frac{\sqrt{3}}{2}\right]^2 + 1$

= 14+12+34+1 $\frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1$

= 52 $\frac{5}{2}$

Q 5. cos²30°+cos²45°+cos²60°+cos²90°$\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90^\circ$

Solution:

$\cos^2 30° + \cos^2 45° + \cos^2 60° + \cos^2 90°$ [1]

We know that by trigonometric ratios we have ,

$\cos 30° = \frac{\sqrt{3}}{2}$ $\cos 45° = \frac{1}{\sqrt{2}}$

$\cos 60° = \frac{1}{2}$ $\cos 90° = 0$

Substituting the values in equation 1 , we get

$[\frac{\sqrt{3}}{2}]^2 + [\frac{1}{\sqrt{2}}]^2 + [\frac{1}{2}]^2 + 0$

$= \frac{3}{4} + \frac{1}{2} + \frac{1}{4}$

$= \frac{3}{2}$

Q 6 . $\tan^2 30° + \tan^2 45° + \tan^2 60°$

Solution:

$\tan^2 30° + \tan^2 45° + \tan^2 60°$ [1]

We know that by trigonometric ratios we have ,

$\tan 30° = \frac{1}{\sqrt{3}}$ $\tan 60° = \sqrt{3}$

$\tan 45° = 1$

Substituting the values in equation 1 , we get

$[\frac{1}{\sqrt{3}}]^2 + [\sqrt{3}]^2 + 1$

$= \frac{1}{3} + 3 + 1$

$= \frac{13}{3}$

Q 7 . $2\sin^2 30° - 3\cos^2 45° + \tan^2 60°$

Solution:

$2\sin^2 30^\circ - 3\cos^2 45^\circ + \tan^2 60^\circ$ [1]

We know that by trigonometric ratios we have ,

$\sin 30^\circ = \frac{1}{2}$ $\cos 45^\circ = \frac{1}{\sqrt{2}}$

$\tan 60^\circ = \sqrt{3}$

Substituting the values in equation 1 , we get

$= 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2$

$= 2\left(\frac{1}{4}\right) - 3\left(\frac{1}{2}\right) + 3$

$= \frac{1-3+6}{2}$

= 2

Q8: $\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2}\sin^2 90^\circ - 2\cos^2 90^\circ + \frac{1}{24}\cos^2 0^\circ$

Solution:

$\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2}\sin^2 90^\circ - 2\cos^2 90^\circ + \frac{1}{24}\cos^2 0^\circ$ [1]

We know that by trigonometric ratios we have ,

$\sin 30^\circ = \frac{1}{2}$ $\cos 45^\circ = \frac{1}{\sqrt{2}}$ $\tan 30^\circ = \frac{1}{\sqrt{3}}$

$\sin 90^\circ = 1$

$\cos 90^\circ = 0$

$\cos 0^\circ = 1$

Substituting the values in equation 1 , we get

$\left[\frac{1}{2}\right]^2 \cdot \left[\frac{1}{\sqrt{2}}\right]^2 + 4\left[\frac{1}{\sqrt{3}}\right]^2 + \frac{1}{2}[1]^2 - 2[0]^2 + \frac{1}{24}[1]^2$

$= \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}$

$= \frac{48}{24} = 2$

Q 9 . $4(\sin^4 60^\circ+\cos^4 30^\circ)-3(\tan^2 60^\circ-\tan^2 45^\circ)+5\cos^2 45^\circ$

$4\left(\sin^4 60^\circ + \cos^4 30^\circ\right) - 3\left(\tan^2 60^\circ - \tan^2 45^\circ\right) + 5\cos^2 45^\circ$

Solution:

$4(\sin^4 60^\circ+\cos^4 30^\circ)-3(\tan^2 60^\circ-\tan^2 45^\circ)+5\cos^2 45^\circ$

$4\left(\sin^4 60^\circ + \cos^4 30^\circ\right) - 3\left(\tan^2 60^\circ - \tan^2 45^\circ\right) + 5\cos^2 45^\circ$ [1]

We know that by trigonometric ratios we have ,

$\sin 60^\circ = \frac{\sqrt{3}}{2}$ $\cos 45^\circ = \frac{1}{\sqrt{2}}$

$\tan 60^\circ = \sqrt{3}$ $\cos 30^\circ = \frac{\sqrt{3}}{2}$

Substituting the values in equation 1 , we get

$4\left(\left[\frac{\sqrt{3}}{2}\right]^4 + \left[\frac{\sqrt{3}}{2}\right]^4\right) - 3(3)^2 - 1^2 + 5\left[\frac{1}{\sqrt{2}}\right]^2$

$= 4 \cdot \frac{18}{16} - 6 + \frac{5}{2}$

$= \frac{1}{4} - 6 + \frac{5}{2}$

$= \frac{14}{2} - 6 = 7 - 6 = 1$

Q 10 . $(\operatorname{cosec}^2 45^\circ \sec^2 30^\circ)(\sin^2 30^\circ+4\cot^2 45^\circ-\sec^2 60^\circ)$

$\left(\operatorname{cosec}^2 45^\circ \sec^2 30^\circ\right)\left(\sin^2 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ\right)$

Solution:

$(\operatorname{cosec}^2 45^\circ \sec^2 30^\circ)(\sin^2 30^\circ+4\cot^2 45^\circ-\sec^2 60^\circ)$

$\left(\operatorname{cosec}^2 45^\circ \sec^2 30^\circ\right)\left(\sin^2 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ\right)$ [1]

We know that by trigonometric ratios we have ,

$\operatorname{cosec} 45^\circ = \sqrt{2}$ $\sec 30^\circ = \frac{2}{\sqrt{3}}$

$\sin 30^\circ = \frac{1}{2}$ $\cot 45^\circ = 1$

$\sec 60^\circ = 2$

Substituting the values in equation 1 , we get

$([\sqrt{2}]^2.[2\sqrt{3}]^2)([12]^2+4(1)(2)^2)([\sqrt{2}]^2.[\frac{2}{\sqrt{3}}]^2)([\frac{1}{2}]^2+4(1)(2)^2)$

$= 3 \cdot 43 \cdot 143 \cdot \frac{4}{3} \cdot \frac{1}{4}$

$= 23\,\frac{2}{3}$

Q11. cosec^{3}30°cos60°tan^{3}45°sin^{2}90°sec^{2}45°cot30°
$\operatorname{cosec}^3 30^\circ \cos 60^\circ \tan^3 45^\circ \sin^2 90^\circ \sec^2 45^\circ \cot 30^\circ$

Solution:

Given,

$= \operatorname{cosec}^3 30^\circ \cos 60^\circ \tan^3 45^\circ \sin^2 90^\circ \sec^2 45^\circ \cot 30^\circ$
$\operatorname{cosec}^3 30^\circ \cos 60^\circ \tan^3 45^\circ \sin^2 90^\circ \sec^2 45^\circ \cot 30^\circ$

$= 2^3(12)(1^3)(1^2)(\sqrt{2}^2)(\sqrt{3})2^3(\frac{1}{2})(1^3)(1^2)(\sqrt{2}^2)(\sqrt{3})$

$= (2)^3\times(12)\times(1^3)\times(1^2)\times(\sqrt{2}^2)\times(\sqrt{3})(2)^3\times(\frac{1}{2})\times(1^3)\times(1^2)\times(\sqrt{2}^2)\times(\sqrt{3})$

$= 8\times(12)\times(1)\times(1)\times(2)\times(\sqrt{3})8\times(\frac{1}{2})\times(1)\times(1)\times(2)\times(\sqrt{3})$

$= 8\sqrt{3}8\sqrt{3}$

Q12. cot^{2}30°−2cos^{2}60°−34sec^{2}45°−4sec^{2}30° $\cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ - 4\sec^2 30^\circ$

Solution:

Given,

$= \cot^2 30^\circ - 2\cos^2 60^\circ - 34\sec^2 45^\circ - 4\sec^2 30^\circ \cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ - 4\sec^2 30^\circ$

$= (\sqrt{3}^2)\times 2(12)^2\times(34\times\sqrt{2}^2)\times(4\times(2\sqrt{3})^2)(\sqrt{3}^2)\times 2(\frac{1}{2})^2\times(\frac{3}{4}\times\sqrt{2}^2)\times(4\times(\frac{2}{\sqrt{3}})^2)$

$= 3-12-32-163\;3-\frac{1}{2}-\frac{3}{2}-\frac{16}{3}$

$= -133\,\frac{-13}{3}$

Q13. (cos0°+sin45°+sin30°)(sin90°−cos45°+cos60°)
$(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

Solution:

Given,

(cos0°+sin45°+sin30°)(sin90°−cos45°+cos60°)(1+1√2+1√2)(1−1√2+1√2)(32+1√2)(32−1√2)

$$(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$$

$$(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}})(1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}})$$

$$(\frac{3}{2} + \frac{1}{\sqrt{2}})(\frac{3}{2} - \frac{1}{\sqrt{2}})$$

((32)2−(1√2)2)94−12 74

$$((\frac{3}{2})^2 - (\frac{1}{\sqrt{2}})^2)\frac{9}{4} - \frac{1}{2}\,\frac{7}{4}$$

Q14. $\frac{\sin 30^\circ - \sin 90^\circ + 2\cos 0^\circ}{\tan 30^\circ \tan 60^\circ}$

Solution:

Given,

$$\frac{\sin 30^\circ - \sin 90^\circ + 2\cos 0^\circ}{\tan 30^\circ \tan 60^\circ}$$

$$\frac{\frac{1}{2} - 1 + 2}{\frac{1}{\sqrt{3}} \times \sqrt{3}}$$

sin30°−sin90°+2cos0°tan30°tan60° 12−1+21√3×√3 32

$$\frac{3}{2}$$

Q15. $\frac{4}{\cot^2 30^\circ} + \frac{1}{\sin^2 60^\circ} - \cos^2 45^\circ$

Solution:

Given,

$$\frac{4}{\cot^2 30^\circ} + \frac{1}{\sin^2 60^\circ} - \cos^2 45^\circ$$

$$= \frac{4}{(\sqrt{3})^2} + \frac{1}{(\frac{\sqrt{3}}{2})^2} - (\frac{1}{\sqrt{2}})^2$$

$$= \frac{4}{3} + \frac{4}{3} - \frac{1}{2}$$

$$= \frac{16-3}{6}$$

4cot²30°+1sin²60°−cos²45°=4(√3)²+1(√32)²−(1√2)²=43+43−12=16–36=136

$$= \frac{13}{6}$$

Q16. $4(\sin^4 30°+\cos^2 60°)-3(\cos^2 45°-\sin^2 90°)-\sin^2 60°$

$$4(\sin^4 30° + \cos^2 60°) - 3(\cos^2 45° - \sin^2 90°) - \sin^2 60°$$

Solution:

Given,

$4(\sin^4 30°+\cos^2 60°)-3(\cos^2 45°-\sin^2 90°)-\sin^2 60°=4((12)^4+(12)^2)-3((1\sqrt{2})^2-1)-$

$$4(\sin^4 30° + \cos^2 60°) - 3(\cos^2 45° - \sin^2 90°) - \sin^2 60°$$

$$= 4((\tfrac{1}{2})^4 + (\tfrac{1}{2})^2) - 3((\tfrac{1}{\sqrt{2}})^2 - 1) - (\tfrac{\sqrt{3}}{2})^2$$

$$= 4(\tfrac{1}{16} + \tfrac{1}{4}) + \tfrac{3}{2} - \tfrac{3}{4}$$

$(\sqrt{3}2)^2=4(116+14)+32-34=84=2$ $= \frac{8}{4} = 2$

Q17. $\tan^2 60°+4\cos^2 45°+3\sec^2 30°+5\cos^2 90° \operatorname{cosec}30°+\sec 60°-\cot^2 30°$ $\dfrac{\tan^2 60°+4\cos^2 45°+3\sec^2 30°+5\cos^2 90°}{\operatorname{cosec}30°+\sec 60°-\cot^2 30°}$

Solution:

Given,

$\tan^2 60°+4\cos^2 45°+3\sec^2 30°+5\cos^2 90° \operatorname{cosec}30°+\sec 60°-\cot^2 30° = (\sqrt{3})^2+4(1\sqrt{2})^2+3(2\sqrt{3})^2+5(0)2+2-(\sqrt{3})^2 = 3+2+4=9$

$$\frac{\tan^2 60°+4\cos^2 45°+3\sec^2 30°+5\cos^2 90°}{\operatorname{cosec}30°+\sec 60°-\cot^2 30°}$$

$$= \frac{(\sqrt{3})^2+4(\frac{1}{\sqrt{2}})^2+3(\frac{2}{\sqrt{3}})^2+5(0)}{2+2-(\sqrt{3})^2}$$

$$= 3 + 2 + 4$$

$$= 9$$

Q18. $\sin 30° \sin 45° + \tan 45° \sec 60° - \sin 60° \cot 45° - \cos 30° \sin 90°$ $\dfrac{\sin 30°}{\sin 45°} + \dfrac{\tan 45°}{\sec 60°} - \dfrac{\sin 60°}{\cot 45°} - \dfrac{\cos 30°}{\sin 90°}$

Solution:

Given,

sin30°sin45° + tan45°sec60° − sin60°cot45° − cos30°sin90° = 12 1√2 + 12 − √32 1 − √32 1 = √22 + 12 − √32 − √32 = √2+1−2√32

$$\frac{\sin 30^\circ}{\sin 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ}$$

$$= \frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}} + \frac{1}{2} - \frac{\frac{\sqrt{3}}{2}}{1} - \frac{\frac{\sqrt{3}}{2}}{1}$$

$$= \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{2}+1-2\sqrt{3}}{2}$$

Q19. tan45°cosec30° + sec60°cot45° + ssin90°2cos0° $\frac{\tan 45^\circ}{\text{cosec}30^\circ} + \frac{\sec 60^\circ}{\cot 45^\circ} + \frac{\text{ssin}90^\circ}{2\cos 0^\circ}$

Solution:

Given,

$$\frac{\tan 45^\circ}{\text{cosec}30^\circ} + \frac{\sec 60^\circ}{\cot 45^\circ} + \frac{\text{ssin}90^\circ}{2\cos 0^\circ}$$

$$= \frac{1}{2} + \frac{2}{1} - \frac{5(1)}{2(1)}$$

$$= \frac{5}{2} - \frac{5}{2}$$

tan45°cosec30° + sec60°cot45° + ssin90°2cos0° = 12 + 21 − 5(1)2(1) = 52 − 52 = 0 $= 0$

Q20. 2sin3x=√3 $2\sin 3x = \sqrt{3}$

Solution:

Given,

$$2\sin 3x = \sqrt{3}$$

$$\Rightarrow \sin 3x = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin 3x = \sin 60^\circ$$

$$\Rightarrow 3x = 60^\circ$$

2sin3x=√3=>sin3x= √32 =>sin3x=sin60°=>3x=60°=>x=20° $\Rightarrow x = 20^\circ$

Q21) 2sin x2 =1,x=? $2\sin\frac{x}{2} = 1,\ x = ?$

Solution:

sin x2 = 12 $\sin\frac{x}{2} = \frac{1}{2}$ sin x2 = sin30^0 $\sin\frac{x}{2} = \sin 30^0$ x2 = 30^0 $\frac{x}{2} = 30^0$

x = 60^0

Q22) √3sinx=cosx $\sqrt{3}\sin x = \cos x$

Solution:

√3tanx=1 $\sqrt{3}\tan x = 1$ tanx=1√3 $\tan x = \frac{1}{\sqrt{3}}$ ∴tanx=tan45^0 $\therefore \tan x = \tan 45^0$

x = 45^0

Q23) Tan x = sin 45^0 cos 45^0 + sin 30^0

Solution:

Tanx=1√2.1√2+12[∵sin45^0=1√2cos45^0=1√2sin30^0=12]
$\text{Tan}x = \frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}} + \frac{1}{2}$ $[\because \sin 45^0 = \frac{1}{\sqrt{2}} \cos 45^0 = \frac{1}{\sqrt{2}} \sin 30^0 = \frac{1}{2}]$ Tanx=12+12 $\text{Tan}x = \frac{1}{2} + \frac{1}{2}$

Tan x = 1

Tan x = 45^0

x = 45^0

Q24) √3Tan2x=cos60^0+sin45^0cos45^0 $\sqrt{3}\,\text{Tan}2x = \cos 60^0 + \sin 45^0 \cos 45^0$

Solution:

√3Tan2x=12+1√2.1√2[∵cos60^0=12sin45^0=cos45^0=1√2]
$\sqrt{3}\,\text{Tan}2x = \frac{1}{2} + \frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}$ $[\because \cos 60^0 = \frac{1}{2} \sin 45^0 = \cos 45^0 = \frac{1}{\sqrt{2}}]$ √3Tan2x=1√3⇒tan2x=tan30^0
$\sqrt{3}\,\text{Tan}2x = \frac{1}{\sqrt{3}} \Rightarrow \tan 2x = \tan 30^0$

2x = 30^0

x = 15^0

Q25) $\cos 2x = \cos 60^0 \cos 30^0 + \sin 60^0 \sin 30^0$

Solution:

$\cos 2x = \frac{1}{2}.\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}.\frac{1}{2}$ $\quad [\because \cos 60^0 = \sin 30^0 = \frac{1}{2} \sin 60^0 = \cos 30^0 = \frac{\sqrt{3}}{2}]$ $\quad \cos 2x = 2.\frac{\sqrt{3}}{4}$ $\quad \cos 2x = \frac{\sqrt{3}}{2}$ $\quad \cos 2x = \cos 30^0$ $\quad 2x = 30^0$ $\quad x = 15^0$

Q26) If $\theta = 30^0$, verify

If $\theta = 30^0$, verify (i) $Tan 2\theta = \frac{2Tan\theta}{1-\tan^2\theta}$

Solution:

$Tan 2\theta = \frac{2Tan\theta}{1-\tan^2\theta} \; \ldots\ldots (i)$

Substitute $\theta = 30^0$ in equation (i)

LHS = Tan 60^0 = $\sqrt{3}$

RHS = $\frac{2Tan30^0}{1+(Tan30^0)^2} = \frac{2-\frac{1}{\sqrt{2}}}{1-(\frac{1}{\sqrt{2}})^2} = \sqrt{3}$

Therefore, LHS = RHS

(ii) $\sin\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

Substitute $\theta = 30^0$

$\sin 60^0 = \frac{2\tan 30^0}{(1-\tan 30^0)^2}$

=> $\frac{\sqrt{3}}{2} = \frac{2.\frac{1}{\sqrt{2}}}{1+(\frac{1}{\sqrt{2}})^2}$

=> $\frac{\sqrt{3}}{2} = \frac{2}{\sqrt{3}}.\frac{3}{4}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

Therefore, LHS = RHS.

(iii) $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

Substitute $\theta = 30^0$

LHS = $\text{cosec}\theta$ RHS = $\frac{1-\tan^2\theta}{1+\tan^2\theta}$

= cos 2(30^0) $= \frac{1-\tan^2 30^0}{1+\tan^2 30^0}$

Cos 60^0 = $\frac{1}{2}$ $= \frac{1-(\frac{1}{\sqrt{2}})^2}{1+(\frac{1}{\sqrt{2}})^2} = \frac{\frac{2}{2}}{\frac{1}{2}} = \frac{1}{2}$

Therefore, LHS = RHS

(iv) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

Solution:

LHS = $\cos 3\theta$

Substitute $\theta = 30^0$

= cos 3 (30^0) = cos 90^0

= 0

RHS = $4\cos^3\theta - 3\cos\theta$

$= 4\cos^3 30^0 - 3\cos 30^0$

$= 4(\frac{\sqrt{3}}{2})^3 - 3.\frac{\sqrt{3}}{2}$

$= 3.\frac{\sqrt{3}}{2} - 3.\frac{\sqrt{3}}{2}$

= 0

Therefore, LHS = RHS.

Q27) If A = B = 60^0. Verify (i) Cos (A – B) = Cos A Cos B + Sin A Sin B

Solution:

Cos (A – B) = Cos A Cos B + Sin A Sin B…….(i)

Substitute A and B in (i)

=>cos (60^0 – 60^0) = cos 60^0 cos 60^0 + sin 60^0 sin 60^0

=>cos 0^0 = $(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2$

=>1 = 14+34 $\frac{1}{4}+\frac{3}{4}$

=>1 = 1

Therefore, LHS = RHS

(ii) Substitute A and B in (i)

=>sin $(60^0 - 60^0)$ = sin 60^0 cos 60^0 – cos 60^0 sin 60^0

=> sin 0^0 = 0

=>0 = 0

Therefore, LHS = RHS

(iii) Tan(A−B)=TanA−TanB1+TanATanB $\text{Tan}(A-B)=\frac{\text{TanA}-\text{TanB}}{1+\text{TanA TanB}}$

A = 60^0, B = 60^0 we get,

Tan(60^0-60^0)=Tan60^0–Tan$60^0$1+Tan60^0Tan60^0 $\text{Tan}(60^0-60^0)=\frac{\text{Tan}60^0-\text{Tan}60^0}{1+\text{Tan}60^0\ \text{Tan}60^0}$

Tan 0^0 = 0

0 = 0

Therefore, LHS = RHS

Q28) If A = 30^0, B = 60^0 verify:

(i) Sin (A + B) = Sin A Cos B + Cos A Sin B

Solution:

A = 30^0, B = 60^0 we get

Sin $(30^0 + 60^0)$ = Sin 30^0 Cos 60^0 + Cos 30^0 Sin 60^0

Sin (90^0) = 12.12+√32.√32 $\frac{1}{2}.\frac{1}{2}+\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}$

Sin (90^0) = 1 => 1 = 1

Therefore, LHS = RHS

(ii) Cos (A + B) = Cos A Cos B – Sin A Sin B

$A = 30^0$, $B = 60^0$ we get

$\text{Cos}(30^0 + 60^0) = \text{Cos } 30^0 \text{ Cos } 60^0 - \text{Sin } 30^0 \text{ Sin } 60^0$

$\text{Cos}(90^0) = 12 . \sqrt{3}2 - \sqrt{3}2 . 12 \frac{1}{2} . \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} . \frac{1}{2}$

$0 = 0$

Therefore, LHS = RHS

Q29. If sin(A+B) = 1 and cos(A-B) = 1, 0°<A+B≤90°$0^\circ < A + B \leq 90^\circ$, A≥B find A and B.

Sol:

Given,

sin(A+B) = 1 this can be written as sin(A+B) = sin(90°)$\sin(90^\circ)$

cos(A-B) = 1 this can be written as cos(A-B) = cos(0°)$\cos(0^\circ)$

=> A + B = 90°90°

A – B = 0°0°

2A = 90°90°

A = 90°2 $\frac{90^\circ}{2}$

A = 45°45°

Substitute A value in A – B = 0°0°

45°45° – B = 0°0°

B =45°45°

Hence, the value of A = 45°45° and B =45°45°

Q30. If tan(A-B) = 1√3 $\frac{1}{\sqrt{3}}$ and tan(A+B) = √3$\sqrt{3}$, 0°<A+B≤90°$0^\circ < A + B \leq 90^\circ$, A>B find A and B

Solution:

Given,

tan(A-B) = 1√3 $\frac{1}{\sqrt{3}}$

A – B = tan⁻¹(1√3)$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$A - B = 30^\circ 30^\circ$ —— 1

$\tan(A+B) = \sqrt{3}\sqrt{3}$

$A + B = \tan^{-1}\sqrt{3}\tan^{-1}\sqrt{3}$

$A + B = 60^\circ 60^\circ$ —— 2

Solve equations 1 and 2

$A + B = 30^\circ 30^\circ$

$A - B = 60^\circ 60^\circ$

$2A = 90^\circ 90^\circ$

$A = 90^\circ 2 \frac{90^\circ}{2}$

$A = 45^\circ 45^\circ$

Substitute the value of A in equation 1

$45^\circ 45^\circ + B = 30^\circ 30^\circ$

$B = 30^\circ 30^\circ - 45^\circ 45^\circ$

$B = 15^\circ 15^\circ$

The value of $A = 45^\circ 45^\circ$ and $B = 15^\circ 15^\circ$

Q31. If sin(A-B) = $12\frac{1}{2}$ and cos(A+B) = $12\frac{1}{2}$, $0^\circ < A+B \le 90^\circ \; 0^\circ < A + B \le 90^\circ$, A<B find A and B.

Solution:

Given,

$\sin(A-B) = 12\frac{1}{2}$

$A - B = \sin^{-1}(12)\sin^{-1}(\frac{1}{2})$

$A - B = 30^\circ 30^\circ$ —— 1

$\cos(A+B) = 12\frac{1}{2}$

$A + B = \cos^{-1}(12)\cos^{-1}(\frac{1}{2})$

$A + B = 60^\circ 60^\circ$ —— 2

Solve equations 1 and 2

A + B = $60^\circ 60^\circ$

A – B = $30^\circ 30^\circ$

2A = $90^\circ 90^\circ$

A = $90^\circ 2 \frac{90^\circ}{2}$

A = $45^\circ 45^\circ$

Substitute the value of A in equation 2

$45^\circ 45^\circ$ + B = $60^\circ 60^\circ$

B = $60^\circ 60^\circ - 45^\circ 45^\circ$

B = $15^\circ 15^\circ$

The value of A = $45^\circ 45^\circ$ and B = $15^\circ 15^\circ$

Q32. In a $\Delta\Delta$ ABC right angled triangle at B, $\angle A = \angle C \angle A = \angle C$ .Find the values of:

1. sinAcosC + cosAsinC

Solution:

since, it is given as $\angle A = \angle C \angle A = \angle C$

the value of A and C is $45^\circ 45^\circ$, the value of angle B is $90^\circ 90^\circ$

because the sum of angles of triangle is $180^\circ 180^\circ$

=> sin($45^\circ 45^\circ$)cos($45^\circ 45^\circ$) + cos($45^\circ 45^\circ$)sin($45^\circ 45^\circ$)

=> $(1\sqrt{2} \times 1\sqrt{2})(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}) + (1\sqrt{2} \times 1\sqrt{2})(\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}})$

=> $12\frac{1}{2} + 12\frac{1}{2}$

=> 1

The value of sinAcosC + cosAsinC is 1

2. sinAsinB + cosAcosB

Solution:

since, it is given as ∠A=∠C $\angle A = \angle C$

the value of A and C is 45° 45°, the value of angle B is 90° 90°

because the sum of angles of triangle is 180° 180°

=> sin(45° 45°)sin(90° 90°) + cos(45° 45°)sin(90° 90°)

=> 1√2 $\frac{1}{\sqrt{2}}$(1) + 1√2 $\frac{1}{\sqrt{2}}$(0)

=> 1√2 $\frac{1}{\sqrt{2}}$ + 0

=> 1√2 $\frac{1}{\sqrt{2}}$

The value of sinAsinB + cosAcosB is 1√2 $\frac{1}{\sqrt{2}}$

Q33. Find the acute angle A and B, if sin(A+2B) = √32 $\frac{\sqrt{3}}{2}$ and cos(A+4B) = 0, A>B.

Solution:

Given,

sin(A+2B) = √32 $\frac{\sqrt{3}}{2}$

A + 2B = sin⁻¹ √32 $\sin^{-1}\frac{\sqrt{3}}{2}$

A + 2B = 60° 60° ——— 1

Cos(A+4B) = 0

A + 4B = sin⁻¹(90) $\sin^{-1}(90)$

A + 4B = 90° 90° ——— 2

Solve equations 1 and 2

A + 2B = 60° 60°

A + 4B = 90° 90°

(-) (-) (-)

-2B = –30° 30°

2B = 30° 30°

B = 30°2 $\frac{30^\circ}{2}$

B = 15° 15°

Substitute B value in eq 2

A + 4B = 90° 90°

A + 4(15° 15°) = 90° 90°

A + 60° 60° = 90° 90°

A = 90° 90° – 60° 60°

A = 30° 30°

The value of A = 30° 30° and B = 15° 15°

Q 34. In ΔPQR ΔPQR , right angled at Q, PQ = 3 cm and PR = 6 cm. Determine ∠∠ P and ∠∠ R.

Solution:

Given,

In ΔPQR ΔPQR , right angled at Q, PQ = 3 cm and PR = 6 cm

By Pythagoras theorem,

$$PR^2 = PQ^2 + QR^2$$
$$=> 6^2 = 3^2 + QR^2$$
$$=> QR^2 = 36 - 9$$
$$=> QR = \sqrt{27}$$
$$=> QR = 3\sqrt{3}$$

PR²=PQ²+QR²=>6²=3²+QR²=>QR²=36–9=>QR=√27=>QR=3√3

sin R = 36=12=sin30° $\frac{3}{6} = \frac{1}{2} = \sin 30^\circ$

∠R=30° $\angle R = 30^\circ$

As we know, Sum of angles in a triangle = 180

$\angle P+\angle Q+\angle R=180^\circ => \angle P+90^\circ+30^\circ=180^\circ => \angle P=180^\circ-120^\circ => \angle P=60^\circ$

$\angle P + \angle Q + \angle R = 180^\circ$

$=> \angle P + 90^\circ + 30^\circ = 180^\circ$

$=> \angle P = 180^\circ - 120^\circ$

$=> \angle P = 60^\circ$

Therefore, $\angle R=30^\circ \angle R = 30^\circ$

And, $\angle P=60^\circ \angle P = 60^\circ$

Q35. If sin(A – B) = sin A cos B – cos A sin B and cos (A – B) = cos A cos B + sin A sin B, find the values of sin 15 and cos 15.

Solution:

Given,

sin(A – B) = sin A cos B – cos A sin B

And, cos (A – B) = cos A cos B + sin A sin B

We need to find, sin 15 and cos 15.

Let A = 45 and B = 30

sin 15 = sin (45- 30) = sin 45 cos 30 – cos 45 sin 30

= $= (\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}) - (\frac{1}{\sqrt{2}} \times \frac{1}{2})$

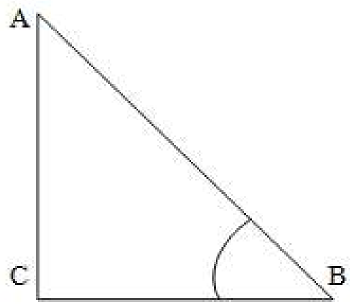
$=(1\sqrt{2} \times \sqrt{3}2)-(1\sqrt{2} \times 12)=\sqrt{3}-12\sqrt{2}= \frac{\sqrt{3}-1}{2\sqrt{2}}$

cos 15 = cos (45- 30) = cos 45 cos 30 – sin 45 sin 30

= $= (\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}) + (\frac{1}{\sqrt{2}} \times \frac{1}{2})$

$=(1\sqrt{2} \times \sqrt{3}2)+(1\sqrt{2} \times 12)=\sqrt{3}+12\sqrt{2}= \frac{\sqrt{3}+1}{2\sqrt{2}}$

Q36. In a right triangle ABC, right angled at C, if $\angle B=60^\circ \angle B = 60^\circ$ and AB=15 units. Find the remaining angles and sides.



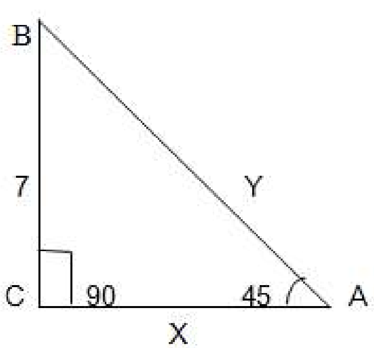
Solution:

$$\sin 60^\circ = \frac{x}{15}$$
$$\frac{\sqrt{3}}{2} = \frac{x}{15}$$
$$x = \frac{15\sqrt{3}}{2}\text{units}$$
$$\cos 60^\circ = \frac{x}{15}$$
$$\frac{1}{2} = \frac{x}{15}$$
$$x = \frac{15}{2}$$
$$x = 7.5\text{units}$$

sin60°=x15 √32=x15x= 15√32 unitscos60°=x15 12=x15x= 152 x=7.5units

Q37. In Δ ABC is a right triangle such that $\angle C = 90^\circ$, $\angle A = 45^\circ$ and BC = 7 units. Find the remaining angles and sides.

Solution:



Here, $\angle C = 90^\circ$ and $\angle A = 45^\circ$

We know that,

∠A+∠B+∠C$\angle A + \angle B + \angle C$ = 180°180°

=> 45°45° + 90°90° + ∠C$\angle C$ = 180°180°

=> 135°135° + ∠C$\angle C$ = 180°180°

=> ∠C$\angle C$ = 180°180° – 135°135°

=> ∠C$\angle C$ = 45°45°

The value of the remaining angle C is 45°45°

Now, we need to find the sides x and y

here,

cos(45) = BCAB $\frac{BC}{AB}$

1√2 $\frac{1}{\sqrt{2}}$ = 7y $\frac{7}{y}$

y = 7√2$7\sqrt{2}$ units

sin(45) = ACAB $\frac{AC}{AB}$

1√2 $\frac{1}{\sqrt{2}}$ = xy $\frac{x}{y}$

1√2 $\frac{1}{\sqrt{2}}$ = x7√2 $\frac{x}{7\sqrt{2}}$

x = 7√2√2 $\frac{7\sqrt{2}}{\sqrt{2}}$

x = 7 units

the value of x = 7 units and y = 7√2$7\sqrt{2}$ units

Q 38 . In a rectangle ABCD , AB = 20 cm , ∠$\angle$BAC = 60°60° , calculate side BC and diagonals AC and BD .

Solution:

Let AC = x cm and CB = y cm

Since , cosθ$\cos\theta$ = basehypotenuse $\frac{\text{base}}{\text{hypotenuse}}$

Therefore , cos60°=20x$\cos 60^\circ = \frac{20}{x}$

⇒12=20x⇒ $\frac{1}{2} = \frac{20}{x}$ [since,cos60°=12$\cos 60^\circ = \frac{1}{2}$]

⇒⇒ x= 40 cm = AC

Similarly BD = 40 cm

Now ,

Since , $\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$

Therefore , $\sin 60^\circ = \frac{BC}{AC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{y}{40} \Rightarrow y = \frac{40\sqrt{3}}{2}$

$\Rightarrow y = 20\sqrt{3}$ cm .

Q39:If A & B are acute angles such that tanA=1/2 tanB=1/3 and tan(A+B)= $\frac{\tan A+\tan B}{1-\tan A\ \tan B}$, find A+B.

Solution:

$\text{Tan}(A+B) = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}.\frac{1}{3}} = \frac{\frac{3+2}{6}}{\frac{5}{6}} = \frac{\frac{5}{6}}{\frac{5}{6}}$ $\text{Tan}(A+B) = \frac{5}{6} \times \frac{6}{5}$ $(A+B) = \text{Tan}^{-1}(1)$

(A+B) = 45^0

Q 40: Prove that : $(\sqrt{3}-1)(3-\cot 30^\circ) = \tan^3 60 - 2\sin 60^\circ$

Ans:

L.H.S => $(\sqrt{3}+1)(3-\cot 30^\circ)$

$=(\sqrt{3}+1)(3-\sqrt{3}) \quad \because \cot 30^\circ = \sqrt{3}$

$=(\sqrt{3}+1)(\sqrt{3}-1)\sqrt{3}$

$= ((\sqrt{3})^2-(1)^2)\sqrt{3}$

$= 2\sqrt{3}$

R.H.S => $\tan^3 60 - 2\sin 60^\circ$

$= (\sqrt{3})^3 - 2\times\frac{\sqrt{3}}{2}$

$= 3\sqrt{3}-\sqrt{3}$

$= 2\sqrt{3}$

L.H.S = R.H.S

Hence Proved