1.) Find the value of Trigonometric ratios in each of the following provided one of the six trigonometric ratios are given.

Sol.

(i) sinA=23 $\sin A = \frac{2}{3}$

Given:

sinA=23 $\sin A = \frac{2}{3}$ ….. (1)

By definition,

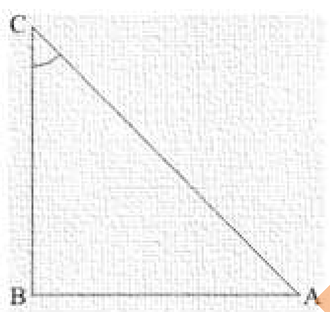
sinA=23 $\sin A = \frac{2}{3}$ = PerpendicularHypotenuse $\frac{\text{Perpendicular}}{\text{Hypotenuse}}$ ….(2)

By Comparing (1) and (2)

We get,

Perpendicular side = 2 and

Hypotenuse = 3



Therefore, by Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Substituting the value of perpendicular side (BC) and hypotenuse (AC) and get the base side

(AB)

Therefore,

$3^2 = AB^2 + 2^2$

$AB^2 = 3^2 – 2^2$

$AB^2 = 9 – 4$

$AB^2 = 5$

AB= √5 $\sqrt{5}$

Hence, Base = √5 $\sqrt{5}$

Now, cosA=BaseHypotenuse $\cos A = \frac{\text{Base}}{\text{Hypotenuse}}$

cosA=√53 $\cos A = \frac{\sqrt{5}}{3}$

Now, cosec A = 1sinA $\frac{1}{\text{sinA}}$

Therefore,

cosec A = HypotenusePerependicular $\frac{\text{Hypotenuse}}{\text{Perependicular}}$

cosec A =32 $\frac{3}{2}$

Now, sec A = HypotenuseBase $\frac{\text{Hypotenuse}}{\text{Base}}$

Therefore,

sec A = 3√5 $\frac{3}{\sqrt{5}}$

Now, tan A = PerependicularBase $\frac{\text{Perependicular}}{\text{Base}}$

tan A = 2√5 $\frac{2}{\sqrt{5}}$

Now, cot A = BasePerpendicular $\frac{\text{Base}}{\text{Perpendicular}}$

Therefore,

cot A = √52 $\frac{\sqrt{5}}{2}$

(ii) cosA=45 $\cos A = \frac{4}{5}$

Given: cosA=45 $\cos A = \frac{4}{5}$ …. (1)

By Definition,

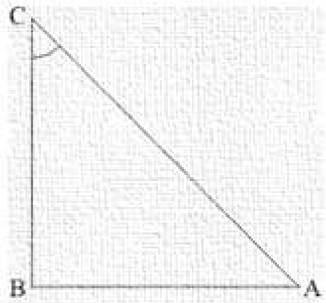
cosA=BaseHypotenuse $\cos A = \frac{\text{Base}}{\text{Hypotenuse}}$ …. (2)

By comparing (1) and (2)

We get,

Base =4 and

Hypotenuse = 5



Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Substituting the value of base (AB) and hypotenuse (AC) and get the perpendicular side

(BC)

$5^2 = 4^2 + BC^2$

$BC^2 = 5^2 – 4^2$

$BC^2 = 25 – 16$

$BC^2 = 9$

BC= 3

Hence, Perpendicular side = 3

Now,

sinA= 23 $\sin A = \frac{2}{3}$ = PerpendicularHypotenuse $\frac{\text{Perpendicular}}{\text{Hypotenuse}}$

Therefore,

sinA= 35 $\sin A = \frac{3}{5}$

Now, cosec A = 1sinA $\frac{1}{\text{sinA}}$

Therefore,

cosec A= 1sinA $\frac{1}{\text{sinA}}$

Therefore,

cosec A = HypotenusePerependicular $\frac{\text{Hypotenuse}}{\text{Perependicular}}$

cosec A = 53 $\frac{5}{3}$

Now, sec A = 1cosA $\frac{1}{\cos A}$

Therefore,

sec A = HypotenuseBase $\frac{\text{Hypotenuse}}{\text{Base}}$

sec A = 54 $\frac{5}{4}$

Now, tan A = PerpendicularBase $\frac{\text{Perpendicular}}{\text{Base}}$

Therefore,

tan A = 34 $\frac{3}{4}$

Now, cot A = 1tanA $\frac{1}{\tan A}$

Therefore,

cot A = BasePerpendicular $\frac{\text{Base}}{\text{Perpendicular}}$

cot A = 43 $\frac{4}{3}$

(iii) tanΘ= 111 $\tan\Theta = \frac{11}{1}$

Given: tanΘ= 111 $\tan\Theta = \frac{11}{1}$ …. (1)

By definition,

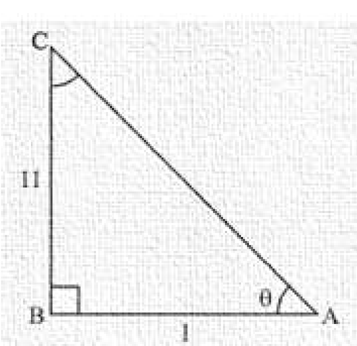
tanΘ= PerpendicularBase $\tan\Theta = \frac{\text{Perpendicular}}{\text{Base}}$ …. (2)

By Comparing (1) and (2)

We get,

Base= 1 and

Perpendicular side= 5



Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Substituting the value of base side (AB) and perpendicular side (BC) and get hypotenuse(AC)

$AC^2 = 1^2 + 11^2$

$AC^2 = 1 + 121$

$AC^2 = 122$

AC= $\sqrt{122}\sqrt{122}$

Now, sinΘ= PerpendicularHypotenuse $\sin\Theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

Therefore,

sinΘ= 11√122 $\sin\Theta = \frac{11}{\sqrt{122}}$

Now, cosecΘ= 1sinΘ $\Theta = \frac{1}{\sin\Theta}$

cosec Θ= √12211 $\Theta = \frac{\sqrt{122}}{11}$

Now, cosΘ= BaseHypotenuse $\cos\Theta = \frac{\text{Base}}{\text{Hypotenuse}}$

Therefore,

cosΘ= 1√122 $\cos\Theta = \frac{1}{\sqrt{122}}$

Now, secΘ= 1cosΘ $\sec\Theta = \frac{1}{\cos\Theta}$

Therefore,

secΘ= HypotenuseBase $\sec\Theta = \frac{\text{Hypotenuse}}{\text{Base}}$ secΘ= √1221 $\sec\Theta = \frac{\sqrt{122}}{1}$ secΘ= √122 $\sec\Theta = \sqrt{122}$

Now, cotΘ= 1tanΘ $\cot\Theta = \frac{1}{\tan\Theta}$

Therefore,

cotΘ= BasePerpendicular $\cot\Theta = \frac{\text{Base}}{\text{Perpendicular}}$ cotΘ= 111 $\cot\Theta = \frac{1}{11}$

(iv) sinΘ= 1115 $\sin\Theta = \frac{11}{15}$

Given: $\sin\Theta = \frac{11}{15}$ …. (1)

By definition,

$\sin\Theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$ …. (2)

By Comparing (1) and (2)

We get,

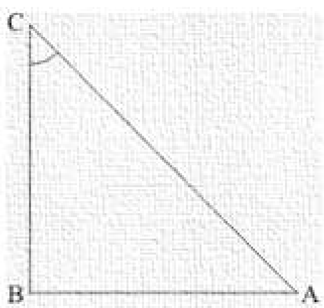
Perpendicular Side = 11 and

Hypotenuse= 15

Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$



Substituting the value of perpendicular side (BC) and hypotenuse (AC) and get the base side (AB)

$15^2 = AB^2 + 11^2$

$AB^2 = 15^2 - 11^2$

$AB^2 = 225 - 121$

$AB^2 = 104$

$AB = \sqrt{104}$

$AB = \sqrt{2\times2\times2\times13}$

$AB = 2\sqrt{2\times13}$

$AB = 2\sqrt{26}$

Hence, Base = $2\sqrt{26}$

Now, $\cos\Theta = \frac{\text{Base}}{\text{Hypotenuse}}$

Therefore,

cosΘ=2√2615 $\cos\Theta = \frac{2\sqrt{26}}{15}$

Now, cosec[Θ=1sinΘ $\Theta = \frac{1}{\sin\Theta}$

Therefore,

cosecΘ=HypotenusePerpendicular $\Theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

cosecΘ=1511 $\Theta = \frac{15}{11}$

Now, secΘ=HypotenuseBase $\Theta = \frac{\text{Hypotenuse}}{\text{Base}}$

Therefore,

secΘ=152√26 $\Theta = \frac{15}{2\sqrt{26}}$

Now, tanΘ=PerpendicularBase $\tan\Theta = \frac{\text{Perpendicular}}{\text{Base}}$

Therefore,

tanΘ=112√26 $\tan\Theta = \frac{11}{2\sqrt{26}}$

Now, cotΘ=BasePerpendicular $\cot\Theta = \frac{\text{Base}}{\text{Perpendicular}}$

Therefore,

cotΘ=2√2611 $\cot\Theta = \frac{2\sqrt{26}}{11}$

(v) tanα=512 $\tan\alpha = \frac{5}{12}$

Given: tanα=512 $\tan\alpha = \frac{5}{12}$ .... (1)

By definition,

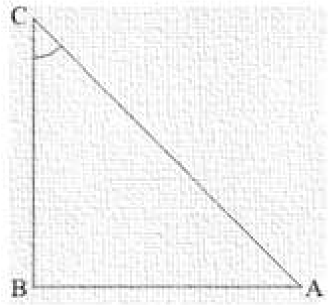
tanα=PerpendicularBase $\tan\alpha = \frac{\text{Perpendicular}}{\text{Base}}$ .... (2)

By comparing (1) and (2)

We get,

Base= 12 and

Perpendicular side = 5



Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Substituting the value of base side (AB) and the perpendicular side (BC) and gte hypotenuse (AC)

$AC^2 = 12^2 + 5^2$

$AC^2 = 144 + 25$

$AC^2 = 169$

AC= 13

Hence Hypotenuse = 13

Now, sinα=PerpendicularHypotenuse $\sin\alpha = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

Therefore,

sinα=513 $\sin\alpha = \frac{5}{13}$

Now, cosecα=HypotenusePerpendicular $\alpha = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

cosecα=135 $\alpha = \frac{13}{5}$

Now, cosα=BaseHypotenuse $\cos\alpha = \frac{\text{Base}}{\text{Hypotenuse}}$

Therefore,

cosα=1213 $\cos\alpha = \frac{12}{13}$

Now, secα=1cosα $\sec\alpha = \frac{1}{\cos\alpha}$

Therefore,

cotα=BasePerpendicular $\cot\alpha = \frac{\text{Base}}{\text{Perpendicular}}$ cotα=125 $\cot\alpha = \frac{12}{5}$

(vi) sinΘ=√**32** $\sin\Theta = \frac{\sqrt{3}}{2}$

Given: sinΘ=√32 $\sin\Theta = \frac{\sqrt{3}}{2}$ …. (1)

By definition,

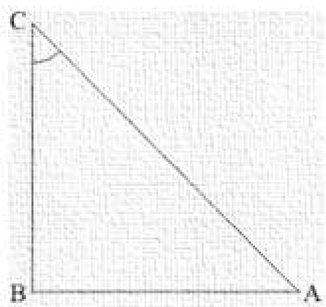
sinΘ= PerpendicularHypotenuse $\sin\Theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$ ….(2)

By comparing (1) and (2)

We get,

Perpendicular Side = √3 $\sqrt{3}$

Hypotenuse = 2



Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Substituting the value of perpendicular side (BC) and hypotenuse (AC) and get the base side (AB)

$2^2 = AB^2 + (\sqrt{3}\sqrt{3})^2$

$AB^2 = 2^2 – (\sqrt{3}\sqrt{3})^2$

$AB^2 = 4 – 3$

$AB^2 = 1$

AB= 1

Hence Base = 1

Now, cosΘ= BaseHypotenuse $\cos\Theta = \frac{\text{Base}}{\text{Hypotenuse}}$

Therefore,

cosΘ= 12 $\cos\Theta = \frac{1}{2}$

Now, cosecΘ= 1sinΘ $\Theta = \frac{1}{\sin \Theta}$

Therefore,

cosecΘ= HypotenusePerpendicualar $\Theta = \frac{\text{Hypotenuse}}{\text{Perpendicualar}}$

cosecΘ= 2√3 $\Theta = \frac{2}{\sqrt{3}}$

Now, secΘ= HypotenuseBase $\sec \Theta = \frac{\text{Hypotenuse}}{\text{Base}}$

Therefore,

secΘ= 21 $\sec \Theta = \frac{2}{1}$

Now, tanΘ= PerpendicularBase $\tan \Theta = \frac{\text{Perpendicular}}{\text{Base}}$

Therefore,

tanΘ= √31 $\tan \Theta = \frac{\sqrt{3}}{1}$

Now, cotΘ= BasePerpendicular $\cot \Theta = \frac{\text{Base}}{\text{Perpendicular}}$

Therefore,

cotΘ= 1√3 $\cot \Theta = \frac{1}{\sqrt{3}}$

(vii) cosΘ= 725 $\cos \Theta = \frac{7}{25}$

Given: cosΘ= 725 $\cos \Theta = \frac{7}{25}$ …. (1)

By definition,

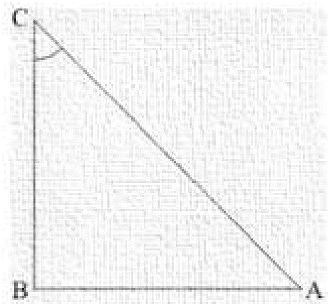
cosΘ= BaseHypotenuse $\cos \Theta = \frac{\text{Base}}{\text{Hypotenuse}}$

By comparing (1) and (2)

We get,

Base = 7 and

Hypotenuse = 25



Therefore

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Substituting the value of base side (AB) and hypotenuse (AC) and get the perpendicular side (BC)

$25^2 = 7^2 + BC^2$

$BC^2 = 25^2 - 7^2$

$BC^2 = 625 - 49$

BC = 576

BC= $\sqrt{576}\sqrt{576}$

BC= 24

Hence, Perpendicular side = 24

Now, sinΘ= perpendicularHypotenuse $\sin\Theta = \frac{\text{perpendicular}}{\text{Hypotenuse}}$

Therefore,

sinΘ= 2425 $\sin\Theta = \frac{24}{25}$

Now, cosecΘ= 1sinΘ $\Theta = \frac{1}{\sin\Theta}$

Therefore,

cosecΘ= HypotenusePerpendicualar $\Theta = \frac{\text{Hypotenuse}}{\text{Perpendicualar}}$

cosecΘ= 2524 $\Theta = \frac{25}{24}$

Now, secΘ= 1cosΘ $\sec\Theta = \frac{1}{\cos\Theta}$

Therefore,

secΘ= HypotenuseBase $\sec\Theta = \frac{\text{Hypotenuse}}{\text{Base}}$ secΘ= 257 $\sec\Theta = \frac{25}{7}$

Now, tanΘ=PerpendicularBase $\tan\Theta = \frac{\text{Perpendicular}}{\text{Base}}$

Therefore,

tanΘ=247 $\tan\Theta = \frac{24}{7}$

Now, cotΘ=1tanΘ $\cot\Theta = \frac{1}{\tan\Theta}$

Therefore,

cotΘ=BasePerpendicular $\cot\Theta = \frac{\text{Base}}{\text{Perpendicular}}$ cotΘ=724 $\cot\Theta = \frac{7}{24}$

(viii) tanΘ=815 $\tan\Theta = \frac{8}{15}$

Given: tanΘ=815 $\tan\Theta = \frac{8}{15}$ …. (1)

By definition,

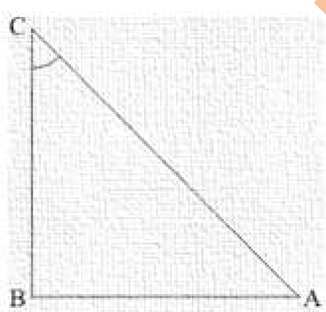
tanΘ=PerpendicularBase $\tan\Theta = \frac{\text{Perpendicular}}{\text{Base}}$ …. (2)

By comparing (1) and (2)

We get,

Base= 15 and

Perpendicular side = 8



Therefore,

By Pythagoras theorem,

$AC^2 = 15^2 + 8^2$

$AC^2 = 225 + 64$

$AC^2 = 289$

AC = √289 $\sqrt{289}$

AC= 17

Hence, Hypotenuse = 17

Now, sinΘ= PerpendicularHypotenuse $\sin\Theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

Therefore,

sinΘ= 817 $\sin\Theta = \frac{8}{17}$

Now, cosecΘ= 1sinΘ $\Theta = \frac{1}{\sin\Theta}$

Therefore,

cosecΘ= HypotenusePerpendicular $\Theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

Θ= 178 $\Theta = \frac{17}{8}$

Now, cosΘ= BaseHypotenuse $\cos\Theta = \frac{\text{Base}}{\text{Hypotenuse}}$

Therefore,

cosΘ= 1517 $\cos\Theta = \frac{15}{17}$

Now, secΘ= 1cosΘ $\sec\Theta = \frac{1}{\cos\Theta}$

Therefore,

secΘ= HypotenuseBase $\sec\Theta = \frac{\text{Hypotenuse}}{\text{Base}}$ secΘ= 1715 $\sec\Theta = \frac{17}{15}$

Now, cotΘ= 1tanΘ $\cot\Theta = \frac{1}{\tan\Theta}$

Therefore,

cotΘ= BasePerpendicular $\cot\Theta = \frac{\text{Base}}{\text{Perpendicular}}$ cotΘ= 158 $\cot\Theta = \frac{15}{8}$

(ix) cotΘ= 125 $\cot\Theta = \frac{12}{5}$

Given: cotΘ= 125 $\cot\Theta = \frac{12}{5}$ …. (1)

By definition,

cotΘ= 1tanΘ $\cot\Theta = \frac{1}{\tan\Theta}$

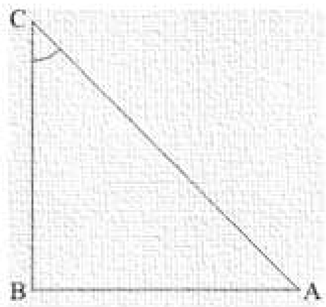
cotΘ= BasePerpendicular $\cot\Theta = \frac{\text{Base}}{\text{Perpendicular}}$ …. (2)

By comparing (1) and (2)

We get,

Base = 12 and

Perpendicular side = 5



Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Substituting the value of base side (AB) and perpendicular side(BC) and get the hypotenuse (AC)

$AC^2 = 12^2 + 5^2$

$AC^2 = 144 + 25$

$AC^2 = 169$

AC = √169 $\sqrt{169}$

AC = 13

Hence, Hypotenuse = 13

Now, sinΘ= PerpendicularHypotenuse $\sin\Theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

Therefore,

sinΘ= 513 $\sin\Theta = \frac{5}{13}$

Now, cosecΘ= 1sinΘ $\Theta = \frac{1}{\sin\Theta}$

Therefore,

cosecΘ= HypotenusePerpendicular $\Theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

cosecΘ= 135 $\Theta = \frac{13}{5}$

Now, $\cos\Theta = \frac{\text{Base}}{\text{Hypotenuse}}$

Therefore,

$\cos\Theta = \frac{12}{13}$

Now, $\sec\Theta = \frac{1}{\cos\Theta}$

Therefore,

$\sec\Theta = \frac{\text{Hypotenuse}}{\text{Base}}$ $\sec\Theta = \frac{13}{12}$

Now, $\tan\Theta = \frac{1}{\cot\Theta}$

Therefore,

$\tan\Theta = \frac{\text{Perpendicular}}{\text{Base}}$ $\tan\Theta = \frac{5}{12}$

(x) $\sec\Theta = \frac{13}{5}$

Given: $\sec\Theta = \frac{13}{5}$ … (1)

By definition,

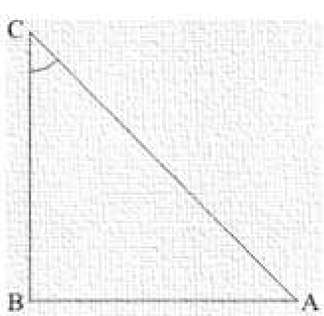
$\sec\Theta = \frac{1}{\cos\Theta}$ …. (2)

By comparing (1) and (2)

We get,

Base=5

Hypotenuse = 13



Therefore,

By Pythagoras theorem,

Substituting the value of base side (AB) and hypotenuse (AC) and get the perpendicular side (BC)

$13^2 = 5^2 + BC^2$

$BC^2 = 13^2 - 5^2$

$BC^2=169 - 25$

$BC^2= 144$

BC= $\sqrt{144}\sqrt{144}$

BC = 12

Hence, Perpendicular side = 12

Now, sinΘ= PerpendicularHypotenuse $\sin\Theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

Therefore,

sinΘ= 1213 $\sin\Theta = \frac{12}{13}$

Now, cosec Θ= 1sinΘ $\Theta = \frac{1}{\sin\Theta}$

Therefore,

cosecΘ= HypotenusePerpendicular $\Theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

cosecΘ= 1312 $\Theta = \frac{13}{12}$

Now, cosΘ= 1secΘ $\cos\Theta = \frac{1}{\sec\Theta}$

Therefore,

cosΘ= BaseHypotenuse $\cos\Theta = \frac{\text{Base}}{\text{Hypotenuse}}$ cosΘ= 513 $\cos\Theta = \frac{5}{13}$

Now, tanΘ= PerpendicularBase $\tan\Theta = \frac{\text{Perpendicular}}{\text{Base}}$

Therefore,

tanΘ= 125 $\tan\Theta = \frac{12}{5}$

Now, cotΘ= 1tanΘ $\cot\Theta = \frac{1}{\tan\Theta}$

Therefore,

cotΘ= BasePerpendicular $\cot\Theta = \frac{\text{Base}}{\text{Perpendicular}}$ cotΘ= 512 $\cot\Theta = \frac{5}{12}$

(xi) cosecΘ=$\sqrt{10}$ $\Theta = \sqrt{10}$

Given: cosecΘ= $\Theta = \frac{\sqrt{10}}{1}$ … (1)

By definition

cosecΘ= $\Theta = \frac{1}{\sin\Theta}$ ….(2)

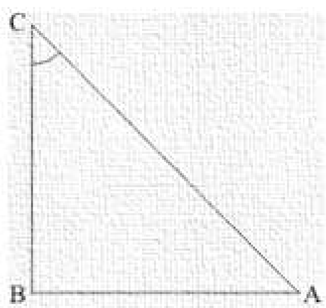
Θ= $\Theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

By comparing (1) and(2)

We get,

Perpendicular side= 1 and

Hypotenuse = $\sqrt{10}$



Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Substituting the value of perpendicular side (BC) and hypotenuse (AC) and get the base side (AB)

$(\sqrt{10})^2 = AB^2 + 1^2$

$AB^2 = (\sqrt{10})^2 - 1^2$

$AB^2 = 10 - 1$

$AB = \sqrt{9}$

AB = 3

Hence, Base side = 3

Now, sinΘ= $\sin\Theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

Therefore,

sinΘ= $\sin\Theta = \frac{1}{\sqrt{10}}$

Now, $\cos\Theta = \frac{\text{Base}}{\text{Hypotenuse}}$

Therefore,

$\cos\Theta = \frac{3}{\sqrt{10}}$

Now, $\sec\Theta = \frac{1}{\cos\Theta}$

Therefore,

$\sec\Theta = \frac{\text{Hypotenuse}}{\text{Base}}$ $\sec\Theta = \frac{\sqrt{10}}{3}$

Now, $\tan\Theta = \frac{\text{Perpendicular}}{\text{Base}}$

Therefore,

$\tan\Theta = \frac{1}{3}$

Now, $\cot\Theta = \frac{1}{\tan\Theta}$

$\cot\Theta = \frac{3}{1}$ $\cot\Theta = 3$

(xii) $\cos\Theta \frac{12}{15}$

Given: $\cos\Theta \frac{12}{15}$ …. (1)

By definition,

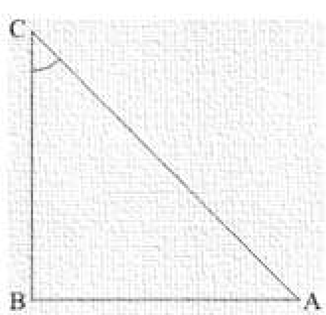
$\cos\Theta = \frac{\text{Base}}{\text{Hypotenuse}}$ …. (2)

By comparing (1) and (2)

We get,

Base=12 and

Hypotenuse = 15



Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Substituting the value of base side (AB) and hypotenuse (AC) and get the perpendicular side (BC)

$15^2 = 12^2 + BC^2$

$BC^2 = 15^2 – 12^2$

$BC^2 = 225 – 144$

$BC^2 = 81$

BC = √81 $\sqrt{81}$

BC = 9

Hence, Perpendicular side = 9

Now, sinΘ=PerpendicularHypotenuse $\sin\Theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

Therefore,

sinΘ=915 $\sin\Theta = \frac{9}{15}$

Now, cosecΘ=1sinΘ $\Theta = \frac{1}{\sin\Theta}$

Therefore,

cosec Θ=HypotenusePerpendicular $\Theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

cosecΘ=159 $\Theta = \frac{15}{9}$

Now, secΘ=1cosΘ $\sec\Theta = \frac{1}{\cos\Theta}$

Therefore,

secΘ=HypotenuseBase $\sec\Theta = \frac{\text{Hypotenuse}}{\text{Base}}$ secΘ=1512 $\sec\Theta = \frac{15}{12}$

Now, tanΘ=PerpendicularBase $\tan\Theta = \frac{\text{Perpendicular}}{\text{Base}}$

Therefore,

tanΘ=912 $\tan\Theta = \frac{9}{12}$

Now, cotΘ=1tanΘ $\cot\Theta = \frac{1}{\tan\Theta}$

Therefore,

cotΘ= BasePerpendicular $\cot\Theta = \frac{\text{Base}}{\text{Perpendicular}}$ cotΘ= 129 $\cot\Theta = \frac{12}{9}$

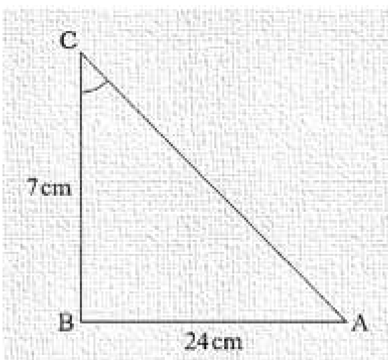
2.) In a ΔΔ ABC, right angled at B , AB – 24 cm , BC= 7 cm , Determine

(i) sin A , cos A

(ii) sin C, cos C

Sol.

(i) The given triangle is below:



Given: In ΔΔ ABC , AB= 24 cm

BC = 7cm

∠ABC∠ABC = 90°

To find: sin A, cos A

In this problem, Hypotenuse side is unknown

Hence we first find hypotenuse side by Pythagoras theorem

By Pythagoras theorem,

We get,

$AC^2 = AB^2 + BC^2$

$AC^2 = 24^2 + 7^2$

$AC^2 = 576 + 49$

$AC^2 = 625$

AC = √625 $\sqrt{625}$

AC= 25

Hypotenuse = 25

By definition,

sinA=Perpendicularsideoppositeto∠AHypotenuse $\sin A = \frac{\text{Perpendicularsideoppositeto}\angle A}{\text{Hypotenuse}}$ sinA=BCAC $\sin A = \frac{BC}{AC}$ sinA=725 $\sin A = \frac{7}{25}$

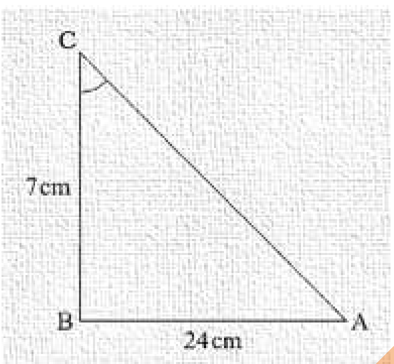
By definition,

cosA=Basesideadjacentto∠AHypotenuse $\cos A = \frac{\text{Basesideadjacentto}\angle A}{\text{Hypotenuse}}$ cosA=ABAC $\cos A = \frac{AB}{AC}$ cosA=2425 $\cos A = \frac{24}{25}$

Answer:

sinA=725 $\sin A = \frac{7}{25}$ **, cosA=2425** $\cos A = \frac{24}{25}$

(ii) The given triangle is below:



Given: In ΔΔ ABC , AB= 24 cm

BC = 7cm

∠ABC∠ABC = 90º

To find: sin C, cos C

In this problem, Hypotenuse side is unknown

Hence we first find hypotenuse side by Pythagoras theorem

By Pythagoras theorem,

We get,

$AC^2 = AB^2 + BC^2$

$AC^2 = 24^2 + 7^2$

$AC^2 = 576 + 49$

$AC^2 = 625$

AC = √625 $\sqrt{625}$

AC= 25

Hypotenuse = 25

By definition,

sinC=Perpendicularsideoppositeto∠CHypotenuse $\sin C = \frac{\text{Perpendicularsideoppositeto}\angle C}{\text{Hypotenuse}}$ sinC=ABAC $\sin C = \frac{AB}{AC}$ sinC=2425 $\sin C = \frac{24}{25}$

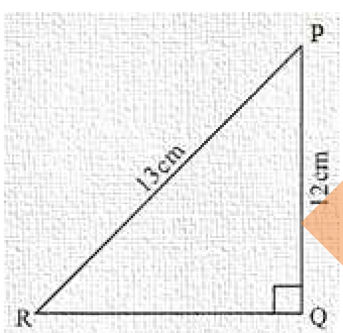
By definition,

cosC=Basesideadjacentto∠CHypotenuse $\cos C = \frac{\text{Basesideadjacentto}\angle C}{\text{Hypotenuse}}$ cosA=BCAC $\cos A = \frac{BC}{AC}$ cosA=725 $\cos A = \frac{7}{25}$

Answer:

sinA=2425 $\sin A = \frac{24}{25}$ **, cosA=725** $\cos A = \frac{7}{25}$

3.) In the below figure, find tan P and cot R. Is tan P = cot R?



To find, tan P, cot R

Sol.

In the given right angled ΔPQR ΔPQR , length of side OR is unknown

Therefore , by applying Pythagoras theorem in ΔPQR ΔPQR

We get,

$PR^2 = PQ^2 + QR^2$

Substituting the length of given side PR and PQ in the above equation

$13^2 = 12^2 + QR^2$

$QR^2 = 13^2 - 12^2$

$QR^2 = 169 - 144$

$QR^2 = 25$

QR = $\sqrt{25}\sqrt{25}$

By definiton, we know that ,

tanP = Perpendicularsideoppositeto∠PBasesideadjacentto∠P $\tan P = \frac{\text{Perpendicularsideoppositeto}\angle P}{\text{Basesideadjacentto}\angle P}$ tanP = QRPQ $\tan P = \frac{QR}{PQ}$

tanP = 512 $\tan P = \frac{5}{12}$ …. (1)

Also, by definition, we know that

cotR = Basesideadjacentto∠RPerpendicularsideoppositeto∠R $\cot R = \frac{\text{Base side adjacent to } \angle R}{\text{Perpendicular side opposite to } \angle R}$

cotR = QRPQ $\cot R = \frac{QR}{PQ}$

cotR = 512 $\cot R = \frac{5}{12}$ …. (2)

Comparing equation (1) ad (2), we come to know that that R.H.S of both the equation are equal.

Therefore, L.H.S of both equations is also equal

tan P = cot R

Answer:

Yes , tan P =cot R = 512 $\frac{5}{12}$

4.) If sin A = 941 $\frac{9}{41}$**, Compute cos A and tan A.**

Sol.

Given: sinA = 941 $\sin A = \frac{9}{41}$ …. (1)

To find: cos A, tan A

By definition,

sinA = Perpendicularsideoppositeto∠AHypotenuse $\sin A = \frac{\text{Perpendicular side opposite to } \angle A}{\text{Hypotenuse}}$ …. (2)

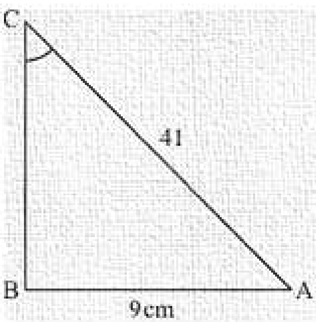
By comparing (1) and (2)

We get ,

Perpendicular side = 9 and

Hypotenuse = 41

Now using the perpendicular side and hypotenuse we can construct $\Delta ABC \Delta ABC$ as shown below



Length of side AB is unknown is right angled $\Delta ABC \Delta ABC$,

To find the length of side AB, we use Pythagoras theorem,

Therefore, by applying Pythagoras theorem in $\Delta ABC \Delta ABC$,

We get,

$AC^2 = AB^2 + BC^2$

$41^2 = AB^2 + 9^2$

$AB^2 = 41^2 – 9^2$

$AB^2 = 168 – 81$

AB= 1600

AB = $\sqrt{1600}\sqrt{1600}$

AB = 40

Hence, length of side AB= 40

Now

By definition,

cosA= Basesideadjacentto∠AHypotenuse $\cos A = \frac{\text{Base side adjacent to } \angle A}{\text{Hypotenuse}}$ cosA= ABAC

$\cos A = \frac{AB}{AC}$ cosA= 4041 $\cos A = \frac{40}{41}$

Now,

By definition,

tanA=Perpendicularsideoppositeto∠ABasesideadjacentto∠A $\tan A = \frac{\text{Perpendicular side opposite to } \angle A}{\text{Base side adjacent to } \angle A}$ tanA=BCAB $\tan A = \frac{BC}{AB}$ tanA=940 $\tan A = \frac{9}{40}$

Answer:

cosA=4041 $\cos A = \frac{40}{41}$ **, tanA=940** $\tan A = \frac{9}{40}$

5.) Given 15cot A=8, find sin A and sec A.

Answer:

Given: 15cot A = 8

To find: sin A, sec A

Since 15 cot A =8

By taking 15 on R.H.S

We get,

cotA=815 $\cot A = \frac{8}{15}$

By definition,

cotA=1tanA $\cot A = \frac{1}{\tan A}$

Hence,

cotA=1Perpendicularsideoppositeto∠ABasesideadjacentto∠A $\cot A = \frac{1}{\frac{\text{Perpendicular side opposite to } \angle A}{\text{Base side adjacent to } \angle A}}$

cotA=Basesideadjacentto∠APerpendicularsideoppositeto∠A $\cot A = \frac{\text{Base side adjacent to } \angle A}{\text{Perpendicular side opposite to } \angle A}$ …. (2)

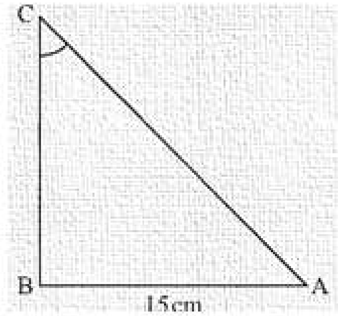
Comparing equation (1) and (2)

We get,

Base side adjacent to ∠A $\angle A$ = 8

Perpendicular side opposite to ∠A $\angle A$ = 15

ΔABC ΔABC can be drawn below using above information



Hypotenuse side is unknown.

Therefore, we find side AC of ΔABC by Pythagoras theorem.

So, by applying Pythagoras theorem to ΔABC

We get,

$AC^2 = AB^2 + BC^2$

Substituting values of sides from the above figure

$AC^2 = 8^2 + 15^2$

$AC^2 = 64 + 225$

$AC^2 = 289$

$AC = \sqrt{289}$

AC = 17

Therefore, hypotenuse =17

Now by definition,

$\sin A = \frac{\text{Perpendicular side opposite to } \angle A}{\text{Hypotenuse}}$

Therefore, $\sin A = \frac{BC}{AC}$

Substituting values of sides from the above figure

$\sin A = \frac{15}{17}$

By definition,

$\sec A = \frac{1}{\cos A}$

Hence,

secA=1Basesideadjacentto∠AHypotenuse $\sec A = \frac{1}{\frac{\text{Base side adjacent to } \angle A}{\text{Hypotenuse}}}$ secA=HypotenuseBasesideadjacentto∠A

$$\sec A = \frac{\text{Hypotenuse}}{\text{Base side adjacent to } \angle A}$$

Substituting values of sides from the above figure

secA=178 $\sec A = \frac{17}{8}$

Answer:

sinA=1517 $\sin A = \frac{15}{17}$ **, secA=178** $\sec A = \frac{17}{8}$

6.) In ΔPQR ΔPQR , right angled at Q, PQ = 4cm and RQ= 3 cm .Find the value of sin P, sin R , sec P and sec R.

Sol.

Given:

ΔPQR ΔPQR is right angled at vertex Q.

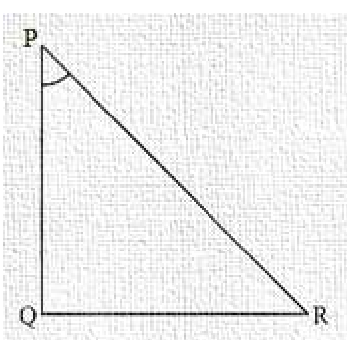
PQ= 4cm

RQ= 3cm

To find,

sin P, sin R , sec P , sec R

Given ΔPQR ΔPQR is as shown below



Hypotenuse side PR is unknown.

Therefore, we find side PR of ΔPQR ΔPQR by Pythagoras theorem

By applying Pythagoras theorem to ΔPQR ΔPQR

We get,

$PR^2 = PQ^2 + RQ^2$

Substituting values of sides from the above figure

$PR^2 = 4^2 + 3^2$

$PR^2 = 16 + 9$

$PR^2 = 25$

PR = √25 $\sqrt{25}$

PR = 5

Hence, Hypotenuse =5

Now by definition,

sinP=PerpendicularsideoppositetoPHypotenuse $\sin P = \frac{\text{Perpendicularsideoppositeto}\angle P}{\text{Hypotenuse}}$ sinP=RQPR

$\sin P = \frac{RQ}{PR}$

Substituting values of sides from the above figure

sinP=35 $\sin P = \frac{3}{5}$

Now by definition,

sinR=Perpendicularsideoppositeto∠RHypotenuse $\sin R = \frac{\text{Perpendicular side opposite to } \angle R}{\text{Hypotenuse}}$ sinR=PQPR

$\sin R = \frac{PQ}{PR}$

Substituting the values of sides from above figure

sinR=45 $\sin R = \frac{4}{5}$

By definition,

secP=1cosP $\sec P = \frac{1}{\cos P}$ secP=1Basesideadjacentto∠pHypotenuse

$\sec P = \frac{1}{\frac{\text{Base side adjacent to } \angle p}{\text{Hypotenuse}}}$ secP=HypotenuseBasesideadjacentto∠P $\sec P = \frac{\text{Hypotenuse}}{\text{Base side adjacent to } \angle P}$

Substituting values of sides from the above figure

secP=PRPQ $\sec P = \frac{PR}{PQ}$ secP=54 $\sec P = \frac{5}{4}$

By definition,

secR=1cosR $\sec R = \frac{1}{\cos R}$ secR=1Basesideadjacentto∠RHypotenuse

$\sec R = \frac{1}{\frac{\text{Base side adjacent to } \angle R}{\text{Hypotenuse}}}$ secR=HypotenuseBasesideadjacentto∠R $\sec R = \frac{\text{Hypotenuse}}{\text{Base side adjacent to } \angle R}$

Substituting values of sides from the above figure secR=PRRQ $\sec R = \frac{PR}{RQ}$ secR=53 $\sec R = \frac{5}{3}$

Answer:

sinP=35 $\sin P = \frac{3}{5}$ **, sinR=45** $\sin R = \frac{4}{5}$ **,**

secP=54 $\sec P = \frac{5}{4}$ **, secR=53** $\sec R = \frac{5}{3}$

7.) If cotΘ=78 $\cot \Theta = \frac{7}{8}$ **, evaluate**

(i) 1+sinΘ×1−sinΘ1+cosΘ×1−cosΘ $\frac{1+\sin \Theta \times 1-\sin \Theta}{1+\cos \Theta \times 1-\cos \Theta}$

(ii) cot²Θ $\cot^2 \Theta$

Sol.

Given: cotΘ=78 $\cot \Theta = \frac{7}{8}$

To evaluate: 1+sinΘ×1−sinΘ1+cosΘ×1−cosΘ $\frac{1+\sin \Theta \times 1-\sin \Theta}{1+\cos \Theta \times 1-\cos \Theta}$

1+sinΘ×1−sinΘ1+cosΘ×1−cosΘ $\frac{1+\sin \Theta \times 1-\sin \Theta}{1+\cos \Theta \times 1-\cos \Theta}$ …(1)

We know the following formula

$(a + b)(a – b) = a^2 – b^2$

By applying the above formula in the numerator of equation (1)

We get,

(1+sinθ)×(1−sinθ)=1−sin²θ.....(2)(Where,a=1andb=sinθ)
$(1 + \sin\theta) \times (1-\sin\theta) = 1-\sin^2\theta.....(2) \quad (\text{Where, } a = 1 \text{ and } b = \sin\theta)$

Similarly,

By applying formula $(a +b) (a – b) = a^2 – b^2$ in the denominator of equation (1).

We get,

(1+cosΘ)(1−cosΘ)=1²−cos²Θ $(1 + \cos \Theta)(1-\cos \Theta) = 1^2-\cos^2 \Theta$ … (Where a=1 and b= cosΘ $\cos \Theta$

(1+cosΘ)(1−cosΘ)=1−cos²Θ $(1 + \cos \Theta)(1-\cos \Theta) = 1-\cos^2 \Theta$ … (Where a=1 and b= cosΘ $\cos \Theta$

Substituting the value of numerator and denominator of equation (1) from equation (2), equation (3).

Therefore,

$(1+\sin\Theta)(1-\sin\Theta)(1+\cos\Theta)(1-\cos\Theta)\ \frac{(1+\sin\Theta)(1-\sin\Theta)}{(1+\cos\Theta)(1-\cos\Theta)} = 1-\sin^2\Theta 1-\cos^2\Theta\ \frac{1-\sin^2\Theta}{1-\cos^2\Theta}$ ….(4)

Since,

$\cos^2\Theta+\sin^2\Theta=1\cos^2\Theta+\sin^2\Theta=1$

Therefore,

$\cos^2\Theta=1-\sin^2\Theta\cos^2\Theta=1-\sin^2\Theta$

Also, $\sin^2\Theta=1-\cos^2\Theta\sin^2\Theta=1-\cos^2\Theta$

Putting the value of $1-\sin^2\Theta 1-\sin^2\Theta$ and $1-\cos^2\Theta 1-\cos^2\Theta$ in equation (4)

We get,

$(1+\sin\Theta)(1-\sin\Theta)(1+\cos\Theta)(1-\cos\Theta)\ \frac{(1+\sin\Theta)(1-\sin\Theta)}{(1+\cos\Theta)(1-\cos\Theta)} = \cos^2\Theta\sin^2\Theta\ \frac{\cos^2\Theta}{\sin^2\Theta}$

We know that, $\cos\Theta\sin\Theta=\cot\Theta\ \frac{\cos\Theta}{\sin\Theta}=\cot\Theta$

$(1+\sin\Theta)(1-\sin\Theta)(1+\cos\Theta)(1-\cos\Theta)\ \frac{(1+\sin\Theta)(1-\sin\Theta)}{(1+\cos\Theta)(1-\cos\Theta)} = (\cot\Theta)^2(\cot\Theta)^2$

Since, it is given that $\cot\Theta=78\cot\Theta=\frac{7}{8}$

Therefore,

$(1+\sin\Theta)(1-\sin\Theta)(1+\cos\Theta)(1-\cos\Theta)\ \frac{(1+\sin\Theta)(1-\sin\Theta)}{(1+\cos\Theta)(1-\cos\Theta)} = (78)^2(\frac{7}{8})^2$

$(1+\sin\Theta)(1-\sin\Theta)(1+\cos\Theta)(1-\cos\Theta)\ \frac{(1+\sin\Theta)(1-\sin\Theta)}{(1+\cos\Theta)(1-\cos\Theta)} = 7^2 8^2\ \frac{7^2}{8^2}$

$(1+\sin\Theta)(1-\sin\Theta)(1+\cos\Theta)(1-\cos\Theta)\ \frac{(1+\sin\Theta)(1-\sin\Theta)}{(1+\cos\Theta)(1-\cos\Theta)} = 4964\ \frac{49}{64}$

(ii) Given: $\cot\Theta=78\cot\Theta=\frac{7}{8}$

To evaluate: $\cot^2\Theta\cot^2\Theta$

$\cot\Theta=78\cot\Theta=\frac{7}{8}$

Squaring on both sides,

We get,

$(\cot\Theta)^2=(78)^2(\cot\Theta)^2=(\frac{7}{8})^2$

$(\cot\Theta)^2(\cot\Theta)^2=4964\ \frac{49}{64}$

Answer:

4964 $\frac{49}{64}$

8.) If 3cotA=4 $3\cot A = 4$ **, check whether** 1–tan²A1+tan²A **=cos²A–sin²A** $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ **or not.**

Sol.

Given: 3cot A =4

To check whether 1–tan²A1+tan²A=cos²A–sin²A $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

3cot A =4

Dividing by 3 on both sides,

We get,

cot A = 43 $\frac{4}{3}$ …. (1)

By definition,

cotA= 1tanA $\cot A = \frac{1}{\tan A}$

Therefore,

cotA= 1 PerpendicularsideoppositetoAngleABasesideadjacenttoAngleA $\cot A = \frac{1}{\frac{\text{Perpendicular side opposite to } \angle A}{\text{Base side adjacent to } \angle A}}$

cotA= BasesideadjacenttoAngleAPerpendicularsideoppositetoAngleA $\cot A = \frac{\text{Base side adjacent to } \angle A}{\text{Perpendicular side opposite to } \angle A}$ …. (2)

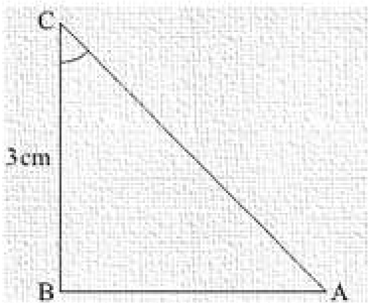
Comparing (1) and (2)

We get,

Base side adjacent to ∠A $\angle A$ = 4

Perpendicular side opposite to ∠A $\angle A$ = 3

Hence ΔABC ΔABC is as shown in figure below



In $\Delta ABC \Delta ABC$, Hypotenuse is unknown

Hence, it can be found by using Pythagoras theorem

Therefore by applying Pythagoras theorem in$\Delta ABC \Delta ABC$

We get

$AC^2 = AB^2 + BC^2$

Substituting the values of sides from the above figure

$AC^2 = 4^2 + 3^2$

$AC^2 = 16 + 9$

$AC^2 = 25$

AC = $\sqrt{25}\sqrt{25}$

AC = 5

Hence, hypotenuse= 5

To check whether

To check whether $1-\tan^2 A 1+\tan^2 A = \cos^2 A - \sin^2 A \frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

We get thee values of tan A , cos A , sin A

By definition,

tan A = $1 \cot A \frac{1}{\cot A}$

Substituting the value of cot A from equation (1)

We get,

tan A = $14 \frac{1}{4}$

tan A = $34 \frac{3}{4}$ …. (3)

Now by definition,

cosA=Basesideadjacentto∠AHypotenuse $\cos A = \frac{\text{Base side adjacent to } \angle A}{\text{Hypotenuse}}$ cosA=ABAC $\cos A = \frac{AB}{AC}$

Substituting the values of sides from the above figure

cosA=45 $\cos A = \frac{4}{5}$ …. (5)

Now we first take L.H.S of equation 1–tan²A1+tan²A=cos²A–sin²A $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

L.H.S = 1–tan²A1+tan²A $\frac{1-\tan^2 A}{1+\tan^2 A}$

Substituting value of **tan A** from equation (3)

We get,

L.H.S= 1–(34)²1+(34)² $\frac{1-(\frac{3}{4})^2}{1+(\frac{3}{4})^2}$

1–tan²A1+tan²A $\frac{1-\tan^2 A}{1+\tan^2 A}$ = 1–(34)²1+(34)² $\frac{1-(\frac{3}{4})^2}{1+(\frac{3}{4})^2}$

1–tan²A1+tan²A $\frac{1-\tan^2 A}{1+\tan^2 A}$ = 1–916 1+916 $\frac{1-\frac{9}{16}}{1+\frac{9}{16}}$

Taking L.C.M on both numerator and denominator

We get,

1–tan²A1+tan²A $\frac{1-\tan^2 A}{1+\tan^2 A}$ = 16–916 16+916 $\frac{\frac{16-9}{16}}{\frac{16+9}{16}}$

1–tan²A1+tan²A $\frac{1-\tan^2 A}{1+\tan^2 A}$ = 725 $\frac{7}{25}$ …. (6)

Now we take R.H.S of equation whether 1–tan²A1+tan²A=cos²A–sin²A $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

R.H.S = cos²A–sin²A $\cos^2 A - \sin^2 A$

Substituting value of sin A and cos A from equation (4) and (5)

We get,

R.H.S= (45)²–(35²) $(\frac{4}{5})^2 - (\frac{3}{5}^2)$

cos²A–sin²A $\cos^2 A - \sin^2 A$ = (45)²–(35²) $(\frac{4}{5})^2 - (\frac{3}{5}^2)$

cos²A–sin²A $\cos^2 A - \sin^2 A$ = 1625–925 $\frac{16}{25} - \frac{9}{25}$

cos²A–sin²A $\cos^2 A - \sin^2 A$ = 16–925 $\frac{16-9}{25}$

cos^{2}A–sin^{2}A$\cos^2 A - \sin^2 A$ = 725 $\frac{7}{25}$ ….(7)

Comparing (6) and (7)

We get.

1–tan^{2}A1+tan^{2}A =cos^{2}A–sin^{2}A $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

Answer:

Yes, 1–tan^{2}A1+tan^{2}A =cos^{2}A–sin^{2}A $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

9.) If tanΘ=ab $\tan\Theta = \frac{a}{b}$ **, find the value of cosΘ+sinΘcosΘ–sinΘ** $\frac{\cos\Theta+\sin\Theta}{\cos\Theta-\sin\Theta}$**.**

Sol.

Given:

tanΘ=ab $\tan\Theta = \frac{a}{b}$ …. (1)

Now, we know that tanΘ=sinΘcosΘ $\tan\Theta = \frac{\sin\Theta}{\cos\Theta}$

Therefore equation (1) become as follows

sinΘcosΘ $\frac{\sin\Theta}{\cos\Theta}$ =ab $\frac{a}{b}$

Now, by applying invertendo

We get,

cosΘsinΘ=ba $\frac{\cos\Theta}{\sin\Theta} = \frac{b}{a}$

Now by applying Componendo – dividendo

We get,

cosΘ+sinΘcosΘ–sinΘ=b+ab–a $\frac{\cos\Theta+\sin\Theta}{\cos\Theta-\sin\Theta} = \frac{b+a}{b-a}$

Therefore,

cosΘ+sinΘcosΘ–sinΘ=b+ab–a $\frac{\cos\Theta+\sin\Theta}{\cos\Theta-\sin\Theta} = \frac{b+a}{b-a}$

10.) If 3tanΘ=4$3\tan\Theta = 4$ **, find the value of 4cosΘ–sinΘ2cosΘ+sinΘ** $\frac{4\cos\Theta-\sin\Theta}{2\cos\Theta+\sin\Theta}$

Sol.

Given: If 3tanΘ=4 $3 \tan \Theta = 4$

Therefore,

tanΘ=43 $\tan \Theta = \frac{4}{3}$ …. (1)

Now, we know that tanΘ=sinΘcosΘ $\tan \Theta = \frac{\sin \Theta}{\cos \Theta}$

Therefore equation (1) becomes

sinΘcosΘ=43 $\frac{\sin \Theta}{\cos \Theta} = \frac{4}{3}$ ….(2)

Now, by applying Invertendo to equation (2)

We get,

cosΘsinΘ=34 $\frac{\cos \Theta}{\sin \Theta} = \frac{3}{4}$ …. (3)

Now, multiplying by 4 on both sides

We get

4× cosΘsinΘ=4× 34 $4 \times \frac{\cos \Theta}{\sin \Theta} = 4 \times \frac{3}{4}$

Therefore

4cosΘ–sinΘsinΘ=3–11 $\frac{4 \cos \Theta - \sin \Theta}{\sin \Theta} = \frac{3-1}{1}$

4cosΘ–sinΘsinΘ=21 $\frac{4 \cos \Theta - \sin \Theta}{\sin \Theta} = \frac{2}{1}$ …. (4)

Now, multiplying by 2 on both sides of equation (3)

We get,

2cosΘsinΘ=32 $\frac{2 \cos \Theta}{\sin \Theta} = \frac{3}{2}$

Now by applying componendo in above equation

2cosΘ+sinΘsinΘ=3+22 $\frac{2 \cos \Theta + \sin \Theta}{\sin \Theta} = \frac{3+2}{2}$

2cosΘ+sinΘsinΘ=52 $\frac{2 \cos \Theta + \sin \Theta}{\sin \Theta} = \frac{5}{2}$ ….(5)

We get,

4cosΘ–sinΘsinΘ 2cosΘ+sinΘsinΘ = 21 52 $\frac{\frac{4 \cos \Theta - \sin \Theta}{\sin \Theta}}{\frac{2 \cos \Theta + \sin \Theta}{\sin \Theta}} = \frac{\frac{2}{1}}{\frac{5}{2}}$

Therefore,

4cosΘ–sinΘsinΘ × sinΘ2cosΘ+sinΘ = 21 × 25 $\frac{4 \cos \Theta - \sin \Theta}{\sin \Theta} \times \frac{\sin \Theta}{2 \cos \Theta + \sin \Theta} = \frac{2}{1} \times \frac{2}{5}$

Therefore, on L.H.S sinΘ$\sin\Theta$ cancels and we get,

4cosΘ–sinΘ2cosΘ+sinΘ=21×25 $\frac{4\cos\Theta - \sin\Theta}{2\cos\Theta + \sin\Theta} = \frac{2}{1} \times \frac{2}{5}$

Therefore,

4cosΘ–sinΘ=4$4\cos\Theta - \sin\Theta = 4$

11.) If 3cotΘ=2$3\cot\Theta = 2$ **, find the value of 4sinΘ–3cosΘ2sinΘ+6cosΘ** $\frac{4\sin\Theta - 3\cos\Theta}{2\sin\Theta + 6\cos\Theta}$

Sol.

Given:

3cotΘ=2$3\cot\Theta = 2$

Therefore,

cotΘ=23$\cot\Theta = \frac{2}{3}$ …. (1)

Now, we know that cotΘ=cosΘsinΘ$\cot\Theta = \frac{\cos\Theta}{\sin\Theta}$

Therefore equation (1) becomes

cosΘsinΘ=23 $\frac{\cos\Theta}{\sin\Theta} = \frac{2}{3}$ ….(2)

Now , by applying invertendo to equation (2)

sinΘcosΘ=32 $\frac{\sin\Theta}{\cos\Theta} = \frac{3}{2}$ ….(3)

Now, multiplying by 43 $\frac{4}{3}$ on both sides,

We get,

43×sinΘcosΘ=43×32 $\frac{4}{3} \times \frac{\sin\Theta}{\cos\Theta} = \frac{4}{3} \times \frac{3}{2}$

Therefore, 3 cancels out on R.H.S and

We get,

4sinΘ3cosΘ=21 $\frac{4\sin\Theta}{3\cos\Theta} = \frac{2}{1}$

Now by applying invertendo dividendo in above equation

We get,

4sinΘ–3cosΘ3cosΘ=2–11 $\frac{4\sin\Theta - 3\cos\Theta}{3\cos\Theta} = \frac{2-1}{1}$

4sinΘ–3cosΘ3cosΘ = 11 $\frac{4\sin\Theta-3\cos\Theta}{3\cos\Theta}=\frac{1}{1}$ ....(4)

Now, multiplying by 26 $\frac{2}{6}$ on both sides of equation (3)

We get,

26 × sinΘcosΘ = 26 × 32 $\frac{2}{6}\times\frac{\sin\Theta}{\cos\Theta}=\frac{2}{6}\times\frac{3}{2}$

Therefore, 2 cancels out on R.H.S and

We get,

2sinΘ6cosΘ = 36 $\frac{2\sin\Theta}{6\cos\Theta}=\frac{3}{6}$ 2sinΘ6cosΘ = 12 $\frac{2\sin\Theta}{6\cos\Theta}=\frac{1}{2}$

Now by applying componendo in above equation

We get,

2cosΘ+6sinΘ6sinΘ = 1+22 $\frac{2\cos\Theta+6\sin\Theta}{6\sin\Theta}=\frac{1+2}{2}$

2cosΘ+6sinΘ6sinΘ = 32 $\frac{2\cos\Theta+6\sin\Theta}{6\sin\Theta}=\frac{3}{2}$ ....(5)

Now, by dividing equation (4) by (5)

We get,

4sinΘ–3cosΘ3sinΘ 2cosΘ+6sinΘ6sinΘ = 11 32 $\frac{\frac{4\sin\Theta-3\cos\Theta}{3\sin\Theta}}{\frac{2\cos\Theta+6\sin\Theta}{6\sin\Theta}}=\frac{\frac{1}{1}}{\frac{3}{2}}$

Therefore,

4sinΘ–3cosΘ3sinΘ × 6sinΘ2cosΘ+6sinΘ = 11 × 23 $\frac{4\sin\Theta-3\cos\Theta}{3\sin\Theta}\times\frac{6\sin\Theta}{2\cos\Theta+6\sin\Theta}=\frac{1}{1}\times\frac{2}{3}$ 4sinΘ–3cosΘ3sinΘ × 2×3sinΘ2cosΘ+6sinΘ = 11 × 23 $\frac{4\sin\Theta-3\cos\Theta}{3\sin\Theta}\times\frac{2\times3\sin\Theta}{2\cos\Theta+6\sin\Theta}=\frac{1}{1}\times\frac{2}{3}$

Therefore, on L.H.S (3 sinΘ$\sin\Theta$) cancels out and we get,

2×4sinΘ–3cosΘ2cosΘ+6sinΘ = 11 × 23 $\frac{2\times4\sin\Theta-3\cos\Theta}{2\cos\Theta+6\sin\Theta}=\frac{1}{1}\times\frac{2}{3}$

Now, by taking 2 in the numerator of L.H.S on the R.H.S

We get,

4sinΘ–3cosΘ2cosΘ+6sinΘ = 23×2 $\frac{4\sin\Theta-3\cos\Theta}{2\cos\Theta+6\sin\Theta}=\frac{2}{3\times2}$

Therefore, 2 cancels out on R.H.S and

We get,

4sinΘ–3cosΘ2cosΘ+6sinΘ = 13 $\frac{4\sin\Theta-3\cos\Theta}{2\cos\Theta+6\sin\Theta}=\frac{1}{3}$

Hence answer,

4sinΘ–3cosΘ2cosΘ+6sinΘ=13 $\frac{4\sin\Theta-3\cos\Theta}{2\cos\Theta+6\sin\Theta} = \frac{1}{3}$

12.) If tanΘ=ab $\tan\Theta = \frac{a}{b}$, prove that asinΘ–bcosΘasinΘ+bcosΘ=a²–b²a²+b²

$\frac{a\sin\Theta-b\cos\Theta}{a\sin\Theta+b\cos\Theta} = \frac{a^2-b^2}{a^2+b^2}$

Sol.

Given:

tanΘ ab $\tan\Theta \frac{a}{b}$ …. (1)

Now, we know that tanΘ= sinΘcosΘ $\tan\Theta = \frac{\sin\Theta}{\cos\Theta}$

Therefore equation (1) becomes

sinΘcosΘ=ab $\frac{\sin\Theta}{\cos\Theta} = \frac{a}{b}$ ….(2)

Now, by multiplying by ab $\frac{a}{b}$ on both sides of equation (2)

We get,

ab × sinΘcosΘ=ab × ab $\frac{a}{b} \times \frac{\sin\Theta}{\cos\Theta} = \frac{a}{b} \times \frac{a}{b}$

Therefore,

asinΘbcosΘ=a²b² $\frac{a\sin\Theta}{b\cos\Theta} = \frac{a^2}{b^2}$ ….(3)

Now by applying dividendo in above equation (3)

We get,

asinΘ–bcosΘbcosΘ=a²–b²b² $\frac{a\sin\Theta-b\cos\Theta}{b\cos\Theta} = \frac{a^2-b^2}{b^2}$ ….(4)

Now by applying componendo in equation (3)

We get,

asinΘ+bcosΘbcosΘ=a²+b²b² $\frac{a\sin\Theta+b\cos\Theta}{b\cos\Theta} = \frac{a^2+b^2}{b^2}$ ….(5)

Now, by dividing equation (4) by equation (5)

We get,

asinΘ–bcosΘbcosΘ asinΘ+bcosΘbcosΘ = a²–b²b² a²+b²b² $\frac{\frac{a\sin\Theta-b\cos\Theta}{b\cos\Theta}}{\frac{a\sin\Theta+b\cos\Theta}{b\cos\Theta}} = \frac{\frac{a^2-b^2}{b^2}}{\frac{a^2+b^2}{b^2}}$

Therefore,

$$\frac{a \sin \Theta - b \cos \Theta}{b \cos \Theta} \times \frac{b \cos \Theta}{a \sin \Theta + b \cos \Theta} = \frac{a^2 - b^2}{b^2} \times \frac{b^2}{a^2 + b^2}$$

Therefore, $b \cos \Theta$ and b^2 cancels on L.H.S and R.H.S respectively

$$\frac{a \sin \Theta - b \cos \Theta}{a \sin \Theta + b \cos \Theta} = \frac{a^2 - b^2}{a^2 + b^2}$$

Hence, it is proved that

$$\frac{a \sin \Theta - b \cos \Theta}{a \sin \Theta + b \cos \Theta} = \frac{a^2 - b^2}{a^2 + b^2}$$

13.) If $\sec \Theta = \frac{13}{5}$, show that $\frac{2 \sin \Theta - 3 \cos \Theta}{4 \sin \Theta - 9 \cos \Theta} = 3$

Sol.

Given:

$\sec \Theta = \frac{13}{5}$

To show that $\frac{2 \sin \Theta - 3 \cos \Theta}{4 \sin \Theta - 9 \cos \Theta} = 3$

Now, we know that $\cos \Theta = \frac{1}{\sec \Theta}$

Therefore,

$\cos \Theta = \frac{1}{\frac{13}{5}}$

Therefore,

$\cos \Theta = \frac{5}{13}$ .... (1)

Now, we know that

$\cos \Theta = \frac{\text{Base side adjacent to } \angle\Theta}{\text{Hypotenuse}}$

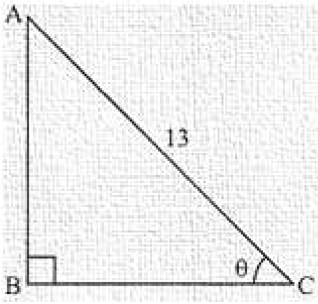
Now, by comparing equation (1) and(2)

We get,

Base side adjacent to $\angle\Theta$ = 5

And

Hypotenuse =13



Therefore from above figure

Base side BC = 5

Hypotenuse AC = 13

Side AB is unknown. It can be determined by using Pythagoras theorem

Therefore by applying Pythagoras theorem

We get,

$AC^2 = AB^2 + BC^2$

Therefore by substituting the values of known sides

We get,

$13^2 = AB^2 + 5^2$

Therefore,

$AB^2 = 13^2 - 5^2$

$AB^2 = 169 - 25$

$AB^2 = 144$

$AB = \sqrt{144}$

Therefore,

AB = 12 …. (3)

Now, we know that

$\sin\Theta = \frac{AB}{AC}$

$\sin\Theta = \frac{12}{13}$ …. (4)

Now L.H.S of the equation to be proved is as follows

L.H.S = $\frac{2\sin\Theta - 3\tan\Theta}{4\sin\Theta - 3\cos\Theta}$

Substituting the value $\cos\Theta$ of $\sin\Theta$ and from equation (1) and (4) respectively

We get,

$$\frac{2\times\frac{12}{13}-3\times\frac{5}{13}}{4\times\frac{12}{13}-9\times\frac{5}{13}}$$

Therefore,

L.H.S = $\frac{2\times12-3\times5}{4\times12-9\times5}$

L.H.S = $\frac{24-15}{48-45}$

L.H.S= $\frac{9}{3}$

L.H.S= 3

Hence proved that,

$\frac{2\sin\Theta-3\tan\Theta}{4\sin\Theta-3\cos\Theta}$ = 3

14.) If $\cos\Theta=\frac{12}{13}$, show that $\sin\Theta(1-\tan\Theta)=\frac{35}{156}$

Sol.

Given: $\cos\Theta=\frac{12}{13}$ …. (1)

To show that $\sin\Theta(1-\tan\Theta)=\frac{35}{156}$

Now we know that $\cos\Theta=\frac{\text{Base side adjacent to }\angle\Theta}{\text{Hypotenuse}}$ ….(2)

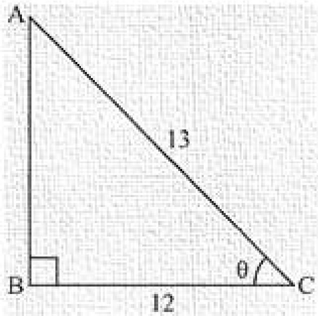
Therefore, by comparing equation (1) and (2)

We get,

Base side adjacent to $\angle\Theta$ = 12

And

Hypotenuse = 13



Therefore from above figure

Base side BC= 12

Hypotenuse AC= 13

Side AB is unknown and it can be determined by using Pythagoras theorem

Therefore by applying Pythagoras theorem

We get,

$AC^2 = AB^2 + BC^2$

Therefore by substituting the values of known sides

We get,

$13^2 = AB^2 + 12^2$

Therefore,

$AB^2 = 13^2 – 12^2$

$AB^2 = 169 – 144$

AB =25

AB= $\sqrt{25}\sqrt{25}$

Therefore,

AB = 5 …. (3)

Now, we know that

sinΘ= Perpendicularsideoppositeto∠ΘHypotenuse $\sin\Theta = \frac{\text{Perpendicular side opposite to } \angle\Theta}{\text{Hypotenuse}}$

Now from figure (a)

We get,

sinΘ= ABAC $\sin\Theta = \frac{AB}{AC}$

Therefore,

sinΘ=512 $\sin\Theta = \frac{5}{12}$ …. (5)

Now L.H.S of the equation to be proved is as follows

L.H.S of the equation to be proved is as follows

L.H.S = sinΘ(1–tanΘ] $\sin\Theta(1-\tan\Theta]$ …. (6)

Substituting the value of sinΘ $\sin\Theta$ and tanΘ $\tan\Theta$ from equation (4) and (5)

We get,

L.H.S = 513 (1− 512) $\frac{5}{13}(1-\frac{5}{12})$

Taking L.C.M inside the bracket

We get,

L.H.S= 513 (1×121×12 − 512) $\frac{5}{13}(\frac{1\times12}{1\times12}-\frac{5}{12})$

Therefore,

L.H.S= 513 (12–512) $\frac{5}{13}(\frac{12-5}{12})$

L.H.S = 513 (712) $\frac{5}{13}(\frac{7}{12})$

Now by opening the bracket and simplifying

We get,

L.H.S = 5×713×12 $\frac{5\times7}{13\times12}$

L.H.S= 35136 $\frac{35}{136}$

From equation (6) and (7) ,it can be shown that

that sinΘ(1–tanΘ) $\sin\Theta(1-\tan\Theta)$ **= 35136** $\frac{35}{136}$

15.) If cotΘ= 1√3 $\cot\Theta = \frac{1}{\sqrt{3}}$ **, show that 1–cos²Θ2–sin²Θ = 35** $\frac{1-\cos^2\Theta}{2-\sin^2\Theta} = \frac{3}{5}$

Sol.

Given: cotΘ= 1√3 $\cot\Theta = \frac{1}{\sqrt{3}}$ …. (1)

To show that 1–cos²Θ2–sin²Θ = 35 $\frac{1-\cos^2\Theta}{2-\sin^2\Theta} = \frac{3}{5}$

Now, we know that cotΘ= 1tanΘ $\cot\Theta = \frac{1}{\tan\Theta}$

Since tanΘ= Perpendicularsideoppositeto∠ΘBasesideadjacentto∠Θ $\tan\Theta = \frac{\text{Perpendicular side opposite to } \angle\Theta}{\text{Base side adjacent to } \angle\Theta}$

Therefore,

$$\tan\Theta = \frac{1}{\frac{\text{Perpendicular side opposite to } \angle\Theta}{\text{Base side adjacent to } \angle\Theta}}$$

Therefore,

$$\cot\Theta = \frac{\text{Base side adjacent to } \angle\Theta}{\text{Perpendicular side opposite to } \angle\Theta} \quad \text{.... (2)}$$

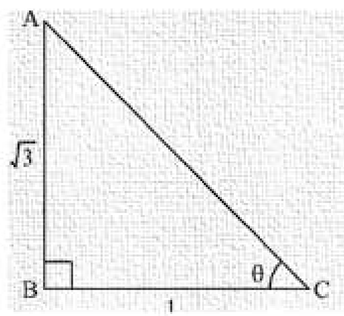
Comparing Equation (1) and (2)

We get.

Base side adjacent to $\angle\Theta$ = 1

Perpendicular side opposite to $\angle\Theta$ = $\sqrt{3}$

Therefore, triangle representing angle $\sqrt{3}$ is as shown below



Therefore, by substituting the values of known sides

We get,

$AC^2 = (\sqrt{3})^2 + 1^2$

Therefore,

$AC^2 = 3 + 1$

$AC^2 = 4$

$AC = \sqrt{4}$

Therefore,

AC = 2 .... (3)

Now, we know that

$$\sin\Theta = \frac{\text{Perpendicular side opposite to } \angle\Theta}{\text{Hypotenuse}}$$

Now from figure (a)

sinΘ=ABAC $\sin\Theta = \frac{AB}{AC}$

Therefore from figure (a) and equation (3),

sinΘ=√32 $\sin\Theta = \frac{\sqrt{3}}{2}$

Now we know that

cosΘ Basesideadjacentto∠ΘHypotenuse $\cos\Theta \frac{\text{Base side adjacent to } \angle\Theta}{\text{Hypotenuse}}$

Now from figure (a)

We get,

BCAC $\frac{BC}{AC}$

Therefore from figure (a) and equation (3),

cosΘ=12 $\cos\Theta = \frac{1}{2}$ …. (5)

Now, L.H.S of the equation to be proved is as follows

L.H.S = 1–cos²Θ2–sin²Θ $\frac{1-\cos^2\Theta}{2-\sin^2\Theta}$

Substituting the value of from equation (4) and (5)

We get,

L.H.S = 1–(12)²2–(√32)² $\frac{1-(\frac{1}{2})^2}{2-(\frac{\sqrt{3}}{2})^2}$

L.H.S = 1–142–34 $\frac{1-\frac{1}{4}}{2-\frac{3}{4}}$

Now by taking L.C.M in numerator as well as denominator

We get,

L.H.S= (4×1)–14 (4×2)–34 $\frac{\frac{(4\times1)-1}{4}}{\frac{(4\times2)-3}{4}}$

Therefore,

L.H.S = 4–14 8–34 $\frac{\frac{4-1}{4}}{\frac{8-3}{4}}$

L.H.S = 34 × 45 $\frac{3}{4} \times \frac{4}{5}$

L.H.S = 35 $\frac{3}{5}$ = R.H.S

Therefore,

$$\frac{1-\cos^2\Theta}{2-\sin^2\Theta} = \frac{3}{5}$$

16.) If $\tan\Theta = \frac{1}{\sqrt{7}}$ **, then show that** $\frac{\operatorname{cosec}^2\Theta - \sec^2\Theta}{\operatorname{cosec}^2\Theta + \sec^2\Theta} = \frac{3}{4}$

Sol.

Given: $\tan\Theta = \frac{1}{\sqrt{7}}$ …. (1)

To show that $\frac{\operatorname{cosec}^2\Theta - \sec^2\Theta}{\operatorname{cosec}^2\Theta + \sec^2\Theta} = \frac{3}{4}$

Now, we know that

Since, $\tan\Theta = \frac{\text{Perpendicular side oposite to } \angle\Theta}{\text{Base side adjacent to } \angle\Theta}$ …. (2)

Therefore,

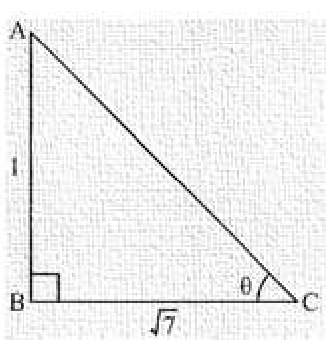
Comparing equation (1) and (2)

We get.

Perpendicular side opposite to $\angle\Theta$ = 1

Base side adjacent to $\angle\Theta$ = $\sqrt{7}$

Therefore, Triangle representing $\angle\Theta$ is shown below



Hypotenuse AC is unknown and it can be found by using Pythagoras theorem

Therefore by applying Pythagoras theorem

We get,

$AC^2 = AB^2 + BC^2$

Therefore by substituting the values of known sides

We get,

$AC^2 = (1)^2 + (\sqrt{7})^2(\sqrt{7})^2$

Therefore,

$AC^2 = 1 + 7$

$AC^2 = 8$

$AC = \sqrt{8}\sqrt{8}$

$AC = \sqrt{2\times2\times2}\sqrt{2\times2\times2}$

Therefore,

$AC = 2\sqrt{2}2\sqrt{2}$ …. (3)

Now we know that

$\sin\Theta = \text{Perpendicularsideoppositeto}\angle\Theta\text{Hypotenuse} \sin\Theta = \frac{\text{Perpendicular side opposite to }\angle\Theta}{\text{Hypotenuse}}$ $\sin\Theta = ABAC$ $\sin\Theta = \frac{AB}{AC}$

$\sin\Theta = 12\sqrt{2} \sin\Theta = \frac{1}{2\sqrt{2}}$ …. (4)

Now, we know that cosec $\Theta = 1\sin\Theta\, \Theta = \frac{1}{\sin\Theta}$

Therefore, from equation (4)

We get,

cosec $\Theta = 2\sqrt{2}\Theta = 2\sqrt{2}$ …. (5)

Now, we know that

$\cos\Theta = \text{Basesideadjacentto}\angle\Theta\text{Hypotenuse} \cos\Theta = \frac{\text{Base side adjacent to }\angle\Theta}{\text{Hypotenuse}}$

Now from figure (a)

We get,

$\cos\Theta = BCAC \cos\Theta = \frac{BC}{AC}$

Therefore from figure (a) and equation (3)

$\cos\Theta = \sqrt{7}2\sqrt{2} \cos\Theta = \frac{\sqrt{7}}{2\sqrt{2}}$ …. (6)

Now we know that $\sec\Theta = 1\cos\Theta \sec\Theta = \frac{1}{\cos\Theta}$

Therefore, from equation (6)

We get,

secΘ= 1√72√2 $\sec\Theta = \frac{1}{\frac{\sqrt{7}}{2\sqrt{2}}}$

secΘ= 2√2√7 $\sec\Theta = \frac{2\sqrt{2}}{\sqrt{7}}$ …. (7)

Now, L.H.S of the equation to be proved is as follows

L.H.S = cosec²Θ–sec²Θcosec²Θ+sec²Θ $\frac{\mathrm{cosec}^2\Theta - \sec^2\Theta}{\mathrm{cosec}^2\Theta + \sec^2\Theta}$

Substituting the value of cosecΘ Θ and secΘ $\sec\Theta$ from equation (6) and (7)

We get,

L.H.S = [(2√2)]²–(2√2√7)²[(2√2)]²+(2√2√7)² $\frac{[(2\sqrt{2})]^2 - (\frac{2\sqrt{2}}{\sqrt{7}})^2}{[(2\sqrt{2})]^2 + (\frac{2\sqrt{2}}{\sqrt{7}})^2}$

L.H.S= (8)–(87)(8)+(87) $\frac{(8)-(\frac{8}{7})}{(8)+(\frac{8}{7})}$

Therefore,

56–87 56+87 $\frac{\frac{56-8}{7}}{\frac{56+8}{7}}$

L.H.S = 487 647 $\frac{\frac{48}{7}}{\frac{64}{7}}$

Therefore,

L.H.S = 4864 $\frac{48}{64}$

L.H.S = 34 $\frac{3}{4}$ = R.H.S

Therefore,

cosec²Θ–sec²Θcosec²Θ+sec²Θ $\frac{\mathrm{cosec}^2\Theta - \sec^2\Theta}{\mathrm{cosec}^2\Theta + \sec^2\Theta}$ = 34 $\frac{3}{4}$

Hence proved that

cosec²Θ–sec²Θcosec²Θ+sec²Θ $\frac{\mathrm{cosec}^2\Theta - \sec^2\Theta}{\mathrm{cosec}^2\Theta + \sec^2\Theta}$ **= 34** $\frac{3}{4}$

17.) If secΘ=54 $\sec\Theta = \frac{5}{4}$ **,find the value of sinΘ–2cosΘtanΘ–cotΘ** $\frac{\sin\Theta - 2\cos\Theta}{\tan\Theta - \cot\Theta}$

Sol.

Given: secΘ=54 $\sec\Theta = \frac{5}{4}$ .... (1)

To find the value of sinΘ–2cosΘtanΘ–cotΘ $\frac{\sin\Theta - 2\cos\Theta}{\tan\Theta - \cot\Theta}$

Now we know that secΘ=1cosΘ $\sec\Theta = \frac{1}{\cos\Theta}$

Therefore,

cosΘ=1secΘ $\cos\Theta = \frac{1}{\sec\Theta}$

Therefore from equation (1)

cosΘ=15 $\cos\Theta = \frac{1}{5}$

cosΘ=45 $\cos\Theta = \frac{4}{5}$ .... (2)

Also, we know that cos2Θ+sin2Θ=1 $\cos^2\Theta + \sin^2\Theta = 1$

Therefore,

sin2Θ=1–cos2Θ $\sin^2\Theta = 1 - \cos^2\Theta$ sinΘ=√1–cos2Θ $\sin\Theta = \sqrt{1 - \cos^2\Theta}$

Substituting the value of cosΘ $\cos\Theta$ from equation (2)

We get,

sinΘ=√1−(45)2 $\sin\Theta = \sqrt{1 - (\frac{4}{5})^2}$

= √1−1625 $\sqrt{1 - \frac{16}{25}}$

= 925 $\frac{9}{25}$

= 35 $\frac{3}{5}$

Therefore,

sinΘ=35 $\sin\Theta = \frac{3}{5}$ .... (3)

Also, we know that

sec2Θ=1+tan2Θ $\sec^2\Theta = 1 + \tan^2\Theta$

Therefore,

tan2Θ=(54)2−1 $\tan^2\Theta = (\frac{5}{4})^2 - 1$ tanΘ=(√916) $\tan\Theta = (\sqrt{\frac{9}{16}})$

Therefore,

$\tan\Theta = \frac{3}{4}$ .... (4)

Also, $\cot\Theta = \frac{1}{\tan\Theta}$

Therefore from equation (4)

We get,

$\cot\Theta = \frac{4}{3}$ .... (5)

Substituting the value of $\cos\Theta$, $\cot\Theta$ and $\tan\Theta$ from the equation (2),(3),(4) and (5) respectively in the expression below

$\frac{\sin\Theta - 2\cos\Theta}{\tan\Theta - \cot\Theta}$

We get,

$\frac{\sin\Theta - 2\cos\Theta}{\tan\Theta - \cot\Theta} = \frac{\frac{3}{5} - 2\left(\frac{4}{5}\right)}{\frac{3}{4} - \frac{4}{3}}$

$= \frac{12}{7}$

Therefore, $\frac{\sin\Theta - 2\cos\Theta}{\tan\Theta - \cot\Theta} = \frac{12}{7}$

18.) If $\sin\Theta = \frac{12}{13}$, find the value of $\frac{2\sin\Theta\cos\Theta}{\cos^2\Theta - \sin^2\Theta}$

Sol.

Given: $\sin\Theta = \frac{12}{13}$ .... (1)

To, find the value of $\frac{2\sin\Theta\cos\Theta}{\cos^2\Theta - \sin^2\Theta}$

Now, we know the following trigonometric identity

$\operatorname{cosec}^2\Theta = 1 + \tan^2\Theta$

Therefore, by substituting the value of $\tan\Theta$ from equation (1)

We get,

$\operatorname{cosec}^2\Theta = 1 + \left(\frac{12}{13}\right)^2$

$= 1 + \frac{12^2}{13^2}$

$= 1 + \frac{144}{169}$

By taking L.C.M on the R.H.S

We get,

cosec^{2}Θ= 169+144169 $\Theta = \frac{169+144}{169}$

= 313169 $\frac{313}{169}$

Therefore

cosecΘ=$\sqrt{313169}$ $\Theta = \sqrt{\frac{313}{169}}$

=Θ= $\sqrt{313}$13 $\Theta = \frac{\sqrt{313}}{13}$

Therefore

cosecΘΘ = Θ= $\sqrt{313}$13 $\Theta = \frac{\sqrt{313}}{13}$ .... (2)

Now, we know that

cosecΘ$\operatorname{cosec}\Theta$ = 1sinΘ $\frac{1}{\sin\Theta}$

sinΘ= 1 $\sqrt{313}$13 $\sin\Theta = \frac{1}{\frac{\sqrt{313}}{13}}$

Therefore

sinΘ= 13$\sqrt{313}$ $\sin\Theta = \frac{13}{\sqrt{313}}$ .... (3)

Now, we know the following trigonometric identity

cos^{2}Θ+sin^{2}Θ=1$\cos^2\Theta + \sin^2\Theta = 1$

Therefore,

cos^{2}Θ=1–sin^{2}Θ$\cos^2\Theta = 1 - \sin^2\Theta$

Now by substituting the value of sinΘ$\sin\Theta$ from equation (3)

We get,

cos^{2}Θ=1–(13$\sqrt{313}$)^2 $\cos^2\Theta = 1 - \left(\frac{13}{\sqrt{313}}\right)^2$

=1– 169313 $1 - \frac{169}{313}$

Therefore, by taking L.C.M on R.H.S

We get,

$\cos^2\Theta=$ 144313 $\cos^2\Theta = \frac{144}{313}$

Now, by taking square root on both sides

We get,

$\cos\Theta=$ $12\sqrt{313}$ $\cos\Theta = \frac{12}{\sqrt{313}}$

Therefore,

$\cos\Theta=$ $12\sqrt{313}$ $\cos\Theta = \frac{12}{\sqrt{313}}$ …. (4)

Substituting the value of sinΘ $\sin\Theta$ and cosΘ $\cos\Theta$ from equation (3) and (4) respectively in the equation below

2sinΘcosΘcos^{2}Θ–sin^{2}Θ $\frac{2\sin\Theta\cos\Theta}{\cos^2\Theta-\sin^2\Theta}$

Therefore,

2sinΘcosΘcos^{2}Θ–sin^{2}Θ $\frac{2\sin\Theta\cos\Theta}{\cos^2\Theta-\sin^2\Theta}$ = 2× $13\sqrt{313}$ × $12\sqrt{313}$ $(13\sqrt{313})^2-(12\sqrt{313})^2$ $\frac{2\times\frac{13}{\sqrt{313}}\times\frac{12}{\sqrt{313}}}{(\frac{13}{\sqrt{313}})^2-(\frac{12}{\sqrt{313}})^2}$

= 312313 25313 $\frac{\frac{312}{313}}{\frac{25}{313}}$

31225 $\frac{312}{25}$

Therefore

2sinΘcosΘcos^{2}Θ–sin^{2}Θ $\frac{2\sin\Theta\cos\Theta}{\cos^2\Theta-\sin^2\Theta}$ =

31225 $\frac{312}{25}$

19.) If cosΘ= 35 $\cos\Theta = \frac{3}{5}$, find the value of sinΘ–1tanΘ2tanΘ $\frac{\sin\Theta-\frac{1}{\tan\Theta}}{2\tan\Theta}$

Sol.

Given: cosΘ= 35 $\cos\Theta = \frac{3}{5}$ …. (1)

To find the value of sinΘ–1tanΘ2tanΘ $\frac{\sin\Theta-\frac{1}{\tan\Theta}}{2\tan\Theta}$

Now we know the following trigonometric identity

cos^{2}Θ+sin^{2}Θ=1 $\cos^2\Theta+\sin^2\Theta = 1$

Therefore by substituting the value of cosΘ $\cos\Theta$ from equation (1)

We get,

(35)2+sin2Θ=1 $(\frac{3}{5})^2 + \sin^2\Theta = 1$

Therefore,

sin2Θ=1–(35)2 $\sin^2\Theta = 1-(\frac{3}{5})^2$ sin2Θ=1–(925) $\sin^2\Theta = 1-(\frac{9}{25})$ sin2Θ= 25–925 $\sin^2\Theta = \frac{25-9}{25}$ sin2Θ= 1625 $\sin^2\Theta = \frac{16}{25}$

Therefore by taking square root on both sides

We get,

sinΘ= 45 $\sin\Theta = \frac{4}{5}$ …. (2)

Now, we know that

tanΘ= sinΘcosΘ $\tan\Theta = \frac{\sin\Theta}{\cos\Theta}$

Therefore by substituting the value of sinΘ $\sin\Theta$ and cosΘ $\cos\Theta$ from equation (2) and (1) respectively

We get,

tanΘ= 45 35 =43 $\tan\Theta = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$ …. (4)

Now, by substituting the value of sinΘ $\sin\Theta$ and of tanΘ $\tan\Theta$ from equation (2) and equation (4) respectively in the expression below

sinΘ–1tanΘ2tanΘ $\frac{\sin\Theta - \frac{1}{\tan\Theta}}{2\tan\Theta}$

We get,

sinΘ–1tanΘ2tanΘ $\frac{\sin\Theta - \frac{1}{\tan\Theta}}{2\tan\Theta}$ = 45–142×43 $\frac{\frac{4}{5}-\frac{1}{4}}{2\times\frac{4}{3}}$

sinΘ–1tanΘ2tanΘ $\frac{\sin\Theta - \frac{1}{\tan\Theta}}{2\tan\Theta}$ = 1620–1520 83 $\frac{\frac{16}{20}-\frac{15}{20}}{\frac{8}{3}}$

sinΘ–1tanΘ2tanΘ $\frac{\sin\Theta - \frac{1}{\tan\Theta}}{2\tan\Theta}$ = 3160 $\frac{3}{160}$

Therefore,

sinΘ–1tanΘ2tanΘ $\frac{\sin\Theta - \frac{1}{\tan\Theta}}{2\tan\Theta}$ **= 3160** $\frac{3}{160}$

20.) If sinΘ=35 $\sin\Theta = \frac{3}{5}$ **, find the value of cosΘ–1tanΘ2cotΘ** $\frac{\cos\Theta - \frac{1}{\tan\Theta}}{2\cot\Theta}$

Sol.

Given:

$\sin\Theta = \frac{3}{5}$ …. (1)

To find the value of $\frac{\cos\Theta - \frac{1}{\tan\Theta}}{2\cot\Theta}$

Now, we know the following trigonometric identity

$\cos^2\Theta + \sin^2\Theta = 1$

Therefore by substituting the value of $\cos\Theta$ from equation (1)

We get,

$\cos^2\Theta + (\frac{3}{5})^2 = 1$

Therefore,

$\cos^2\Theta = 1 - (\frac{3}{5})^2$ $\cos^2\Theta = 1 - \frac{9}{25}$

Now by taking L.C.M

We get,

$\cos^2\Theta = \frac{25-9}{25}$ $\cos^2\Theta = \frac{25-9}{25}$

Therefore, by taking square roots on both sides

We get,

$\cos\Theta = \frac{4}{5}$

Therefore,

$\cos\Theta = \frac{4}{5}$ …. (2)

Now we know that

$\tan\Theta = \frac{\sin\Theta}{\cos\Theta}$

Therefore by substituting the value of $\sin\Theta$ and $\cos\Theta$ from equation (1) and (2) respectively

We get,

$\tan\Theta = \frac{\frac{3}{5}}{\frac{4}{5}}$

$\tan\Theta = \frac{3}{4}$ …. (3)

Also, we know that

$$\cot\Theta = \frac{1}{\tan\Theta}$$

Therefore from equation (3)

We get,

$$\cot\Theta = \frac{1}{\frac{3}{4}}$$

$$\cot\Theta = \frac{4}{3} \quad \ldots. (4)$$

Now by substituting the value of $\cos\Theta$, $\tan\Theta$ and $\cot\Theta$ from equation (2) ,(3) and (4) respectively from the expression below

$$\frac{\cos\Theta - \frac{1}{\tan\Theta}}{2\cot\Theta}$$

We get,

$$\frac{\cos\Theta - \frac{1}{\tan\Theta}}{2\cot\Theta} = \frac{\frac{4}{5} - \frac{1}{3}}{2 \times \frac{4}{3}}$$

$$\frac{\cos\Theta - \frac{1}{\tan\Theta}}{2\cot\Theta} = \frac{\frac{12}{15} - \frac{20}{15}}{\frac{8}{3}}$$

$$= \frac{\frac{-8}{15}}{\frac{8}{3}}$$

$$= \frac{-1}{5}$$

Therefore, $\frac{\cos\Theta - \frac{1}{\tan\Theta}}{2\cot\Theta} = \frac{-1}{5}$

21.) If $\tan\Theta = \frac{24}{7}$, find that $\sin\Theta + \cos\Theta$

Sol.

Given:

$$\tan\Theta = \frac{24}{7} \quad \ldots. (1)$$

To find,

$$\sin\Theta + \cos\Theta$$

Now we know that $\tan\Theta$ is defined as follows

$$\tan\Theta = \frac{\text{Perpendicular side opposite to } \angle\Theta}{\text{Base side adjacent to } \angle\Theta} \quad \ldots. (2)$$

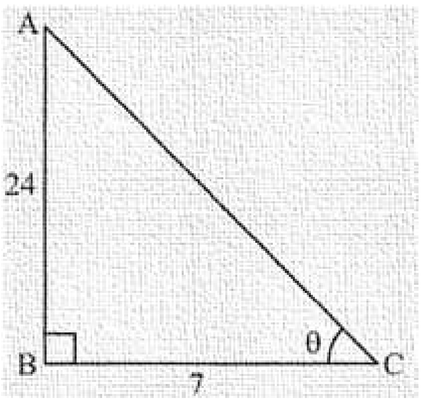
Now by comparing equation (1) and (2)

We get,

Perpendicular side opposite to $\angle\Theta\angle\Theta$ = 24

Base side adjacent to $\angle\Theta\angle\Theta$ = 7

Therefore triangle representing $\angle\Theta\angle\Theta$ is as shown below



Side AC is unknown and can be found by using Pythagoras theorem

Therefore,

$AC^2 = AB^2 + BC^2$

Now by substituting the value of unknown sides from figure

We get,

$AC^2 = 24^2 + 7^2$

AC = 576 + 49

AC= 625

Now by taking square root on both sides,

We get,

AC = 25

Therefore H hy

Hypotenuse side AC = 25 …. (3)

Now we know $\sin\Theta \sin\Theta$ is defined as follows

$\sin\Theta =$ Perpendicularsideoppositeto∠ΘHypotenuse $\sin\Theta = \frac{\text{Perpendicular side opposite to } \angle\Theta}{\text{Hypotenuse}}$

Therefore from figure (a) and equation (3)

We get,

$\sin\Theta = \frac{AB}{AC}$

$\sin\Theta = \frac{24}{25}$ …. (4)

Now we know that $\cos\Theta$ is defined as follows

$\cos\Theta = \frac{\text{Base side adjacent to } \angle\Theta}{\text{Hypotenuse}}$

Therefore by substituting the value of $\sin\Theta$ and $\cos\Theta$ from equation (4) and (5) respectively, we get

$\sin\Theta + \cos\Theta = \frac{24}{25} + \frac{7}{25}$

$\sin\Theta + \cos\Theta = \frac{31}{25}$

Hence, $\sin\Theta + \cos\Theta = \frac{31}{25}$

22.) If $\sin\Theta = \frac{a}{b}$, find $\sec\Theta + \tan\Theta$ in terms of a and b.

Sol.

Given:

$\sin\Theta = \frac{a}{b}$ …. (1)

To find: $\sec\Theta + \tan\Theta$

Now we know, $\sin\Theta$ is defined as follows

$\sin\Theta = \frac{\text{Perpendicular side opposite to } \angle\Theta}{\text{Hypotenuse}}$ …. (2)

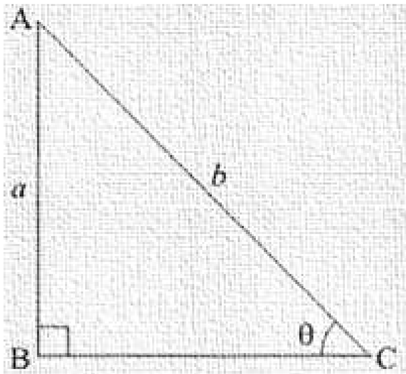
Now by comparing equation (1) and (2)

We get,

Perpendicular side opposite to $\angle\Theta$ = a

Hypotenuse = b

Therefore triangle representing $\angle\Theta$ is as shown below



Hence side BC is unknown

Now we find BC by applying Pythagoras theorem to right angled ΔABC

Therefore,

$AC^2 = AB^2 + BC^2$

Now by substituting the value of sides AB and AC from figure (a)

We get,

$b^2 = a^2 + BC^2$

Therefore,

$BC^2 = b^2 - a^2$

Now by taking square root on both sides

We get,

$BC = \sqrt{b^2 - a^2}$

Therefore,

Base side BC = $\sqrt{b^2 - a^2}$ …. (3)

Now we know $\cos\Theta$ is defined as follows

$\cos\Theta = \frac{\text{Base side adjacent to } \angle\Theta}{\text{Hypotenuse}}$

Therefore from figure (a) and equation (3)

We get,

$\cos\Theta = \frac{BC}{AC}$

$= \frac{\sqrt{b^2 - a^2}}{b}$

$\cos\Theta = \frac{BC}{AC}$

$$= \frac{\sqrt{b^2-a^2}}{b} \quad .... (4)$$

Now we know, $\sec\Theta = \frac{1}{\cos\Theta}$

Therefore,

$$\sec\Theta = \frac{b}{\sqrt{b^2-a^2}} \quad .... (5)$$

Now we know, $\tan\Theta = \frac{\sin\Theta}{\cos\Theta}$

Now by substituting the values from equation (1) and (3)

We get,

$$\tan\Theta = \frac{\frac{a}{b}}{\frac{\sqrt{b^2-a^2}}{b}} \qquad \tan\Theta = \frac{a}{\sqrt{b^2-a^2}}$$

Therefore,

$$\tan\Theta = \frac{a}{\sqrt{b^2-a^2}} \quad .... (6)$$

Now we need to find $\sec\Theta + \tan\Theta$

Now by substituting the values of $\sec\Theta$ and $\tan\Theta$ from equation (5) and (6) respectively

We get,

$$\sec\Theta + \tan\Theta = \frac{b}{\sqrt{b^2-a^2}} + \frac{a}{\sqrt{b^2-a^2}}$$

$$\sec\Theta + \tan\Theta = \frac{b+a}{\sqrt{b^2-a^2}} \quad .... (7)$$

We get,

$$\sec\Theta + \tan\Theta = \frac{b+a}{\sqrt{b+a}-\sqrt{b-a}}$$

Now by substituting the value in above expression

We get,

$$\sec\Theta + \tan\Theta = \frac{\sqrt{b+a}\times\sqrt{b+a}}{\sqrt{b+a}-\sqrt{b-a}}$$

Now, $\sqrt{b+a}$ present in the numerator as well as denominator of above denominator of above expression gets cancels we get,

$$\sec\Theta + \tan\Theta = \frac{\sqrt{b+a}}{\sqrt{b-a}}$$

Square root is present in the numerator as well as denominator of above expression

Therefore we can place both numerator and denominator under a common square root sign

Therefore, $\sec\Theta+\tan\Theta=\frac{\sqrt{b+a}}{\sqrt{b-a}}$

23.) If $8\tan A=15$ **, find** $\sin A-\cos A$

Sol.

Given:

$8\tan A=15$

Therefore,

$\tan A=\frac{15}{8}$ .... (1)

To find:

$\sin A-\cos A$

Now we know tan A is defined as follows

$\tan A=\frac{\text{Perpendicular side opposite to }\angle A}{\text{Base side adjacent to }\angle A}$ .... (2)

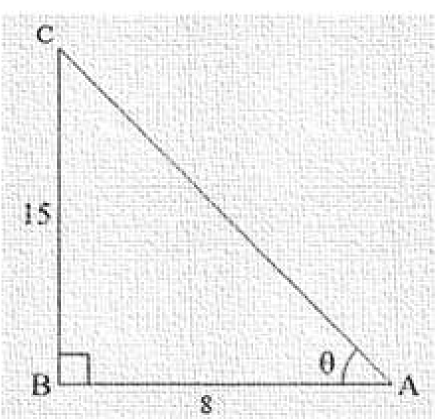
Now by comparing equation (1) and (2)

We get

Perpendicular side opposite to $\angle A$ = 15

Base side adjacent to $\angle A$ = 8

Therefore triangle representing angle A is as shown below



Side AC= is unknown and can be found by using Pythagoras theorem

Therefore,

$AC^2= AB^2 + BC^2$

Now by substituting the value of known sides from figure (a)

We get,

$AC^2 = 15^2 + 8^2$

$AC^2 = 225 + 64$

AC = 289

Now by taking square root on both sides

We get,

AC = √289 $\sqrt{289}$

AC = 17

Therefore Hypotenuse side AC=17 …. (3)

Now we know, sin A is defined as follows

sinA=Perpendicularsideoppositeto∠AHypotenuse $\sin A = \frac{\text{Perpendicular side opposite to } \angle A}{\text{Hypotenuse}}$

Therefore from figure (a) and equation (3)

We get,

sinA=BCAC $\sin A = \frac{BC}{AC}$

sinA=1517 $\sin A = \frac{15}{17}$ …. (4)

Now we know, cos A is defined as follows

cosA=Basesideadjacentto∠AHypotenuse $\cos A = \frac{\text{Base side adjacent to } \angle A}{\text{Hypotenuse}}$

Therefore from figure (a) and equation (3)

We get,

cosA=ABAC $\cos A = \frac{AB}{AC}$

cosA=817 $\cos A = \frac{8}{17}$ …. (5)

Now we find the value of expression sinA–cosA $\sin A - \cos A$

Therefore by substituting the value the value of sinA $\sin A$ and cosA $\cos A$ from equation (4) and (5) respectively , we get,

sinA–cosA=1517–817 $\sin A - \cos A = \frac{15}{17} - \frac{8}{17}$ sinA–cosA=15–817 $\sin A - \cos A = \frac{15-8}{17}$ sinA–cosA=717 $\sin A - \cos A = \frac{7}{17}$

Hence, $\sin A - \cos A = \frac{7}{17}$

24.) **If $\tan \Theta = \frac{20}{21}$, show that $\frac{1 - \sin \Theta - \cos \Theta}{1 + \sin \Theta + \cos \Theta} = \frac{3}{7}$**

Sol.
Given:

$\tan \Theta = \frac{20}{21}$

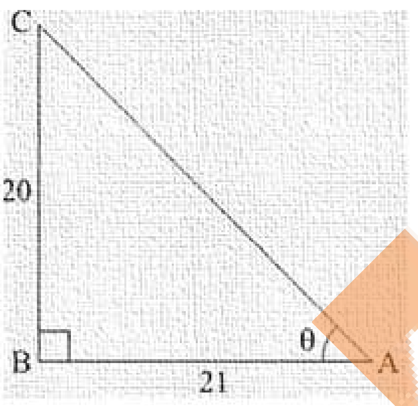
To show that $\frac{1 - \sin \Theta + \cos \Theta}{1 + \sin \Theta + \cos \Theta} = \frac{3}{7}$

Now we know that

$\tan \Theta = \frac{\text{Perpendicular side opposite to } \angle\Theta}{\text{Base side adjacent to } \angle\Theta}$

Therefore,

$\tan \Theta = \frac{20}{21}$



Side AC be the hypotenuse and can be found by applying Pythagoras theorem

Therefore,

$AC^2 = AB^2 + BC^2$

$AC^2 = 21^2 + 20^2$

$AC^2 = 441 + 400$

$AC^2 = 841$

Now by taking square root on both sides

We get,

$AC = \sqrt{841}$

AC= 29

Therefore Hypotenuse side AC= 29

Now we know, sinΘ$\sin\Theta$ is defined as follows,

sinAΘ= Perpendicularsideoppositeto∠ΘHypotenuse $\sin A\Theta = \frac{\text{Perpendicular side opposite to } \angle\Theta}{\text{Hypotenuse}}$

Therefore from figure and above equation

We get,

sinΘ= ABAC $\sin\Theta = \frac{AB}{AC}$ sinΘ= 2029 $\sin\Theta = \frac{20}{29}$

Now we know cosΘ$\cos\Theta$ is defined as follows

cosΘ= Basesideadjacentto∠ΘHypotenuse $\cos\Theta = \frac{\text{Base side adjacent to } \angle\Theta}{\text{Hypotenuse}}$

Therefore from figure and above equation

We get,

cosΘ= ABAC $\cos\Theta = \frac{AB}{AC}$ cosΘ= 2129 $\cos\Theta = \frac{21}{29}$

Now we need to find the value of expression 1–sinΘ+cosΘ1+sinΘ+cosΘ $\frac{1-\sin\Theta+\cos\Theta}{1+\sin\Theta+\cos\Theta}$

Therefore by substituting the value of sinΘ$\sin\Theta$ and cosΘ$\cos\Theta$from above equations, we get

1–sinΘ+cosΘ1+sinΘ+cosΘ $\frac{1-\sin\Theta+\cos\Theta}{1+\sin\Theta+\cos\Theta}$ =

29–20+2129 7029 $\frac{\frac{29-20+21}{29}}{\frac{70}{29}}$

Therefore after evaluating we get,

1–sinΘ+cosΘ1+sinΘ+cosΘ $\frac{1-\sin\Theta+\cos\Theta}{1+\sin\Theta+\cos\Theta}$ = 37 $\frac{3}{7}$

Hence,

1–sinΘ+cosΘ1+sinΘ+cosΘ $\frac{1-\sin\Theta+\cos\Theta}{1+\sin\Theta+\cos\Theta}$ **=**

37 $\frac{3}{7}$

25.) If cosecA=2$\operatorname{cosec}A = 2$ **, find 1tanA + sinA1+cosA** $\frac{1}{\tan A} + \frac{\sin A}{1+\cos A}$

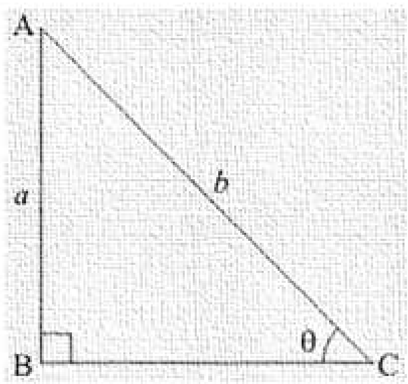
Sol.

Given:

cosecA=2$\operatorname{cosec}A = 2$

To find $\frac{1}{\tan A} + \frac{\sin A}{1+\cos A}$

Now cosec A = $\frac{\text{Hypotenuse}}{\text{Oppositeside}} = \frac{2}{1}$



Here BC is the adjacent side,

By applying Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$4 = 1 + BC^2$

$BC^2 = 3$

$BC = \sqrt{3}$

Now we know that

$\sin A = \frac{1}{\text{cosec} A}$

$\sin A = \frac{1}{2}$ …. (1)

$\tan A = \frac{AB}{BC}$

$\tan A = \frac{1}{\sqrt{3}}$ …. (2)

$\cos A = \frac{BC}{AC}$

$\cos A = \frac{\sqrt{3}}{2}$ …. (3)

Substitute all the values of $\sin A$, $\cos A$ and $\tan A$ from the equations(1) ,(2) and (3) respectively

We get.

$$\frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \frac{1}{\frac{1}{\sqrt{3}}} + \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}}$$

= √3+ 12+√3 $\sqrt{3} + \frac{1}{2+\sqrt{3}}$

= 2(2+√3)2+√3 $\frac{2(2+\sqrt{3})}{2+\sqrt{3}}$

= 2

Hence,

1tanA + sinA1+cosA $\frac{1}{\tan A} + \frac{\sin A}{1+\cos A}$ **= 2**

26.) If ∠A$\angle A$ and ∠B$\angle B$ are acute angles such that cos A =cos B , then show that ∠A $\angle A$= ∠B$\angle B$

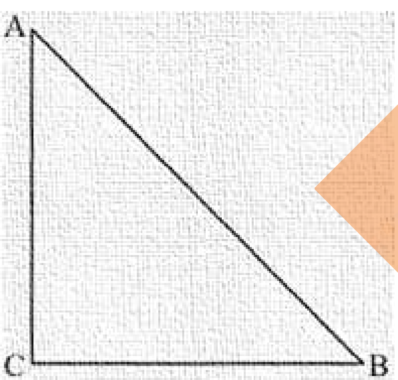
Sol.

Given:

∠A$\angle A$ and ∠B$\angle B$ are acute angles

cos A = cos B such that ∠A$\angle A$ = ∠B$\angle B$

Let us consider right angled triangle ACB



Now since cos A = cos B

Therefore

ACAB = BCAB $\frac{AC}{AB} = \frac{BC}{AB}$

Now observe that denominator of above equality is same that is AB

Hence ACAB = BCAB $\frac{AC}{AB} = \frac{BC}{AB}$ only when AC=BC

Therefore AC=BC

We know that when two sides of triangle are equal, then opposite of the sides are also

Equal.

Therefore

We can say that

Angle opposite to side AC = angle opposite to side BC

Therefore,

$\angle B \angle B = \angle A \angle A$

Hence, $\angle A \angle A = \angle B \angle B$

27.) In a $\Delta ABC \Delta ABC$, right angled triangle at A, if tan C = $\sqrt{3}\sqrt{3}$, find the value of sin B cos C + cos B sin C.

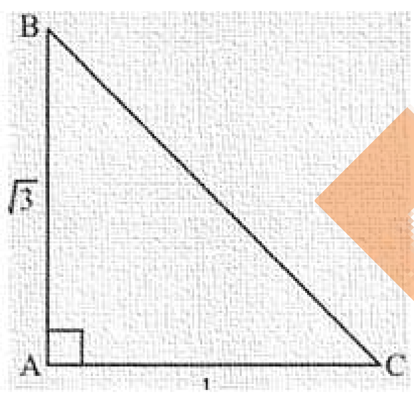
Sol.

Given:

$\Delta ABC \Delta ABC$

To find : sin B cos C + cos B sin C

The given a $\Delta ABC \Delta ABC$ is as shown in figure



Side BC is unknown and can be found using Pythagoras theorem,

Therefore,

$BC^2 = AB^2 + AC^2$

$BC^2 = \sqrt{3}^2 \sqrt{3}^2 + 1^2$

$BC^2 = 3 + 1$

$BC^2 = 4$

Now by taking square root on both sides

We get,

BC = √4 $\sqrt{4}$

BC= 2

Therefore Hypotenuse side BC= 2 …. (1)

Now, sin B = Perpendicularsideoppositeto∠BHypotenuse $\frac{\text{Perpendicular side opposite to } \angle B}{\text{Hypotenuse}}$

Therefore,

sinB= ACBC $\sin B = \frac{AC}{BC}$

Now by substituting the values from equation (1) and figure

We get,

sin B = 12 $\frac{1}{2}$ …. (2)

Now, cos B= basesideadjacentto∠BHypotenuse $\frac{\text{base side adjacent to } \angle B}{\text{Hypotenuse}}$

Therefore,

cos B = ABBC $\frac{AB}{BC}$

Now substituting the value from equation

cos B= √32 $\frac{\sqrt{3}}{2}$ …. (3)

Similarly

sin C = √32 $\frac{\sqrt{3}}{2}$ …. (4)

Now by definition,

tanC= sinCcosC $\tan C = \frac{\sin C}{\cos C}$

So by evaluating

cosC= 12 $\cos C = \frac{1}{2}$ …. (5)

Now, by substituting the value of sinB, cosB,sin C and cosC from equation (2) ,(3) ,(4) and (5) respectively in sinB cosC + cosB sin C

sinB cosC + cosB sin C= 12 × 12 + √32 × √32 $\frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

= 14 + 34 $\frac{1}{4} + \frac{3}{4}$

= 1

Hence,

sinB cosC + cosB sin C = 1

28.) State whether the following are true or false. Justify your answer.

(i) The value od tan A is always less than 1.

(ii) sec A = 125 $\frac{12}{5}$ for some value of $\angle A \angle A$.

(iii) cos A is the abbreviation used for the cosecant of $\angle A \angle A$.

(iv) sinΘ=43 $\sin \Theta = \frac{4}{3}$ for some angle ΘΘ.

Sol.

(i) tan A << 1

Value of tan A at 45^{O} i.e… tan 45 = 1

As value os A increases to 90^{O}

Tan A becomes infinite

So given statement is false.

(ii) sec A = 125 $\frac{12}{5}$ for some value of angle if

M-I

sec A = 2.4

sec A > 1

So given statements is true.

M- II

For sec A = 125 $\frac{12}{5}$ we get adjacent side = 13

Subtending 9i at B.

So, given statement is true.

(iii) Cos A is the abbreviation used for cosecant of angle A.

The given statement is false.

As such cos A is the abbreviation used for cos of angle A , not as cosecant of angle A.

(iv) Cot A is the product of cot A and A

Given statement is false

∵∵ cot A is a co-tangent of angle A and co-tangent of angle A = adjacentsideOpositeside $\frac{\text{adjacent side}}{\text{Oposite side}}$.

(v) sinΘ=43 $\sin\Theta = \frac{4}{3}$ for some angle ΘΘ.

Given statement is false

Since value of sinΘ$\sin\Theta$ is less than(or) equal to one.

Here value of sinΘ$\sin\Theta$ exceeds one,

So given statement is false.

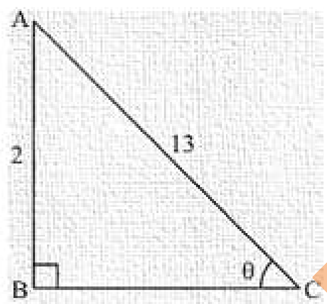
29.) If sinΘ=1213 $\sin\Theta = \frac{12}{13}$ find sin²Θ−cos²Θ2sinΘcosΘ × 1tan²Θ $\frac{\sin^2\Theta - \cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$

Sol.

Given: sinΘ=1213 $\sin\Theta = \frac{12}{13}$

To Find: sin²Θ−cos²Θ2sinΘcosΘ × 1tan²Θ $\frac{\sin^2\Theta - \cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$

As shown in figure



Here BC is the adjacent side,

By applying Pythagoras theorem,

$AC^2=AB^2+BC^2$

$169 = 144 + BC^2$

$BC^2 = 169 - 144$

$BC^2 = 25$

BC = 5

Now we know that,

cosΘ=basesideadjacentto∠ΘHypotenuse $\cos\Theta = \frac{\text{base side adjacent to } \angle\Theta}{\text{Hypotenuse}}$ cosΘ=BCAC

$\cos\Theta = \frac{BC}{AC}$ cosΘ=513 $\cos\Theta = \frac{5}{13}$

We also know that,

tanΘ= sinΘcosΘ $\tan\Theta = \frac{\sin\Theta}{\cos\Theta}$

Therefore, substituting the value of sinΘ $\sin\Theta$ and cosΘ $\cos\Theta$ from above equations

We get,

tanΘ= 125 $\tan\Theta = \frac{12}{5}$

Now substitute all the values of sinΘ $\sin\Theta$, cosΘ $\cos\Theta$ and tanΘ $\tan\Theta$ from above equations in sin²Θ−cos²Θ2sinΘcosΘ × 1tan²Θ $\frac{\sin^2\Theta - \cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$

We get,

sin²Θ−cos²Θ2sinΘcosΘ × 1tan²Θ $\frac{\sin^2\Theta - \cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$ = (1213)²−(513)²2×(1213)×(513) × 1(125)²

$$\frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2\times\left(\frac{12}{13}\right)\times\left(\frac{5}{13}\right)} \times \frac{1}{\left(\frac{12}{5}\right)^2}$$

Therefore by further simplifying we get,

sin²Θ−cos²Θ2sinΘcosΘ × 1tan²Θ $\frac{\sin^2\Theta - \cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$ = 119169 × 169120 × 25144 $\frac{119}{169} \times \frac{169}{120} \times \frac{25}{144}$

Therefore,

sin²Θ−cos²Θ2sinΘcosΘ × 1tan²Θ $\frac{\sin^2\Theta - \cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$ = 5953456 $\frac{595}{3456}$

Hence,

sin²Θ−cos²Θ2sinΘcosΘ × 1tan²Θ $\frac{\sin^2\Theta - \cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$ **= 5953456** $\frac{595}{3456}$

30.) If cosΘ= 513 $\cos\Theta = \frac{5}{13}$ **, find the value of sin²Θ−cos²Θ2sinΘcosΘ × 1tan²Θ** $\frac{\sin^2\Theta - \cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$

Sol.

Given: If cosΘ= 513 $\cos\Theta = \frac{5}{13}$

To find:

The value of expression sin²Θ−cos²Θ2sinΘcosΘ × 1tan²Θ $\frac{\sin^2\Theta - \cos^2\Theta}{2\sin\Theta\cos\Theta} \times \frac{1}{\tan^2\Theta}$

Now we know that

cosΘ $\cos\Theta$ = basesideadjacentto∠ΘHypotenuse $\frac{\text{base side adjacent to }\angle\Theta}{\text{Hypotenuse}}$ …. (2)

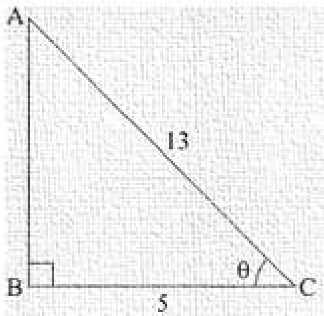
Now when we compare equation (1) and (2)

We get,

Base side adjacent to ∠Θ∠Θ = 5

Hypotenuse = 13

Therefore, Triangle representing ∠Θ∠Θ is as shown below



Perpendicular side AB is unknown and it can be found by using Pythagoras theorem

Therefore by applying Pythagoras theorem

We get,

$AC^2 = AB^2 + BC^2$

Therefore by substituting the values ogf known sides,

$AB^2 = 13^2 – 5^2$

$AB^2 = 169 - 25$

$AB^2 = 144$

AB = 12 …. (3)

Now we know from figure and equation,

sinΘ=1213 $\sin \Theta = \frac{12}{13}$ …. (4)

Now we know that,

tanΘ=sinΘcosΘ $\tan\Theta = \frac{\sin \Theta}{\cos \Theta}$

tanΘ=125 $\tan\Theta = \frac{12}{5}$ …. (5)

Now w substitute all the values from equation (1), (4) and (5) in the expression below,

sin²Θ−cos²Θ2sinΘcosΘ × 1tan²Θ $\frac{\sin^2 \Theta - \cos^2 \Theta}{2 \sin \Theta \cos \Theta} \times \frac{1}{\tan^2 \Theta}$

Therefore

We get,

$$\frac{\sin^2\Theta-\cos^2\Theta}{2\sin\Theta\cos\Theta}\times\frac{1}{\tan^2\Theta}=\frac{\left(\frac{12}{13}\right)^2-\left(\frac{5}{13}\right)^2}{2\times\left(\frac{12}{13}\right)\times\left(\frac{5}{13}\right)}\times\frac{1}{\left(\frac{12}{5}\right)^2}$$

Therefore by further simplifying we get,

$$\frac{\sin^2\Theta-\cos^2\Theta}{2\sin\Theta\cos\Theta}\times\frac{1}{\tan^2\Theta}=\frac{119}{169}\times\frac{169}{120}\times\frac{25}{144}$$

Therefore,

$$\frac{\sin^2\Theta-\cos^2\Theta}{2\sin\Theta\cos\Theta}\times\frac{1}{\tan^2\Theta}=\frac{595}{3456}$$

Hence,

$$\boldsymbol{\frac{\sin^2\Theta-\cos^2\Theta}{2\sin\Theta\cos\Theta}\times\frac{1}{\tan^2\Theta}=\frac{595}{3456}}$$

31.) If sec A = $\frac{17}{8}$ **, verify that** $\frac{3-4\sin^2 A}{4\cos^2 A-3}=\frac{3-\tan^2 A}{1-3\tan^2 A}$

Sol.

Given: sec A = $\frac{17}{8}$

To verify: $\frac{3-4\sin^2 A}{4\cos^2 A-3}=\frac{3-\tan^2 A}{1-3\tan^2 A}$

Now we know that $\cos A=\frac{1}{\sec A}$

Now, by substituting the value of sec A

We get,

$$\cos A=\frac{8}{17}$$

Now we also know that,

$$\sin^2 A+\cos^2 A=1$$

Therefore

$$\sin^2 A=1-\cos^2 A$$

$$=\left(\frac{8}{17}\right)^2$$

$$=\frac{225}{289}$$

Now by taking square root on both sides,

We get,

sinA=1517 $\sin A = \frac{15}{17}$

We also know that , tanA=sinAcosA $\tan A = \frac{\sin A}{\cos A}$

Now by substituting the value of all the terms ,

We get,

tanA=158 $\tan A = \frac{15}{8}$

Now from the expression of above equation which we want to prove:

L.H.S = 3–4sin^{2}A4cos^{2}A–3 $\frac{3–4\sin^2 A}{4\cos^2 A–3}$

Now by substituting the value of cos A ad sin A from equation (3) and (4)

We get,

L.H.S = 3–4225289 4–64289–3 $\frac{3–4\frac{225}{289}}{4–\frac{64}{289}–3}$

= 867–900256–867 $\frac{867–900}{256–867}$

= 33611 $\frac{33}{611}$

From expression

R.H.S = frac3–tan^{2}A1–3tan^{2}Afrac3– $\tan^2 A$1–3 $\tan^2 A$

Now by substituting the value of tan A from above equation

We get,

R.H.S= 3–(158)^{2}1–3(158)^2 $\frac{3–(\frac{15}{8})^2}{1–3(\frac{15}{8})^2}$

= −3364 −61164 $\frac{\frac{-33}{64}}{\frac{-611}{64}}$

= 33611 $\frac{33}{611}$

Therefore,

We can see that,

3–4sin^{2}A4cos^{2}A–3 = 3–tan^{2}A1–3tan^{2}A $\frac{3–4\sin^2 A}{4\cos^2 A–3} = \frac{3–\tan^2 A}{1–3\tan^2 A}$

32.) If sinΘ=34 $\sin\Theta = \frac{3}{4}$, prove that $\sqrt{\text{cosec}^2\Theta\text{–cot}^2\Theta\text{sec}^2\text{–1}}$ = √73 $\sqrt{\frac{\text{cosec}^2\Theta-\cot^2\Theta}{\sec^2-1}} = \frac{\sqrt{7}}{3}$

Sol.

Given: sinΘ=34 $\sin\Theta = \frac{3}{4}$ …. (1)

To prove:

$\sqrt{\text{cosec}^2\Theta\text{–cot}^2\Theta\text{sec}^2\text{–1}}$ = √73 $\sqrt{\frac{\text{cosec}^2\Theta-\cot^2\Theta}{\sec^2-1}} = \frac{\sqrt{7}}{3}$ …. (2)

By definition,

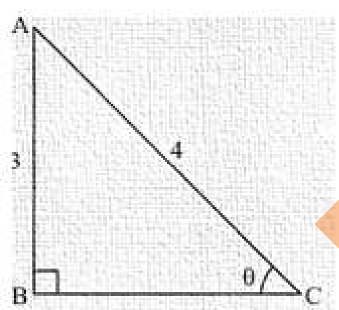
sin A = Perpendicularsideoppositeto∠AHypotenuse $\frac{\text{Perpendicular side opposite to }\angle A}{\text{Hypotenuse}}$ …. (3)

By comparing (1) and (3)

We get,

Perpendicular side = 3 and

Hypotenuse = 4



Side BC is unknown.

So we find BC by applying Pythagoras theorem to right angled ΔABCΔABC

Hence,

$AC^2 = AB^2 + BC^2$

Now we substitute the value of perpendicular side (AB) and hypotenuse (AC) and get the base side (BC)

Therefore,

$4^2 = 3^2 + BC^2$

$BC^2 = 16 – 9$

$BC^2 = 7$

BC = √7√ 7

Hence, Base side BC =√7√ 7 …. (3)

Now cos A = BCAC $\frac{BC}{AC}$

√74 $\frac{\sqrt{7}}{4}$ …. (4)

Now , cosecA= 1sinA $\operatorname{cosec}A = \frac{1}{\sin A}$

Therefore, from fig and equation (1)

cosecA= HypotenusePerpendicular $\operatorname{cosec}A = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

cosecA= 43 $\operatorname{cosec}A = \frac{4}{3}$ …. (5)

Now, similarly

secA= 4√7 $\sec A = \frac{4}{\sqrt{7}}$ …. (6)

Further we also know that

cotA= cosAsinA $\cot A = \frac{\cos A}{\sin A}$

Therefore by substituting th values from equation (1) and (4),

We get,

cotA= √73 $\cot A = \frac{\sqrt{7}}{3}$ …. (7)

Now by substituting the value of cosec A, sec A and cot A from the equations (5), (6), and (7) in the L.H.S of expression (2)

√cosec²Θ–cot²Θsec²–1 $\sqrt{\frac{\operatorname{cosec}^2\Theta - \cot^2\Theta}{\sec^2 - 1}}$ = □□ □ $\sqrt{(43)^2-(\sqrt{7}3)^2(4\sqrt{7})^2-1}$ $\sqrt{\frac{\left(\frac{4}{3}\right)^2-\left(\frac{\sqrt{7}}{3}\right)^2}{\left(\frac{4}{\sqrt{7}}\right)^2-1}}$

= 169−79 167−1 $\frac{\frac{16}{9}-\frac{7}{9}}{\frac{16}{7}-1}$

= √73 $\frac{\sqrt{7}}{3}$

Hence it is proved that,

√cosec²Θ–cot²Θsec²–1 = √73 $\sqrt{\frac{\operatorname{cosec}^2\Theta - \cot^2\Theta}{\sec^2 - 1}} = \frac{\sqrt{7}}{3}$

33.) If secA=178 $\sec A = \frac{17}{8}$ **, verify that 3–4sin²A4cos²A–3=3−tan²A1–3tan²A** $\frac{3-4\sin^2 A}{4\cos^2 A-3} = \frac{3-\tan^2 A}{1-3\tan^2 A}$

Sol.

Given: secA=178 $\sec A = \frac{17}{8}$ …. (1)

To verify:

3–4sin²A4cos²A–3=3−tan²A1–3tan²A $\frac{3-4\sin^2 A}{4\cos^2 A-3} = \frac{3-\tan^2 A}{1-3\tan^2 A}$ …. (2)

Now we know that sec A = 1cosA $\frac{1}{\cos A}$

Therefore cosA=1secA $\cos A = \frac{1}{\sec A}$

We get,

cosA=817 $\cos A = \frac{8}{17}$ …. (3)

Similarly we can also get,

sin A= sinA=1517 $\sin A = \frac{15}{17}$ …. (4)

An also we know that tanA=sinAcosA $\tan A = \frac{\sin A}{\cos A}$

tanA=158 $\tan A = \frac{15}{8}$ …. (5)

Now from the expression of equation (2)

L.H.S: Missing close brace Missing close brace

Now by substituting the value of cos A and sin A from equation (3) and (4)

We get,

L.H.S = 3–4(1517)²4(817)²–3 $\frac{3-4\left(\frac{15}{17}\right)^2}{4\left(\frac{8}{17}\right)^2-3}$

= 867−900289 256–867289 $\frac{\frac{867-900}{289}}{\frac{256-867}{289}}$

= 33611 $\frac{33}{611}$ …. (6)

R.H.S = 3−tan²A1–3tan²A $\frac{3-\tan^2 A}{1-3\tan^2 A}$

Now by substituting the value of tan A from equation (5)

We get,

R.H.S= $3–(1518)^2 1–3(158)^2$ $\frac{3-(\frac{15}{18})^2}{1-3(\frac{15}{8})^2}$

$-3364\ -61164$ $\frac{\frac{-33}{64}}{\frac{-611}{64}}$

$= 33611$ $\frac{33}{611}$ …. (7)

Now by comparing equation (6) and (7)

We get,

$3–4\sin^2A4\cos^2A–3 = 3−\tan^2A1–3\tan^2A$ $\frac{3-4\sin^2 A}{4\cos^2 A-3} = \frac{3-\tan^2 A}{1-3\tan^2 A}$

34.) If cotΘ= 34 $\cot\Theta = \frac{3}{4}$ **, prove that secΘ–cosecΘsecΘ+cosecΘ = 1√7** $\frac{\sec\Theta - \text{cosec}\Theta}{\sec\Theta + \text{cosec}\Theta} = \frac{1}{\sqrt{7}}$

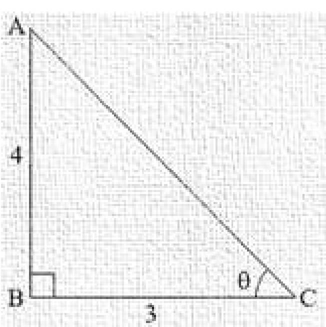
Sol.

Given: cotΘ= 34 $\cot\Theta = \frac{3}{4}$

Prove that: secΘ–cosecΘsecΘ+cosecΘ = 1√7 $\frac{\sec\Theta - \text{cosec}\Theta}{\sec\Theta + \text{cosec}\Theta} = \frac{1}{\sqrt{7}}$

Now we know that

secΘ–cosecΘsecΘ+cosecΘ = 1√7 $\frac{\sec\Theta - \text{cosec}\Theta}{\sec\Theta + \text{cosec}\Theta} = \frac{1}{\sqrt{7}}$



Here AC is the hypotenuse and we can find that by applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$AC^2 = 16 + 9$

$AC^2 = 25$

AC = 5

Similarly

$\sec\Theta = \frac{AC}{BC}$ $\sec\Theta = \frac{5}{3}$ $\text{cosec} = \frac{AC}{AB}$ $\text{cosec} = \frac{5}{4}$

Now on substituting the values in equations we get,

$$\frac{\sec\Theta - \text{cosec}\Theta}{\sec\Theta + \text{cosec}\Theta} = \frac{1}{\sqrt{7}}$$

Therefore,

$$\mathbf{\frac{\sec\Theta - \text{cosec}\Theta}{\sec\Theta + \text{cosec}\Theta} = \frac{1}{\sqrt{7}}}$$

35.) If $3\cos\Theta - 4\sin\Theta = 2\cos\Theta + \sin\Theta$,find $\tan\Theta$

Sol.

Given: $3\cos\Theta - 4\sin\Theta = 2\cos\Theta + \sin\Theta$

To find: $\tan\Theta$

We can write this as:

$3\cos\Theta - 4\sin\Theta = 2\cos\Theta + \sin\Theta$ $\cos\Theta = 5\sin\Theta$

Dividing both the sides by $\cos\Theta$,

We get,

$\frac{\cos\Theta}{\cos\Theta} = \frac{5\sin\Theta}{\cos\Theta}$ $1 = 5\tan\Theta$ $\tan\Theta = 1$

Hence,

$\tan\Theta = 1$

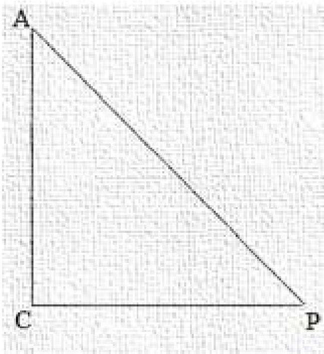
36.) If $\angle A$ and $\angle P$ are acute angles such that tan A = tan P, then show $\angle A = \angle P$

Sol.

Given: A and P are acute angles tan A =tan P

Prove that: $\angle A = \angle P$

Let us consider right angled triangle ACP



We know tanΘ= oppositesideadjacentside $\tan\Theta = \frac{\text{oppositeside}}{\text{adjacentside}}$

tan A = PCAC $\frac{PC}{AC}$

tan P = ACPC $\frac{AC}{PC}$

∴∴ tan A =tan P

PCAC = ACPC $\frac{PC}{AC} = \frac{AC}{PC}$

PC =AC [∵∵ Angle opposite to equal sides are equal]

∠A=∠P $\angle A = \angle P$