1. If the sides of a triangle are 3 cm, 4 cm, and 6 cm long, determine whether the triangle is a right-angled triangle.

Sol:

We have,

Sides of triangle

AB = 3 cm

BC = 4 cm

AC = 6 cm

$\therefore AB^2 = 3^2 = 9$

$BC^2 = 4^2 = 16$

$AC^2 = 6^2 = 36$

Since, $AB^2 + BC^2 \neq AC^2$

Then, by converse of Pythagoras theorem, triangle is not a right triangle.

2. The sides of certain triangles are given below. Determine which of them right triangles are.

(i) a = 7 cm, b = 24 cm and c = 25 cm

(ii) a = 9 cm, b = 16 cm and c = 18 cm

(iii) a = 1.6 cm, b = 3.8 cm and c = 4 cm

(iv) a = 8 cm, b = 10 cm and c = 6 cm

Sol:

We have,

a = 7 cm, b = 24 cm and c = 25 cm
$\therefore a^2 = 49, b^2 = 576 \text{ and } c^2 = 625$
Since, $a^2 + b^2 = 49 + 576$
$= 625$
$= c^2$
Then, by converse of Pythagoras theorem, given triangle is a right triangle.

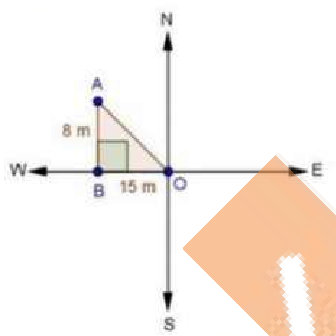
We have,
a = 9 cm, b = 16 cm and c = 18 cm
$\therefore a^2 = 81, b^2 = 256 \text{ and } c^2 = 324$
Since, $a^2 + b^2 = 81 + 256 = 337$
$\neq c^2$
Then, by converse of Pythagoras theorem, given triangle is not a right triangle.

We have,
a = 1.6 cm, b = 3.8 cm and C = 4 cm
$\therefore a^2 = 64, b^2 = 100 \text{ and } c^2 = 36$
Since, $a^2 + c^2 = 64 + 36 = 100 = b^2$

Then, by converse of Pythagoras theorem, given triangle is a right triangle.

3. A man goes 15 metres due west and then 8 metres due north. How far is he from the starting point?

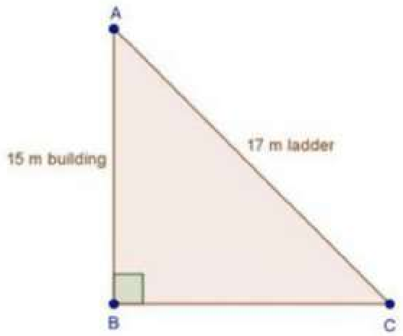
Sol:



Let the starting point of the man be O and final point be A.
$\therefore$ In ΔABO, by Pythagoras theorem $AO^2 = AB^2 + BO^2$
$\Rightarrow AO^2 = 8^2 + 15^2$
$\Rightarrow AO^2 = 64 + 225 = 289$
$\Rightarrow AO = \sqrt{289} = 17m$
$\therefore$ He is 17m far from the starting point.

4. A ladder 17 m long reaches a window of a building 15 m above the ground. Find the distance of the foot of the ladder from the building.

Sol:



In ΔABC, by Pythagoras theorem

$AB^2 + BC^2 = AC^2$

$\Rightarrow 15^2 + BC^2 = 17^2$

$\Rightarrow 225 + BC^2 = 17^2$

$\Rightarrow BC^3 = 289 - 225$

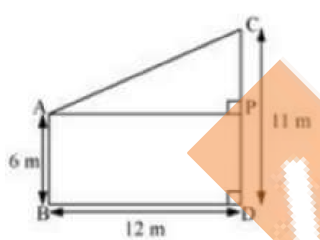
$\Rightarrow BC^2 = 64$

$\Rightarrow BC = 8\,m$

$\therefore Distance\ of\ the\ foot\ of\ the\ ladder\ from\ building = 8\,m$

5. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

Sol:



Let CD and AB be the poles of height 11 and 6 m.

Therefore CP = 11 – 6 = 5 m

From the figure we may observe that AP = 12m

In triangle APC, by applying Pythagoras theorem

$AP^2 + PC^2 = AC^2$

$12^2 + 5^2 = AC^2$

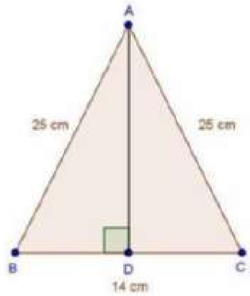
$AC^2 = 144 + 25 = 169$

AC = 13

Therefore distance between their tops = 13m.

6. In an isosceles triangle ABC, AB = AC = 25 cm, BC = 14 cm. Calculate the altitude from A on BC.

Sol:



We have

AB = AC = 25 cm and BC = 14 cm

In ΔABD and ΔACD

∠ADB = ∠ADC [Each 90°]

AB = AC [Each 25 cm]

AD = AD [Common]

Then, ΔABD ≅ ΔACD [By RHS condition]

∴ BD = CD = 7 cm [By c.p.c.t]

In ΔADB, by Pythagoras theorem

$AD^2 + BD^2 = AB^2$

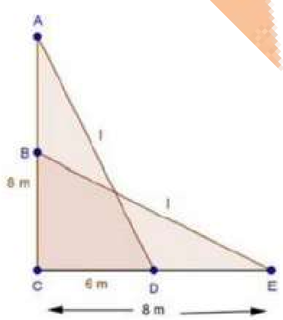
$\Rightarrow AD^2 + 7^2 = 25^2$

$\Rightarrow AD^2 = 625 - 49 = 576$

$\Rightarrow AD = \sqrt{576} = 24\ cm$

7. The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its tip reach?

Sol:



Let, length of ladder be AD = BE = l m

In ΔACD, by Pythagoras theorem

$AD^2 = AC^2 + CD^2$

$\Rightarrow l^2 = 8^2 + 6^2$ ....(i)

$In\ \Delta BCE, by\ pythagoras\ theorem$

$BE^2 = BC^2 + CE^2$

$\Rightarrow l^2 = BC^2 + 8^2$ ....(ii)

$Compare\ (i) and\ (ii)$

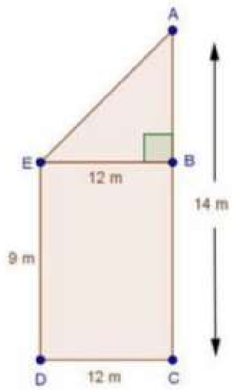
$BC^2 + 8^2 = 8^2 + 6^2$

$\Rightarrow BC^2 = 6^2$

$\Rightarrow BC = 6m$

8. Two poles of height 9 m and 14 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

Sol:



We have,

AC = 14 m, DC = 12m and ED = BC = 9m

Construction: Draw EB ⊥ AC

∴ AB = AC – BC = 14 – 9 = 5m

And, EB = DC = 12 m

In ΔABE, by Pythagoras theorem,

$AE^2 = AB^2 + BE^2$

$\Rightarrow AE^2 = 5^2 + 12^2$

$\Rightarrow AE^2 = 25 + 144 = 169$

$\Rightarrow AE = \sqrt{169} = 13\ m$

$\therefore Distance\ between\ their\ tops = 13\ m$

9. Using Pythagoras theorem determine the length of AD in terms of b and c shown in Fig. 4.219

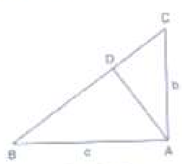


Fig. 4.219

Sol:

We have,

In ΔBAC, by Pythagoras theorem

$BC^2 = AB^2 + AC^2$

$\Rightarrow BC^2 = c^2 + b^2$

$\Rightarrow BC = \sqrt{c^2 + b^2}$ ...(i)

$In\ \Delta ABD\ and\ \Delta CBA$

∠B = ∠B [Common]

$\angle ADB = \angle BAC$ [Each 90°]

Then, $\Delta ABD \sim \Delta CBA$ [By AA similarity]

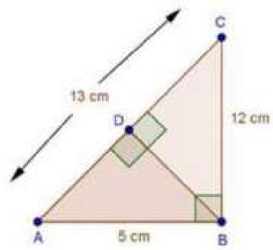
$\therefore \frac{AB}{CB} = \frac{AD}{CA}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{c}{\sqrt{c^2+b^2}} = \frac{AD}{b}$

$\Rightarrow AD = \frac{bc}{\sqrt{c^2+b^2}}$

10. A triangle has sides 5 cm, 12 cm and 13 cm. Find the length to one decimal place, of the perpendicular from the opposite vertex to the side whose length is 13 cm.

Sol:



Let, AB = 5cm, BC = 12 cm and AC = 13 cm. Then, $AC^2 = AB^2 + BC^2$. This proves that ΔABC is a right triangle, right angles at B. Let BD be the length of perpendicular from B on AC.

Now, Area $\Delta ABC = \frac{1}{2}(BC \times BA)$

$= \frac{1}{2}(12 \times 5)$

$= 30 \text{ cm}^2$

Also, Area of $\Delta ABC = \frac{1}{2} AC \times BD = \frac{1}{2}(13 \times BD)$

$\Rightarrow (13 \times BD) = 30 \times 2$

$\Rightarrow BD = \frac{60}{13}$ cm