

**Q.1) The numerator of a fraction is 4 less than the denominator. If the numerator is decreased by 2 and denominator is increased by 1, then the denominator is eight times the numerator. Find the fraction.**

**Soln:**

Let the numerator and denominator of the fraction be  $x$  and  $y$  respectively. Then the fraction is  $\frac{x}{y}$

The numerator of the fraction is 4 less the denominator. Thus, we have

$$x = y - 4$$

$$\Rightarrow x - y = -4 \Rightarrow x - y = -4$$

If the numerator is decreased by 2 and denominator is increased by 1, then the denominator is 8 times the numerator. Thus, we have

$$y + 1 = 8(x - 2)$$

$$\Rightarrow y + 1 = 8x - 16 \Rightarrow y + 1 = 8x - 16 \Rightarrow 8x - y = 1 + 16 \Rightarrow 8x - y = 17 \Rightarrow 8x - y = 17$$

So, we have two equations

$$x - y = -4$$

$$8x - y = 17$$

Here  $x$  and  $y$  are unknowns. We have to solve the above equations for  $x$  and  $y$ . Subtracting the second equation from the first equation, we get

$$(x - y) - (8x - y) = -4 - 17$$

$$\Rightarrow x - y - 8x + y = -21 \Rightarrow x - y - 8x + y = -21 \Rightarrow -7x = -21 \Rightarrow -7x = -21 \Rightarrow x = 217 \Rightarrow x = \frac{21}{7}$$

$$\Rightarrow 217 \Rightarrow \frac{21}{7} x = 3$$

Substituting the value of  $x$  in the first equation, we have

$$3 - y = -4$$

$$\Rightarrow y = 3 + 4 \Rightarrow y = 3 + 4 \Rightarrow y = 7 \Rightarrow y = 7$$

Hence the fraction is  $37 \frac{3}{7}$

**Q.2) A fraction becomes  $911 \frac{9}{11}$  if 2 is added to both numerator and the denominator. If 3 is added to both the numerator and the denominator it becomes  $56 \frac{5}{6}$ . Find the fraction**

**Soln:**

Let the numerator and denominator of the fraction be  $x$  and  $y$  respectively. Then the fraction is  $xy \frac{x}{y}$

If 2 is added to both numerator and the denominator, the fraction becomes  $911 \frac{9}{11}$ . Thus, we have

$$\begin{aligned}x+2y+2 &= 911 \frac{x+2}{y+2} = \frac{9}{11} \Rightarrow 11(x+2)=9(y+2) \Rightarrow 11(x+2) = 9(y+2) \Rightarrow 11x+22=9y+18 \\ \Rightarrow 11x+22 &= 9y+18 \Rightarrow 11x-9y=18-22 \Rightarrow 11x-9y = 18-22 \Rightarrow 11x-9y+4=0 \\ \Rightarrow 11x-9y &+4 = 0\end{aligned}$$

If 3 is added to both numerator and the denominator, the fraction becomes  $56 \frac{5}{6}$

$$\begin{aligned}x+3y+3 &= 56 \frac{x+3}{y+3} = \frac{5}{6} \Rightarrow 6(x+3)=5(y+3) \Rightarrow 6(x+3) = 5(y+3) \Rightarrow 6x+18=5y+15 \\ \Rightarrow 6x+18 &= 5y+15 \Rightarrow 6x-5y=15-18 \Rightarrow 6x-5y = 15-18 \Rightarrow 6x-5y+3=0 \Rightarrow 6x-5y+3 = 0\end{aligned}$$

So, we have two equations

$$11x - 9y + 4 = 0$$

$$6x - 5y + 3 = 0$$

Here  $x$  and  $y$  are unknowns. We have to solve the above equations for  $x$  and  $y$ .

By using cross-multiplication, we have

$$x-9*3-(-5)*4 \frac{x}{-9*3-(-5)*4} = -y11*3-6*4 \frac{-y}{11*3-6*4} = 111*(-5)-6*(-9) \frac{1}{11*(-5)-6*(-9)}$$

$$\Rightarrow x-27+20 \Rightarrow \frac{x}{-27+20} = \Rightarrow -y33-24 \Rightarrow \frac{-y}{33-24} = 1-55+54 \frac{1}{-55+54}$$

$$\Rightarrow x-7 \Rightarrow \frac{x}{-7} = -y9 \frac{-y}{9} = 1-1 \frac{1}{-1}$$

$$\Rightarrow x7 = y9 = 1 \Rightarrow \frac{x}{7} = \frac{y}{9} = 1$$

$$x = 7, y = 9$$

The fraction is  $79 \frac{7}{9}$

**Q.3) A fraction becomes  $13 \frac{1}{3}$  if 1 is subtracted from both its numerator and denominator. If 1 is added to both the numerator and denominator, it becomes  $12 \frac{1}{2}$ . Find the fraction.**

**Soln:**

Let the numerator and denominator of the fraction be  $x$  and  $y$  respectively. Then the fraction is  $xy \frac{x}{y}$

If 1 is subtracted from both numerator and the denominator, the fraction becomes  $13\frac{1}{3}$ . Thus, we have

$$x-1y-1=13\frac{x-1}{y-1}=\frac{1}{3}\Rightarrow 3(x-1)=(y-1)\Rightarrow 3(x-1)=(y-1)\Rightarrow 3x-3=y-1\Rightarrow 3x-3=y-1\Rightarrow 3x-y-2=0\Rightarrow 3x-y-2=0$$

If 1 is added to both numerator and the denominator, the fraction becomes  $12\frac{1}{2}$ . Thus, we have

$$x+1y+1=12\frac{x+1}{y+1}=\frac{1}{2}\Rightarrow 2(x+1)=(y+1)\Rightarrow 2(x+1)=(y+1)\Rightarrow 2x+2=y+1\Rightarrow 2x+2=y+1\Rightarrow 2x-y+1=0\Rightarrow 2x-y+1=0$$

So, we have two equations

$$3x - y - 2 = 0$$

$$2x - y + 1 = 0$$

Here x and y are unknowns. We have to solve the above equations for x and y.

By using cross-multiplication, we have

$$x(-1)*1-(-1)*(-2)\frac{x}{(-1)*1-(-1)*(-2)}=-y3*1-2*(-2)\frac{-y}{3*1-2*(-2)}=13*(-1)-2*(-1)\frac{1}{3*(-1)-2*(-1)}$$

$$\Rightarrow x-1-2\Rightarrow \frac{x}{-1-2}=\Rightarrow -y3+4\Rightarrow \frac{-y}{3+4}=1-3+2\frac{1}{-3+2}$$

$$\Rightarrow x-3\Rightarrow \frac{x}{-3}=-y7\frac{-y}{7}=1-1\frac{1}{-1}$$

$$\Rightarrow x3=y7=1\Rightarrow \frac{x}{3}=\frac{y}{7}=1\Rightarrow x=3,y=7\Rightarrow x=3,y=7$$

The fraction is  $37\frac{3}{7}$

**Q.4) If we add 1 to the numerator and subtract 1 from the denominator, a fraction becomes 1. It also becomes  $12\frac{1}{2}$  if we only add 1 to the denominator. What is the fraction?**

**Soln:**

Let the numerator and denominator of the fraction be x and y respectively. Then the fraction is  $xy\frac{x}{y}$

If 1 is added to the numerator and 1 is subtracted from the denominator, the fraction becomes 1. Thus, we have

$$x+1y-1=1\frac{x+1}{y-1}=1\Rightarrow(x+1)=(y-1)\Rightarrow(x+1)=(y-1)\Rightarrow x+1-y+1=0\Rightarrow x+1-y+1=0\Rightarrow x-y+2=0\Rightarrow x-y+2=0$$

If 1 is added to the denominator, the fraction becomes  $12\frac{1}{2}$ . Thus, we have

$$xy+1=12\frac{x}{y+1}=\frac{1}{2}\Rightarrow 2x=(y+1)\Rightarrow 2x=y+1\Rightarrow 2x-y-1=0\Rightarrow 2x-y-1=0$$

So, we have two equations

$$x-y+2=0$$

$$2x-y-1=0$$

Here x and y are unknowns. We have to solve the above equations for x and y.

By using cross-multiplication, we have

$$x(-1)*(-1)-(-1)*2\frac{x}{(-1)*(-1)-(-1)*2}=-y1*(-1)-2*2\frac{-y}{1*(-1)-2*2}=11*(-1)-2*(-1)\frac{1}{1*(-1)-2*(-1)}$$

$$\Rightarrow x1+2\Rightarrow \frac{x}{1+2}=\Rightarrow -y-1-4\Rightarrow \frac{-y}{-1-4}=1-1+2\frac{1}{-1+2}$$

$$\Rightarrow x3\Rightarrow \frac{x}{3}=-y-5\frac{-y}{-5}=11\frac{1}{1}$$

$$\Rightarrow x3=y5=1\Rightarrow \frac{x}{3}=\frac{y}{5}=1\Rightarrow x=3,y=5\Rightarrow x=3,y=5$$

The fraction is  $35\frac{3}{5}$

**5) The sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes  $12\frac{1}{2}$ . Find the fraction.**

**Soln:**

Let the numerator and denominator of the fraction be x and y respectively. Then the fraction is  $xy\frac{x}{y}$

The sum of the numerator and denominator of the fraction is 12. Thus, we have

$$x+y=12$$

$$\Rightarrow x+y-12=0\Rightarrow x+y-12=0$$

If the denominator is increased by 3, the fraction becomes  $12\frac{1}{2}$ . Thus, we have

$$xy+3=12\frac{x}{y+3}=\frac{1}{2}\Rightarrow 2x=(y+3)\Rightarrow 2x=y+3\Rightarrow 2x-y-3=0\Rightarrow 2x-y-3=0$$

So, we have two equations

$$x+y-12=0$$

$$2x-y-3=0$$

Here x and y are unknowns. We have to solve the above equations for x and y.

By using cross-multiplication, we have

$$x(1)*(-3)-(-1)*-12 \frac{x}{(1)*(-3)-(-1)*-12} = -y1*(-3)-2*-12 \frac{-y}{1*(-3)-2*-12} = 11*(-1)-2*(1) \frac{1}{1*(-1)-2*(1)}$$

$$\Rightarrow x-3-12 \Rightarrow \frac{x}{-3-12} = \Rightarrow -y-3+24 \Rightarrow \frac{-y}{-3+24} = 1-1-2 \frac{1}{-1-2}$$

$$\Rightarrow x-15 \Rightarrow \frac{x}{-15} = -y21 \frac{-y}{21} = 1-3 \frac{1}{-3}$$

$$\Rightarrow x15=y21=13 \Rightarrow \frac{x}{15} = \frac{y}{21} = \frac{1}{3} \Rightarrow x=15 \cdot \frac{1}{3}, y=21 \cdot \frac{1}{3} \Rightarrow x=5, y=7 \Rightarrow x=5, y=7$$

The fraction is  $57 \frac{5}{7}$

**7) The sum of a numerator and denominator of a fraction is 18. If the denominator is increased by 2, the fraction reduces to  $13 \frac{1}{3}$ . Find the fraction.**

**Soln:**

Let the numerator and denominator of the fraction be  $x$  and  $y$  respectively. Then the fraction is  $xy \frac{x}{y}$

The sum of the numerator and denominator of the fraction is 18. Thus, we have

$$x + y = 18$$

$$\Rightarrow x+y-18=0 \Rightarrow x + y - 18 = 0$$

If the denominator is increased by 2, the fraction becomes  $13 \frac{1}{3}$ . Thus, we have

$$xy+2=13 \frac{x}{y+2} = \frac{1}{3} \Rightarrow 3x=(y+2) \Rightarrow 3x = (y + 2) \Rightarrow 3x-y-2=0 \Rightarrow 3x-y-2 = 0$$

So, we have two equations

$$x + y - 18 = 0$$

$$3x - y - 2 = 0$$

Here  $x$  and  $y$  are unknowns. We have to solve the above equations for  $x$  and  $y$ .

By using cross-multiplication, we have

$$x(1)*(-2)-(-1)*-18 \frac{x}{(1)*(-2)-(-1)*-18} = -y1*(-2)-3*-18 \frac{-y}{1*(-2)-3*-18} = 11*(-1)-3*(1) \frac{1}{1*(-1)-3*(1)}$$

$$\Rightarrow x-2-18 \Rightarrow \frac{x}{-2-18} = \Rightarrow -y-2+54 \Rightarrow \frac{-y}{-2+54} = 1-1-3 \frac{1}{-1-3}$$

$$\Rightarrow x-20 \Rightarrow \frac{x}{-20} = -y52 \frac{-y}{52} = 1-4 \frac{1}{-4}$$

$$\Rightarrow x^{20} = y^{52} = 14 \Rightarrow \frac{x}{20} = \frac{y}{52} = \frac{1}{4} \Rightarrow x = 20 \cdot \frac{1}{4}, y = 52 \cdot \frac{1}{4} \Rightarrow x = 5, y = 13 \Rightarrow x = 5, y = 13$$

The fraction is  $5 \frac{13}{13}$

**8) If 2 is added to the numerator of a fraction, it reduces to  $12 \frac{1}{2}$  and if 1 is subtracted from the denominator, it reduces to  $13 \frac{1}{3}$ . Find the fraction.**

**Soln:**

Let the numerator and denominator of the fraction be  $x$  and  $y$  respectively. Then the fraction is  $\frac{x}{y}$

If 2 is added to the numerator of the fraction, it reduces to  $12 \frac{1}{2}$ . Thus we have

$$x+2y \frac{x+2}{y} = 12 \frac{1}{2}$$

$$\Rightarrow 2(x+2)=y \Rightarrow 2(x+2) = y \Rightarrow 2x+4=y \Rightarrow 2x+4 = y \Rightarrow 2x-y+4=0 \Rightarrow 2x-y+4 = 0$$

If 1 is subtracted from the denominator, the fraction reduces to  $13 \frac{1}{3}$ . Thus, we have

$$xy-1 = 13 \frac{x}{y-1} = \frac{1}{3} \Rightarrow 3x=(y-1) \Rightarrow 3x = (y-1) \Rightarrow 3x-y+1=0 \Rightarrow 3x-y+1 = 0$$

So, we have two equations

$$2x - y + 4 = 0$$

$$3x - y + 1 = 0$$

Here  $x$  and  $y$  are unknowns. We have to solve the above equations for  $x$  and  $y$ .

By using cross-multiplication, we have

$$x \frac{(-1) \cdot (1) - (-1) \cdot 4}{(-1) \cdot (1) - (-1) \cdot 4} = -y \frac{2 \cdot (1) - 3 \cdot 4}{2 \cdot (1) - 3 \cdot 4} = 12 \frac{(-1) \cdot 3 - (-1) \cdot (-1)}{2 \cdot (-1) - 3 \cdot (-1)}$$

$$\Rightarrow x-1+4 \Rightarrow \frac{x}{-1+4} = \Rightarrow -y-12 \Rightarrow \frac{-y}{2-12} = 1-2+3 \frac{1}{-2+3}$$

$$\Rightarrow x3 \Rightarrow \frac{x}{3} = -y-10 \frac{-y}{-10} = 11 \frac{1}{1}$$

$$\Rightarrow x3 = y10 = 1 \Rightarrow \frac{x}{3} = \frac{y}{10} = 1$$

$$\Rightarrow x=3, y=10 \Rightarrow x = 3, y = 10$$

The fraction is  $310 \frac{3}{10}$

**9) The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2:3. Determine the fraction.**

**Soln:**

Let the numerator and denominator of the fraction be  $x$  and  $y$  respectively. Then the fraction is  $xy \frac{x}{y}$

The sum of the numerator and denominator of the fraction is 4 more than twice the numerator. Thus, we have

$$x + y = 2x + 4$$

$$\Rightarrow 2x + 4 - x - y = 0 \Rightarrow 2x + 4 - x - y = 0$$

$$\Rightarrow x - y + 4 = 0 \Rightarrow x - y + 4 = 0$$

If the numerator and denominator are increased by 3, they are in the ratio 2:3. Thus we have

$$x + 3 : y + 3 = 2 : 3$$

$$\Rightarrow x + 3y + 3 = 23 \Rightarrow \frac{x+3}{y+3} = \frac{2}{3} \Rightarrow 3(x+3) = 2(y+3) \Rightarrow 3(x+3) = 2(y+3) \Rightarrow 3x+9=2y+6$$

$$\Rightarrow 3x + 9 = 2y + 6 \Rightarrow 3x - 2y + 3 = 0 \Rightarrow 3x - 2y + 3 = 0$$

So, we have two equations

$$x - y + 4 = 0$$

$$3x - 2y + 3 = 0$$

Here  $x$  and  $y$  are unknowns. We have to solve the above equations for  $x$  and  $y$ .

By using cross-multiplication, we have

$$x(-1) \cdot (3) - (-2) \cdot 4 \frac{x}{(-1) \cdot (3) - (-2) \cdot 4} = -y1 \cdot (3) - 3 \cdot 4 \frac{-y}{1 \cdot (3) - 3 \cdot 4} = 11 \cdot (-2) - 3 \cdot (-1) \frac{1}{1 \cdot (-2) - 3 \cdot (-1)}$$

$$\Rightarrow x-3+8 \Rightarrow \frac{x}{-3+8} = \Rightarrow -y3-12 \Rightarrow \frac{-y}{3-12} = 1-2+3 \frac{1}{-2+3}$$

$$\Rightarrow x5 \Rightarrow \frac{x}{5} = -y-9 \frac{-y}{-9} = 1-2+3 \frac{1}{-2+3}$$

$$\Rightarrow x5=y9=1 \Rightarrow \frac{x}{5} = \frac{y}{9} = 1 \Rightarrow x=5, y=9 \Rightarrow x = 5, y = 9$$

The fraction is  $59\frac{5}{9}$

**10) If the numerator of a fraction is multiplied by 2 and the denominator is reduced by 5 the fraction becomes  $65\frac{6}{5}$ . And, if the denominator is doubled and the numerator is increased by 8, the fraction becomes  $25\frac{2}{5}$ . Find the fraction.**

**Soln:**

Let the numerator and denominator of the fraction be  $x$  and  $y$  respectively. Then the fraction is  $xy\frac{x}{y}$

If the numerator is multiplied by 2 and denominator is reduced by 5, the fraction becomes  $65\frac{6}{5}$ . Thus, we have

$$\begin{aligned}\Rightarrow 2xy-5 &= 65 \Rightarrow \frac{2x}{y-5} = \frac{6}{5} \Rightarrow 10x=6(y-5) \Rightarrow 10x = 6(y-5) \Rightarrow 10x-6y+30=0 \\ \Rightarrow 10x-6y+30 &= 0 \Rightarrow 2(5x-3y+15)=0 \Rightarrow 2(5x-3y+15) = 0 \Rightarrow 5x-3y+15=0 \\ \Rightarrow 5x-3y+15 &= 0\end{aligned}$$

If the denominator is doubled and the numerator are increased by 8, the fraction becomes  $25\frac{2}{5}$ .. Thus we have

$$\begin{aligned}\Rightarrow x+8y &= 25 \Rightarrow \frac{x+8}{2y} = \frac{2}{5} \Rightarrow 5(x+8)=4y \Rightarrow 5(x+8) = 4y \Rightarrow 5x+40=4y \Rightarrow 5x+40 = 4y \Rightarrow 5x-4y+40=0 \\ \Rightarrow 5x-4y+40 &= 0\end{aligned}$$

So, we have two equations

$$5x - 3y + 15 = 0$$

$$5x - 4y + 40 = 0$$

Here  $x$  and  $y$  are unknowns. We have to solve the above equations for  $x$  and  $y$ .

By using cross-multiplication, we have

$$x(-3)*(40)-(-4)*15 \frac{x}{(-3)*(40)-(-4)*15} = -y5*(40)-5*15 \frac{-y}{5*(40)-5*15} = 15*(-4)-5*(-3) \frac{1}{5*(-4)-5*(-3)}$$

$$\Rightarrow x-120+60 \Rightarrow \frac{x}{-120+60} = \Rightarrow -y200-75 \Rightarrow \frac{-y}{200-75} = 1-20+15 \frac{1}{-20+15}$$

$$\Rightarrow x-60 \Rightarrow \frac{x}{-60} = -y125 \frac{-y}{125} = 1-5 \frac{1}{-5}$$

$$\Rightarrow x60 \Rightarrow \frac{x}{60} = y125 \frac{y}{125} = 15 \frac{1}{5}$$

$$x=605, y=1255 \Rightarrow x = \frac{60}{5}, y = \frac{125}{5} \Rightarrow x=12, y=25 \Rightarrow x = 12, y = 25$$

The fraction is  $1225\frac{12}{25}$



11) The sum of the numerator and denominator of a fraction is 3 less than twice the denominator. If the numerator and denominator are decreased by 1, the numerator becomes half the denominator. Determine the fraction

**Soln:**

Let the numerator and denominator of the fraction be  $x$  and  $y$  respectively. Then the fraction is  $\frac{x}{y}$

The sum of the numerator and denominator of a fraction is 3 less than twice the denominator. Thus, we have

$$x + y = 2y - 3$$

$$\Rightarrow x + y - 2y + 3 = 0 \Rightarrow x - y + 3 = 0 \Rightarrow x - y + 3 = 0$$

If the numerator and denominator are decreased by 1, the numerator becomes half the denominator. Thus, we have

$$\begin{aligned} x - 1 &= \frac{1}{2}(y - 1) \Rightarrow 2(x - 1) = (y - 1) \\ \Rightarrow 2x - 2 &= (y - 1) \Rightarrow 2x - 2 = (y - 1) \Rightarrow 2x - y - 1 = 0 \end{aligned}$$

So, we have two equations

$$x - y + 3 = 0$$

$$2x - y - 1 = 0$$

Here  $x$  and  $y$  are unknowns. We have to solve the above equations for  $x$  and  $y$ .

By using cross-multiplication, we have

$$x(-1)(-1) - (-1)(-1) * 3 \frac{x}{(-1)(-1) - (-1)(-1) * 3} = -y(1)(-1) - 2 * 3 \frac{-y}{1(-1) - 2 * 3} = 11(-1) - 2(-1) \frac{1}{1(-1) - 2(-1)}$$

$$\Rightarrow x1 + 3 \Rightarrow \frac{x}{1 + 3} = \Rightarrow -y - 1 - 6 \Rightarrow \frac{-y}{-1 - 6} = 1 - 1 + 2 \frac{1}{-1 + 2}$$

$$\Rightarrow x4 \Rightarrow \frac{x}{4} = -y - 7 \frac{-y}{-7} = 11 \frac{1}{1}$$

$$\Rightarrow x4 \Rightarrow \frac{x}{4} = y7 \frac{y}{7} = 11$$

$$\Rightarrow x = 4, y = 7 \Rightarrow x = 4, y = 7$$

The fraction is  $\frac{4}{7}$