Q.1: The sum of two numbers is 8. If their sum is four times their difference, find the numbers.

Soln: Let the numbers are x and y. One of them must be greater than or equal to the other. Let us assume that x is greater than or equal to y.

The sum of the two numbers is 8. Thus, we have x+y = 8

The sum of the two numbers is four times their difference. Thus, we have

x+y = 4(x— y)

=>x+y= 4x-4y

=>4x — 4y—x—y = 0

=> 3x-5y = 0

So, we have two equations

x+y=8

3x-5y = 0

Here x and y are unknowns.

We have to solve the above equations for x and y.

Multiplying the first equation by 5 and then adding with the second equation, we have

5(x + y)+(3x — 5y) = 5×8+0

=>5x+5y+3x-5y =40

=>8x = 40

=> x = 5

x = Substituting the value of x in the first equation, we have

5+y=8

=>y=8-5

=>y=3

Hence, the numbers are 5 and 3.

Q. 2: The sum of digits of a two digit number is 13. If the number is subtracted from the one obtained by interchanging the digits, the result is 45. What is the number?

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y+x.

The sum of the digits of the number is 13. Thus, we have x+y =13

After interchanging the digits, the number becomes10x+y.

The difference between the number obtained by interchanging the digits and the original number is 45. Thus, we have

(10x + y) — (10y + x)= 45

=> 110x+y-10y—x= 45

=> 9x-9y=45

=> 9(x – y) = 45

=>.x-y=5

So, we have two equations

x+y=13

x- y =5

Here x and y are unknowns. We have to solve the above equations for x and y.

Adding the two equations, we have

(x+y)+(x—y)=13+5

=>x+y+x-y=18

=> 2x=18

=> x= 9

Substituting the value of x in the first equation, we have

9+y = 13

=> y = 13 – 9

=> y = 4

Hence, the number is 10 x 4 + 9 = 49

Q.3: A number consists of two digits whose sum is five. When the digits are reversed, the number becomes greater by nine. Find the number.

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y+x.

The sum of the digits of the number is 5. Thus, we have x+ y = 5

After interchanging the digits, the number becomes 10x+ y.

The number obtained by interchanging the digits is greater by 9 from the original number. Thus, we have

10x+ y =10y + x +9

=> 10x + y – 10y – x = 9

=> 9x – 9y = 9

=> 9(x – y) = 9

=> x – y = 1

So, we have two equations

x +y=5

x-y=1

Here x and y are unknowns. We have to solve the above equations for x and y.

Adding the two equations, we have

(x + y)+(x – y) = 5+1

=> x + y + x – y = 5+1

=> 2x = 6

=> x = 6/2

=> x = 3

Substituting the value of x in the first equation, we have

3 + y = 5

=> y = 5-3

=> y = 2

Hence, the number is 10 x 2 + 3 = 23

Q.4: The sum of digits of a two digit number is 15. The number obtained by reversing the order of digits of the given number exceeds the given number by 9. Find the given number.

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y+x.

The sum of the digits of the number is 15. Thus, we have x+ y = 15

After interchanging the digits, the number becomes 10x + y.

The number obtained by interchanging the digits is exceeding by 9 from the original number. Thus, we have

10x+y=10y+x+ 9

=> 10x + y – 10y –x = 9

=>9x- 9y = 9

=>9(x -y) = 9

=> x-y = 9/9

=>x-y = I

So, we have two equations

x+y =15

x-y = I

Here x and y are unknowns. We have to solve the above equations for x and y. Adding the two equations, we have

(x+ y)+(x – y)=15+1

=>x+y+x-y=16

=>2x = 16

=> x = 16/2

=> x = 8

Substituting the value of x in the first equation, we have

8+ y = 5

=> y = 15-8

=> y = 7

Hence, the number is 10 x 7 + 8 = 78

Q.5: The sum of two- digit number and the number formed by reversing the order of digits is 66.If the two digits differ by 2, find the number. How many such numbers are there?

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y+x.

The two digits of the number are differing by 2. Thus, we have x-y=±2

After interchanging the digits, the number becomes 10x + y.

The sum of the numbers obtained by interchanging the digits and the original number is 66. Thus, we have

(10x+ y)+(10y+x)=66

=> 10x+y+10y+x = 66

=> 11x +11y= 66

=> 11(x + y) = 66

=> x + y = 66/11

=> x + y = 6

So, we have two systems of simultaneous equations

x-y = 2,

x +y = 6

x – y = -2,

x + y = 6

Here x and y are unknowns. We have to solve the above systems of equations for x and y.

(i) First, we solve the system

x-y= 2,

x+y=6

Adding the two equations, we have

(x— y)+(x + y) = 2+6

=>x-y+x+y=8

=>2x =8

=> x = 8/2

=> x = 4

Substituting the value of x in the first equation, we have

4-y=2

=>y=4-2

=> y=2

Hence, the number is 10 x 2 +4 = 24

(ii) Now, we solve the system

x-y= -2,

x+y=6

Adding the two equations, we have

(x – y)+(x + y)= -2 + 6

=>x-y+x+y=4

=> 2x =4

=> x = 4/2

=> x=2

Substituting the value of x in the first equation, we have

2-y = -2

=> y=2+2

=> y=4

Hence, the number is 10×4+ 2 = 42

There are two such numbers.

6. The sum of two numbers is 1000 and the difference between their square is 256000. Find the numbers.

Soln: Let the numbers are x and y. One of them must be greater than or equal to the other. Let us assume that x is greater than or equal to y.

The sum of the two numbers is 1000. Thus, we have x+y = 1000

The difference between the squares of the two numbers is 256000. Thus, we have

$x^2 – y^2 = 256000$

=>(x+y)(x-y)= 256000

=>1000(x-y)= 256000

=> x-y = 256000/1000

=>x-y =256

So, we have two equations

x+y =1000

x- y= 256

Here x and y are unknowns. We have to solve the above equations for x and y. Adding the two equations, we have

(x+ y)+(x- y)=1000 + 256

=>x+y+x-y =1256

=> 2x=1256

=> x= 1256/ 2

x=628

Substituting the value of x in the first equation.we have

628+y =1000

=> y= 1000-628

=>y = 372

Hence, the numbers are 628 and 372

7. The sum of a two digit number and the number obtained by reversing the order of its digits is 99.If the digits differ by 3, find the number.

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y + x.

The two digits of the number are differing by 3. Thus, we have x —y = ±3

After interchanging the digits, the number becomes 10x + y.

The sum of the numbers obtained by interchanging the digits and the original number is 99. Thus, we have

(10x+y)+(I0y+x)= 99

=> 10x+y+10y+x = 99

=>11x+11y = 99

=> 11(x + y)= 99

=>x+y = 99/11

=>x = 9

So, we have two systems of simultaneous equations

x-y= 3,

x+y = 9

x-y = -3,

x+y=9

Adding the two equations, we have

(x – y)+(x + y)= 3 + 9

=>x–y+x+y=12

=> 2x =12

=> x= 12/2

=>x =6

Substituting the value of x in the first equation, we have

6–y=3

=>y= 6-3

=> y=3

Hence, the number is 10×3+ 6 =36

(ii) Now, we solve the system

x –y = –3,

x+y=9

Adding the two equations we have

(x – y)+(x + y)= –3 +9

=> x–y+x+y= 6

=> 2x = 6

=>x=3

Substituting the value of x in the first equation, we have

3-y = -3

=>y=3+3

=>y=6

Hence, the number is 10×6+3=63

Note that there are two such numbers.

8. A two- digit number is 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y+x.

The number is 4 times the sum of the two digits. Thus, we have

10y+x = 4(x + y)

=>10y+x = 4x+ 4y

=> 4x +4y -10y -x =0

=> 3x-6y = 0

=> 3(x — 2y) = 0

=>x-2y=0

After interchanging the digits, the number becomes10x+y.

If 18 is added to the number, the digits are reversed. Thus, we have

(10y+x)+18=10x+y

=>10x+y-10y-x=18

=> 9x-9y =18

=> 9(x -y)=18

=>x-y= 18/9

=>x-y =2

So, we have the systems of equations

x- 2y =0,

x-y=2

Here x and y are unknowns. We have to solve the above systems of equations for x and y. Subtracting the first equation from the second, we have

(x- y)- (x – 2y) = 2-0

=> x-y-x+ 2y =2

=> y=2

Substituting the value of y in the first equation, we have

x-2×2=0

=> x-4=0

=> x = 4

Hence, the number is 10x 2 +4 = 24

9. A two-digit number is 3 more than 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y + x.

The number is 3 more than 4 times the sum of the two digits. Thus, we have

10y+x=4(x+y)+3

=>10y+x=4x+4y+3

=> 4x+4y-10y-x=-3

=> 3x – 6y = -3

=> 3(x – 2y) = -3

=>x-2y= -3/3

=>x-2y= -1

Alter interchanging the digits, the number becomes10x+ y.

If 18 is added to the number, the digits are reversed. Thus, we have

(10y+x)+18 =10x+y

=>10x+y-10y-x =18

=>9x-9y =18

=> 9(x — y) =18

=> x-y=18/9

=> x-y= 2

So, we have the systems of equations x-2y=-1,

x-y= 2

Here x and y are unknowns. We have to solve the above systems of equations for x and y. Subtracting the first equation from the second, we have

(x- y)-(x-2y) = 2 – (-1)

=> x- y-x+2y =3

=> y=3

Substituting the value of y in the first equation, we have

x-2×3=-1

=>x-6=-1

=> x=-1+6

=> x =5

Hence the number is 10 x3 + 5 = 35

10. A two-digit number is 4 more than 6 times the sum of its digits. If 18 is subtracted from the number, the digits are reversed. Find the number.

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y+x.

The number is 4 more than 6 times the sum of the two digits. Thus, we have

10y+ x = 6(x + y)+ 4

=> 10y+x=6x+6y+4

=> 6x +6y -10y – x = -4

=> 5x-4y = -4

After interchanging the digits, the number becomes 10x + y.

If 18 is subtracted from the number, the digits are reversed. Thus, we have

(10y+x)-18=10x+y

=> 10x+y-10y-x=-18

=> 9x-9y=-18

=> 9(x -y)= -18

=>x-y = -18/9

=>x-y = -2

So, we have the systems of equations

5x – 4y = -4,

x – y = -2

Here x and y are unknowns. We have to solve the above systems of equations for x and y. Multiplying the second equation by 5 and then subtracting from the first, we have

(5x – 4y)- (5x -5y) = -4 – (-2 x 5)

=> 5x – 4y – 5x + 5y= -4 +10

=> y = 6

Substituting the value of y in the second equation, we have

x-6 = -2

=> x = 6 – 2

=> x = 4

Hence, the number is 10 x 6 + 4 = 64

11. A two-digit number is 4 times the sum of its digits and twice the product of the digits. Find the number.

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y + x.

The number is 4 times the sum of the two digits. Thus, we have

10y+ x = 4(x + y)

=> 10y + x = 4x + 4y

=> 4x +4y -10y – x = 0

=> 3x – 6y = 0

3(x – 2y) = 0

=> x – 2y = 0

=> x = 2y

After interchanging the digits, the number becomes 10x + y.

The number is twice the product of the digits. Thus, we have l0y+x = 2xy

So, we have the systems of equations

x = 2y,

10y + x = 2xy

Here x and y are unknowns. We have to solve the above systems of equations for x and y. Substituting x = 2y in the second equation, we get

10y+2y=2x2yxy

=>12y = $4y^2$

=> $4y^2$ -12y = 0

=> 4y(y -3) = 0

=>y(y — 3) = 0

=> y=0 or y=3

Substituting the value of y in the first equation, we have

y	0	3
x	0	6

Hence, the number is 10 x 3 +6 = 36

Note that the first pair of solution does not give a two digit number

12. A two-digit number is such that the product of its digits is 20. If 9 is added to the number, the digits interchange their places. Find the number.

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y+ x.

The product of the two digits of the number is 20. Thus, we have xy = 20

After interchanging the digits, the number becomes 10x+ y

If 9 is added to the number, the digits interchange their places. Thus, we have

(10y + x)+ 9 =10x + y

=>10y+x+9=10x+y

=> 10x+y-10y-x = 9

=> 9x- 9y = 9

=> 9(x – y) = 9

=> x-y = 9/9

=> x-y=1

So, we have the systems of equations

xy = 20,

x-y=1

Here x and y are unknowns. We have to solve the above systems of equations for x and y.

Substituting x = I + y from the second equation to the first equation, we get

(1+ y) y = 20

=> y +y^2 = 20

=> y^2+y-20=0

=> y^2 +5y-4y -20 = 0

=>y(y + 5)- 4(y + 5) = 0

=>(y+5) (y-4)= 0

=> y=-5 or y=4

Substituting the value of y in the second equation, we have

y	-5	4
x	-4	5

Hence, the number is 10 x 4 +5 = 45

Note that in the first pair of solution the values of x and y are both negative. But the digits of the number can't be negative. So, we must remove this pair.

13. The difference between two numbers is 26 and one number is three times the other. Find them.

Soln: Let the numbers are x and y. One of them must be greater than or equal to the other. Let us assume that x is greater than or equal to y.

The difference between the two numbers is 26. Thus, we have x -y = 26

One of the two numbers is three times the other number. Here, we are assuming that x is greater than or equal to y. Thus, we have x = 3y

So, we have two equations

x-y = 26

x = 3y

Here x and y are unknowns. We have to solve the above equations for x and y.

Substituting x = 3y from the second equation in the first equation, we get

3y – y = 26

=> 2y = 26

=> y = 13

Substituting the value of y in the first equation, we have

x-13 =26

=> x=13+ 26

=> x = 39

Hence.the numbers are 39 and 13.

14. The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Soln: Let the digits at units and tens place of the given number be x and y respectively Thus, the number is 10y+x.

The sum of the two digits of the number is 9.

Thus, we have x + y = 9

After interchanging the digits, the number becomes 10x + y.

Also, 9 times the number is equal to twice the number obtained by reversing the order of the digits. Thus, we have

9(10y+ x) = 2(10x + y)

=> 90y+ 9x = 20x + 2y

=> 20x + 2y – 90y – 9x = 0

=>11x -88y = 0

=>11(x -8y) = 0

=>x-8y=0

So, we have the systems of equations

x+ y = 9,

x – 8y = 0

Here x and y are unknowns. We have to solve the above systems of equations for x and y. Substituting x = 8y from the second equation to the first equation, we get

8y + y = 9

=>9y = 9

=> y =9/9

=> y = 1

Substituting the value of y in the second equation, we have

x-8×1=0

=> x-8 = 0

=> x = 8

Hence, the number is 10 x 1 +8 = 18

15. Seven times a two-digit number is equal to four times the number obtained by reversing the digits. If the difference between the digits is 3. Find the number.

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y+x.

The difference between the two digits of the number is 3. Thus, we have x-y = ±3

After interchanging the digits, the number becomes 10x + y.

Seven times the number is equal to four times the number obtained by reversing the order of the digits. Thus, we have

7(10y + x) = 4(10x+ y)

=> 70y + 7x = 40x + 4y

=> 40x + 4y – 70y-7x = 0

=> 33x – 66y = 0

=>33(x — 2y) = 0

=> x – 2y = 0

So, we have two systems of simultaneous equations

x – y = 3,

x-2y=0

x – y = -3,

x – 2y = 0

Here x and y are unknowns. We have to solve the above systems of equations for x and y.

(i) First, we solve the system

x – y = 3,

x – 2y = 0

Multiplying the first equation by 2 and then subtracting from the second equation, we have

(x -2y)- 2(x – y) = 0 – 2 x 3

=> x – 2y – 2x + 2y = -6

=> -x = -6

=> x =6

Substituting the value of x in the first equation, we have

6-y=3

=>y=6 – 3

=> y=3

Hence, the number is 10 x3 + 6 = 36

(ii) Now, we solve the system

x – y = -3,

x-2y=0

Multiplying the first equation by 2 and then subtracting from the second equation we have

(x -2y)- 2(x – y) = 0 – (-3 x 2)

=> x- 2y – 2x + 2y =6

=> -x = 6

=> x = -6

Substituting the value of x in the first equation, we have

-6 – y = -3

=> y = -6 + 3

=> y = -3

But, the digits of the number can’t be negative. Hence, the second case must be removed.

16. Two numbers are in the ratio 5: 6. If 8 is subtracted from each of the numbers the ratio becomes 4: 5. Find the numbers.

Soln: Let the numbers be 5x and 6x

Now subtracting 8 we get the numbers as

5x – 8 and 6x– 8

Thus, (5x – 8)/ (6x – 8) = 4: 5

By cross multiplying we get,

5(5x – 8) = 4(6x – 8)

=> 25x – 40 = 24x – 32

=> x = 8

Hence, the numbers are

5x = 5 x 8 = 40

6x = 6 x 8 = 48

17. A two-digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number.

Soln: Let the unit digit and ten's digit of the number be x and y respectively.

Therefore the number = 10y + x

Sum of digits = x + y

10y +x = 8(x + y) – 5

10y + x = 8x + 8y – 5

7x – 2y = 5……(1)

Difference of the digits = y – x [if x < y]

10y + x = 16(y – x) + 3

10y + x = 16y – 16x +3

17x – 6y = 3……(2)

Multiply equation (1) and (2) and subtracting equation (2)

21x – 6y = 15

17x – 6y = 3

4x = 12

x = 12/4 = 3

Putting the value of x = 3 in equation (1)

7 x 3 – 2y = 5

2y = 21 – 5

2y = 16

y = 16/2 = 8

Thus the unit digit of the number is 3 and ten's digit is 8.

Therefore the number is 83.

Exercise 3.7

Q.1: The sum of two numbers is 8. If their sum is four times their difference, find the numbers.

Soln: Let the numbers are x and y. One of them must be greater than or equal to the other. Let us assume that x is greater than or equal to y.

The sum of the two numbers is 8. Thus, we have x+y = 8

The sum of the two numbers is four times their difference. Thus, we have

x+y = 4(x— y)

=>x+y= 4x-4y

=>4x — 4y—x—y = 0

=> 3x-5y = 0

So, we have two equations

x+y=8

3x-5y = 0

Here x and y are unknowns.

We have to solve the above equations for x and y.

Multiplying the first equation by 5 and then adding with the second equation, we have

5(x + y)+(3x — 5y) = 5×8+0

=>5x+5y+3x-5y =40

=>8x = 40

=> x = 5

x = Substituting the value of x in the first equation, we have

5+y=8

=>y=8-5

=>y=3

Hence, the numbers are 5 and 3.

Q. 2: The sum of digits of a two digit number is 13. If the number is subtracted from the one obtained by interchanging the digits, the result is 45. What is the number?

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y+x.

The sum of the digits of the number is 13. Thus, we have x+y =13

After interchanging the digits, the number becomes10x+y.

The difference between the number obtained by interchanging the digits and the original number is 45. Thus, we have

(10x + y) — (10y + x)= 45

=> 110x+y-10y—x= 45

=> 9x-9y=45

=> 9(x – y) = 45

=>.x-y=5

So, we have two equations

x+y=13

x- y =5

Here x and y are unknowns. We have to solve the above equations for x and y.

Adding the two equations, we have

(x+y)+(x—y)=13+5

=>x+y+x-y=18

=> 2x=18

=> x= 9

Substituting the value of x in the first equation, we have

9+y = 13

=> y = 13 – 9

=> y = 4

Hence, the number is 10 x 4 + 9 = 49

Q.3: A number consists of two digits whose sum is five. When the digits are reversed, the number becomes greater by nine. Find the number.

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y+x.

The sum of the digits of the number is 5. Thus, we have x+ y = 5

After interchanging the digits, the number becomes 10x+ y.

The number obtained by interchanging the digits is greater by 9 from the original number. Thus, we have

10x+ y =10y + x +9

=> 10x + y – 10y – x = 9

=> 9x – 9y = 9

=> 9(x – y) = 9

=> x – y = 1

So, we have two equations

x +y=5

x-y=1

Here x and y are unknowns. We have to solve the above equations for x and y.

Adding the two equations, we have

(x + y)+(x – y) = 5+1

=> x + y + x – y = 5+1

=> 2x = 6

=> x = 6/2

=> x = 3

Substituting the value of x in the first equation, we have

3 + y = 5

=> y = 5-3

=> y = 2

Hence, the number is 10 x 2 + 3 = 23

Q.4: The sum of digits of a two digit number is 15. The number obtained by reversing the order of digits of the given number exceeds the given number by 9. Find the given number.

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y+x.

The sum of the digits of the number is 15. Thus, we have x+ y = 15

After interchanging the digits, the number becomes 10x + y.

The number obtained by interchanging the digits is exceeding by 9 from the original number. Thus, we have

10x+y=10y+x+ 9

=> 10x + y – 10y –x = 9

=>9x- 9y = 9

=>9(x -y) = 9

=> x-y = 9/9

=>x-y = I

So, we have two equations

x+y =15

x-y = I

Here x and y are unknowns. We have to solve the above equations for x and y. Adding the two equations, we have

(x+ y)+(x – y)=15+1

=>x+y+x-y=16

=>2x = 16

=> x = 16/2

=> x = 8

Substituting the value of x in the first equation, we have

8+ y = 5

=> y = 15-8

=> y = 7

Hence, the number is 10 x 7 + 8 = 78

Q.5: The sum of two- digit number and the number formed by reversing the order of digits is 66.If the two digits differ by 2, find the number. How many such numbers are there?

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y+x.

The two digits of the number are differing by 2. Thus, we have x-y=±2

After interchanging the digits, the number becomes 10x + y.

The sum of the numbers obtained by interchanging the digits and the original number is 66. Thus, we have

(10x+ y)+(10y+x)=66

=> 10x+y+10y+x = 66

=> 11x +11y= 66

=> 11(x + y) = 66

=> x + y = 66/11

=> x + y = 6

So, we have two systems of simultaneous equations

x-y = 2,

x +y = 6

x – y = -2,

x + y = 6

Here x and y are unknowns. We have to solve the above systems of equations for x and y.

(i) First, we solve the system

x-y= 2,

x+y=6

Adding the two equations, we have

(x— y)+(x + y) = 2+6

=>x-y+x+y=8

=>2x =8

=> x = 8/2

=> x = 4

Substituting the value of x in the first equation, we have

4-y=2

=>y=4-2

=> y=2

Hence, the number is 10 x 2 +4 = 24

(ii) Now, we solve the system

x-y= -2,

x+y=6

Adding the two equations, we have

(x – y)+(x + y)= -2 + 6

=>x-y+x+y=4

=> 2x =4

=> x = 4/2

=> x=2

Substituting the value of x in the first equation, we have

2-y = -2

=> y=2+2

=> y=4

Hence, the number is 10×4+ 2 = 42

There are two such numbers.

6. The sum of two numbers is 1000 and the difference between their square is 256000. Find the numbers.

Soln: Let the numbers are x and y. One of them must be greater than or equal to the other. Let us assume that x is greater than or equal to y.

The sum of the two numbers is 1000. Thus, we have x+y = 1000

The difference between the squares of the two numbers is 256000. Thus, we have

$x^2 - y^2 = 256000$

=>(x+y)(x-y)= 256000

=>1000(x-y)= 256000

=> x-y = 256000/1000

=>x-y =256

So, we have two equations

x+y =1000

x- y= 256

Here x and y are unknowns. We have to solve the above equations for x and y. Adding the two equations, we have

(x+ y)+(x- y)=1000 + 256

=>x+y+x-y =1256

=> 2x=1256

=> x= 1256/ 2

x=628

Substituting the value of x in the first equation.we have

628+y =1000

=> y= 1000-628

=>y = 372

Hence, the numbers are 628 and 372

7. The sum of a two digit number and the number obtained by reversing the order of its digits is 99.If the digits differ by 3, find the number.

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y + x.

The two digits of the number are differing by 3. Thus, we have x —y = ±3

After interchanging the digits, the number becomes 10x + y.

The sum of the numbers obtained by interchanging the digits and the original number is 99. Thus, we have

(10x+y)+(l0y+x)= 99

=> 10x+y+10y+x = 99

=>11x+11y = 99

=> 11(x + y)= 99

=>x+y = 99/11

=>x = 9

So, we have two systems of simultaneous equations

x-y= 3,

x+y = 9

x-y = -3,

x+y=9

Adding the two equations, we have

(x – y)+(x + y)= 3 + 9

=>x–y+x+y=12

=> 2x =12

=> x= 12/2

=>x =6

Substituting the value of x in the first equation, we have

6–y=3

=>y= 6-3

=> y=3

Hence, the number is 10×3+ 6 =36

(ii) Now, we solve the system

x –y = –3,

x+y=9

Adding the two equations we have

(x – y)+(x + y)= –3 +9

=> x–y+x+y= 6

=> 2x = 6

=>x=3

Substituting the value of x in the first equation, we have

3-y = -3

=>y=3+3

=>y=6

Hence, the number is 10×6+3=63

Note that there are two such numbers.

8. A two- digit number is 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y+x.

The number is 4 times the sum of the two digits. Thus, we have

10y+x = 4(x + y)

=>10y+x = 4x+ 4y

=> 4x +4y -10y -x =0

=> 3x-6y = 0

=> 3(x — 2y) = 0

=>x-2y=0

After interchanging the digits, the number becomes10x+y.

If 18 is added to the number, the digits are reversed. Thus, we have

(10y+x)+18=10x+y

=>10x+y-10y-x=18

=> 9x-9y =18

=> 9(x -y)=18

=>x-y= 18/9

=>x-y =2

So, we have the systems of equations

x- 2y =0,

x-y=2

Here x and y are unknowns. We have to solve the above systems of equations for x and y. Subtracting the first equation from the second, we have

(x- y)- (x – 2y) = 2-0

=> x-y-x+ 2y =2

=> y=2

Substituting the value of y in the first equation, we have

x-2×2=0

=> x-4=0

=> x = 4

Hence, the number is 10x 2 +4 = 24

9. A two-digit number is 3 more than 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y + x.

The number is 3 more than 4 times the sum of the two digits. Thus, we have

10y+x=4(x+y)+3

=>10y+x=4x+4y+3

=> 4x+4y-10y-x=-3

=> 3x – 6y = -3

=> 3(x – 2y) = -3

=>x-2y= -3/3

=>x-2y= -1

Alter interchanging the digits, the number becomes10x+ y.

If 18 is added to the number, the digits are reversed. Thus, we have

(10y+x)+18 =10x+y

=>10x+y-10y-x =18

=>9x-9y =18

=> 9(x — y) =18

=> x-y=18/9

=> x-y= 2

So, we have the systems of equations x-2y=-1,

x-y= 2

Here x and y are unknowns. We have to solve the above systems of equations for x and y. Subtracting the first equation from the second, we have

(x- y)-(x-2y) = 2 – (-1)

=> x- y-x+2y =3

=> y=3

Substituting the value of y in the first equation, we have

x-2×3=-1

=>x-6=-1

=> x=-1+6

=> x =5

Hence the number is 10 x3 + 5 = 35

10. A two-digit number is 4 more than 6 times the sum of its digits. If 18 is subtracted from the number, the digits are reversed. Find the number.

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y+x.

The number is 4 more than 6 times the sum of the two digits. Thus, we have

$10y+ x = 6(x + y)+ 4$

$\Rightarrow 10y+x=6x+6y+4$

$\Rightarrow 6x +6y -10y – x = -4$

$\Rightarrow 5x-4y = -4$

After interchanging the digits, the number becomes 10x + y.

If 18 is subtracted from the number, the digits are reversed. Thus, we have

$(10y+x)-18=10x+y$

$\Rightarrow 10x+y-10y-x=-18$

$\Rightarrow 9x-9y=-18$

$\Rightarrow 9(x -y)= -18$

$\Rightarrow x-y = -18/9$

$\Rightarrow x-y = -2$

So, we have the systems of equations

$5x – 4y = -4,$

$x – y = -2$

Here x and y are unknowns. We have to solve the above systems of equations for x and y. Multiplying the second equation by 5 and then subtracting from the first, we have

$(5x – 4y)- (5x -5y) = -4 – (-2 \times 5)$

$\Rightarrow 5x – 4y – 5x + 5y= -4 +10$

$\Rightarrow y = 6$

Substituting the value of y in the second equation, we have

$x-6 = -2$

$\Rightarrow x = 6 – 2$

$\Rightarrow x = 4$

Hence, the number is $10 \times 6 + 4 = 64$

11. A two-digit number is 4 times the sum of its digits and twice the product of the digits. Find the number.

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y + x.

The number is 4 times the sum of the two digits. Thus, we have

$10y+ x = 4(x + y)$

$\Rightarrow 10y + x = 4x + 4y$

$\Rightarrow 4x +4y -10y – x = 0$

$\Rightarrow 3x – 6y = 0$

$3(x – 2y) = 0$

$\Rightarrow x – 2y = 0$

$\Rightarrow x = 2y$

After interchanging the digits, the number becomes 10x + y.

The number is twice the product of the digits. Thus, we have $10y+x = 2xy$

So, we have the systems of equations

$x = 2y,$

$10y + x = 2xy$

Here x and y are unknowns. We have to solve the above systems of equations for x and y. Substituting x = 2y in the second equation, we get

$10y+2y=2x2yxy$

$\Rightarrow 12y = 4y^2$

$\Rightarrow 4y^2 -12y = 0$

$\Rightarrow 4y(y -3) = 0$

$\Rightarrow y(y — 3) = 0$

$\Rightarrow y=0$ or $y=3$

Substituting the value of y in the first equation, we have

y	0	3
x	0	6

Hence, the number is 10 x 3 +6 = 36

Note that the first pair of solution does not give a two digit number

12. A two-digit number is such that the product of its digits is 20. If 9 is added to the number, the digits interchange their places. Find the number.

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y+ x.

The product of the two digits of the number is 20. Thus, we have xy = 20

After interchanging the digits, the number becomes 10x+ y

If 9 is added to the number, the digits interchange their places. Thus, we have

(10y + x)+ 9 =10x + y

=>10y+x+9=10x+y

=> 10x+y-10y-x = 9

=> 9x- 9y = 9

=> 9(x – y) = 9

=> x-y = 9/9

=> x-y=1

So, we have the systems of equations

xy = 20,

x-y=1

Here x and y are unknowns. We have to solve the above systems of equations for x and y.

Substituting x = I + y from the second equation to the first equation, we get

(1+ y) y = 20

=> y +y^2 = 20

=> y^2+y-20=0

=> y^2 +5y-4y -20 = 0

=>y(y + 5)- 4(y + 5) = 0

=>(y+5) (y-4)= 0

=> y=-5 or y=4

Substituting the value of y in the second equation, we have

y	-5	4
x	-4	5

Hence, the number is 10 x 4 +5 = 45

Note that in the first pair of solution the values of x and y are both negative. But the digits of the number can't be negative. So, we must remove this pair.

13. The difference between two numbers is 26 and one number is three times the other. Find them.

Soln: Let the numbers are x and y. One of them must be greater than or equal to the other. Let us assume that x is greater than or equal to y.

The difference between the two numbers is 26. Thus, we have x -y = 26

One of the two numbers is three times the other number. Here, we are assuming that x is greater than or equal to y. Thus, we have x = 3y

So, we have two equations

x-y = 26

x = 3y

Here x and y are unknowns. We have to solve the above equations for x and y.

Substituting x = 3y from the second equation in the first equation, we get

3y – y = 26

=> 2y = 26

=> y = 13

Substituting the value of y in the first equation, we have

x-13 =26

=> x=13+ 26

=> x = 39

Hence.the numbers are 39 and 13.

14. The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Soln: Let the digits at units and tens place of the given number be x and y respectively Thus, the number is 10y+x.

The sum of the two digits of the number is 9.

Thus, we have x + y = 9

After interchanging the digits, the number becomes 10x + y.

Also, 9 times the number is equal to twice the number obtained by reversing the order of the digits. Thus, we have

9(10y+ x) = 2(10x + y)

=> 90y+ 9x = 20x + 2y

=> 20x + 2y – 90y – 9x = 0

=>11x -88y = 0

=>11(x -8y) = 0

=>x-8y=0

So, we have the systems of equations

x+ y = 9,

x – 8y = 0

Here x and y are unknowns. We have to solve the above systems of equations for x and y. Substituting x = 8y from the second equation to the first equation, we get

8y + y = 9

=>9y = 9

=> y =9/9

=> y = 1

Substituting the value of y in the second equation, we have

x-8×1=0

=> x-8 = 0

=> x = 8

Hence, the number is 10 x 1 +8 = 18

15. Seven times a two-digit number is equal to four times the number obtained by reversing the digits. If the difference between the digits is 3. Find the number.

Soln: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y+x.

The difference between the two digits of the number is 3. Thus, we have x-y = ±3

After interchanging the digits, the number becomes 10x + y.

Seven times the number is equal to four times the number obtained by reversing the order of the digits. Thus, we have

7(10y + x) = 4(10x+ y)

=> 70y + 7x = 40x + 4y

=> 40x + 4y – 70y-7x = 0

=> 33x – 66y = 0

=>33(x — 2y) = 0

=> x – 2y = 0

So, we have two systems of simultaneous equations

x – y = 3,

x-2y=0

x – y = -3,

x – 2y = 0

Here x and y are unknowns. We have to solve the above systems of equations for x and y.

(i) First, we solve the system

x – y = 3,

x – 2y = 0

Multiplying the first equation by 2 and then subtracting from the second equation, we have

(x -2y)- 2(x – y) = 0 – 2 x 3

=> x – 2y – 2x + 2y = -6

=> -x = -6

=> x =6

Substituting the value of x in the first equation, we have

6-y=3

=>y=6 – 3

=> y=3

Hence, the number is 10 x3 + 6 = 36

(ii) Now, we solve the system

x – y = -3,

x-2y=0

Multiplying the first equation by 2 and then subtracting from the second equation we have

(x -2y)- 2(x – y) = 0 – (-3 x 2)

=> x- 2y – 2x + 2y =6

=> -x = 6

=> x = -6

Substituting the value of x in the first equation, we have

-6 – y = -3

=> y = -6 + 3

=> y = -3

But, the digits of the number can't be negative. Hence, the second case must be removed.

16. Two numbers are in the ratio 5: 6. If 8 is subtracted from each of the numbers the ratio becomes 4: 5. Find the numbers.

Soln: Let the numbers be 5x and 6x

Now subtracting 8 we get the numbers as

5x – 8 and 6x– 8

Thus, (5x – 8)/ (6x – 8) = 4: 5

By cross multiplying we get,

5(5x – 8) = 4(6x – 8)

=> 25x – 40 = 24x – 32

=> x = 8

Hence, the numbers are

5x = 5 x 8 = 40

6x = 6 x 8 = 48

17. A two-digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number.

Soln: Let the unit digit and ten's digit of the number be x and y respectively.

Therefore the number = 10y + x

Sum of digits = x + y

10y +x = 8(x + y) – 5

10y + x = 8x + 8y – 5

7x – 2y = 5……(1)

Difference of the digits = y – x [if x < y]

10y + x = 16(y – x) + 3

10y + x = 16y – 16x +3

17x – 6y = 3……(2)

Multiply equation (1) and (2) and subtracting equation (2)

21x – 6y = 15

17x – 6y = 3

4x = 12

x = 12/4 = 3

Putting the value of x = 3 in equation (1)

7 x 3 – 2y = 5

2y = 21 – 5

2y = 16

y = 16/2 = 8

Thus the unit digit of the number is 3 and ten's digit is 8.

Therefore the number is 83.