

In each of the following systems of equation determine whether the system has a unique solution, no solution or infinite solutions. In case there is a unique solution, find it from 1 to 4:

$$(1) \quad x-3y-3=0 \quad x-3y-3=0$$

$$3x-9y-2=0 \quad 3x-9y-2=0$$

Soln:

The given system may be written as

$$x-3y-3=0 \quad x-3y-3=0 \quad 3x-9y-2=0 \quad 3x-9y-2=0$$

The given system of equation is of the form

$$a_1x+b_1y-c_1=0 \quad a_1x+b_1y-c_1=0 \quad a_2x+b_2y-c_2=0 \quad a_2x+b_2y-c_2=0$$

$$\text{Where, } a_1=1, b_1=-3, c_1=-3 \quad a_1=1, b_1=-3, c_1=-3$$

$$a_2=3, b_2=-9, c_2=-2 \quad a_2=3, b_2=-9, c_2=-2$$

We have,

$$a_1a_2=13 \quad \frac{a_1}{a_2} = \frac{1}{3} \quad b_1b_2=-3-9=13 \quad \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}$$

$$\text{And, } c_1c_2=-3-2=-5 \quad \frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$$

$$a_1a_2 \neq b_1b_2 \neq c_1c_2 \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, the given equation has no solution.

$$(2) \quad 2x+y-5=0 \quad 2x+y-5=0$$

$$4x+2y-10=0 \quad 4x+2y-10=0$$

Soln:

The given system may be written as

$$2x+y-5=0 \quad 2x+y-5=0 \quad 4x+2y-10=0 \quad 4x+2y-10=0$$

The given system of equation is of the form

$$a_1x+b_1y-c_1=0 \quad a_1x+b_1y-c_1=0 \quad a_2x+b_2y-c_2=0 \quad a_2x+b_2y-c_2=0$$

$$\text{Where, } a_1=2, b_1=1, c_1=-5 \quad a_1=2, b_1=1, c_1=-5$$

$$a_2=4, b_2=2, c_2=-10 \quad a_2=4, b_2=2, c_2=-10$$

We have,

$$a_1a_2 = 24 = 12 \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2} \quad b_1b_2 = 12 \frac{b_1}{b_2} = \frac{1}{2}$$

$$\text{And, } c_1c_2 = -5-10 = 12 \frac{c_1}{c_2} = \frac{-5}{-10} = \frac{1}{2}$$

$$\text{So, } a_1a_2 = b_1b_2 = c_1c_2 \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, the given equation has infinitely many solution.

$$(3) \quad 3x - 5y = 20$$

$$6x - 10y = 40$$

Soln:

The given system may be written as

$$3x - 5y = 20 \quad 6x - 10y = 40$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0$$

$$\text{Where, } a_1 = 3, b_1 = -5, c_1 = -20$$

$$a_2 = 6, b_2 = -10, c_2 = -40$$

We have,

$$a_1a_2 = 36 = 12 \frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2} \quad b_1b_2 = -5-10 = 12 \frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}$$

$$\text{And, } c_1c_2 = -20-40 = 12 \frac{c_1}{c_2} = \frac{-20}{-40} = \frac{1}{2}$$

$$\text{So, } a_1a_2 = b_1b_2 = c_1c_2 \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, the given equation has infinitely many solution.

$$(4) \quad x - 2y - 8 = 0$$

$$5x - 10y - 10 = 0$$

Soln:

The given system may be written as

$$x - 2y - 8 = 0 \quad 5x - 10y - 10 = 0$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0$$

$$\text{Where, } a_1 = 1, b_1 = -2, c_1 = -8$$

$$a_2=5, b_2=-10, c_2=-10 \quad a_2 = 5, b_2 = -10, c_2 = -10$$

We have,

$$a_1 a_2 = 15 \frac{a_1}{a_2} = \frac{1}{5} \quad b_1 b_2 = -2 \cdot -10 = 20 \quad \frac{b_1}{b_2} = \frac{-2}{-10} = \frac{1}{5}$$

$$\text{And, } c_1 c_2 = -8 \cdot -10 = 80 \quad \frac{c_1}{c_2} = \frac{-8}{-10} = \frac{4}{5}$$

$$a_1 a_2 = b_1 b_2 \neq c_1 c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, the given equation has no solution.

Find the value of k for each of the following system of equations which have a unique solution (5-8)

(5) $kx+2y-5=0$ $kx + 2y - 5 = 0$

$3x+y-1=0$ $3x + y - 1 = 0$

Soln:

The given system may be written as

$$kx+2y-5=0 \quad kx + 2y - 5 = 0 \quad 3x+y-1=0 \quad 3x + y - 1 = 0$$

The given system of equation is of the form

$$a_1x+b_1y-c_1=0 \quad a_1x + b_1y - c_1 = 0 \quad a_2x+b_2y-c_2=0 \quad a_2x + b_2y - c_2 = 0$$

Where, $a_1=k, b_1=2, c_1=-5$ $a_1 = k, b_1 = 2, c_1 = -5$

$a_2=3, b_2=1, c_2=-1$ $a_2 = 3, b_2 = 1, c_2 = -1$

For unique solution, we have

$$a_1 a_2 \neq b_1 b_2 \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad k \neq 2 \cdot 1 \quad \frac{k}{3} \neq \frac{2}{1} \Rightarrow k \neq 6 \Rightarrow k \neq 6$$

Therefore, the given system will have unique solution for all real values of k other than 6.

(6) $4x+ky+8=0$ $4x + ky + 8 = 0$

$2x+2y+2=0$ $2x + 2y + 2 = 0$

Soln:

The given system may be written as

$$4x+ky+8=0 \quad 4x + ky + 8 = 0 \quad 2x+2y+2=0 \quad 2x + 2y + 2 = 0$$

The given system of equation is of the form

$$a_1x+b_1y-c_1=0 \quad a_1x + b_1y - c_1 = 0 \quad a_2x+b_2y-c_2=0 \quad a_2x + b_2y - c_2 = 0$$

Where, $a_1=4, b_1=k, c_1=8$
 $a_1 = 4, b_1 = k, c_1 = 8$

$a_2=2, b_2=2, c_2=2$
 $a_2 = 2, b_2 = 2, c_2 = 2$

For unique solution, we have

$$a_1 a_2 \neq b_1 b_2 \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{4}{2} \neq \frac{k}{2} \Rightarrow k \neq 4$$

Therefore, the given system will have unique solution for all real values of k other than 4.

(7) $4x - 5y = k$

$2x - 3y = 12$

Soln:

The given system may be written as

$$4x - 5y - k = 0 \quad 2x - 3y - 12 = 0$$

The given system of equation is of the form

$$a_1 x + b_1 y - c_1 = 0 \quad a_2 x + b_2 y - c_2 = 0$$

Where, $a_1=4, b_1=-5, c_1=-k$
 $a_1 = 4, b_1 = -5, c_1 = -k$

$a_2=2, b_2=-3, c_2=-12$
 $a_2 = 2, b_2 = -3, c_2 = -12$

For unique solution, we have

$$a_1 a_2 \neq b_1 b_2 \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{4}{2} \neq \frac{-5}{-3}$$

$\Rightarrow k \Rightarrow k$ can have any real values.

Therefore, the given system will have unique solution for all real values of k .

(8) $x + 2y = 3$

$5x + ky + 7 = 0$

Soln:

The given system may be written as

$$x + 2y = 3 \quad 5x + ky + 7 = 0$$

The given system of equation is of the form

$$a_1 x + b_1 y - c_1 = 0 \quad a_2 x + b_2 y - c_2 = 0$$

Where, $a_1=1, b_1=2, c_1=-3$
 $a_1 = 1, b_1 = 2, c_1 = -3$

$a_2=5, b_2=k, c_2=7$
 $a_2 = 5, b_2 = k, c_2 = 7$

For unique solution, we have

$$a_1 a_2 \neq b_1 b_2 \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow 15 \neq 2k \Rightarrow \frac{1}{5} \neq \frac{2}{k} \Rightarrow k \neq 10 \Rightarrow k \neq 10$$

Therefore, the given system will have unique solution for all real values of k other than 10.

**Find the value of k for which each of the following system of equations having infinitely many solution:
(9-19)**

(9) $2x+3y-5=0$ $2x + 3y - 5 = 0$

$6x-ky-15=0$ $6x - ky - 15 = 0$

Soln:

The given system may be written as

$$2x+3y-5=0 \quad 2x + 3y - 5 = 0 \quad 6x-ky-15=0 \quad 6x - ky - 15 = 0$$

The given system of equation is of the form

$$a_1x+b_1y-c_1=0 \quad a_1x + b_1y - c_1 = 0 \quad a_2x+b_2y-c_2=0 \quad a_2x + b_2y - c_2 = 0$$

Where, $a_1=2, b_1=3, c_1=-5$ $a_1 = 2, b_1 = 3, c_1 = -5$

$a_2=6, b_2=k, c_2=-15$ $a_2 = 6, b_2 = k, c_2 = -15$

For unique solution, we have

$$a_1 a_2 = b_1 b_2 = c_1 c_2 \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow 26 \neq 3k \Rightarrow \frac{2}{6} \neq \frac{3}{k} \Rightarrow k=9 \Rightarrow k = 9$$

Therefore, the given system of equation will have infinitely many solutions, if $k=9$.

(10) $4x+5y=3$ $4x + 5y = 3$

$kx+15y=9$ $kx + 15y = 9$

Soln:

The given system may be written as

$$4x+5y=3 \quad 4x + 5y = 3 \quad kx+15y=9 \quad kx + 15y = 9$$

The given system of equation is of the form

$$a_1x+b_1y-c_1=0 \quad a_1x + b_1y - c_1 = 0 \quad a_2x+b_2y-c_2=0 \quad a_2x + b_2y - c_2 = 0$$

Where, $a_1=4, b_1=5, c_1=3$ $a_1 = 4, b_1 = 5, c_1 = 3$

$a_2=k, b_2=15, c_2=9$ $a_2 = k, b_2 = 15, c_2 = 9$

For unique solution, we have

$$a_1 a_2 = b_1 b_2 = c_1 c_2 \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow 4k = 15 \cdot 3 = -3 \cdot 9 \Rightarrow \frac{4}{k} = \frac{5}{15} = \frac{-3}{-9} \Rightarrow 4k = 13 \Rightarrow \frac{4}{k} = \frac{1}{3} \Rightarrow k \neq 12 \Rightarrow k \neq 12$$

Therefore, the given system will have infinitely many solutions if $k=12$.

$$(11) \quad kx - 2y + 6 = 0$$

$$4x + 3y + 9 = 0$$

Soln:

The given system may be written as

$$kx - 2y + 6 = 0 \quad 4x + 3y + 9 = 0$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0$$

$$\text{Where, } a_1 = k, b_1 = -2, c_1 = 6$$

$$a_2 = 4, b_2 = -3, c_2 = 9$$

For unique solution, we have

$$a_1a_2 = b_1b_2 = c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad k \cdot 4 = -2 \cdot -3 = 6 \quad \frac{k}{4} = \frac{-2}{-3} = \frac{2}{3} \Rightarrow k = 8 \Rightarrow k = \frac{8}{3}$$

Therefore, the given system of equations will have infinitely many solutions, if $k = 8$.

$$(12) \quad 8x + 5y = 9$$

$$kx + 10y = 19$$

Soln:

The given system may be written as

$$8x + 5y = 9 \quad kx + 10y = 19$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0$$

$$\text{Where, } a_1 = 8, b_1 = 5, c_1 = -9$$

$$a_2 = k, b_2 = 10, c_2 = -18$$

For unique solution, we have

$$a_1a_2 = b_1b_2 = c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad 8k = 5 \cdot 10 = -9 \cdot -18 = 162 \quad \frac{8}{k} = \frac{5}{10} = \frac{-9}{-18} = \frac{1}{2} \Rightarrow k = 16 \Rightarrow k = 16$$

Therefore, the given system of equations will have infinitely many solutions, if $k = 16$.

$$(13) \quad 2x - 3y = 7$$

$$(k+2)x - (2k+1)y = 3(2k-1) \quad (k+2)x - (2k+1)y = 3(2k-1)$$

Soln:

The given system may be written as

$$2x - 3y = 7 \quad 2x - 3y = 7 \quad (k+2)x - (2k+1)y = 3(2k-1) \quad (k+2)x - (2k+1)y = 3(2k-1)$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0 \quad a_2x + b_2y - c_2 = 0$$

$$\text{Where, } a_1 = 2, b_1 = -3, c_1 = -7 \quad a_1 = 2, b_1 = -3, c_1 = -7$$

$$a_2 = k, b_2 = -(2k+1), c_2 = -3(2k-1) \quad a_2 = k, b_2 = -(2k+1), c_2 = -3(2k-1)$$

For unique solution, we have

$$a_1a_2 = b_1b_2 = c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \frac{2}{k+2} = \frac{-3}{-(2k+1)} = \frac{-7}{-3(2k-1)} \quad \frac{2}{k+2} = \frac{-3}{-(2k+1)} \text{ and } \frac{-7}{-3(2k-1)} = \frac{-3}{-(2k+1)}$$

$$\Rightarrow 2(2k+1) = 3(k+2) \text{ and } 3 \times 3(2k-1) = 7(2k+1)$$

$$\Rightarrow 4k+2 = 3k+6 \text{ and } 18k-9 = 14k+7 \quad \Rightarrow k=4 \text{ and } 4k=16 \Rightarrow k=4 \Rightarrow k=4 \text{ and } 4k=16 \Rightarrow k=4$$

Therefore, the given system of equations will have infinitely many solutions, if $k=4$.

$$(14) \quad 2x + 3y = 2 \quad 2x + 3y = 2$$

$$(k+2)x + (2k+1)y = 2(k-1) \quad (k+2)x + (2k+1)y = 2(k-1)$$

Soln:

The given system may be written as

$$2x + 3y = 2 \quad 2x + 3y = 2 \quad (k+2)x + (2k+1)y = 2(k-1) \quad (k+2)x + (2k+1)y = 2(k-1)$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0 \quad a_2x + b_2y - c_2 = 0$$

$$\text{Where, } a_1 = 2, b_1 = 3, c_1 = -2 \quad a_1 = 2, b_1 = 3, c_1 = -2$$

$$a_2 = (k+2), b_2 = (2k+1), c_2 = -2(k-1) \quad a_2 = (k+2), b_2 = (2k+1), c_2 = -2(k-1)$$

For unique solution, we have

$$a_1a_2 = b_1b_2 = c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \frac{2}{k+2} = \frac{3}{2k+1} = \frac{-2}{-2(k-1)} \quad \frac{2}{k+2} = \frac{3}{2k+1} \text{ and } \frac{-2}{-2(k-1)} = \frac{3}{2k+1}$$

$$\frac{2}{k+2} = \frac{3}{2k+1} \text{ and } \frac{3}{2k+1} = \frac{2}{2(k-1)} \Rightarrow 2(2k+1) = 3(k+2) \text{ and } 3(k-1) = 2(k+1)$$

$$\Rightarrow 4k+2 = 3k+6 \text{ and } 3k-3 = 2k+1 \quad \Rightarrow k=4 \text{ and } k=4 \Rightarrow k=4 \text{ and } k=4$$

Therefore, the given system of equations will have infinitely many solutions, if $k=4$ $k = 4$.

$$(15) \quad x+(k+1)y=4 \quad x+(k+1)y = 4$$

$$(k+1)x+9y=(5k+2) \quad (k+1)x+9y = (5k+2)$$

Soln:

The given system may be written as

$$x+(k+1)y=4 \quad x+(k+1)y = 4 \quad (k+1)x+9y=(5k+2) \quad (k+1)x+9y = (5k+2)$$

The given system of equation is of the form

$$a_1x+b_1y-c_1=0 \quad a_1x+b_1y-c_1 = 0 \quad a_2x+b_2y-c_2=0 \quad a_2x+b_2y-c_2 = 0$$

$$\text{Where, } a_1=1, b_1=(k+1), c_1=-4 \quad a_1 = 1, b_1 = (k+1), c_1 = -4$$

$$a_2=(k+1), b_2=9, c_2=-(5k+2) \quad a_2 = (k+1), b_2 = 9, c_2 = -(5k+2)$$

For unique solution, we have

$$a_1a_2 = b_1b_2 = c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad 1 \cdot (k+1) = (k+1) \cdot 9 = -4 \cdot -(5k+2) \quad \frac{1}{k+1} = \frac{(k+1)}{9} = \frac{-4}{-(5k+2)} \quad 1 \cdot (k+1) = (k+1) \cdot 9 \text{ and } (k+1) \cdot 9 = 45k+2$$

$$\frac{1}{k+1} = \frac{k+1}{9} \text{ and } \frac{k+1}{9} = \frac{4}{5k+2} \Rightarrow 9=(k+1)^2 \text{ and } (k+1)(5k+2)=36$$

$$\Rightarrow 9 = (k+1)^2 \text{ and } (k+1)(5k+2) = 36 \Rightarrow 9=k^2+2k+1 \text{ and } 5k^2+2k+5k+2=36$$

$$\Rightarrow 9 = k^2 + 2k + 1 \text{ and } 5k^2 + 2k + 5k + 2 = 36 \Rightarrow k^2+2k-8=0 \text{ and } 5k^2+7k-34=0$$

$$\Rightarrow k^2 + 2k - 8 = 0 \text{ and } 5k^2 + 7k - 34 = 0 \Rightarrow k^2+4k-2k-8=0 \text{ and } 5k^2+17k-10k-34=0$$

$$\Rightarrow k^2 + 4k - 2k - 8 = 0 \text{ and } 5k^2 + 17k - 10k - 34 = 0 \Rightarrow k(k+4)-2(k+4)=0 \text{ and } (5k+17)-2(5k+17)=0$$

$$\Rightarrow k(k+4)-2(k+4)=0 \text{ and } (5k+17)-2(5k+17)=0 \Rightarrow (k+4)(k-2)=0 \text{ and } (5k+17)(k-2)=0$$

$$\Rightarrow (k+4)(k-2)=0 \text{ and } (5k+17)(k-2)=0 \Rightarrow k=-4 \text{ or } k=2 \text{ and } k=-17/5 \text{ or } k=2$$

$$\Rightarrow k = -4 \text{ or } k = 2 \text{ and } k = \frac{-17}{5} \text{ or } k = 2$$

thus, $k=2$ satisfies both the condition.

Therefore, the given system of equations will have infinitely many solutions, if $k=2$ $k = 2$.

$$(16) \quad kx+3y=2k+1 \quad kx+3y = 2k+1$$

$$2(k+1)x+9y=(7k+1) \quad 2(k+1)x+9y = (7k+1)$$

Soln:

The given system may be written as

$$kx+3y=2k+1 \quad kx+3y = 2k+1 \quad 2(k+1)x+9y=(7k+1) \quad 2(k+1)x+9y = (7k+1)$$

The given system of equation is of the form

$$a_1x+b_1y-c_1=0 \quad a_1x+b_1y-c_1 = 0 \quad a_2x+b_2y-c_2=0 \quad a_2x+b_2y-c_2 = 0$$

$$\text{Where, } a_1=k, b_1=3, c_1=-(2k+1) \quad a_1 = k, b_1 = 3, c_1 = -(2k+1)$$

$$a_2=2(k+1), b_2=9, c_2=-(7k+1) \quad a_2 = 2(k+1), b_2 = 9, c_2 = -(7k+1)$$

For unique solution, we have

$$a_1 a_2 = b_1 b_2 = c_1 c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad 12(k+2) = 39 = -(2k+1) - (7k+1) \frac{1}{2(k+2)} = \frac{3}{9} = \frac{-(2k+1)}{-(7k+1)} \quad 12(k+2) = 39 \text{ and } 39 = (2k+1)$$

$$(7k+1) \frac{1}{2(k+2)} = \frac{3}{9} \text{ and } \frac{3}{9} = \frac{(2k+1)}{(7k+1)} \Rightarrow 9k = 3 \times 2(k+1) \text{ and } 3(7k+1) = 9(2k+1)$$

$$\Rightarrow 9k = 3 \times 2(k+1) \text{ and } 3(7k+1) = 9(2k+1) \quad \Rightarrow 9k - 6k = 6 \text{ and } 21k - 18k = 9 - 3$$

$$\Rightarrow 9k - 6k = 6 \text{ and } 21k - 18k = 9 - 3 \quad \Rightarrow 3k = 6 \text{ and } 3k = 6 \Rightarrow 3k = 6 \text{ and } 3k = 6 \quad \Rightarrow k = 2 \text{ and } k = 2$$

$$\Rightarrow k = 2 \text{ and } k = 2$$

Therefore, the given system of equations will have infinitely many solutions, if $k=2$.

$$(17) \quad 2x + (k-2)y = k \quad 2x + (k-2)y = k$$

$$6x + (2k-1)y = (2k+5) \quad 6x + (2k-1)y = (2k+5)$$

Soln:

The given system may be written as

$$2x + (k-2)y = k \quad 2x + (k-2)y = k \quad 6x + (2k-1)y = (2k+5) \quad 6x + (2k-1)y = (2k+5)$$

The given system of equation is of the form

$$a_1 x + b_1 y - c_1 = 0 \quad a_1 x + b_1 y - c_1 = 0 \quad a_2 x + b_2 y - c_2 = 0 \quad a_2 x + b_2 y - c_2 = 0$$

$$\text{Where, } a_1 = 2, b_1 = (k-2), c_1 = -k \quad a_1 = 2, b_1 = (k-2), c_1 = -k$$

$$a_2 = 6, b_2 = (2k-1), c_2 = -(2k+5) \quad a_2 = 6, b_2 = (2k-1), c_2 = -(2k+5)$$

For unique solution, we have

$$a_1 a_2 = b_1 b_2 = c_1 c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad 26 = k - 22k - 1 = -k - 2(2k+5) \quad \frac{2}{6} = \frac{k-2}{2k-1} = \frac{-k}{-(2k+5)} \quad 26 = k - 22k - 1 \text{ and } k - 22k - 1 = k(2k+5)$$

$$\frac{2}{6} = \frac{k-2}{2k-1} \text{ and } \frac{k-2}{2k-1} = \frac{k}{(2k+5)} \quad 13 = k - 22k - 1 \text{ and } 2k^2 + 5k - 4k - 10 = 2k^2 - k$$

$$\frac{1}{3} = \frac{k-2}{2k-1} \text{ and } 2k^2 + 5k - 4k - 10 = 2k^2 - k \quad \Rightarrow 2k - 3k = -6 + 1 \text{ and } k + k = 10$$

$$\Rightarrow 2k - 3k = -6 + 1 \text{ and } k + k = 10 \quad \Rightarrow -k = -5 \text{ and } 2k = 10 \Rightarrow -k = -5 \text{ and } 2k = 10 \quad \Rightarrow k = 5 \text{ and } k = 5$$

$$\Rightarrow k = 5 \text{ and } k = 5$$

Therefore, the given system of equations will have infinitely many solutions, if $k=5$.

$$(18) \quad 2x + 3y = 7 \quad 2x + 3y = 7$$

$$(k+1)x + (2k-1)y = (4k+1) \quad (k+1)x + (2k-1)y = (4k+1)$$

Soln:

The given system may be written as

$$2x + 3y = 7 \quad 2x + 3y = 7 \quad (k+1)x + (2k-1)y = (4k+1) \quad (k+1)x + (2k-1)y = (4k+1)$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0$$

Where, $a_1=2, b_1=3, c_1=-7$

$$a_2=k+1, b_2=2k-1, c_2=-(4k+1)$$

For unique solution, we have

$$a_1a_2 = b_1b_2 = c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad 2k+1 = 3(2k-1) = -7-(4k+1) \quad \frac{2}{k+1} = \frac{3}{2k-1} = \frac{-7}{-(4k+1)} \quad 2k+1 = 3(2k-1) \text{ and } 3(2k-1) = 7(4k+1)$$

$$\frac{2}{k+1} = \frac{3}{2k-1} \text{ and } \frac{3}{2k-1} = \frac{7}{(4k+1)} \quad \text{Extra close brace or missing open brace}$$

$$\Rightarrow 4k-2=3k+3 \text{ and } 12k+3=14k-7$$

$$\Rightarrow 4k-2 = 3k+3 \text{ and } 12k+3 = 14k-7 \quad \Rightarrow k=5 \text{ and } 2k=10 \Rightarrow k=5 \text{ and } k=5$$

$$\Rightarrow k=5 \text{ and } k=5$$

Therefore, the given system of equations will have infinitely many solutions, if $k=5$.

(19) $2x+3y=k$

$$(k-1)x + (k+2)y = 3k$$

Soln:

The given system may be written as

$$2x+3y=k \quad (k-1)x + (k+2)y = 3k$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0$$

Where, $a_1=2, b_1=3, c_1=-k$

$$a_2=k-1, b_2=k+2, c_2=-3k$$

For unique solution, we have

$$a_1a_2 = b_1b_2 = c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad 2k-1 = 3(k+2) = -k-3k \quad \frac{2}{k-1} = \frac{3}{k+2} = \frac{-k}{-3k} \quad 2k-1 = 3(k+2) \text{ and } 3(k+2) = -k-3k$$

$$\frac{2}{k-1} = \frac{3}{k+2} \text{ and } \frac{3}{k+2} = \frac{-k}{-3k} \quad \text{Extra close brace or missing open brace}$$

$$2k+4=3k-3 \text{ and } 9=k+2$$

$$\Rightarrow \Rightarrow k=7 \text{ and } k=7 \Rightarrow k=7 \text{ and } k=7$$

Therefore, the given system of equations will have infinitely many solutions, if $k=7$.

Find the value of k for which the following system of equation has no solution : (20-25)

(20) $kx-5y=2$

$$6x+2y=7 \quad 6x+2y = 7$$

Soln:

The given system may be written as

$$kx-5y=2 \quad 6x+2y=7$$

The given system of equation is of the form

$$a_1x+b_1y-c_1=0 \quad a_2x+b_2y-c_2=0$$

$$\text{Where, } a_1=k, b_1=-5, c_1=-2 \quad a_2=6, b_2=2, c_2=-7$$

$$a_1a_2 \neq b_1b_2 \neq c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad \frac{k}{6} = \frac{-5}{2} \neq \frac{-2}{-7} \Rightarrow k \neq -15 \Rightarrow k = -15$$

For no solution, we have

$$a_1a_2 \neq b_1b_2 \neq c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad k \neq -15 \Rightarrow k = -15$$

Therefore, the given system of equations will have no solutions, if $k = -15$.

$$(21) \quad x+2y=0 \quad x+2y = 0$$

$$2x+ky=5 \quad 2x+ky = 5$$

Soln:

The given system may be written as

$$x+2y=0 \quad 2x+ky=5$$

The given system of equation is of the form

$$a_1x+b_1y-c_1=0 \quad a_2x+b_2y-c_2=0$$

$$\text{Where, } a_1=1, b_1=2, c_1=0 \quad a_2=2, b_2=k, c_2=-5$$

$$a_1a_2 \neq b_1b_2 \neq c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad \frac{1}{2} = \frac{2}{k} \neq \frac{0}{-5} \Rightarrow k = 4$$

For no solution, we have

$$a_1a_2 \neq b_1b_2 \neq c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad 1 \neq \frac{2}{k} \neq \frac{0}{-5} \Rightarrow k = 4$$

Therefore, the given system of equations will have no solutions, if $k = 4$.

$$(22) \quad 3x-4y+7=0 \quad 3x-4y+7 = 0$$

$$kx+3y-5=0 \quad kx+3y-5 = 0$$

Soln:

The given system may be written as

$$3x-4y+7=0 \quad kx+3y-5=0$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0$$

$$\text{Where, } a_1 = 3, b_1 = -4, c_1 = 7 \quad a_1 = 3, b_1 = -4, c_1 = 7$$

$$a_2 = k, b_2 = 3, c_2 = -5 \quad a_2 = k, b_2 = 3, c_2 = -5$$

For no solution, we have

$$a_1a_2 = b_1b_2 \neq c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad 3k = -4 \cdot 3 = -12 \Rightarrow k = -4 \Rightarrow k = -4$$

Therefore, the given system of equations will have no solutions, if $k = -4$.

$$(23) \quad 2x - ky + 3 = 0 \quad 2x - ky + 3 = 0$$

$$3x + 2y - 1 = 0 \quad 3x + 2y - 1 = 0$$

Soln:

The given system may be written as

$$2x - ky + 3 = 0 \quad 3x + 2y - 1 = 0$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0$$

$$\text{Where, } a_1 = 2, b_1 = -k, c_1 = 3 \quad a_1 = 2, b_1 = -k, c_1 = 3$$

$$a_2 = 3, b_2 = 2, c_2 = -1 \quad a_2 = 3, b_2 = 2, c_2 = -1$$

For no solution, we have

$$a_1a_2 = b_1b_2 \neq c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad 2 \cdot 3 = -k \cdot 2 = -2k \Rightarrow k = -3 \Rightarrow k = -3$$

Therefore, the given system of equations will have no solutions, if $k = -3$.

$$(24) \quad 2x + ky - 11 = 0 \quad 2x + ky - 11 = 0$$

$$5x - 7y - 5 = 0 \quad 5x - 7y - 5 = 0$$

Soln:

The given system may be written as

$$2x + ky - 11 = 0 \quad 5x - 7y - 5 = 0$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0$$

Where, $a_1=2, b_1=k, c_1=-11$
 $a_1 = 2, b_1 = k, c_1 = -11$

$a_2=5, b_2=-7, c_2=-5$
 $a_2 = 5, b_2 = -7, c_2 = -5$

For no solution, we have

$$a_1 a_2 = b_1 b_2 \neq c_1 c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad 25 = -k-7 \quad \frac{2}{5} = \frac{-k}{-7} \Rightarrow k = -145 \Rightarrow k = \frac{-14}{5}$$

Therefore, the given system of equations will have no solutions, if $k = -145$ or $k = \frac{-14}{5}$.

(25) $kx+3y=3$
 $kx + 3y = 3$

$12x+ky=6$
 $12x + ky = 6$

Soln:

The given system may be written as

$$kx+3y=3 \quad 12x+ky=6$$

The given system of equation is of the form

$$a_1x+b_1y-c_1=0 \quad a_2x+b_2y-c_2=0$$

Where, $a_1=k, b_1=3, c_1=-3$
 $a_1 = k, b_1 = 3, c_1 = -3$

$a_2=12, b_2=k, c_2=-6$
 $a_2 = 12, b_2 = k, c_2 = -6$

For no solution, we have

$$a_1 a_2 = b_1 b_2 \neq c_1 c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$k \cdot 12 = 3k \neq 36 \quad \frac{k}{12} = \frac{3}{k} \neq \frac{3}{6} \quad \dots \dots (i)$$

$$\Rightarrow k^2 = 36 \Rightarrow k^2 = 36 \Rightarrow k = +6 \text{ or } -6 \Rightarrow k = +6 \text{ or } -6$$

From (i)

$$k \cdot 12 \neq 36 \quad \frac{k}{12} \neq \frac{3}{6} \Rightarrow k \neq 6 \Rightarrow k \neq 6$$

Therefore, the given system of equations will have no solutions, if $k = -6$ or $k = -6$.

(26) For what value of a, the following system of equation will be inconsistent?

$4x+6y-11=0$
 $4x + 6y - 11 = 0$

$2x+ay-7=0$
 $2x + ay - 7 = 0$

Soln:

The given system may be written as

$$4x+6y-11=0 \quad 2x+ay-7=0$$

The given system of equation is of the form

$$a_1x+b_1y-c_1=0 \quad a_2x+b_2y-c_2=0$$

$$\text{Where, } a_1=4, b_1=6, c_1=-11$$

$$a_2=2, b_2=a, c_2=-7$$

For unique solution, we have

$$a_1a_2 \neq b_1b_2 \neq c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad \frac{4}{2} = \frac{6}{a} \neq \frac{-11}{-7} \Rightarrow a=3 \Rightarrow a=3$$

Therefore, the given system of equations will be inconsistent, if $a=3$.

(27) For what value of a, the following system of equation have no solution?

$$ax+3y=a-3$$

$$12x+ay=a$$

Soln:

The given system may be written as

$$ax+3y=a-3 \quad 12x+ay=a$$

The given system of equation is of the form

$$a_1x+b_1y-c_1=0 \quad a_2x+b_2y-c_2=0$$

$$\text{Where, } a_1=a, b_1=3, c_1=-(a-3)$$

$$a_2=12, b_2=a, c_2=-a$$

For unique solution, we have

$$a_1a_2 \neq b_1b_2 \neq c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad \frac{a}{12} = \frac{3}{a} \neq \frac{-(a-3)}{-a} \Rightarrow a \neq 6 \Rightarrow a \neq 6$$

And,

$$a12=3a \frac{a}{12} = \frac{3}{a} \Rightarrow a^2=36 \Rightarrow a^2=36 \Rightarrow a=+6 \text{ or } -6$$

$$\therefore a \neq 6$$

$$\Rightarrow a = -6$$

Therefore, the given system of equations will have no solution, if $a=-6$.

(28) Find the value of a, for which the following system of equation have

(i) Unique solution

(ii) No solution

$$kx + 2y = 5$$

$$3x + y = 1$$

Soln:

The given system may be written as

$$kx + 2y - 5 = 0 \quad 3x + y - 1 = 0$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0$$

$$\text{Where, } a_1 = k, b_1 = 2, c_1 = -5$$

$$a_2 = 3, b_2 = 1, c_2 = -1$$

(i) For unique solution, we have

$$a_1 a_2 \neq b_1 b_2 \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad k \neq 3 \quad \frac{k}{3} \neq \frac{2}{1} \quad k \neq 6$$

Therefore, the given system of equations will have unique solution, if $k \neq 6$.

(ii) For no solution, we have

$$a_1 a_2 = b_1 b_2 \neq c_1 c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad k \cdot 3 = 2 \cdot 1 \neq -5 \cdot -1 \quad \frac{k}{3} = \frac{2}{1} \neq \frac{-5}{-1} \quad k \cdot 3 = 2 \cdot 1 \Rightarrow k = \frac{2}{3} \Rightarrow k = \frac{2}{3}$$

Therefore, the given system of equations will have no solution, if $k = \frac{2}{3}$.

(29) For what value of c , the following system of equation have infinitely many solution (where $c \neq 0$)?

$$6x + 3y = c - 3$$

$$12x + cy = c$$

Soln:

The given system may be written as

$$6x + 3y - (c - 3) = 0 \quad 12x + cy - c = 0$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0$$

Where, $a_1=6, b_1=3, c_1=-(c-3)$ $a_1 = 6, b_1 = 3, c_1 = -(c-3)$

$a_2=12, b_2=c, c_2=-ca_2 = 12, b_2 = c, c_2 = -c$

For infinitely many solution, we have

$$a_1 a_2 = b_1 b_2 = c_1 c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad 6 \cdot 12 = 3c = -(c-3) - c \quad \frac{6}{12} = \frac{3}{c} = \frac{-(c-3)}{-c} \quad 12 = 3c \text{ and } 3c = -(c-3) - c$$
$$\frac{1}{2} = \frac{3}{c} \text{ and } \frac{3}{c} = \frac{-(c-3)}{-c} \Rightarrow c=6 \text{ and } c-3=3 \Rightarrow c=6 \text{ and } c-3=3 \Rightarrow c=6 \text{ and } c=6$$

Therefore, the given system of equations will have infinitely many solution, if $c=6$.

(30) Find the value of k, for which the following system of equation have

(i) Unique solution

(ii) No solution

(iii) Infinitely many solution

$$2x + ky = 1$$

$$3x - 5y = 7$$

Soln:

The given system may be written as

$$2x + ky = 1 \quad 3x - 5y = 7$$

The given system of equation is of the form

$$a_1 x + b_1 y - c_1 = 0 \quad a_2 x + b_2 y - c_2 = 0$$

Where, $a_1=2, b_1=k, c_1=-1$

$a_2=3, b_2=-5, c_2=-7$

(i) For unique solution, we have

$$a_1 a_2 \neq b_1 b_2 \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad 2 \cdot 3 \neq -k \cdot 5 \quad \frac{2}{3} \neq \frac{-k}{-5} \quad k \neq -10$$

Therefore, the given system of equations will have unique solution, if $k \neq -10$.

(ii) For no solution, we have

$$a_1 a_2 = b_1 b_2 \neq c_1 c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad 2 \cdot 3 = k \cdot 5 \text{ and } -1 \cdot 7 \neq \frac{k}{-5} \quad 6 = 5k \text{ and } k \neq -10$$
$$\frac{2}{3} = \frac{k}{5} \text{ and } \frac{k}{5} \neq \frac{-1}{-5} \Rightarrow k = 10 \text{ and } k \neq -10 \Rightarrow k = 10$$

Therefore, the given system of equations will have no solution, if $k = 10$.

(iii) For the given system to have infinitely many solution, we have

$$a_1 a_2 = b_1 b_2 = c_1 c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad 23 = k - 5 = -1 - 7 \frac{2}{3} = \frac{k}{-5} = \frac{-1}{-7}$$

Clearly $a_1 a_2 \neq c_1 c_2 \quad \frac{a_1}{a_2} \neq \frac{c_1}{c_2}$,

So there is no value of k for which the given system of equation has infinitely many solution.

(31) For what value of k, the following system of equation will represent the coincident lines?

$$x + 2y + 7 = 0 \quad x + 2y + 7 = 0$$

$$2x + ky + 14 = 0 \quad 2x + ky + 14 = 0$$

Soln:

The given system may be written as

$$x + 2y + 7 = 0 \quad x + 2y + 7 = 0 \quad 2x + ky + 14 = 0 \quad 2x + ky + 14 = 0$$

The given system of equation is of the form

$$a_1 x + b_1 y - c_1 = 0 \quad a_1 x + b_1 y - c_1 = 0 \quad a_2 x + b_2 y - c_2 = 0 \quad a_2 x + b_2 y - c_2 = 0$$

Where, $a_1 = 1, b_1 = 2, c_1 = 7$ $a_1 = 1, b_1 = 2, c_1 = 7$

$a_2 = 2, b_2 = k, c_2 = 14$ $a_2 = 2, b_2 = k, c_2 = 14$

The given system of equation will represent the coincident lines if they have infinitely many solution.

$$a_1 a_2 = b_1 b_2 = c_1 c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad 12 = 2k = 7 \cdot 14 \quad \frac{1}{2} = \frac{2}{k} = \frac{7}{14} \quad 12 = 2k = 12 \quad \frac{1}{2} = \frac{2}{k} = \frac{1}{2} \Rightarrow k = 4 \Rightarrow k = 4$$

Therefore, the given system of equations will have infinitely many solution, if $k = 4$.

(32) (30) Find the value of k, for which the following system of equation have unique solution.

$$ax + by = c \quad ax + by = c$$

$$lx + my = n \quad lx + my = n$$

Soln:

The given system may be written as

$$ax + by - c = 0 \quad ax + by - c = 0 \quad lx + my - n = 0 \quad lx + my - n = 0$$

The given system of equation is of the form

$$a_1 x + b_1 y - c_1 = 0 \quad a_1 x + b_1 y - c_1 = 0 \quad a_2 x + b_2 y - c_2 = 0 \quad a_2 x + b_2 y - c_2 = 0$$

Where, $a_1 = a, b_1 = b, c_1 = -c$ $a_1 = a, b_1 = b, c_1 = -c$

$$a_2=1, b_2=m, c_2=-na_2 = 1, b_2 = m, c_2 = -n$$

For unique solution, we have

$$a_1a_2 \neq b_1b_2 \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow a_1 \neq b_1m \Rightarrow \frac{a_1}{1} \neq \frac{b_1}{m} \Rightarrow am \neq b_1 \Rightarrow am \neq b_1$$

Therefore, the given system of equations will have unique solution, if $am \neq b_1$.

(33) Find the value of a and b such that the following system of linear equation have infinitely many solution:

$$(2a-1)x+3y-5=0 \quad (2a-1)x + 3y - 5 = 0$$

$$3x+(b-1)y-2=0 \quad 3x + (b-1)y - 2 = 0$$

Soln:

The given system of equation may be written as,

$$(2a-1)x+3y-5=0 \quad (2a-1)x + 3y - 5 = 0 \quad 3x+(b-1)y-2=0 \quad 3x + (b-1)y - 2 = 0$$

The given system of equation is of the form

$$a_1x+b_1y-c_1=0 \quad a_1x + b_1y - c_1 = 0 \quad a_2x+b_2y-c_2=0 \quad a_2x + b_2y - c_2 = 0$$

$$\text{Where, } a_1=(2a-1), b_1=3, c_1=-5 \quad a_1 = (2a-1), b_1 = 3, c_1 = -5$$

$$a_2=3, b_2=b-1, c_2=-2 \quad a_2 = 3, b_2 = b-1, c_2 = -2$$

The given system of equation will have infinitely many solution, if

$$a_1a_2 = b_1b_2 = c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad (2a-1)3 = 3b-1 = -5-2 \frac{(2a-1)}{3} = \frac{3}{b-1} = \frac{-5}{-2} \quad (2a-1)3 = 52 \text{ and } 3b-1 = 52$$

$$\frac{(2a-1)}{3} = \frac{5}{2} \text{ and } \frac{3}{b-1} = \frac{5}{2} \Rightarrow 2(2a-1)=15 \text{ and } 6=5(b-1)$$

$$\Rightarrow 2(2a-1) = 15 \text{ and } 6 = 5(b-1) \Rightarrow 4a-2=15 \text{ and } 6=5b-5$$

$$\Rightarrow 4a-2 = 15 \text{ and } 6 = 5b-5 \Rightarrow 4a=17 \text{ and } 5b=11 \Rightarrow 4a = 17 \text{ and } 5b = 11 \Rightarrow a = \frac{17}{4} \text{ and } b = \frac{11}{5}$$

$$\Rightarrow a = \frac{17}{4} \text{ and } b = \frac{11}{5}$$

(34) Find the value of a and b such that the following system of linear equation have infinitely many solution:

$$2x-3y=7 \quad 2x - 3y = 7$$

$$(a+b)x-(a+b-3)y=4a+b \quad (a+b)x - (a+b-3)y = 4a+b$$

Soln:

The given system of equation may be written as,

$$2x-3y-7=0 \quad 2x - 3y - 7 = 0 \quad (a+b)x-(a+b-3)y-(4a+b)=0 \quad (a+b)x - (a+b-3)y - (4a+b) = 0$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0$$

$$\text{Where, } a_1 = 2, b_1 = -3, c_1 = -7 \quad a_2 = (a+b), b_2 = -(a+b-3), c_2 = -(4a+b)$$

$$a_2 = (a+b), b_2 = -(a+b-3), c_2 = -(4a+b)$$

The given system of equation will have infinitely many solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \frac{2}{(a+b)} = \frac{-3}{-(a+b-3)} = \frac{-7}{-(4a+b)}$$

$$\frac{2}{(a+b)} = \frac{-3}{-(a+b-3)} = \frac{-7}{-(4a+b)} \quad 2(a+b) = -3-(a+b-3) = -7-(4a+b)$$

$$\frac{2}{(a+b)} = \frac{3}{(a+b-3)} \quad \text{and} \quad \frac{3}{(a+b-3)} = \frac{7}{(4a+b)} \Rightarrow 2(a+b-3) = 3(a+b) \text{ and } 3(4a+b) = 7(a+b-3)$$

$$\Rightarrow 2(a+b-3) = 3(a+b) \text{ and } 3(4a+b) = 7(a+b-3) \quad \Rightarrow 2a+2b-6=3a+3b \text{ and } 12a+3b=7a+7b-21$$

$$\Rightarrow 2a+2b-6=3a+3b \text{ and } 12a+3b=7a+7b-21 \quad \Rightarrow a+b=-6 \text{ and } 5a-4b=-21$$

$$\Rightarrow a+b = -6 \text{ and } 5a-4b = -21$$

$$a+b = -6$$

$$\Rightarrow a = -6 - b$$

Substituting the value of a in $5a-4b=-21$ we have

$$5(-b-6)-4b=-21$$

$$\Rightarrow -5b-30-4b=-21 \Rightarrow -5b-30-4b=-21 \Rightarrow -9b=-9 \Rightarrow 9b=-9 \Rightarrow b=-1 \Rightarrow b=-1$$

$$\text{As } a = -6 - b$$

$$\Rightarrow a = -6 + 1 = -5 \Rightarrow a = -6 + 1 = -5$$

Hence the given system of equation will have infinitely many solution if

$$a = -5 \text{ and } b = -1.$$

(35) Find the value of p and q such that the following system of linear equation have infinitely many solution:

$$2x - 3y = 9 \quad 2x - 3y = 9$$

$$(p+q)x + (2p-q)y = 3(p+q+1) \quad (p+q)x + (2p-q)y = 3(p+q+1)$$

Soln:

The given system of equation may be written as,

$$2x - 3y - 9 = 0 \quad (p+q)x + (2p-q)y - 3(p+q+1) = 0$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0$$

$$\text{Where, } a_1 = 2, b_1 = 3, c_1 = -9 \quad a_2 = (p+q), b_2 = (2p-q), c_2 = -3(p+q+1)$$

$$a_2 = (p+q), b_2 = (2p-q), c_2 = -3(p+q+1)$$

The given system of equation will have infinitely many solution, if

$$a_1 a_2 = b_1 b_2 = c_1 c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad 2(p+q) = 3(2p-q) = -9-3(p+q+1)$$

$$\frac{2}{(p+q)} = \frac{3}{(2p-q)} = \frac{-9}{-3(p+q+1)} \quad 2(p+q) = 3(2p-q) \text{ and } 3(2p-q) = 3(p+q+1)$$

$$\frac{2}{(p+q)} = \frac{3}{(2p-q)} \text{ and } \frac{3}{(2p-q)} = \frac{3}{(p+q+1)} \quad 2(2p-q) = 3(p+q) \text{ and } (p+q+1) = 2p-q$$

$$2(2p-q) = 3(p+q) \text{ and } (p+q+1) = 2p-q \quad \Rightarrow 4p-2q=3p+3q \text{ and } -p+2q=-1$$

$$\Rightarrow 4p-2q=3p+3q \text{ and } -p+2q=-1 \quad \Rightarrow p=5q \text{ and } p-2q=1 \Rightarrow p=5q \text{ and } p-2q=1$$

Substituting the value of p in p-2q=1, we have

$$3q=1$$

$$\Rightarrow q = \frac{1}{3} \Rightarrow p = 5q = \frac{5}{3}$$

Substituting the value of p in p=5q, we have

$$p = 5q = \frac{5}{3}$$

Hence the given system of equation will have infinitely many solution if

$$p = \frac{5}{3} \text{ and } q = \frac{1}{3}.$$

(36) Find the values of a and b for which the following system of equation has infinitely many solution:

$$(i) (2a-1)x+3y=5 \quad (2a-1)x+3y=5$$

$$3x+(b-2)y=3 \quad 3x+(b-2)y=3$$

Soln:

The given system of equation may be written as,

$$(2a-1)x+3y-5=0 \quad (2a-1)x+3y-5=0 \quad 3x+(b-2)y-3=0 \quad 3x+(b-2)y-3=0$$

The given system of equation is of the form

$$a_1 x + b_1 y - c_1 = 0 \quad a_1 x + b_1 y - c_1 = 0 \quad a_2 x + b_2 y - c_2 = 0 \quad a_2 x + b_2 y - c_2 = 0$$

$$\text{Where, } a_1 = 2a-1, b_1 = 3, c_1 = -5 \quad a_1 = 2a-1, b_1 = 3, c_1 = -5$$

$$a_2 = 3, b_2 = b-2, c_2 = -3 \quad a_2 = 3, b_2 = b-2, c_2 = -3$$

The given system of equation will have infinitely many solution, if

$$a_1 a_2 = b_1 b_2 = c_1 c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad 2a-13 = -3b-2 = -5-3 \frac{2a-1}{3} = \frac{-3}{b-2} = \frac{-5}{-3} \quad 2a-13 = 5 \text{ and } -3b-2 = 5$$

$$\frac{2a-1}{3} = \frac{5}{3} \text{ and } \frac{-3}{b-2} = \frac{5}{3} \quad 2a-1=5 \text{ and } -9=5(b-2) \quad 2a-1=5 \text{ and } -9=5(b-2) \quad \Rightarrow a=3 \text{ and } -9=5b-10$$

$$\Rightarrow a = 3 \text{ and } -9 = 5b - 10 \quad \Rightarrow a=3 \text{ and } b = \frac{1}{5}$$

Hence the given system of equation will have infinitely many solution if

$$a=3 \text{ and } b = \frac{1}{5}.$$

$$(ii) \quad 2x - (2a+5)y = 5 \quad 2x - (2a+5)y = 5$$

$$(2b+1)x - 9y = 15 \quad (2b+1)x - 9y = 15$$

Soln:

The given system of equation may be written as,

$$2x - (2a+5)y = 5 \quad (2b+1)x - 9y = 15$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0$$

$$\text{Where, } a_1 = 2, b_1 = -(2a+5), c_1 = -5 \quad a_1 = 2, b_1 = -(2a+5), c_1 = -5$$

$$a_2 = (2b+1), b_2 = -9, c_2 = -15 \quad a_2 = (2b+1), b_2 = -9, c_2 = -15$$

The given system of equation will have infinitely many solution, if

$$a_1a_2 = b_1b_2 = c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad 2(2b+1) = -(2a+5)(-9) = -5(-15) \quad \frac{2}{2b+1} = \frac{-5}{-15} \quad 2(2b+1) = 13 \text{ and } (2a+5)9 = 13$$

$$\frac{2}{2b+1} = \frac{1}{3} \text{ and } \frac{(2a+5)}{9} = \frac{1}{3} \Rightarrow 6 = 2b+1 \text{ and } 2a+5 = 3 \Rightarrow 6 = 2b+1 \text{ and } 2a+5 = 3 \Rightarrow b = \frac{5}{2} \text{ and } a = -1$$

Hence the given system of equation will have infinitely many solution if

$$a = -1 \quad a = -1 \quad \text{and } b = \frac{5}{2} \quad b = \frac{5}{2}$$

$$(iii) \quad (a-1)x + 3y = 2 \quad (a-1)x + 3y = 2$$

$$6x + (1-2b)y = 6 \quad 6x + (1-2b)y = 6$$

Soln:

The given system of equation may be written as,

$$(a-1)x + 3y = 2 \quad 6x + (1-2b)y = 6$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0$$

$$\text{Where, } a_1 = a-1, b_1 = 3, c_1 = -2 \quad a_1 = a-1, b_1 = 3, c_1 = -2$$

$$a_2 = 6, b_2 = 1-2b, c_2 = -6 \quad a_2 = 6, b_2 = 1-2b, c_2 = -6$$

The given system of equation will have infinitely many solution, if

$$a_1a_2 = b_1b_2 = c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad a-16 = 3(1-2b) = -6 \quad \frac{a-1}{6} = \frac{3}{1-2b} = \frac{-2}{-6} \quad a-16 = 13 \text{ and } 3(1-2b) = -6$$

$$\frac{a-1}{6} = \frac{1}{3} \text{ and } \frac{3}{1-2b} = \frac{1}{3} \Rightarrow a-1 = 2 \text{ and } 1-2b = 9 \Rightarrow a-1 = 2 \text{ and } 1-2b = 9 \Rightarrow a = 3 \text{ and } b = -4$$

$$\Rightarrow a = 3 \text{ and } b = -4$$

Hence the given system of equation will have infinitely many solution if

$$a=3a = 3 \text{ and } b=-4b = -4 .$$

(iv) $3x+4y=12$ $3x + 4y = 12$

$$(a+b)x+2(a-b)y=5a-1 \quad (a+b)x + 2(a-b)y = 5a - 1$$

Soln:

The given system of equation may be written as,

$$3x+4y-12=0 \quad 3x + 4y - 12 = 0 \quad (a+b)x+2(a-b)y-(5a-1)=0 \quad (a+b)x + 2(a-b)y - (5a-1) = 0$$

The given system of equation is of the form

$$a_1x+b_1y-c_1=0 \quad a_1x + b_1y - c_1 = 0 \quad a_2x+b_2y-c_2=0 \quad a_2x + b_2y - c_2 = 0$$

Where, $a_1=3, b_1=4, c_1=-12$ $a_1 = 3, b_1 = 4, c_1 = -12$

$$a_2=(a+b), b_2=2(a-b), c_2=-(5a-1) \quad a_2 = (a+b), b_2 = 2(a-b), c_2 = -(5a-1)$$

The given system of equation will have infinitely many solution, if

$$a_1a_2 = b_1b_2 = c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad 3a+b = 2(a-b) = 125a-1 \quad \frac{3}{a+b} = \frac{4}{2(a-b)} = \frac{12}{5a-1} \quad 3a+b = 2a+b \text{ and } 2a+b = 125a-1$$

$$\frac{3}{a+b} = \frac{2}{a+b} \text{ and } \frac{2}{a+b} = \frac{12}{5a-1} \Rightarrow 3(a-b) = 2a+2b \text{ and } 2(5a-1) = 12(a-b)$$

$$\Rightarrow 3(a-b) = 2a + 2b \text{ and } 2(5a-1) = 12(a-b) \Rightarrow a=5b \text{ and } -2a = -12b+2$$

$$\Rightarrow a = 5b \text{ and } -2a = -12b + 2$$

Substituting $a=5b$ in $-2a=-12b+2$, we have

$$-2(5b) = -12b + 2$$

$$\Rightarrow -10b = -12b + 2 \Rightarrow -10b = -12b + 2 \Rightarrow b = 1 \Rightarrow b = 1$$

Thus $a=5$

Hence the given system of equation will have infinitely many solution if

$$a=5a = 5 \text{ and } b=1b = 1 .$$

(v) $2x+3y=7$ $2x + 3y = 7$

$$(a-1)x+(a+1)y=3a-1 \quad (a-1)x + (a+1)y = 3a - 1$$

Soln:

The given system of equation may be written as,

$$2x+3y-7=0 \quad 2x + 3y - 7 = 0 \quad (a-1)x+(a+1)y-(3a-1)=0 \quad (a-1)x + (a+1)y - (3a-1) = 0$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0$$

$$\text{Where, } a_1=2, b_1=3, c_1=-7 \quad a_2=2, b_2=3, c_2=-7$$

$$a_2=(a-1), b_2=(a+1), c_2=-(3a-1) \quad a_2=(a-1), b_2=(a+1), c_2=-(3a-1)$$

The given system of equation will have infinitely many solution, if

$$\begin{aligned} a_1a_2 = b_1b_2 = c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad 2a-b=3a+1) = -7 \quad \frac{2}{a-b} = \frac{3}{a+1} = \frac{-7}{3a-1} \quad 2a-b=3a+1) \text{ and } 3a+1) = -7 \quad 3a-1 \\ \frac{2}{a-b} = \frac{3}{a+1} \text{ and } \frac{3}{a+1} = \frac{-7}{3a-1} \Rightarrow 2(a+1)=3(a-1) \text{ and } 3(3a-1)=7(a+1) \\ \Rightarrow 2(a+1) = 3(a-1) \text{ and } 3(3a-1) = 7(a+1) \quad \Rightarrow 2a-3a = -3-2 \text{ and } 9a-3 = 7a+7 \\ \Rightarrow 2a-3a = -3-2 \text{ and } 9a-3 = 7a+7 \quad \Rightarrow a=5 \text{ and } a=5 \Rightarrow a=5 \text{ and } a=5 \end{aligned}$$

Hence the given system of equation will have infinitely many solution if

$$a=5 \quad a=5 \quad \text{and } b=1 \quad b=1.$$

$$\text{(vi) } 2x+3y=7 \quad 2x+3y=7$$

$$(a-1)x+(a+2)y=3a \quad (a-1)x+(a+2)y=3a$$

Soln:

The given system of equation may be written as,

$$2x+3y-7=0 \quad 2x+3y-7=0 \quad (a-1)x+(a+2)y-3a=0 \quad (a-1)x+(a+2)y-3a=0$$

The given system of equation is of the form

$$a_1x + b_1y - c_1 = 0 \quad a_2x + b_2y - c_2 = 0 \quad a_2x + b_2y - c_2 = 0$$

$$\text{Where, } a_1=2, b_1=3, c_1=-7 \quad a_1=2, b_1=3, c_1=-7$$

$$a_2=(a-1), b_2=(a+2), c_2=-3a \quad a_2=(a-1), b_2=(a+2), c_2=-3a$$

The given system of equation will have infinitely many solution, if

$$\begin{aligned} a_1a_2 = b_1b_2 = c_1c_2 \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad 2a-b=3a+2) = -7-3a \quad \frac{2}{a-b} = \frac{3}{a+2} = \frac{-7}{-3a} \quad 2a-b=3a+2) \text{ and } 3a+2) = 7 \quad 3a \\ \frac{2}{a-b} = \frac{3}{a+2} \text{ and } \frac{3}{a+2} = \frac{7}{3a} \Rightarrow 2(a+2)=3(a-1) \text{ and } 3(3a)=7(a+2) \\ \Rightarrow 2(a+2) = 3(a-1) \text{ and } 3(3a) = 7(a+2) \quad \Rightarrow 2a+4=3a-3 \text{ and } 9a=7a+14 \\ \Rightarrow 2a+4 = 3a-3 \text{ and } 9a = 7a+14 \quad \Rightarrow a=7 \text{ and } a=7 \Rightarrow a=7 \text{ and } a=7 \end{aligned}$$

Hence the given system of equation will have infinitely many solution if

$$a=7 \quad a=7 \quad \text{and } b=1 \quad b=1.$$