RD SHARMA
Solutions
Class 10 Maths
Chapter 2
Ex 2.2

Q.1: Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also, verify the relationship between the zeroes and coefficients in each of the following cases:

(i) **f(x)=2x³+x²–5x+2; 12,1,−2** $f(x) = 2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$

(ii) **g(x)=x³–4x²+5x–2;2,1,1** $g(x) = x^3 - 4x^2 + 5x - 2; 2, 1, 1$

Sol:

(i) f(x)=2x³+x²–5x+2; 12,1,−2 $f(x) = 2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$

(a) By putting x = 12 $\frac{1}{2}$ in the above equation, we will get

f(12)=2(12)³+(12)²–5(12)+2 $f(\frac{1}{2}) = 2(\frac{1}{2})^3 + (\frac{1}{2})^2 - 5(\frac{1}{2}) + 2$

= f(12)=2(18)+14−52+2 $f(\frac{1}{2}) = 2(\frac{1}{8}) + \frac{1}{4} - \frac{5}{2} + 2$

= f(12)=14+14−52+2 $f(\frac{1}{2}) = \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$ = 0

(b) By putting x = 1 in the above equation, we will get

f(1)=2(1)³+(1)²–5(1)+2 $f(1) = 2(1)^3 + (1)^2 - 5(1) + 2$

= 2 + 1 – 5 + 2 = 0

(c) By putting x = -2 in the above equation, we will get

f(−2)=2(−2)³+(−2)²–5(−2)+2 $f(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$

= -16 + 4 + 10 + 2 = -16 + 16 = 0

Now,

Sum of zeroes = α+β+γ $\alpha + \beta + \gamma$ = −ba $-\frac{b}{a}$

⇒ 12+1−2=−12 $\Rightarrow \frac{1}{2} + 1 - 2 = \frac{-1}{2}$ −12=−12 $\frac{-1}{2} = \frac{-1}{2}$

Product of the zeroes = αβ+βγ+αγ $\alpha\beta + \beta\gamma + \alpha\gamma$ = ca $\frac{c}{a}$

$\frac{1}{2} \times 1 + 1 \times (-2) + (-2) \times \frac{1}{2} = \frac{-5}{2}$

$\frac{1}{2} - 2 - 1 = \frac{-5}{2}$

$\frac{-5}{2} = \frac{-5}{2}$

Hence, verified.

(ii) $g(x) = x^3 - 4x^2 + 5x - 2; 2, 1, 1$

(a) By putting x = 2 in the given equation, we will get

$g(2) = (2)^3 - 4(2)^2 + 5(2) - 2$

= 8 – 16 + 10 – 2 = 18 – 18 = 0

(b) By putting x = 1 in the given equation, we will get

$g(1) = (1)^3 - 4(1)^2 + 5(1) - 2$

= 1 – 4 + 5 – 2 = 0

Now,

Sum of zeroes = $\alpha + \beta + \gamma = -\frac{b}{a}$

$\Rightarrow 2 + 1 + 1 = -(-4)$

4 = 4

Product of the zeroes = $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$

$2 \times 1 + 1 \times 1 + 1 \times 2 = 5$

2 + 1 + 2 = 5

5 = 5

$\alpha\beta\gamma = -(-2)$

$2 \times 1 \times 1 = 2$

2 = 2

Hence, verified.

Q.2: Find a cubic polynomial with the sum, sum of the product of its zeroes is taken two at a time, and product of its zeroes as 3, -1 and -3 respectively.

Sol:

Any cubic polynomial is of the form ax^3+bx^2+cx+d :

$= x^3$ – (sum of the zeroes) x^2 + (sum of the products of its zeroes) x – (product of the zeroes)

$= x^3 - 3x^2 + (-1)x + (-3)$

$= k[x^3 - 3x^2 - x - 3]$

k is any non-zero real numbers.

Q.3: If the zeroes of the polynomial $f(x) = 2x^3 - 15x^2 + 37x - 30$, find them.

Sol:

Let, $\alpha = a - d,\ \beta = a \text{ and } \gamma = a + d$ be the zeroes of the polynomial.

$f(x) = 2x^3 - 15x^2 + 37x - 30$

$\alpha + \beta + \gamma = -(\frac{-15}{2}) = \frac{15}{2}$ $\alpha\beta\gamma = -(\frac{-30}{2}) = 15$

$a - d + a + a + d = \frac{15}{2}$ and $a(a-d)(a+d) = 15$

So, $3a = \frac{15}{2}$

$a = \frac{5}{2}$

And, $a(a^2 + d^2) = 15$

$d^2 = \frac{1}{4}$ $d = \frac{1}{2}$

Therefore, $\alpha = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$

$\beta = \frac{5}{2}$

$\gamma = \frac{5}{2} + \frac{1}{2}$ = 3

Q.4: Find the condition that the zeroes of the polynomial $f(x)=x^3+3px^2+3qx+r$ $f(x) = x^3 + 3px^2 + 3qx + r$ may be in A.P.

Sol:

$f(x)=x^3+3px^2+3qx+r$ $f(x) = x^3 + 3px^2 + 3qx + r$

Let, a – d, a, a + d be the zeroes of the polynomial.

Then,

The sum of zeroes = $-ba$ $\frac{-b}{a}$

a + a – d + a + d = -3p

3a = -3p

a = -p

Since, a is the zero of the polynomial f(x),

Therefore, f(a) = 0

$f(a)=a^3+3pa^2+3qa+r$ $f(a) = a^3 + 3pa^2 + 3qa + r$ = 0

Therefore, $f(a)=0$ $f(a) = 0$

= $\Rightarrow a^3+3pa^2+3qa+r=0$ $\Rightarrow a^3 + 3pa^2 + 3qa + r = 0$

= $\Rightarrow (-p)^3+3p(-p)^2+3q(-p)+r=0$ $\Rightarrow (-p)^3 + 3p(-p)^2 + 3q(-p) + r = 0$

= $-p^3+3p^3-pq+r=0$ $-p^3 + 3p^3 - pq + r = 0$

= $2p^3-pq+r=0$ $2p^3 - pq + r = 0$

Q.5: If zeroes of the polynomial $f(x)=ax^3+3bx^2+3cx+d$ $f(x) = ax^3 + 3bx^2 + 3cx + d$ are tin A.P., prove that $2b^3-3abc+a^2d=0$ $2b^3 - 3abc + a^2d = 0$.

Sol:

$f(x)=x^3+3px^2+3qx+r$ $f(x) = x^3 + 3px^2 + 3qx + r$

Let, a – d, a, a + d be the zeroes of the polynomial.

Then,

The sum of zeroes = −ba $\frac{-b}{a}$

a + a – d + a + d = −3ba $\frac{-3b}{a}$

⇒3a=−3ba⇒ $3a = -\frac{3b}{a}$ ⇒a=−3b3a=−ba⇒ $a = \frac{-3b}{3a} = \frac{-b}{a}$ Since,f(a)=0

$\text{Since, } f(a) = 0$ ⇒$a(a^2)+3b(a)^2+3c(a)+d=0$

$\Rightarrow a(a^2) + 3b(a)^2 + 3c(a) + d = 0$ ⇒$a(-ba)^3$+$3b^2a^2$−3bca+d=0

$\Rightarrow a\left(\frac{-b}{a}\right)^3 + \frac{3b^2}{a^2} - \frac{3bc}{a} + d = 0$ $2b^3a^2$−3bca+d=0$\frac{2b^3}{a^2} - \frac{3bc}{a} + d = 0$ $2b^3$–3abc+a^2da^2=0

$\frac{2b^3-3abc+a^2d}{a^2} = 0$ $2b^3$–3abc+a^2d=0$2b^3 - 3abc + a^2d = 0$

Q.6: If the zeroes of the polynomial $f(x)=x^3–12x^2+39x+k$ $f(x) = x^3 - 12x^2 + 39x + k$ are in A.P., find the value of k.

Sol:

$f(x)=x^3–12x^2+39x+k$ $f(x) = x^3 - 12x^2 + 39x + k$

Let, a-d, a, a+d be the zeroes of the polynomial f(x).

The sum of the zeroes = 12

3a = 12

a = 4

Now,

f(a) = 0

$f(a)=a^3–12a^2+39a+k$ $f(a) = a^3 - 12a^2 + 39a + k$ $f(4)=4^3–12(4)^2+39(4)+k=0$

$f(4) = 4^3 - 12(4)^2 + 39(4) + k = 0$

64 – 192 + 156 + k = 0

k = -28