RD SHARMA
Solutions
Class 10 Maths
Chapter 2
Ex 2.1

Q.1: Find the zeroes of each of the following quadratic polynomials and verify the relationship between the zeroes and their coefficients:

(i) $f(x)=x^2-2x-8$ $f(x) = x^2-2x-8$

(ii) $g(s)=4s^2-4s+1$ $g(s) = 4s^2-4s+1$

(iii) $6x^2-3-7x$ $6x^2-3-7x$

(iv) $h(t)=t^2-15$ $h(t) = t^2-15$

(v) $p(x)=x^2+2\sqrt{2}x-6$ $p(x) = x^2+2\sqrt{2}x-6$

(vi) $q(x)=\sqrt{3}x^2+10x+7\sqrt{3}$ $q(x) = \sqrt{3}x^2+10x+7\sqrt{3}$

(vii) $f(x)=x^2-(\sqrt{3}+1)x+\sqrt{3}$ $f(x) = x^2-(\sqrt{3}+1)x+\sqrt{3}$

(viii) $g(x)=a(x^2+1)-x(a^2+1)$ $g(x) = a(x^2+1)-x(a^2+1)$

Solution:

(i) $f(x)=x^2-2x-8$ $f(x) = x^2-2x-8$

We have,

$f(x)=x^2-2x-8$ $f(x) = x^2-2x-8$

$= x^2-4x+2x-8$ $x^2-4x+2x-8$

$= x(x-4)+2(x-4)$ $x(x-4)+2(x-4)$

$= (x+2)(x-4)$ $(x+2)(x-4)$

Zeroes of the polynomials are -2 and 4.

Now,

Sum of the zeroes = −coefficientofxcoefficientofx $\frac{-\text{ coefficient of } x}{\text{coefficient of } x}$

-2 + 4 = −(−2)1 $\frac{-(-2)}{1}$

2 = 2

Product of the zeroes = constanttermCoefficientofx $\frac{\text{constant term}}{\text{Coefficient of x}}$

-8 = −81 $\frac{-8}{1}$

-8 = -8

Hence, the relationship is verified.

(ii) g(s)=4s²–4s+1 $g(s) = 4s^2 - 4s + 1$

We have,

g(s)=4s²–4s+1 $g(s) = 4s^2 - 4s + 1$

= 4s²–2s–2s+1 $4s^2 - 2s - 2s + 1$

= 2s(2s–1)−1(2s–1) $2s(2s-1) - 1(2s-1)$

= (2s–1)(2s–1) $(2s-1)(2s-1)$

Zeroes of the polynomials are 12 $\frac{1}{2}$ and 12 $\frac{1}{2}$.

Sum of zeroes = −coefficientofscoefficientofs² $\frac{-\text{coefficient of s}}{\text{coefficient of } s^2}$

12 + 12 = −(−4)4 $\frac{1}{2} + \frac{1}{2} = \frac{-(-4)}{4}$

1 = 1

Product of zeroes = constanttermCoefficientofx $\frac{\text{constant term}}{\text{Coefficient of x}}$

12 × 12 = 14 $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ 14 = 14 $\frac{1}{4} = \frac{1}{4}$

Hence, the relationship is verified.

(iii) 6s²−3−7x $6s^2 - 3 - 7x$

= 6s²−7x−3 $6s^2 - 7x - 3$ = (3x + 11)(2x – 3)

Zeros of the polynomials are 32 and −13 $\frac{3}{2}$ and $\frac{-1}{3}$

Sum of the zeros = −coefficientofxcoefficientofx² $\frac{-\text{coefficient of x}}{\text{coefficientofx}^2}$

−13 + 32 = −(−7)6 $\frac{-1}{3}+\frac{3}{2}=\frac{-(-7)}{6}$ 76 = 76 $\frac{7}{6}=\frac{7}{6}$

Product of the zeroes = constanttermcoefficientofx² $\frac{\text{constant term}}{\text{coefficient of } x^2}$

−13 × 32 = −36 $\frac{-1}{3}\times\frac{3}{2}=\frac{-3}{6}$ −36 = −36 $\frac{-3}{6}=\frac{-3}{6}$

Hence, the relationship is verified.

(iv) h(t)=t²–15$h(t)=t^2-15$

We have,

h(t)=t²–15$h(t)=t^2-15$

= t²–√15$t^2-\sqrt{15}$

= (t+√15)(t–√15)$(t+\sqrt{15})(t-\sqrt{15})$

Zeroes of the polynomials are **−√15and√15**$-\sqrt{15}$ and $\sqrt{15}$

Sum of the zeroes = 0

−√15+√15$-\sqrt{15}+\sqrt{15}$ = 0

0 = 0

Product of zeroes = constanttermCoefficientofx $\frac{\text{constant term}}{\text{Coefficient of } x}$ = −151 $\frac{-15}{1}$

−√15×√15=−15$-\sqrt{15}\times\sqrt{15}=-15$

-15 = -15

Hence, the relationship verified.

(v) p(x)=x²+2√2x–6$p(x)=x^2+2\sqrt{2}x-6$

We have,

p(x)=x²+2√2x–6$p(x)=x^2+2\sqrt{2}x-6$

= x²+3√2x+3√2x–6$x^2+3\sqrt{2}x+3\sqrt{2}x-6$

$= x(x+3\sqrt{2})-\sqrt{2}(x+3\sqrt{2})x(x+3\sqrt{2})-\sqrt{2}(x+3\sqrt{2})$

$= (x+3\sqrt{2})(x-\sqrt{2})(x+3\sqrt{2})(x-\sqrt{2})$

Zeroes of the polynomials are $3\sqrt{2}3\sqrt{2}$ and $-3\sqrt{2}-3\sqrt{2}$

Sum of the zeroes = $-2\sqrt{2}1 \frac{-2\sqrt{2}}{1}$

$\sqrt{2}-3\sqrt{2}=-2\sqrt{2}\sqrt{2}-3\sqrt{2}=-2\sqrt{2}$ $-2\sqrt{2}=-2\sqrt{2}-2\sqrt{2}=-2\sqrt{2}$

Product of the zeroes = constanttermCoefficientofx $\frac{\text{constant term}}{\text{Coefficient of x}}$

$\sqrt{2}\times-3\sqrt{2}=-61\sqrt{2}\times-3\sqrt{2}=\frac{-6}{1}$

-6 = -6

Hence, the relationship is verified.

(vi) q(x) = $\sqrt{3}x^2+10x+7\sqrt{3}\sqrt{3}x^2+10x+7\sqrt{3}$

$= \sqrt{3}x^2+7x+3x+7\sqrt{3}\sqrt{3}x^2+7x+3x+7\sqrt{3}$

$= \sqrt{3}x(x+\sqrt{3})7(x+sqrt3)\sqrt{3}x(x+\sqrt{3})7(x+sqrt3)$

$= (x+\sqrt{3})(7+sqrt3)(x+\sqrt{3})(7+sqrt3)$

Zeros of the polynomials are $-\sqrt{3}$and$-7\sqrt{3}-\sqrt{3}$and$\frac{-7}{\sqrt{3}}$

Sum of zeros = $-10\sqrt{3}\frac{-10}{\sqrt{3}}$

$-\sqrt{3}-7\sqrt{3}=-10\sqrt{3}-\sqrt{3}-\frac{7}{\sqrt{3}}=\frac{-10}{\sqrt{3}}$ $-10\sqrt{3}=-10\sqrt{3}\frac{-10}{\sqrt{3}}=\frac{-10}{\sqrt{3}}$

Product of the polynomials are $-\sqrt{3}, -7\sqrt{3}-\sqrt{3}, \frac{-7}{\sqrt{3}}$

7 = 7

Hence, the relationship is verified.

(vii) h(x) = $x^2-(\sqrt{3}+1)x+\sqrt{3}x^2-(\sqrt{3}+1)x+\sqrt{3}$

$= x^2 - \sqrt{3} - x + \sqrt{3} x^2 - \sqrt{3} - x + \sqrt{3}$

$= x(x - \sqrt{3}) - 1(x - \sqrt{3}) x(x - \sqrt{3}) - 1(x - \sqrt{3})$

$= (x - \sqrt{3})(x - 1)(x - \sqrt{3})(x - 1)$

Zeros of the polynomials are 1 and $\sqrt{3}\sqrt{3}$

Sum of zeros = −coefficientofxcoefficientofx² $= -[-\sqrt{3} - 1] \frac{-\text{co efficient of } x}{\text{co efficient of } x^2} = -[-\sqrt{3} - 1]$

$1 + \sqrt{3} = \sqrt{3} + 1 1 + \sqrt{3} = \sqrt{3} + 1$

Product of zeros = −coefficientofxcoefficientofx² $= \sqrt{3} \frac{-\text{co efficient of } x}{\text{co efficient of } x^2} = \sqrt{3}$

$\sqrt{3} = \sqrt{3} \sqrt{3} = \sqrt{3}$

Hence, the relationship is verified

(viii) g(x) = $a[(x^2+1) - x(a^2+1)]^2$ $a[(x^2 + 1) - x(a^2 + 1)]^2$

$= ax^2 + a - a^2x - x ax^2 + a - a^2x - x$

$= ax^2 - [(a^2x + 1)] + a ax^2 - [(a^2x + 1)] + a$

$= ax^2 - a^2x - x + a ax^2 - a^2x - x + a$

$= ax(x - a) - 1(x - a) ax(x - a) - 1(x - a) = (x - a)(ax - 1)(x - a)(ax - 1)$

Zeros of the polynomials are 1a and1 $\frac{1}{a}$ and 1

Sum of the zeros = a[−a²−1]a $\frac{a[-a^2-1]}{a}$

1a $+a=$ a²+1a a²+1a $=$ a²+1a $\frac{1}{a} + a = \frac{a^2+1}{a} \ \frac{a^2+1}{a} = \frac{a^2+1}{a}$

Product of zeros = a/a

1a $\times a =$ aa $\frac{1}{a} \times a = \frac{a}{a}$

1 = 1

Hence, the relationship is verified.

Q.2: If αandβα and β are the zeroes of the quadratic polynomial f(x)=x²–5x+4 $f(x) = x^2 - 5x + 4$, find the value of 1α+1β–2αβ$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$.

Solution: We have,

αandβα and β are the roots of the quadratic polynomial.

f(x)=x²–5x+4$f(x) = x^2 - 5x + 4$

Sum of the roots = α+β$\alpha + \beta$ = 5

Product of the roots = αβ$\alpha\beta$ = 4

So,

1α+1β–2αβ$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$ = β+ααβ–2αβ$\frac{\beta+\alpha}{\alpha\beta} - 2\alpha\beta$

= 54–2×4=54–8=-274 $\frac{5}{4} - 2 \times 4 = \frac{5}{4} - 8 = \frac{-27}{4}$

Q.3: If αandβα and β are the zeroes of the quadratic polynomial f(x)=x²–5x+4 $f(x) = x^2 - 5x + 4$, find the value of 1α+1β–2αβ$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$.

Solution:

Since, αandβα and β are the zeroes of the quadratic polynomial.

p(y)=x²–5x+4$p(y) = x^2 - 5x + 4$

Sum of the zeroes = α+β$\alpha + \beta$ = 5

Product of the roots = αβ$\alpha\beta$ = 4

So,

1α+1β–2αβ$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

= β+α–2α²β²αβ $\frac{\beta+\alpha-2\alpha^2\beta^2}{\alpha\beta}$

= (α+β)–2(αβ)²αβ $\frac{(\alpha+\beta)-2(\alpha\beta)^2}{\alpha\beta}$

= (5)–2(4)²4 $\frac{(5)-2(4)^2}{4}$

$= 5-2\times164\ \frac{5-2\times16}{4} = 5-324\ \frac{5-32}{4} = -274\ \frac{-27}{4}$

Q.4: If αandβ α and β are the zeroes of the quadratic polynomial p(y)=5y²–7y+1 $p(y) = 5y^2 - 7y + 1$, find the value of 1α+1β $\frac{1}{\alpha} + \frac{1}{\beta}$.

Solution: Since, αandβ α and β are the zeroes of the quadratic polynomial.

p(y)=5y²–7y+1 $p(y) = 5y^2 - 7y + 1$

Sum of the zeroes = α+β $\alpha + \beta$ = 7

Product of the roots = αβ $\alpha\beta$ = 1

So,

1α+1β $\frac{1}{\alpha} + \frac{1}{\beta}$ = α+βαβ $\frac{\alpha+\beta}{\alpha\beta}$ = 71 $\frac{7}{1}$ = 7

Q.5: If αandβ α and β are the zeroes of the quadratic polynomial f(x)=x²–x–4 $f(x) = x^2 - x - 4$, find the value of 1α+1β–αβ $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$.

Solution:

Since, αandβ α and β are the zeroes of the quadratic polynomial.

We have,

f(x)=x²–x–4 $f(x) = x^2 - x - 4$

Sum of zeroes = α+β $\alpha + \beta$ = 1

Product of the zeroes = αβ $\alpha\beta$ = -4

So,

1α+1β–αβ $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$ = α+βαβ–αβ $\frac{\alpha+\beta}{\alpha\beta} - \alpha\beta$

= 1–4–(–4)=–14+4 $\frac{1}{-4} - (-4) = \frac{-1}{4} + 4$

= –1+164 $\frac{-1+16}{4}$ = 154 $\frac{15}{4}$

Q.6: If αandβ α and β are the zeroes of the quadratic polynomial f(x)=x²+x–2 $f(x) = x^2 + x - 2$, find the value of 1α−1β $\frac{1}{\alpha} - \frac{1}{\beta}$.

Solution:

Since, αandβ α and β are the zeroes of the quadratic polynomial.

We have,

f(x)=x²+x–2 $f(x) = x^2 + x - 2$

Sum of zeroes = α+β $\alpha + \beta$ = 1

Product of the zeroes = αβ $\alpha\beta$ = -2

So,

1α+1β−αβ $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$ = α+βαβ $\frac{\alpha+\beta}{\alpha\beta}$

= β−ααβ $\frac{\beta-\alpha}{\alpha\beta}$

= β−ααβ × (α−β)αβ = $\frac{\beta-\alpha}{\alpha\beta} \times \frac{(\alpha-\beta)}{\alpha\beta}$ = √(α+β)²−4αβαβ = $\frac{\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}}{\alpha\beta}$ = √1+82 = √92 = 32

$= \frac{\sqrt{1+8}}{2} = \frac{\sqrt{9}}{2} = \frac{3}{2}$

Q.7: If one of the zero of the quadratic polynomial f(x)=4x²–8kx–9 $f(x) = 4x^2 - 8kx - 9$ is negative of the other, then find the value of k.

Solution:

Let, the two zeroes of the polynomial f(x)=4x²–8kx–9 $f(x) = 4x^2 - 8kx - 9$ be αand−α α and $-\alpha$.

Product of the zeroes = α×−α $\alpha \times -\alpha$ = -9

Sum of the zeroes = α+(−α) $\alpha + (-\alpha)$ = -8k = 0 Since,α–α=0 Since, $\alpha - \alpha = 0$

⇒8k=0 $\Rightarrow 8k = 0$ ⇒k=0 $\Rightarrow k = 0$

Q.8: If the sum of the zeroes of the quadratic polynomial f(t)=kt²+2t+3k $f(t) = kt^2 + 2t + 3k$ is equal to their product, then find the value of k.

Solution: Let the two zeroes of the polynomial $f(t) = kt^2 + 2t + 3k$ be α and β.

Sum of the zeroes = $\alpha + \beta$ = 2

Product of the zeroes = $\alpha \times \beta$ = 3k

Now,

$\frac{-2}{k} = \frac{3k}{k}$ $\Rightarrow 3k = -2$ $\Rightarrow k = \frac{-2}{3}$

So, k = 0 and $\Rightarrow k = \frac{-2}{3}$

Q.9: If α and β are the zeroes of the quadratic polynomial $p(x) = 4x^2 - 5x - 1$, find the value of $\alpha^2\beta + \alpha\beta^2$.

Solution:

Since, α and β are the zeroes of the quadratic polynomial $p(x) = 4x^2 - 5x - 1$

So, Sum of the zeroes = $\alpha + \beta$ = $\frac{5}{4}$

Product of the zeroes = $\alpha \times \beta$ = $\frac{-1}{4}$

Now,

$\alpha^2\beta + \alpha\beta^2$ = $\alpha\beta(\alpha + \beta)$

= $\frac{5}{4}(\frac{-1}{4})$

= $\frac{-5}{16}$

Q.10: If α and β are the zeroes of the quadratic polynomial $f(t) = t^2 - 4t + 3$, find the value of $\alpha^4\beta^3 + \alpha^3\beta^4$.

Solution:

Since, α and β are the zeroes of the quadratic polynomial $f(t) = t^2 - 4t + 3$

So, Sum of the zeroes = $\alpha+\beta\alpha+\beta$ = 4

Product of the zeroes = $\alpha\times\beta\alpha\times\beta$ = 3

Now,

$\alpha^4\beta^3+\alpha^3\beta^4\alpha^4\beta^3+\alpha^3\beta^4$ = $\alpha^3\beta^3(\alpha+\beta)\alpha^3\beta^3(\alpha+\beta)$

= $(3)^3(4)(3)^3(4)$ = 108

Q.11: If $\alpha$and$\beta\alpha$ and β are the zeroes of the quadratic polynomial $f(x)=6x^2+x-2$ $f(x)=6x^2+x-2$, find the value of $\alpha\beta+\beta\alpha\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$.

Solution:

Since, $\alpha$and$\beta\alpha$ and β are the zeroes of the quadratic polynomial $f(x)=6x^2+x-2$ $f(x)=6x^2+x-2$.

Sum of the zeroes = $\alpha+\beta\alpha+\beta$ = $-16\frac{-1}{6}$

Product of the zeroes = $\alpha\times\beta\alpha\times\beta$ = $-13\frac{-1}{3}$

Now,

$\alpha\beta+\beta\alpha\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

= $(\alpha^2+\beta^2)-2\alpha\beta\alpha\beta\frac{(\alpha^2+\beta^2)-2\alpha\beta}{\alpha\beta}$

By substitution the values of the sum of zeroes and products of the zeroes, we will get

= $-2512\frac{-25}{12}$

Q.12: If $\alpha$and$\beta\alpha$ and β are the zeroes of the quadratic polynomial $f(x)=6x^2+x-2$ $f(x)=6x^2+x-2$, find the value of $\alpha\beta+2[1\alpha+1\beta]+3\alpha\beta\frac{\alpha}{\beta}+2[\frac{1}{\alpha}+\frac{1}{\beta}]+3\alpha\beta$.

Solution:

Since, $\alpha$and$\beta\alpha$ and β are the zeroes of the quadratic polynomial $f(x)=6x^2+x-2$ $f(x)=6x^2+x-2$.

Sum of the zeroes = α+β$\alpha + \beta$ = 63$\frac{6}{3}$

Product of the zeroes = α×β$\alpha \times \beta$ = 43$\frac{4}{3}$

Now,

αβ+2[1α+1β]+3αβ$\frac{\alpha}{\beta} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta$.

= α²+β²αβ+2[1α+1β]+3αβ$\frac{\alpha^2+\beta^2}{\alpha\beta} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta$

= (α+β)²−2αβαβ+2[1α+1β]+3αβ$\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta$

By substituting the values of sum and product of the zeroes, we will get

αβ+2[1α+1β]+3αβ$\frac{\alpha}{\beta} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta$ = 8

Q.13: If the squared difference of the zeroes of the quadratic polynomial f(x)=x²+px+45 $f(x) = x^2 + px + 45$ is equal to 144, find the value of p.

Solution:

Let the two zeroes of the polynomial be αandβα and β .

We have,

f(x)=x²+px+45$f(x) = x^2 + px + 45$

Now,

Sum of the zeroes = α+β$\alpha + \beta$ =-p

Product of the zeroes = α×β$\alpha \times \beta$ = 45

So,

(α+β)²−4αβ=144$(\alpha+\beta)^2 - 4\alpha\beta = 144$ (p)²−4×45=144$(p)^2 - 4 \times 45 = 144$ (p)²=144+180 $(p)^2 = 144 + 180$ (p)²=324$(p)^2 = 324$ p=√324$p = \sqrt{324}$ p=±18$p = \pm 18$

Thus, in the given equation, p will be either 18 or -18.

Q.14: If αandβ α and β are the zeroes of the quadratic polynomial f(x)=x²–px+q $f(x) = x^2 - px + q$, prove that α²β²+β²α²=p⁴q²−4p²q+2 $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$.

Solution:

Since, αandβ α and β are the roots of the quadratic polynomial given in the question.

f(x)=x²–px+q $f(x) = x^2 - px + q$

Now,

Sum of the zeroes = p = α+β $\alpha + \beta$

Product of the zeroes = q = α×β $\alpha \times \beta$

LHS = α²β²+β²α² $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}$

= α⁴+β⁴α²β² $\frac{\alpha^4+\beta^4}{\alpha^2\beta^2}$

= (α²+β²)²–2(αβ)²(αβ)² $\frac{(\alpha^2+\beta^2)^2 - 2(\alpha\beta)^2}{(\alpha\beta)^2}$

= [(α+β)²–2αβ]²–2(αβ)²(αβ)² $\frac{[(\alpha+\beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2}{(\alpha\beta)^2}$

= [(p)²–2q]²–2(q)²(q)² $\frac{[(p)^2 - 2q]^2 - 2(q)^2}{(q)^2}$

= (p⁴+4q²–4p²q)–2q²q² $\frac{(p^4+4q^2-4p^2q) - 2q^2}{q^2}$

= p⁴+2q²–4p²qq² $\frac{p^4+2q^2-4p^2q}{q^2}$

= p²q²+2–4p²q $\frac{p^2}{q^2} + 2 - \frac{4p^2}{q}$

= p²q²−4p²q+2 $\frac{p^2}{q^2} - \frac{4p^2}{q} + 2$

LHS = RHS

Hence, proved.

Q.15: If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - p(x+1) - c$, show that $(\alpha+1)(\beta+1) = 1 - c$.

Solution:

Since, α and β are the zeroes of the quadratic polynomial

$f(x) = x^2 - p(x+1) - c$

Now,

Sum of the zeroes = $\alpha + \beta$ = p

Product of the zeroes = $\alpha \times \beta$ = (- p – c)

So,

$(\alpha+1)(\beta+1)$

$= \alpha\beta + \alpha + \beta + 1$

$= \alpha\beta + (\alpha + \beta) + 1$

$= (-p - c) + p + 1$

= 1 – c = RHS

So, LHS = RHS

Hence, proved.

Q.16: If α and β are the zeroes of the quadratic polynomial such that $\alpha + \beta = 24$ and $\alpha - \beta = 8$, find a quadratic polynomial having α and β as its zeroes.

Solution:

We have,

$\alpha + \beta = 24$ …………E-1

$\alpha - \beta = 8$ …………E-2

By solving the above two equations accordingly, we will get

$2\alpha = 32$ $\alpha = 16$

Substitute the value of αα, in any of the equation. Let we substitute it in E-2, we will get

$\beta=16-8\beta = 16-8$ $\beta=8\beta = 8$

Now,

Sum of the zeroes of the new polynomial = $\alpha+\beta\alpha + \beta$ = 16 + 8 = 24

Product of the zeroes = $\alpha\beta\alpha\beta$ = $16\times 8 16 \times 8$ = 128

Then, the quadratic polynomial is-

K x^2-(sumofthezeroes)x+(productofthezeroes)
x^2-(sum of the zeroes) x + (product of the zeroes) = $x^2-24x+128 x^2-24x+128$

Hence, the required quadratic polynomial is $f(x)=x^2+24x+128 f(x) = x^2+24x+128$

Q.17: If αandβ α and β are the zeroes of the quadratic polynomial $f(x)=x^2-1 f(x) = x^2-1$, find a quadratic polynomial whose zeroes are 2αβ and 2βα $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$.

Solution:

We have,

$f(x)=x^2-1 f(x) = x^2-1$

Sum of the zeroes = $\alpha+\beta\alpha+\beta$ = 0

Product of the zeroes = $\alpha\beta\alpha\beta$ = -1

From the question,

Sum of the zeroes of the new polynomial = 2αβ and 2βα $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$

= $2\alpha^2+2\beta^2\alpha\beta$ $\frac{2\alpha^2+2\beta^2}{\alpha\beta}$

= $2(\alpha^2+\beta^2)\alpha\beta$ $\frac{2(\alpha^2+\beta^2)}{\alpha\beta}$

= $2((\alpha+\beta)^2-2\alpha\beta)\alpha\beta$ $\frac{2((\alpha+\beta)^2-2\alpha\beta)}{\alpha\beta}$

= $2(2)1-1$ $\frac{2(2)1}{-1}$ { By substituting the value of the sum and products of the zeroes }

As given in the question,

Product of the zeroes = (2α)(2β)αβ $\frac{(2\alpha)(2\beta)}{\alpha\beta}$ = 4αβαβ $\frac{4\alpha\beta}{\alpha\beta}$ = 4

Hence, the quadratic polynomial is

x²–(sumofthezeroes)x+(productofthezeroes)
$x^2 - (\text{sum of the zeroes})\,x + (\text{product of the zeroes})$

= kx²–(−4)x+4 $x^2 - (-4)\,x + 4$ = x²+4x+4 $x^2 + 4x + 4$

Hence, the required quadratic polynomial is f(x)=x²+4x+4 $f(x) = x^2 + 4x + 4$

Q.18: If αandβ α and β are the zeroes of the quadratic polynomial f(x)=x²–3x–2 $f(x) = x^2 - 3x - 2$, find a quadratic polynomial whose zeroes are 12α+β and 12β+α $\frac{1}{2\alpha+\beta}$ and $\frac{1}{2\beta+\alpha}$.

Solution:

We have,

f(x)=x²–3x–2 $f(x) = x^2 - 3x - 2$

Sum of the zeroes = α+β $\alpha + \beta$ = 3

Product of the zeroes = αβ $\alpha\beta$ = -2

From the question,

Sum of the zeroes of the new polynomial = 12α+β + 12β+α $\frac{1}{2\alpha+\beta} + \frac{1}{2\beta+\alpha}$

= 2β+α+2α+β(2α+β)(2β+α) $\frac{2\beta+\alpha+2\alpha+\beta}{(2\alpha+\beta)(2\beta+\alpha)}$

= 3α+3β2(α²+β²)+5αβ $\frac{3\alpha+3\beta}{2(\alpha^2+\beta^2)+5\alpha\beta}$

= 3×32[2(α+β)²–2αβ+5×(−2)] $\frac{3\times 3}{2[2(\alpha+\beta)^2 - 2\alpha\beta + 5\times(-2)]}$

= 92[9–(−4)]–10 $\frac{9}{2[9-(-4)]-10}$

= 92[13]–10 $\frac{9}{2[13]-10}$

= 926–10 $\frac{9}{26-10}$ = 916 $\frac{9}{16}$

Product of the zeroes = 12α+β × 12β+α $\frac{1}{2\alpha+\beta} \times \frac{1}{2\beta+\alpha}$

= 1(2α+β)(2β+α) $\frac{1}{(2\alpha+\beta)(2\beta+\alpha)}$

= 14αβ+2α²+2β²+αβ $\frac{1}{4\alpha\beta+2\alpha^2+2\beta^2+\alpha\beta}$

= 15αβ+2(α²+β²) $\frac{1}{5\alpha\beta+2(\alpha^2+\beta^2)}$

= 15αβ+2((α+β)²–2αβ) $\frac{1}{5\alpha\beta+2((\alpha+\beta)^2-2\alpha\beta)}$

= 15×(−2)+2((3)²–2×(−2)) $\frac{1}{5\times(-2)+2((3)^2-2\times(-2))}$

= 1−10+26 $\frac{1}{-10+26}$ = 116 $\frac{1}{16}$

So, the quadratic polynomial is,

x²–(sumofthezeroes)x+(productofthezeroes)
$x^2-(\text{sum of the zeroes})\,x+(\text{product of the zeroes})$

= k (x²+ 916x+ 116)$(x^2+\frac{9}{16}x+\frac{1}{16})$

Hence, the required quadratic polynomial is k (x²+ 916x+ 116)$(x^2+\frac{9}{16}x+\frac{1}{16})$.

Q.19: If αandβ α and β **are the zeroes of the quadratic polynomial f(x)=x²+px+q** $f(x)=x^2+px+q$ **, form a polynomial whose zeroes are (α+β)²and(α–β)²** $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$.

Solution:

We have,

f(x)=x²+px+q$f(x)=x^2+px+q$

Sum of the zeroes = α+β$\alpha+\beta$ = -p

Product of the zeroes = αβ$\alpha\beta$ = q

From the question,

Sum of the zeroes of new polynomial = (α+β)²+(α–β)²$(\alpha+\beta)^2+(\alpha-\beta)^2$

= (α+β)²+α²+β²–2αβ$(\alpha+\beta)^2+\alpha^2+\beta^2-2\alpha\beta$

$= (\alpha+\beta)^2+(\alpha+\beta)^2-2\alpha\beta-2\alpha\beta(\alpha+\beta)^2+(\alpha+\beta)^2-2\alpha\beta-2\alpha\beta$

$= (-p)^2+(-p)^2-2\times q-2\times q(-p)^2+(-p)^2-2\times q-2\times q$

$= p^2+p^2-4qp^2+p^2-4q$

$= 2p^2-4q2p^2-4q$

Product of the zeroes of new polynomial = $(\alpha+\beta)^2(\alpha-\beta)^2(\alpha+\beta)^2(\alpha-\beta)^2$

$= (-p)^2((-p)^2-4q)(-p)^2((-p)^2-4q)$

$= p^2(p^2-4q)p^2(p^2-4q)$

So, the quadratic polynomial is ,

$x^2-(\text{sumofthezeroes})x+(\text{productofthezeroes})$
$x^2-(\text{sum of the zeroes})\,x+(\text{product of the zeroes})$

$= x^2-(2p^2-4q)x+p^2(p^2-4q)x^2-(2p^2-4q)\,x+p^2(p^2-4q)$

Hence, the required quadratic polynomial is $f(x)=k(x^2-(2p^2-4q)x+p^2(p^2-4q))$
$f(x)=k(x^2-(2p^2-4q)\,x+p^2(p^2-4q))$.

Q.20: If αandβ α and β are the zeroes of the quadratic polynomial $f(x)=x^2-2x+3$ $f(x)=x^2-2x+3$, find a polynomial whose roots are:

(i) $\alpha+2,\beta+2$ $\alpha+2,\beta+2$

(ii) $\alpha-1\alpha+1$, $\beta-1\beta+1$ $\frac{\alpha-1}{\alpha+1},\frac{\beta-1}{\beta+1}$.

Solution:

We have,

$f(x)=x^2-2x+3f(x)=x^2-2x+3$

Sum of the zeroes = $\alpha+\beta\alpha+\beta$ = 2

Product of the zeroes = $\alpha\beta\alpha\beta$ = 3

(i) Sum of the zeroes of new polynomial = $(\alpha+2)+(\beta+2)$

$= \alpha+\beta+4$

$= 2 + 4 = 6$

Product of the zeroes of new polynomial = $(\alpha+1)(\beta+1)$

$= \alpha\beta+2\alpha+2\beta+4$

$= \alpha\beta+2(\alpha+\beta)+4 \quad = 3+2(2)+4 = 11$

So, quadratic polynomial is:

$x^2-(\text{sum of the zeroes})\,x+(\text{product of the zeroes})$

$= x^2-6x+11$

Hence, the required quadratic polynomial is $f(x)=k(x^2-6x+11)$

(ii) Sum of the zeroes of new polynomial = $\frac{\alpha-1}{\alpha+1}+\frac{\beta-1}{\beta+1}$

$= \frac{(\alpha-1)(\beta+1)+(\beta-1)(\alpha+1)}{(\alpha+1)(\beta+1)}$

$= \frac{\alpha\beta+\alpha-\beta-1+\alpha\beta+\beta-\alpha-1}{(\alpha+1)(\beta+1)}$

$= \frac{3-1+3-1}{3+1+2} = \frac{4}{6} = \frac{2}{3}$

Product of the zeroes of new polynomial = $\frac{\alpha-1}{\alpha+1}+\frac{\beta-1}{\beta+1}$

$= \frac{2}{6} = \frac{1}{3}$

So, the quadratic polynomial is,

$x^2-(\text{sum of the zeroes})\,x+(\text{product of the zeroes})$

$= x^2-\frac{2}{3}x+\frac{1}{3}$

Thus, the required quadratic polynomial is $f(x)=k(x^2-\frac{2}{3}x+\frac{1}{3})$.

Q.21: If α and β are the zeroes of the quadratic polynomial $f(x) = ax^2 + bx + c$, then evaluate:

(i) $\alpha - \beta$

(ii) $\frac{1}{\alpha} - \frac{1}{\beta}$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

(iv) $\alpha^2\beta + \alpha\beta^2$

(v) $\alpha^4 + \beta^4$

(vi) $\frac{1}{a\alpha+b} + \frac{1}{a\beta+b}$

(vii) $\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b}$

(viii) $a\left[\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right] + b\left[\frac{\alpha}{a} + \frac{\beta}{a}\right]$

Solution:

$f(x) = ax^2 + bx + c$

Here,

Sum of the zeroes of polynomial = $\alpha + \beta$ = $\frac{-b}{a}$

Product of zeroes of polynomial = $\alpha\beta$ = $\frac{c}{a}$

Since, $\alpha + \beta$ are the roots (or) zeroes of the given polynomial, so

(i) $\alpha - \beta$

The two zeroes of the polynomials are-

$$\frac{-b+\sqrt{b^2-4ac}}{2a} - \left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$$= \frac{-b+\sqrt{b^2-4ac}+b+\sqrt{b^2-4ac}}{2a}$$

= 2√b²–4ac2a $\frac{2\sqrt{b^2-4ac}}{2a}$ = √b²–4aca $\frac{\sqrt{b^2-4ac}}{a}$

(ii) 1α−1β $\frac{1}{\alpha}-\frac{1}{\beta}$

= β–ααβ=−(α–β)αβ $\frac{\beta-\alpha}{\alpha\beta} = \frac{-(\alpha-\beta)}{\alpha\beta}$ ….. E.1

From previous question we know that,

α–βα–β = √b²–4aca $\frac{\sqrt{b^2-4ac}}{a}$

Also,

αβαβ = ca $\frac{c}{a}$

Putting the values in E.1, we will get

−(√b²–4aca ca)−($\frac{\frac{\sqrt{b^2-4ac}}{a}}{\frac{c}{a}}$)

= −(√b²–4acc)−($\frac{\sqrt{b^2-4ac}}{c}$)

(iii) 1α+1β–2αβ $\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta$

= [1α+1β]–2αβ[$\frac{1}{\alpha}+\frac{1}{\beta}$]−2αβ

= [α+βαβ]–2αβ[$\frac{\alpha+\beta}{\alpha\beta}$]−2αβ …….. E-1

Since,

Sum of the zeroes of polynomial = α+βα+β = −ba $\frac{-b}{a}$

Product of zeroes of polynomial = αβαβ = ca $\frac{c}{a}$

After substituting it in E-1, we will get

−ba×ac–2 ca $\frac{-b}{a}\times\frac{a}{c}-2\frac{c}{a}$

= −bc–2 ca $-\frac{b}{c}-2\frac{c}{a}$

= −ab–2c²ac $\frac{-ab-2c^2}{ac}$

$= -[bc + 2ca] - [\frac{b}{c} + \frac{2c}{a}]$

(iv) $\alpha^2\beta + \alpha\beta^2 \alpha^2\beta + \alpha\beta^2$

$= \alpha\beta(\alpha+\beta)\alpha\beta(\alpha+\beta)$ …….. E-1.

Since,

Sum of the zeroes of polynomial = $\alpha+\beta\alpha+\beta$ = $-ba \frac{-b}{a}$

Product of zeroes of polynomial = $\alpha\beta\alpha\beta$ = $ca \frac{c}{a}$

After substituting it in E-1, we will get

$ca(-ba)\frac{c}{a}(\frac{-b}{a})$

$= -bca^2 \frac{-bc}{a^2}$

(v) $\alpha^4+\beta^4\alpha^4+\beta^4$

$= (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2(\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2$

$= ((\alpha+\beta)^2 - 2\alpha\beta)^2 - (2\alpha\beta)^2((\alpha+\beta)^2 - 2\alpha\beta)^2 - (2\alpha\beta)^2$ ……. E- 1

Since,

Sum of the zeroes of polynomial = $\alpha+\beta\alpha+\beta$ = $-ba \frac{-b}{a}$

Product of zeroes of polynomial = $\alpha\beta\alpha\beta$ = $ca \frac{c}{a}$

After substituting it in E-1, we will get

$[(-ba) - 2(ca)]^2 - [2(ca)^2][(-\frac{b}{a}) - 2(\frac{c}{a})]^2 - [2(\frac{c}{a})^2]$

$= [b^2 - 2aca^2]^2 - 2c^2a^2[\frac{b^2-2ac}{a^2}]^2 - \frac{2c^2}{a^2}$

$= (b^2-2ac)^2 - 2a^2c^2a^4 \frac{(b^2-2ac)^2 - 2a^2c^2}{a^4}$

(vi) $1a\alpha+b + 1a\beta+b \frac{1}{a\alpha+b} + \frac{1}{a\beta+b}$

= aβ+b+aα+b(aα+b)(aβ+b) $\frac{a\beta+b+a\alpha+b}{(a\alpha+b)(a\beta+b)}$

= a(α+β)+2ba²αβ+abα+abβ+b² $\frac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab\alpha+ab\beta+b^2}$

= a(α+β)+2ba²αβ+ab(α+β)+b² $\frac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$

Since,

Sum of the zeroes of polynomial = α+βα + β = −ba $\frac{-b}{a}$

Product of zeroes of polynomial = αβαβ = ca $\frac{c}{a}$

After substituting it , we will get

bac−b²+b² $\frac{b}{ac-b^2+b^2}$

= bac $\frac{b}{ac}$

(vii) βaα+b + αaβ+b $\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b}$

= β(aβ+b)+α(aα+b)(aα+b)(aβ+b) $\frac{\beta(a\beta+b)+\alpha(a\alpha+b)}{(a\alpha+b)(a\beta+b)}$

= aβ²+bβ+αa²+bαa²αβ+abα+abβ+b² $\frac{a\beta^2+b\beta+\alpha a^2+b\alpha}{a^2\alpha\beta+ab\alpha+ab\beta+b^2}$

= aα²+bβ²+bα+bβa²×ca+ab(α+β)+b² $\frac{a\alpha^2+b\beta^2+b\alpha+b\beta}{a^2\times\frac{c}{a}+ab(\alpha+\beta)+b^2}$

Since,

Sum of the zeroes of polynomial = α+βα + β = −ba $\frac{-b}{a}$

Product of zeroes of polynomial = αβαβ = ca $\frac{c}{a}$

After substituting it , we will get

a[(α+β)²+b(α+β)]ac $\frac{a[(\alpha+\beta)^2+b(\alpha+\beta)]}{ac}$

= a[(α+β)²−2αβ]−b²aac $\frac{a[(\alpha+\beta)^2-2\alpha\beta]-\frac{b^2}{a}}{ac}$

= a[b²a−2ca]−b²aac $\frac{a[\frac{b^2}{a}-\frac{2c}{a}]-\frac{b^2}{a}}{ac}$

$= a[b^2 - 2ca] - b^2 a ac \dfrac{a[\frac{b^2-2c}{a}] - \frac{b^2}{a}}{ac}$

$= a[b^2 - 2c - b^2 a] ac \dfrac{a[\frac{b^2-2c-b^2}{a}]}{ac}$

$= b^2 - 2c - b^2 ac \dfrac{b^2-2c-b^2}{ac}$

$= -2cac \dfrac{-2c}{ac} = -2a \dfrac{-2}{a}$

(viii) $a[\alpha^2\beta + \beta^2\alpha] + b[\alpha a + \beta a] a\left[\dfrac{\alpha^2}{\beta} + \dfrac{\beta^2}{\alpha}\right] + b\left[\dfrac{\alpha}{a} + \dfrac{\beta}{a}\right]$

$= a[\alpha^2 + \beta^2 \alpha\beta] + b(\alpha^2 + \beta^2 \alpha\beta) a\left[\dfrac{\alpha^2+\beta^2}{\alpha\beta}\right] + b\left(\dfrac{\alpha^2+\beta^2}{\alpha\beta}\right)$

$= a[(\alpha+\beta)^2 - 2\alpha\beta] + b((\alpha+\beta)^2 - 2\alpha\beta)\alpha\beta \dfrac{a[(\alpha+\beta)^2 - 2\alpha\beta] + b((\alpha+\beta)^2 - 2\alpha\beta)}{\alpha\beta}$

Since,

Sum of the zeroes of polynomial = $\alpha+\beta\alpha+\beta = -ba \dfrac{-b}{a}$

Product of zeroes of polynomial = $\alpha\beta\alpha\beta = ca \dfrac{c}{a}$

After substituting it , we will get

$a[(-ba)^2 - 3 \times ca] + b((-ba)^2 - 2ca)ca \dfrac{a[(\frac{-b}{a})^2 - 3 \times \frac{c}{a}] + b((\frac{-b}{a})^2 - 2\frac{c}{a})}{\frac{c}{a}}$

$= a^2c[-b^2a^2 + 3bca^2 + b^2a^2 - 2bca^2] \dfrac{a^2}{c}\left[\dfrac{-b^2}{a^2} + \dfrac{3bc}{a^2} + \dfrac{b^2}{a^2} - \dfrac{2bc}{a^2}\right]$

$= [-b^2a^2a^2c + 3bca^2a^2c + b^2a^2a^2c - 2bca^2a^2c]\left[\dfrac{-b^2a^2}{a^2c} + \dfrac{3bca^2}{a^2c} + \dfrac{b^2a^2}{a^2c} - \dfrac{2bca^2}{a^2c}\right]$

$= -b^2ac + 3b + b^2ac - 2b \dfrac{-b^2}{ac} + 3b + \dfrac{b^2}{ac} - 2b = b$