RD SHARMA

Solutions

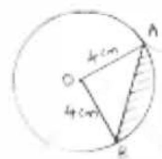
Class 10 Maths

Chapter 15

Ex15.4

1. AB is a chord of a circle with centre O and radius 4cm. AB is length 4cm and divides circle into two segments. Find the area of minor segment

Sol:



Radius of circle r = 4cm = OA = OB

Length of chord AB = 4cm

OAB is equilateral triangle ∠AOB = 60° ⟶ θ

Angle subtended at centre $\theta = 60°$

Area of segment (shaded region) = (area of sector) – (area of ΔAOB)

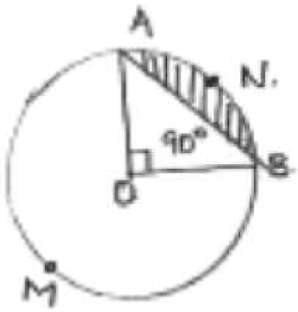
$= \frac{\theta}{360°} \times \pi r^2 = \frac{\sqrt{3}}{4}(side)^2$

$= \frac{60}{360} \times \frac{22}{7} \times 4 \times 4 = \frac{\sqrt{2}}{4} \times 4 \times 4$

$= \frac{176}{3} - 4\sqrt{3} = 58.67 - 6.92 = 51.75\ cm^2$

2. A chord of circle of radius 14cm makes a right angle at the centre. Find the areas of minor and major segments of the circle.

Sol:



Radius (r) = 14cm

$\theta = 90°$

= OA = OB

Area of minor segment (ANB)

$= (area\ of\ ANB\ sector) - (area\ of\ \Delta AOB)$

$= \frac{\theta}{360°} \times \pi r^2 - \frac{1}{2} \times OA \times OB$

$= \frac{90}{360} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{2} \times 14 \times 14$

$= 154 - 98 = 56cm^2$

Area of major segment (other than shaded)

= area of circle – area of segment ANB

$= \pi r^2 - 56$

$= \frac{22}{7} \times 14 \times 14 - 56$

= 616 – 56

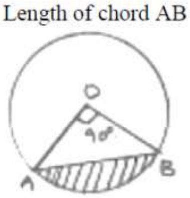
= 560 cm^2.

3. A chord 10 cm long is drawn in a circle whose radius is $5\sqrt{2}$ cm. Find the area of both segments

Sol:

Given radius = r = $5\sqrt{2}$ cm = OA = OB

Length of chord AB = 10cm



In ΔOAB, OA = OB = $5\sqrt{2}\ cm$ AB = 10cm

$OA^2 + OB^2 = (5\sqrt{2})^2 + (5\sqrt{2})^2 = 50 + 50 = 100 = (AB)^2$

Pythagoras theorem is satisfied OAB is right triangle

θ = angle subtended by chord = ∠AOB = 90°

Area of segment (minor) = shaded region

$= area\ of\ sector - area\ of\ \Delta OAB$

$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times OA \times OB$

$= \frac{90}{360} \times \frac{22}{7}\left(5\sqrt{2}\right)^2 - \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2}$

$= \frac{275}{7} - 25 - \frac{100}{7}\ cm^2$

Area of major segment = (area of circle) – (area of minor segment)

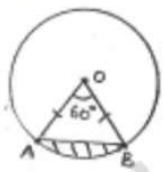
$= \pi r^2 2 - \frac{100}{7}$

$= \frac{22}{7} \times \left(5\sqrt{2}\right)^2 - \frac{100}{7}$

$= \frac{1100}{7} - \frac{100}{7} = \frac{1000}{7}\ cm^2$

4. A chord AB of circle, of radius 14cm makes an angle of 60° at the centre. Find the area of minor segment of circle.

Sol:



Given radius (r) = 14cm = OA = OB

θ = angle at centre = 60°

In ΔAOB, ∠A = ∠B [angles opposite to equal sides OA and OB] = x

By angle sum property ∠A + ∠B + ∠O = 180°

x + x + 60° = 180° ⇒ 2x = 120° ⇒ x = 60°

All angles are 60°, OAB is equilateral OA = OB = AB

Area of segment = area of sector – area Δle OAB

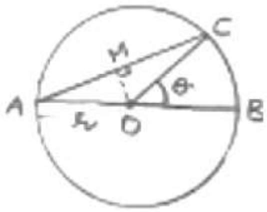
$= \frac{\theta}{360^\circ} \times \pi r^2 - \frac{\sqrt{3}}{4} \times (-AB)^2$

$= \frac{60}{360} \times \frac{22}{7} \times 14 \times 14 - \frac{\sqrt{3}}{4} \times 14 \times 14$

$= \frac{308}{3} - 49\sqrt{3} = \frac{308 - 147\sqrt{3}}{3}\ cm^2$

5. AB is the diameter of a circle, centre O. C is a point on the circumference such that ∠COB $= \theta$. The area of the minor segment cutoff by AC is equal to twice the area of sector BOC. Prove that $\sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2} = \pi\left(\frac{1}{2} - \frac{\theta}{120^\circ}\right)$

Sol:



Given AB is diameter of circle with centre O

∠COB = θ

Area of sector BOC = $\frac{\theta}{360^\circ} \times \pi r^2$

Area of segment cut off, by AC = (area of sector) – (area of ΔAOC)

∠AOC = 180 – θ [∠AOC and ∠BOC form linear pair]

Area of sector = $\frac{(180-\theta)}{360^\circ} \times \pi r^2 = \frac{\pi r^2}{2} - \frac{\pi \theta r^2}{360^\circ}$

In ΔAOC, drop a perpendicular AM, this bisects ∠AOC and side AC.

Now, In ΔAMO, sin∠AOM = $\frac{AM}{DA} \Rightarrow \sin\left(\frac{180-\theta}{2}\right) = \frac{AM}{R}$

$\Rightarrow$ AM = R $\sin\left(90 - \frac{\theta}{2}\right) = R.\cos\frac{\theta}{2}$

cos ∠ADM = $\frac{OM}{OA} \Rightarrow \cos\left(90 - \frac{\theta}{2}\right) = \frac{OM}{Y} \Rightarrow OM = R.Sin\frac{\theta}{2}$

Area of segment = $\frac{\pi r^2}{2} - \frac{\pi \theta r^2}{360^\circ} - \frac{1}{2}(AC \times OM)\ [AC = 2\ AM]$

$= \frac{\pi r^2}{2} - \frac{\pi \theta r^2}{360^\circ} - \frac{1}{2} \times \left(2\ R\cos\frac{\theta}{2}\ R\sin\frac{\theta}{2}\right)$

$= r^2\left[\frac{\pi}{2} - \frac{\pi\theta}{360^\circ} - \cos\frac{\theta}{2}\sin\frac{\theta}{2}\right]$

Area of segment by AC = 2 (Area of sector BDC)

$r^2\left[\frac{\pi}{2} - \frac{\pi\theta}{360^\circ} - \cos\frac{\theta}{2}.\sin\frac{\theta}{2}\right] = 2r^2\left[\frac{\pi\theta}{360^\circ}\right]$

$\cos\frac{\theta}{2}.\sin\frac{\theta}{2} = \frac{\pi}{2} - \frac{\pi\theta}{360} - \frac{2\pi\theta}{360^\circ}$

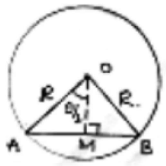
$= \frac{\pi}{2} - \frac{\pi\theta}{360^\circ}[1+2]$

$= \frac{\pi}{2} - \frac{\pi\theta}{360^\circ} = \pi\left(\frac{1}{2} - \frac{\theta}{120^\circ}\right)$

$\cos\frac{\theta}{2}.\sin\frac{\theta}{2} = \pi\left(\frac{1}{2} - \frac{\theta}{120^\circ}\right)$

6. A chord of a circle subtends an angle θ at the centre of circle. The area of the minor segment cut off by the chord is one eighth of the area of circle. Prove that 8 $\sin\frac{\theta}{2}.\cos\frac{\theta}{2} + \pi = \frac{\pi\theta}{45}$

Sol:



Let radius of circle = r

Area of circle = πr^2

AB is a chord, OA, OB are joined drop OM ⊥ AB. This OM bisects AB as well as ∠AOB.

$\angle AOM = \angle MOB = \frac{1}{2}(0) = \frac{\theta}{2}$ AB = 2AM

In ΔAOM, ∠AMO = 90°

$\text{Sin}\frac{\theta}{2} = \frac{AM}{AD} \Rightarrow AM = R.\sin\frac{\theta}{2}$ $AB = 2R\sin\frac{\theta}{2}$

$\text{Cos}\frac{\theta}{2} = \frac{OM}{AD} \Rightarrow OM = R\cos\frac{\theta}{2}$

Area of segment cut off by AB = (area of sector) – (area of triangles)

$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times AB \times OM$

$= r^2\left[\frac{\pi\theta}{360°} - \frac{1}{2}.2r\sin\frac{\theta}{2}.R\cos\frac{\theta}{2}\right]$

$= R^2\left[\frac{\pi\theta}{360°} - \sin\frac{\theta}{2}.\cos\frac{\theta}{2}\right]$

Area of segment $= \frac{1}{2}(area\ of\ circle)$

$$r^2\left[\frac{\pi\theta}{360} - \sin\frac{\theta}{2}.\cos\frac{\theta}{2}\right] = \frac{1}{8}\pi r^2$$

$$\frac{8\pi\theta}{360°} - 8\sin\frac{\theta}{2}.\cos\frac{\theta}{2} = \pi$$

$$8\sin\frac{\theta}{2}.\cos\frac{\theta}{2} + \pi = \frac{\pi\theta}{45}$$