

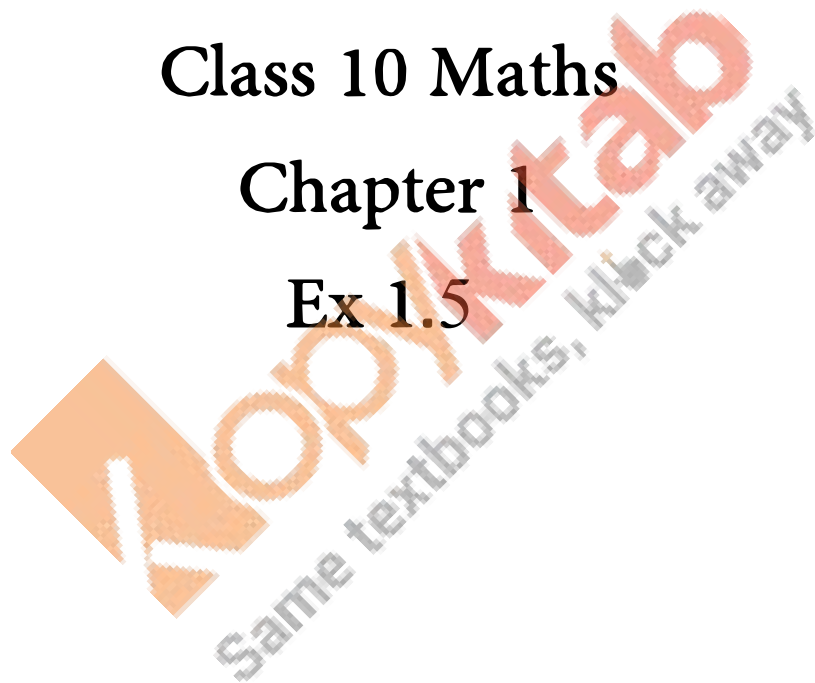
RD SHARMA

Solutions

Class 10 Maths

Chapter 1

Ex 1.5



Q.1: Show that the following numbers are irrational.

(i) $7\sqrt{57}\sqrt{5}$

Let us assume that $7\sqrt{57}\sqrt{5}$ is rational. Then, there exist positive co primes a and b such that

$$7\sqrt{57}\sqrt{5} = ab \frac{a}{b}$$

$$\sqrt{5}\sqrt{5} = a^2b \frac{a}{7b}$$

We know that $\sqrt{5}\sqrt{5}$ is an irrational number

Here we see that $\sqrt{5}\sqrt{5}$ is a rational number which is a contradiction.

(ii) $6+\sqrt{26} + \sqrt{2}$

Let us assume that $6+\sqrt{26} + \sqrt{2}$ is rational. Then, there exist positive co primes a and b such that

$$6+\sqrt{26} + \sqrt{2} = ab \frac{a}{b}$$

$$\sqrt{2}\sqrt{2} = ab - 6 \frac{a}{b} - 6$$

$$\sqrt{2}\sqrt{2} = a - 6bb \frac{a-6b}{b}$$

Here we see that $\sqrt{2}\sqrt{2}$ is a rational number which is a contradiction as we know that $\sqrt{2}\sqrt{2}$ is an irrational number

Hence $6+\sqrt{26} + \sqrt{2}$ is an irrational number

(iii) $3-\sqrt{53} - \sqrt{5}$

Let us assume that $3-\sqrt{53} - \sqrt{5}$ is rational. Then, there exist positive co primes a and b such that

$$3-\sqrt{53} - \sqrt{5} = ab \frac{a}{b}$$

$$\sqrt{5}\sqrt{5} = 3 - ab - \frac{a}{b}$$

$$\sqrt{5}\sqrt{5} = 3b - ab - \frac{3b-a}{b}$$

Here we see that $\sqrt{5}\sqrt{3}$ is a rational number which is a contradiction as we know that $\sqrt{5}\sqrt{3}$ is an irrational number

Hence $3 - \sqrt{53} - \sqrt{5}$ is an irrational number.

Q.2: Prove that the following numbers are irrationals.

Sol:

(i) $2\sqrt{7} \frac{2}{\sqrt{7}}$

Let us assume that $2\sqrt{7} \frac{2}{\sqrt{7}}$ is rational. Then, there exist positive co primes a and b such that

$$2\sqrt{7} \frac{2}{\sqrt{7}} = ab \frac{a}{b}$$

$$\sqrt{7} \frac{2}{\sqrt{7}} = 2ba \frac{2b}{a}$$

$\sqrt{7} \frac{2}{\sqrt{7}}$ is rational number which is a contradiction

Hence $2\sqrt{7} \frac{2}{\sqrt{7}}$ is an irrational number

(ii) $32\sqrt{5} \frac{3}{2\sqrt{5}}$

Let us assume that $32\sqrt{5} \frac{3}{2\sqrt{5}}$ is rational. Then, there exist positive co primes a and b such that

$$32\sqrt{5} \frac{3}{2\sqrt{5}} = ab \frac{a}{b}$$

$$\sqrt{5} \frac{3}{\sqrt{5}} = 3b2a \frac{3b}{2a}$$

$\sqrt{5} \frac{3}{\sqrt{5}}$ is rational number which is a contradiction

Hence $32\sqrt{5} \frac{3}{2\sqrt{5}}$ is irrational.

(iii) $4 + \sqrt{24} + \sqrt{2}$

Let us assume that $4 + \sqrt{24} + \sqrt{2}$ is rational. Then, there exist positive co primes a and b such that

$$4 + \sqrt{24} + \sqrt{2} = ab \frac{a}{b}$$

$$\sqrt{2}\sqrt{2} = ab - 4 \frac{a}{b} - 4$$

$$\sqrt{2}\sqrt{2} = a - 4bb \frac{a-4b}{b}$$

$\sqrt{2}\sqrt{2}$ is rational number which is a contradiction

Hence $4 + \sqrt{24} + \sqrt{2}$ is irrational.

(iv) $5\sqrt{25}\sqrt{2}$

Let us assume that $5\sqrt{25}\sqrt{2}$ is rational. Then, there exist positive co primes a and b such that

$$5\sqrt{25}\sqrt{2} = ab \frac{a}{b}$$

$$\sqrt{2}\sqrt{2} = ab - 5 \frac{a}{b} - 5$$

$$\sqrt{2}\sqrt{2} = a - 5bb \frac{a-5b}{b}$$

$\sqrt{2}\sqrt{2}$ is rational number which is a contradiction

Hence $5\sqrt{25}\sqrt{2}$ is irrational

Q.3: Show that $2 - \sqrt{3}2 - \sqrt{3}$ is an irrational number.

Sol:

Let us assume that $2 - \sqrt{3}2 - \sqrt{3}$ is rational. Then, there exist positive co primes a and b such that

$$2 - \sqrt{3}2 - \sqrt{3} = ab \frac{a}{b}$$

$$\sqrt{3}\sqrt{3} = 2 - ab2 - \frac{a}{b}$$

Here we see that $\sqrt{3}\sqrt{3}$ is a rational number which is a contradiction

Hence $2 - \sqrt{3}2 - \sqrt{3}$ is irrational

Q.4: Show that $3 + \sqrt{23} + \sqrt{2}$ is an irrational number.

Sol:

Let us assume that $3 + \sqrt{2}3 + \sqrt{2}$ is rational. Then, there exist positive co primes a and b such that

$$3 + \sqrt{2}3 + \sqrt{2} = ab \frac{a}{b}$$

$$\sqrt{2}\sqrt{2} = ab - 3\frac{a}{b} - 3$$

$$\sqrt{2}\sqrt{2} = a - 3bb \frac{a-3b}{b}$$

Here we see that $\sqrt{2}\sqrt{2}$ is a irrational number which is a contradiction

Hence $3 + \sqrt{2}3 + \sqrt{2}$ is irrational

Q.5: Prove that $4 - 5\sqrt{2}4 - 5\sqrt{2}$ is an irrational number.

Sol:

Let us assume that $4 - 5\sqrt{2}4 - 5\sqrt{2}$ is rational. Then, there exist positive co primes a and b such that

$$4 - 5\sqrt{2}4 - 5\sqrt{2} = ab \frac{a}{b}$$

$$5\sqrt{2}5\sqrt{2} = ab - 4\frac{a}{b} - 4$$

$$\sqrt{2} = ab - 45\sqrt{2} = \frac{a-4}{5}$$

$$\sqrt{2}\sqrt{2} = a - 4b5b \frac{a-4b}{5b}$$

This contradicts the fact that $\sqrt{2}\sqrt{2}$ is an irrational number

Hence $4 - 5\sqrt{2}4 - 5\sqrt{2}$ is irrational

Q.6: Show that $5 - 2\sqrt{3}5 - 2\sqrt{3}$ is an irrational number.

Sol.

Let us assume that $5 - 2\sqrt{3}5 - 2\sqrt{3}$ is rational. Then, there exist positive co primes a and b such that

$$5 - 2\sqrt{3}5 - 2\sqrt{3} = ab \frac{a}{b}$$

$$2\sqrt{3}2\sqrt{3} = ab - 5\frac{a}{b} - 5$$

$$\sqrt{3} = \frac{a-5b}{2b} \quad \sqrt{3} = \frac{a-5b}{2b}$$

This contradicts the fact that $\sqrt{3}\sqrt{3}$ is an irrational number

Hence $5 - 2\sqrt{3}5 - 2\sqrt{3}$ is irrational

Q.7: Prove that $2\sqrt{3} - 12\sqrt{3} - 1$ is an irrational number.

Sol:

Let us assume that $2\sqrt{3} - 12\sqrt{3} - 1$ is rational. Then, there exist positive co primes a and b such that

$$2\sqrt{3} - 12\sqrt{3} - 1 = ab \frac{a}{b}$$

$$2\sqrt{3}2\sqrt{3} = ab + 1\frac{a}{b} + 1$$

$$\sqrt{3} = \frac{a+1}{2b} \quad \sqrt{3} = \frac{a+1}{2b}$$

This contradicts the fact that $\sqrt{3}\sqrt{3}$ is an irrational number

Hence $5 - 2\sqrt{3}5 - 2\sqrt{3}$ is irrational

Q.8: Prove that $2 - 3\sqrt{5}2 - 3\sqrt{5}$ is an irrational number.

Sol:

Let us assume that $2 - 3\sqrt{5}2 - 3\sqrt{5}$ is rational. Then, there exist positive co primes a and b such that

$$2 - 3\sqrt{5}2 - 3\sqrt{5} = ab \frac{a}{b}$$

$$3\sqrt{5}3\sqrt{5} = ab - 2\frac{a}{b} - 2$$

$$3\sqrt{5} = \frac{a-2}{3b} \quad \sqrt{5} = \frac{a-2}{3b}$$

This contradicts the fact that $\sqrt{5}\sqrt{5}$ is an irrational number

Hence $2 - 3\sqrt{5}2 - 3\sqrt{5}$ is irrational

Q.9: Prove that $\sqrt{5} + \sqrt{3}\sqrt{5} + \sqrt{3}$ is irrational.

Sol:

Let us assume that $\sqrt{5} + \sqrt{3}\sqrt{5} + \sqrt{3}$ is rational. Then, there exist positive co primes a and b such that

$$\sqrt{5} + \sqrt{3}\sqrt{5} + \sqrt{3} = ab \frac{a}{b}$$

$$\sqrt{5} = ab - \sqrt{3}\sqrt{5} = \frac{a}{b} - \sqrt{3} \quad (\sqrt{5})^2 = (ab - \sqrt{3})^2 (\sqrt{5})^2 = \left(\frac{a}{b} - \sqrt{3}\right)^2 5 = (ab)^2 - 2a\sqrt{3}b + 3$$

$$5 = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{3}}{b} + 3 \Rightarrow 5 - 3 = (ab)^2 - 2a\sqrt{3}b \Rightarrow 5 - 3 = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{3}}{b} \Rightarrow 2 = (ab)^2 - 2a\sqrt{3}b$$

$$\Rightarrow 2 = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{3}}{b} \Rightarrow (ab)^2 - 2 = 2a\sqrt{3}b \Rightarrow \left(\frac{a}{b}\right)^2 - 2 = \frac{2a\sqrt{3}}{b} \Rightarrow a^2 - 2b^2 = 2a\sqrt{3}b$$

$$\Rightarrow \frac{a^2 - 2b^2}{b^2} = \frac{2a\sqrt{3}}{b} \Rightarrow (a^2 - 2b^2)(b^2a) = \sqrt{3} \Rightarrow \left(\frac{a^2 - 2b^2}{b^2}\right) \left(\frac{b}{2a}\right) = \sqrt{3} \Rightarrow (a^2 - 2b^2) = \sqrt{3}$$

$$\Rightarrow \left(\frac{a^2 - 2b^2}{2ab}\right) = \sqrt{3}$$

Here we see that $\sqrt{3}\sqrt{3}$ is a rational number which is a contradiction as we know that $\sqrt{3}\sqrt{3}$ is an irrational number

Hence $\sqrt{5} + \sqrt{3}\sqrt{5} + \sqrt{3}$ is an irrational number

Q.10: Prove that $\sqrt{3} + \sqrt{4}\sqrt{3} + \sqrt{4}$ is irrational.

Sol:

Let us assume that $\sqrt{3} + \sqrt{4}\sqrt{3} + \sqrt{4}$ is rational. Then, there exist positive co primes a and b such that

$$\sqrt{3} + \sqrt{4}\sqrt{3} + \sqrt{4} = ab \frac{a}{b}$$

$$\sqrt{4} = ab - \sqrt{3}\sqrt{4} = \frac{a}{b} - \sqrt{3} \quad (\sqrt{4})^2 = (ab - \sqrt{3})^2 (\sqrt{4})^2 = \left(\frac{a}{b} - \sqrt{3}\right)^2 4 = (ab)^2 - 2a\sqrt{3}b + 3$$

$$4 = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{3}}{b} + 3 \Rightarrow 4 - 3 = (ab)^2 - 2a\sqrt{3}b \Rightarrow 4 - 3 = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{3}}{b} \Rightarrow 1 = (ab)^2 - 2a\sqrt{3}b$$

$$\Rightarrow 1 = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{3}}{b} \Rightarrow (ab)^2 - 1 = 2a\sqrt{3}b \Rightarrow \left(\frac{a}{b}\right)^2 - 1 = \frac{2a\sqrt{3}}{b} \Rightarrow a^2 - b^2 = 2a\sqrt{3}b$$

$$\Rightarrow \frac{a^2 - b^2}{b^2} = \frac{2a\sqrt{3}}{b} \Rightarrow (a^2 - b^2)(b^2a) = \sqrt{3} \Rightarrow \left(\frac{a^2 - b^2}{b^2}\right) \left(\frac{b}{2a}\right) = \sqrt{3} \Rightarrow (a^2 - b^2) = \sqrt{3}$$

$$\Rightarrow \left(\frac{a^2 - b^2}{2ab}\right) = \sqrt{3}$$

Here we see that $\sqrt{3}\sqrt{3}$ is a rational number which is a contradiction as we know that $\sqrt{3}\sqrt{3}$ is an irrational number

Hence $\sqrt{3} + \sqrt{4}\sqrt{3} + \sqrt{4}$ is an irrational number

Q.11: Prove that for any prime positive integer p , $\sqrt{p}\sqrt{p}$ is an irrational number.

Sol:

Let us assume that $\sqrt{p}\sqrt{p}$ is rational. Then, there exist positive co primes a and b such that

$$\sqrt{p}\sqrt{p} = ab \frac{a}{b}$$

$$pp = (ab)^2 \left(\frac{a}{b}\right)^2$$

$$\Rightarrow pp = a^2 b^2 \frac{a^2}{b^2}$$

$$\Rightarrow pb^2 = a^2 \Rightarrow pb^2 = a^2 \Rightarrow p|a^2 \Rightarrow p|a^2 \Rightarrow p|a \Rightarrow a = pc \text{ for some positive integer } c$$

$$\Rightarrow b^2 p = a^2 \Rightarrow b^2 p = a^2$$

$$\Rightarrow b^2 p = a^2 \Rightarrow b^2 p = p^2 c^2 p^2 c^2 \quad (\because a = pc)$$

$$\Rightarrow p|b^2 \text{ (since } p|c^2 p) \Rightarrow p|b^2 \text{ (since } p|c^2 p) \Rightarrow p|b \Rightarrow p|a \text{ and } p|b \Rightarrow p|a \text{ and } p|b$$

This contradicts the fact that a and b are co primes

Hence $\sqrt{p}\sqrt{p}$ is irrational

Q.12: If p, q are prime positive integers, prove that $\sqrt{p} + \sqrt{q}\sqrt{p} + \sqrt{q}$ is an irrational number.

Sol:

Let us assume that $\sqrt{p} + \sqrt{q}\sqrt{p} + \sqrt{q}$ is rational. Then, there exist positive co primes a and b such that

$$\sqrt{p} + \sqrt{q}\sqrt{p} + \sqrt{q} = ab \frac{a}{b}$$

$$\sqrt{p} = ab - \sqrt{q} \sqrt{p} = \frac{a}{b} - \sqrt{q} \quad (\sqrt{p})^2 = (ab - \sqrt{q})^2 \quad (\sqrt{p})^2 = \left(\frac{a}{b} - \sqrt{q}\right)^2 \quad p = (ab)^2 - 2a\sqrt{q}b + q$$

$$p = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{q}}{b} + q \quad p - q = (ab)^2 - 2a\sqrt{q}b \quad p - q = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{q}}{b} \quad (ab)^2 - (p - q) = 2a\sqrt{q}b$$

$$\left(\frac{a}{b}\right)^2 - (p - q) = \frac{2a\sqrt{q}}{b} \quad a^2 - b^2(p - q)b^2 = 2a\sqrt{q}b \quad \frac{a^2 - b^2(p - q)}{b^2} = \frac{2a\sqrt{q}}{b} \quad (a^2 - b^2(p - q)b^2)(b^2a) = \sqrt{q}$$

$$\left(\frac{a^2 - b^2(p - q)}{b^2}\right)\left(\frac{b}{2a}\right) = \sqrt{q} \quad \sqrt{q} = \frac{a^2 - b^2(p - q)}{2ab}$$

Here we see that $\sqrt{q}\sqrt{q}$ is a rational number which is a contradiction as we know that $\sqrt{q}\sqrt{q}$ is an irrational number

Hence $\sqrt{p} + \sqrt{q}\sqrt{p} + \sqrt{q}$ is an irrational number

