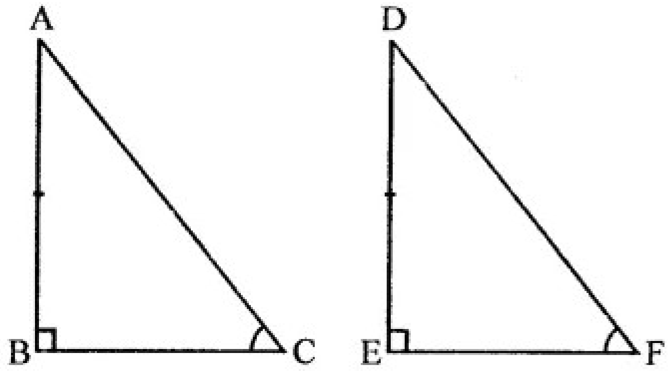
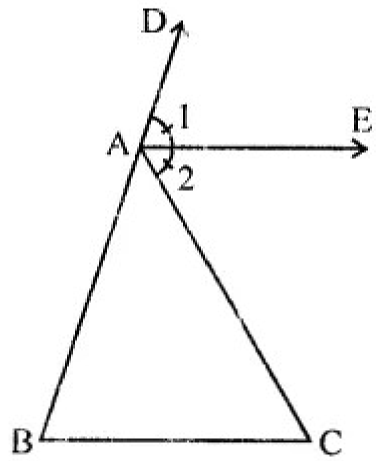
RD Sharma Class 9 Solutions Chapter 12 Heron's Formula Ex 12.3

Question 1.
In two right triangles one side an acute angle of one are equal to the corresponding side and angle of the other. Prove that the triangles are congruent.
Solution:
Given : In ΔABC and ΔDEF,
∠B = ∠E = 90°
∠C = ∠F
AB = DE



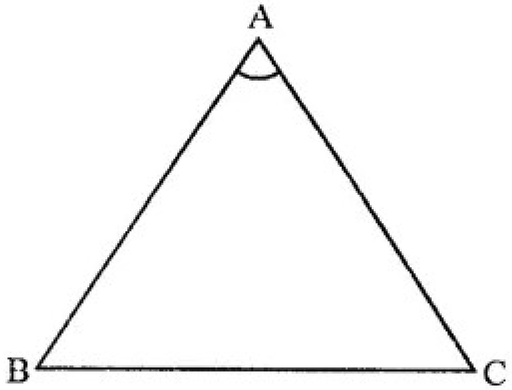
To prove : ΔABC = ΔDEF
Proof : In ΔABC and ΔDEF,
∠B = ∠E (Each = 90°)
∠C = ∠F (Given)
AB = DE (Given)
ΔABC = ΔDEF (AAS axiom)

Question 2.
If the bisector of the exterior vertical angle of a triangle be parallel to the base. Show that the triangle is isosceles.
Solution:
Given : In ΔABC, AE is the bisector of vertical exterior ∠A and AE || BC

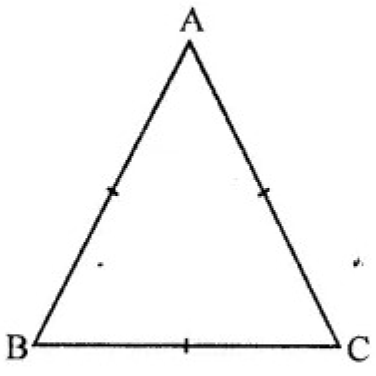


To prove : ΔABC is an isosceles

Proof: ∵ AE || BC
∴ ∠1 = ∠B (Corresponding angles)
∠2 = ∠C (Alternate angle)
But ∠1 = ∠2 (∵ AE is the bisector of ∠CAD)
∴ ∠B = ∠C
∴ AB = AC (Sides opposite to equal angles)
∴ ΔABC is an isosceles triangle
Question 3.
In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.
Solution:
Given : In ΔABC, AB = AC
∠A = 2(∠B + ∠C)

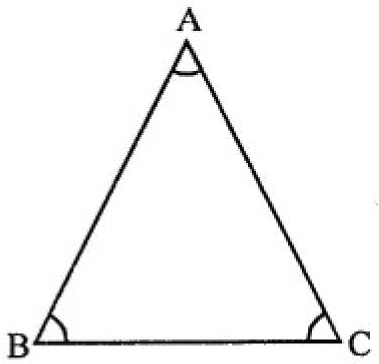


To calculate: Base angles,
Let ∠B = ∠C = x
Then ∠A = 2(∠B + ∠C)
= 2(x + x) = 2 x 2x = 4x
∵ Sum of angles of a triangle = 180°
∴ 4x + x + x – 180° ⇒ 6x = 180°
⇒ x= 180∘6 = 30° o
∴ ∠B = ∠C = 30 and ∠A = 4 x 30° = 120
Question 4.
Prove that each angle of an equilateral triangle is 60°. [NCERT]
Solution:
Given : ΔABC is an equilateral triangle



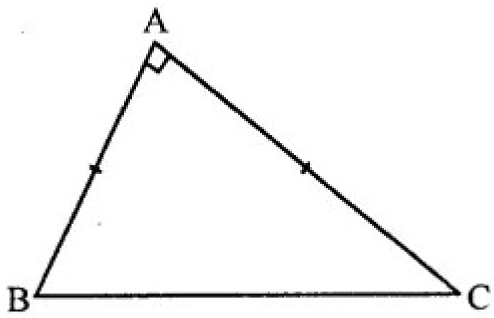
Proof: In ΔABC,
AB = AC (Sides of an equilateral triangle)

∴ ∠C = ∠B …(i)
(Angles opposite to equal angles)
Similarly, AB = BC
∴ ∠C = ∠A …(ii)
From (i) and (ii),
∠A = ∠B = ∠C
But ∠A + ∠B + ∠C = 180° (Sum of angles of a triangle)
∴ ∠A = ∠B = ∠C = $\frac{180^{\circ}}{3}$= 60°
Question 5.
Angles A, B, C of a triangle ABC are equal to each other. Prove that ΔABC is equilateral.
Solution:
Given : In ΔABC, ∠A = ∠B = ∠C



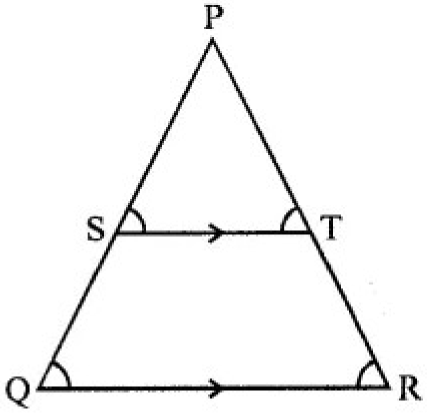
To prove : ΔABC is an equilateral
Proof: In ΔABC,
∴ ∠B = ∠C (Given)
∴ AC = AB …(i) (Sides opposite to equal angles)
Similarly, ∠C = ∠A
∴ BC =AB …(ii)
From (i) and (ii)
AB = BC = CA
Hence ΔABC is an equilateral triangle

Question 6.
ABC is a right angled triangle in which ∠A = 90° and AB = AC. Find ∠B and ∠C.
Solution:
In ΔABC, ∠A = 90°



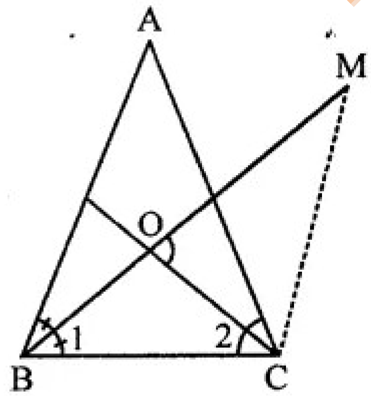
AB =AC (Given)
∴ ∠C = ∠B (Angles opposite to equal sides)

But ∠B + ∠C = 90° (∵ ∠B = 90°)
∴ ∠B = ∠C = 90∘2 = 45°
Hence ∠B = ∠C = 45°
Question 7.
PQR is a triangle in which PQ = PR and S is any point on the side PQ. Through S, a line is drawn parallel to QR and intersecting PR at T. Prove that PS = PT.
Solution:
Given : In ΔPQR, PQ = PR
S is a point on PQ and PT || QR



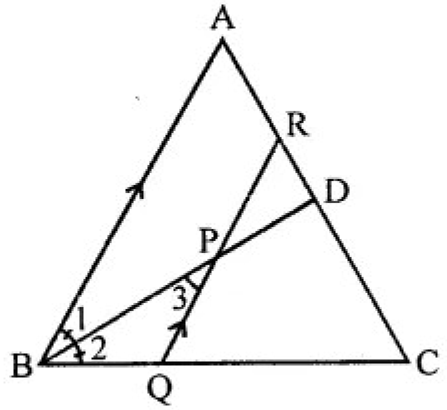
To prove : PS = PT
Proof : ∵ST || QR
∴ ∠S = ∠Q and ∠T = ∠R (Corresponding angles)
But ∠Q = ∠R (∵ PQ = PR)
∴ PS = PT (Sides opposite to equal angles)

Question 8.
In a ΔABC, it is given that AB = AC and the bisectors of ∠B and ∠C intersect at O. If M is a point on BO produced, prove that ∠MOC = ∠ABC.
Solution:
Given : In ΔABC, AB = AC the bisectors of ∠B and ∠C intersect at O. M is any point on BO produced.



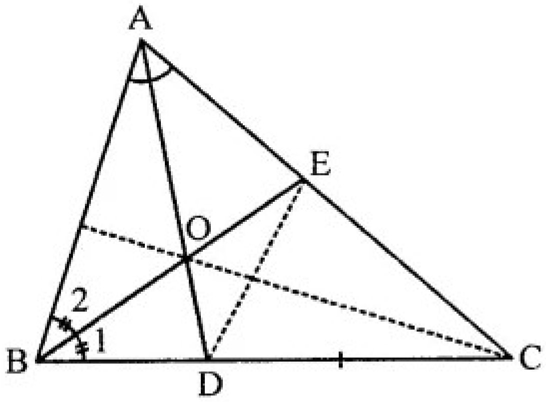
To prove : ∠MOC = ∠ABC
Proof: In ΔABC, AB = BC
∴ ∠C = ∠B
∵ OB and OC are the bisectors of ∠B and ∠C
∴ ∠1 =∠2 = 12∠B

Now in ∠OBC,
Ext. ∠MOC = Interior opposite angles ∠1 + ∠2
= ∠1 + ∠1 = 2∠1 = ∠B
Hence ∠MOC = ∠ABC
Question 9.
P is a point on the bisector of an angle ∠ABC. If the line through P parallel to AB meets BC at Q, prove that triangle BPQ is isosceles.
Solution:
Given : In ΔABC, P is a point on the bisector of ∠B and from P, RPQ || AB is draw which meets BC in Q



To prove : ΔBPQ is an isosceles
Proof : ∵ BD is the bisectors of CB
∴ ∠1 = ∠2
∵ RPQ || AB
∴ ∠1 = ∠3 (Alternate angles)
But ∠1 == ∠2 (Proved)
∴ ∠2 = ∠3
∴ PQ = BQ (sides opposite to equal angles)
∴ ΔBPQ is an isosceles

Question 10.
ABC is a triangle in which ∠B = 2∠C, D is a point on BC such that AD bisects ∠BAC = 72°.
Solution:
Given: In ΔABC,
∠B = 2∠C, AD is the bisector of ∠BAC AB = CD
To prove : ∠BAC = 72°
Construction : Draw bisector of ∠B which meets AD at O



Proof : AC at E join ED

In ΔABC,

$\angle B = 2\angle C$

OB is the bisector of $\angle B$

$\therefore \angle C = \frac{1}{2}\angle B = \angle 1$

$\therefore$ In ΔBCE,

$\angle 1 = C$

$\therefore$ BE = EC (Opposite sides of equal angles)

Now in ΔABE and ΔCDE,

AB = CD (Given)

$\angle 2 = \angle C$

BE = EC

$\therefore \Delta ABE \cong \Delta CDE$ (SAS axiom)

$\therefore \angle BAE = \angle CDE$ (c.p.c.t.)

$\therefore$ AE = ED (Opposite sides)

Let $\angle CDE = 2x$, and $\angle ADE = \angle DAE = x$

($\because \angle A = 2x$)

In ΔABD, we have,

$\angle ADC = \angle ABD + \angle BAD \Rightarrow x + 2x = 2y + x$

$\Rightarrow 2x = 2y \Rightarrow x = y$

In ΔABC,

$\angle A + \angle B + \angle C = 180°$

(Angles of a triangle)

$\Rightarrow 2x + 2y + y = 180°$

$\Rightarrow 2x + 2x + x = 180° \Rightarrow 5x = 180°$

$$\Rightarrow x = \frac{180^\circ}{5} = 36^\circ$$

$$\therefore \angle BAC = 2x = 2 \times 36^\circ = 72^\circ$$

Hence proved.