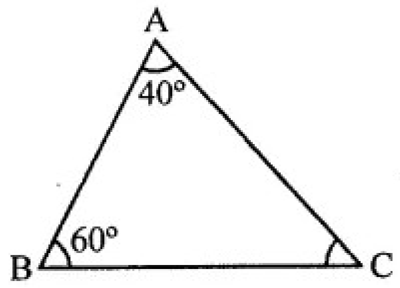
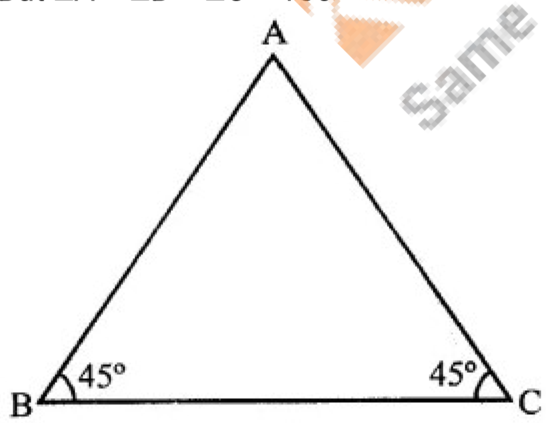
# RD Sharma Class 9 Solutions Chapter 12 Heron's Formula Ex 12.6

Question 1.
In ΔABC, if ∠A = 40° and ∠B = 60°. Determine the longest and shortest sides of the triangle.
Solution:
In ΔABC, ∠A = 40°, ∠B = 60°
But ∠A + ∠B + ∠C = 180°



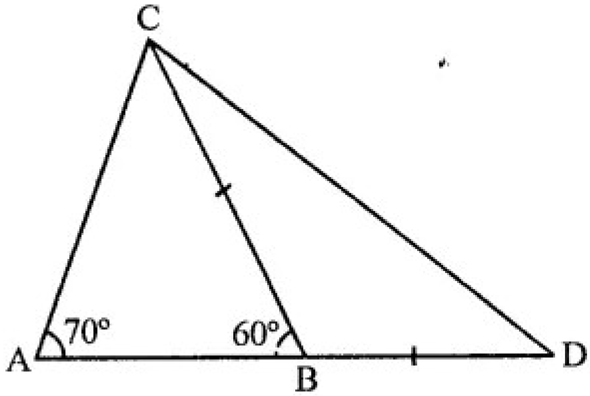

⇒ 40° + 60° + ∠C = 180°
⇒ ∠C = 180° = (40° + 60°)
= 180° – 100° = 80°
∵ ∠C = 80°, which is the greatest angle and
∠A = 40° is the smallest angle
∴ Side AB which is opposite to the greatest angle is the longest and side BC which is opposite to the smallest angle is the shortest.

Question 2.
In a ΔABC, if ∠B = ∠C = 45°. which is the longest side?
Solution:
In ΔABC, ∠B = ∠C = 45°
But ∠A + ∠B + ∠C = 180°




⇒ ∠A + 45° + 45° = 180°
⇒ ∠A + 90° = 180°
∴ ∠A = 180°-90° = 90°
∴∠A is the greatest
∴ Side BC opposite to it is the longest

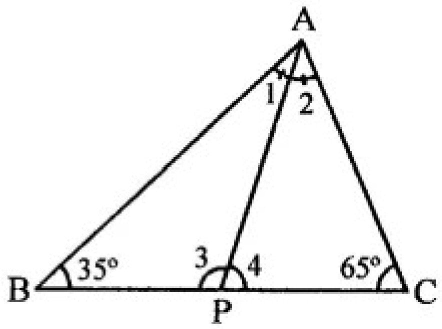
Question 3.
In ΔABC, side AB is produced to D so that BD = BC. If ∠B = 60° and ∠A = 70°, prove that :
(i) AD > CD
(ii) AD > AC
Solution:
Given : In AABC, side BC is produced to D such that BD = BC
∠A = 70° and ∠B = 60°



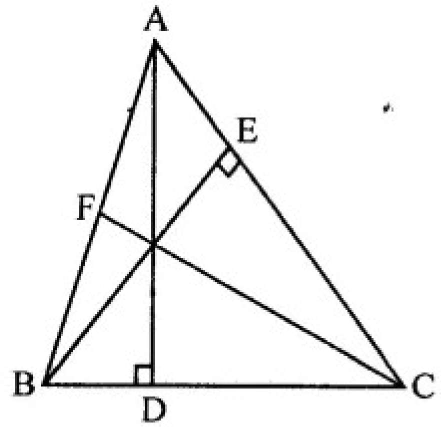

To prove :
(i) AD > CD (ii) AD > AC
Proof: In ΔABC,
∠A = 70°, ∠B = 60°
But Ext. ∠CBD + ∠CBA = 180° (Linear pair)
∠CBD + 60° = 180° 3
⇒ ∠CBD = 180° – 60° = 120°
But in ΔBCD,
BD = BC
∴ ∠D = ∠BCD
But ∠D + ∠BCD = 180° – 120° = 60°
∴∠D = ∠BCD = $\frac{60^\circ}{2}$ = 30°
and in ΔABC,
∠A + ∠B + ∠C = 180°
⇒ 70° + 60° + ∠C = 180°
⇒ 130° + ∠C = 180°
∴ ∠C =180°- 130° = 50°
Now ∠ACD = ∠ACB + ∠BCD = 50° + 30° = 80°
(i) Now in ΔACB,
∠ACD = 80° and ∠A = 70°
∴ Side AD > CD
(Greater angle has greatest side opposite to it)
(ii) ∵ ∠ACD = 80° and ∠D = 30°
∴ AD > AC
Question 4.
Is it possible to draw a triangle with sides of length 2 cm, 3 cm and 7 cm?
Solution:
We know that in a triangle, sum of any two sides is greater than the third side and 2

cm + 3 cm = 5 cm and 5 cm < 7 cm
∴ This triangle is not possible to draw

Question 5.
In ΔABC, ∠B = 35°, ∠C = 65° and the bisector of ∠BAC meets BC in P. Arrange AP, BP and CP in descending order.
Solution:
In ΔABC, ∠B = 35°, ∠C = 65° and AP is the bisector of ∠BAC which meets BC in P.
Arrange PA, PB and PC in descending order In ΔABC,
∠A + ∠B + ∠C = 180° (Sum of angles of a triangle)

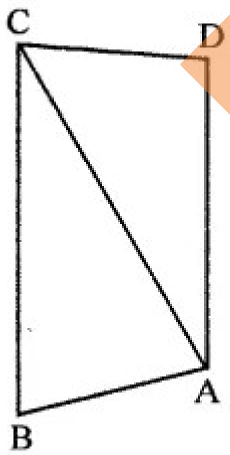



⇒ ∠A + 35° + 65° = 180°
∠A + 100°= 180°
∴ ∠A =180°- 100° = 80°
∵ PA is a bisector of ∠BAC
∴ ∠1 = ∠2 = $\frac{80^\circ}{2}$ = 40°
Now in ΔACP, ∠ACP > ∠CAP
⇒ ∠C > ∠2
∴ AP > CP …(i)
Similarly, in ΔABP,
∠BAP > ∠ABP ⇒ ∠1 > ∠B
∴ BP > AP …(ii)
From (i) and (ii)
BP > AP > CP
Question 6.
Prove that the perimeter of a triangle is greater than the sum of its altitudes
Solution:
Given : In ΔABC,
AD, BE and CF are altitudes




To prove : AB + BC + CA > AD + BC + CF
Proof : We know that side opposite to greater angle is greater.
In $\Delta ABD$, $\angle D = 90°$
$\therefore \angle D > \angle B$
$\therefore$ AB >AD ...(i)
Similarly, we can prove that
BC > BE and
CA > CF
Adding we get,
AB + BC + CA > AD + BE + CF

Question 7.
In the figure, prove that:
(i) CD + DA + AB + BC > 2AC
(ii) CD + DA + AB > BC






Solution:
Given : In the figure, ABCD is a quadrilateral and AC is joined
To prove :
(i) CD + DA + AB + BC > 2AC
(ii) CD + DA + AB > BC
Proof:
(i) In $\Delta ABC$,
AB + BC > AC ...(i)
(Sum of two sides of a triangle is greater than its third side)

Similarly in $\Delta ADC$,
CD + DA > AC ...(ii)
Adding (i) and (ii)
CD + DA + AB + BC > AC + AC
⇒ CD + DA + AB + BC > 2AC
(ii) In $\Delta ACD$,
CD + DA > CA
(Sum of two sides of a triangle is greater than its third side)
Adding AB to both sides,
CD + DA + AB > CA + AB
But CA + AB > BC (in $\Delta ABC$)
∴ CD + DA + AD > BC

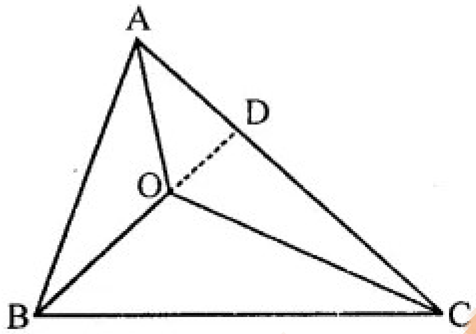
Question 8.
Which of the following statements are true (T) and which are false (F)?
(i) Sum of the three sides of a triangle is less than the sum of its three altitudes.
(ii) Sum of any two sides of a triangle is greater than twice the median drawn to the third side.
(iii) Sum of any two sides of a triangle is greater than the third side.
(iv) Difference of any two sides of a triangle is equal to the third side.
(v) If two angles of a triangle are unequal, then the greater angle has the larger side opposite to it.
(vi) Of all the line segments that can be drawn from a point to a line not containing it, the perpendicular line segment is the shortest one.
Solution:
(i) False. Sum of three sides of a triangle is greater than the sum of its altitudes.
(ii) True.
(iii) True.
(iv) False. Difference of any two sides is less than the third side.
(v) True.
(vi) True.



Question 9.
Fill in the blanks to make the following statements true.
(i) In a right triangle, the hypotenuse is the ....... side.
(ii) The sum of three altitudes of a triangle is ....... than its perimeter.
(iii) The sum of any two sides of a triangle is ........ than the third side.
(iv) If two angles of a triangle are unequal, then the smaller angle has the ..... side opposite to it.
(v) Difference of any two sides of a triangle is....... than the third side.
(vi) If two sides of a triangle are unequal, then the larger side has ......... angle opposite to it.
Solution:
(i) In a right triangle, the hypotenuse is the longest side.
(ii) The sum of three altitudes of a triangle is less than its perimeter.
(iii) The sum of any two sides of a triangle is greater than the third side.
(iv) If two angles of a triangle are unequal, then the smaller angle has the smaller

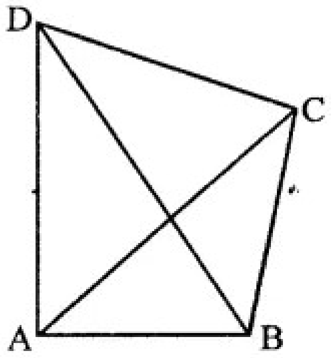
side opposite to it.
(v) Difference of any two sides of a triangle is less than the third side.
(vi) If two sides of a triangle are unequal, then the larger side has greater angle opposite to it.

Question 10.
O is any point in the interior of ΔABC. Prove that
(i) AB + AC > OB + OC
(ii) AB + BC + CA > OA + OB + OC
(iii) OA + OB + OC > 12 (AB + BC + CA)
Solution:
Given : In ΔABC, O is any point in the interior of the ΔABC, OA, OB and OC are joined
To prove :
(i) AB + AC > OB + OC
(ii) AB + BC + CA > OA + OB + OC
(iii) OA + OB + OC > 12 (AB + BC + CA)
Construction : Produce BO to meet AC in D.




Proof: In ΔABD,
(i) AB + AD > BD (Sum of any two sides of a triangle is greater than third)
⇒ AB + AD > BO + OD …(i)
Similarly, in ΔODC,
OD + DC > OC …(ii)
Adding (i) and (ii)
AB + AD + OD + DC > OB + OD + OC
⇒ AB + AD + DC > OB + OC
⇒ AB + AC > OB + OC
(ii) Similarly, we can prove that
BC + AB > OA + OC
and CA + BC > OA + OB
(iii) In ΔOAB, AOBC and ΔOCA,
OA + OB > AB
OB + OC > BC
and OC + OA > CA
Adding, we get
2(OA + OB + OC) > AB + BC + CA
∴ OA + OB + OO > 12 (AB + BC + CA)
Question 11.
Prove that in a quadrilateral the sum of all the sides is greater than the sum of its

diagonals.
Solution:
Given : In quadrilateral ABCD, AC and BD are its diagonals,




To prove : AB + BC + CD + DA > AC + BD
Proof: In ΔABC,
AB + BC > AC …(i)
(Sum of any two sides of a triangle is greater than its third side)
Similarly, in ΔADC,
DA + CD > AC …(ii)
In ΔABD,
AB + DA > BD …(iii)
In ΔBCD,
BC + CD > BD …(iv)
Adding (i), (ii), (iii) and (iv)
2(AB + BC + CD + DA) > 2AC + 2BD
⇒ 2(AB + BC + CD + DA) > 2(AC + BD)
∴ AB + BC + CD + DA > AC + BD