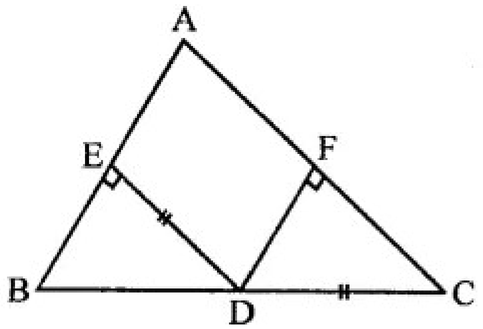
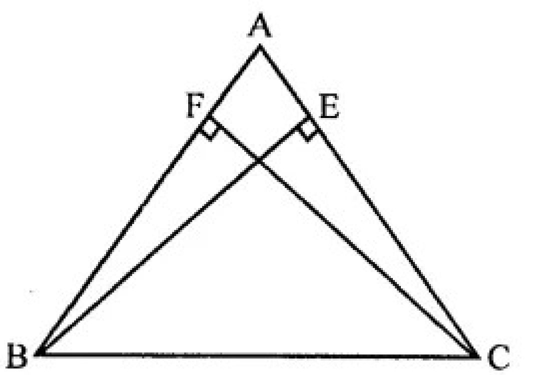
RD Sharma Class 9 Solutions Chapter 12 Heron’s Formula Ex 12.5

Question 1.
ABC is a triangle and D is the mid-point of BC. The perpendiculars from D to AB and AC are equal. Prove that the triangle is isosceles.
Solution:
Given : In ΔABC, D is mid-point of BC and DE ⊥ AB, DF ⊥ AC and DE = DF



To Prove : ΔABC is an isosceles triangle
Proof : In right ΔBDE and ΔCDF,
Side DE = DF
Hyp. BD = CD
∴ ΔBDE ≅ ΔCDF (RHS axiom)
∴ ∠B = ∠C (c.p.c.t.)
Now in ΔABC,
∠B = ∠C (Prove)
∴ AC = AB (Sides opposite to equal angles)
∴ AABC is an isosceles triangle

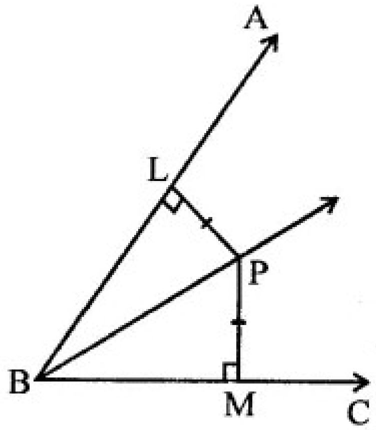
Question 2.
ABC is a triangle in which BE and CF are, respectively, the perpendiculars to the sides AC and AB. If BE = CF, prove that ΔABC is an isosceles.
Solution:
Given : In ΔABC,
BE ⊥ AC and CF ⊥ AB
BE = CF



To prove : AABC is an isosceles triangle

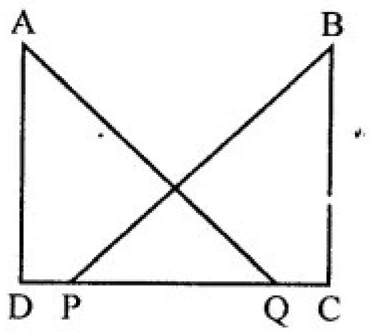
Proof : In right ABCE and ABCF Side
BE = CF (Given)
Hyp. BC = BC (Common)
∴ ΔBCE ≅ ΔBCF (RHS axiom)
∴ ∠BCE = ∠CBF (c.p.c.t.)
∴ AB = AC (Sides opposite to equal angles)
∴ ΔABC is an isosceles triangle

Question 3.
If perpendiculars from any point within an angle on its arms are congruent, prove that it lies on the bisector of that angle.
Solution:
Given : A point P lies in the angle ABC and PL ⊥ BA and PM ⊥ BC and PL = PM. PB is joined



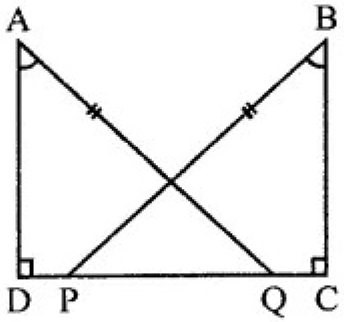
To prove : PB is the bisector ∠ABC,
Proof : In right ΔPLB and ΔPMB
Side PL = PM (Given)
Hyp. PB = PB (Common)
∴ ΔPLB ≅ ΔPMB (RHS axiom)
∴ ΔPBL = ΔPBM (c.p.c.t.)
∴ PB is the bisector of ∠ABC

Question 4.
In the figure, AD ⊥ CD and CB ⊥ CD. If AQ = BP and DP = CQ, prove that ∠DAQ = ∠CBP.



Solution:
Given : In the figure,

AD ⊥ CD and CB ⊥ CD, AQ = BP and DP = CQ

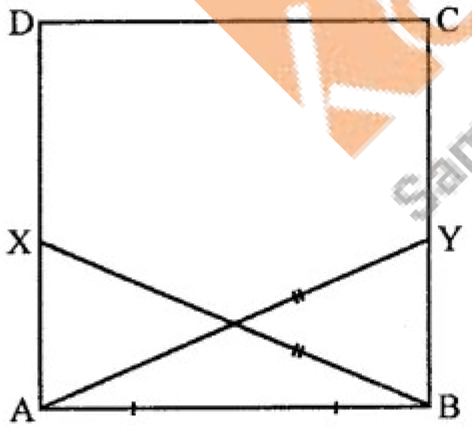


To prove : ∠DAQ = ∠CBP
Proof : ∵ DP = CQ
∴ DP + PQ = PQ + QC
⇒ DQ = PC
Now in right ΔADQ and ΔBCP
Side DQ = PC (Proved)
Hyp. AQ = BP
∴ ΔADQ ≅ ΔBCP (RHS axiom)
∴ ∠DAQ = ∠CBP (c.p.c.t.)

Question 5.
Which of the following statements are true (T) and which are false (F):
(i) Sides opposite to equal angles of a triangle may be unequal.
(ii) Angles opposite to equal sides of a triangle are equal.
(iii) The measure of each angle of an equilateral triangle is 60°.
(iv) If the altitude from one vertex of a triangle bisects the opposite side, then the triangle may be isosceles.
(v) The bisectors of two equal angles of a triangle are equal.
(vi) If the bisector of the vertical angle of a triangle bisects the base, then the triangle may be isosceles.
(vii) The two altitudes corresponding to two equal sides of a triangle need not be equal.
(viii)If any two sides of a right triangle are respectively equal to two sides of other right triangle, then the two triangles are congruent.
(ix) Two right triangles are congruent if hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle.
Solution:
(i) False : Sides opposite to equal angles of a triangle are equal.
(ii) True.
(iii) True.
(iv) False : The triangle is an isosceles triangle.
(v) True.
(vi) False : The triangle is an isosceles.
(vii) False : The altitude an equal.
(viii) False : If one side and hypotenuse of one right triangle on one side and hypotenuse of the other right triangle are equal, then triangles are congruent.
(ix) True.

Question 6.
Fill in the blanks in the following so that each of the following statements is true.
(i) Sides opposite to equal angles of a triangle are …….
(ii) Angle opposite to equal sides of a triangle are …….
(iii) In an equilateral triangle all angles are …….
(iv) In a ΔABC if ∠A = ∠C, then AB = …….
(v) If altitudes CE and BF of a triangle ABC are equal, then AB = ……..
(vi) In an isosceles triangle ABC with AB = AC, if BD and CE are its altitudes, then BD is ……… CE.
(vii) In right triangles ABC and DEF, if hypotenuse AB = EF and side AC = DE, then ΔABC ≅ Δ……
Solution:
(i) Sides opposite to equal angles of a triangle are equal.
(ii) Angle opposite to equal sides of a triangle are equal.
(iii) In an equilateral triangle all angles are equal.
(iv) In a ΔABC, if ∠A = ∠C, then AB = BC.
(v) If altitudes CE and BF of a triangle ABC are equal, then AB = AC.
(vi) In an isosceles triangle ABC with AB = AC, if BD and CE are its altitudes, then BD is equal to CE.
(vii) In right triangles ABC and DEF, it hypotenuse AB = EF and side AC = DE, then ΔABC ≅ ΔEFD.

Question 7.
ABCD is a square, X and Y are points on sides AD and BC respectively such that AY = BX. Prove that BY = AX and ∠BAY = ∠ABX.
Solution:
Given : In square ABCD, X and Y are points on side AD and BC respectively and AY = BX



To prove : BY = AX
∠BAY = ∠ABX
Proof: In right ΔBAX and ΔABY
AB =AB (Common)
Hyp. BX = AY (Given)
∴ ΔBAX ≅ ΔABY (RHS axiom)
∴ AX = BY (c.p.c.t.)

∠ABX = ∠BAY (c.p.c.t.)
Hence, BY = AX and ∠BAY = ∠ABX.