Exercise 2.1: Polynomials

Q.1: Find the zeroes of each of the following quadratic polynomials and verify the relationship between the zeroes and their coefficients:

(i) $f(x)=x^2-2x-8$ $f(x)=x^2-2x-8$

(ii) $g(s)=4s^2-4s+1$ $g(s)=4s^2-4s+1$

(iii) $6x^2-3-7x$ $6x^2-3-7x$

(iv) $h(t)=t^2-15$ $h(t)=t^2-15$

(v) $p(x)=x^2+2\sqrt{2}x-6$ $p(x)=x^2+2\sqrt{2}x-6$

(vi) $q(x)=\sqrt{3}x^2+10x+7\sqrt{3}$ $q(x)=\sqrt{3}x^2+10x+7\sqrt{3}$

(vii) $f(x)=x^2-(\sqrt{3}+1)x+\sqrt{3}$ $f(x)=x^2-\left(\sqrt{3}+1\right)x+\sqrt{3}$

(viii) $g(x)=a(x^2+1)-x(a^2+1)$ $g(x)=a\left(x^2+1\right)-x\left(a^2+1\right)$

Solution:

(i) $f(x)=x^2-2x-8$ $f(x)=x^2-2x-8$

We have,

f(x)=x^2–2x–8 $f(x) = x^2 - 2x - 8$

= x^2–4x+2x–8 $x^2 - 4x + 2x - 8$

= x(x–4)+2(x–4) $x(x-4) + 2(x-4)$

= (x+2)(x–4) $(x+2)(x-4)$

Zeroes of the polynomials are -2 and 4.

Now,

Sum of the zeroes = −coefficientofxcoefficientofx $\frac{-\ coefficient\ of\ x}{coefficient\ of\ x}$

-2 + 4 = −(−2)1 $\frac{-(-2)}{1}$

2 = 2

Product of the zeroes = constanttermCoefficientofx $\frac{constant\ term}{Coefficient\ of\ x}$

-8 = −81 $\frac{-8}{1}$

-8 = -8

Hence, the relationship is verified.

(ii) g(s)=4s^2–4s+1 $g(s) = 4s^2 - 4s + 1$

We have,

g(s)=4s^2–4s+1 $g(s) = 4s^2 - 4s + 1$

= 4s^2–2s–2s+1 $4s^2 - 2s - 2s + 1$

= 2s(2s–1)−1(2s–1) $2s(2s-1) - 1(2s-1)$

= (2s–1)(2s–1) $(2s-1)(2s-1)$

Zeroes of the polynomials are 12 $\frac{1}{2}$ and 12 $\frac{1}{2}$.

Sum of zeroes = −coefficientofscoefficientofs2 $\frac{-coefficient\ of\ s}{coefficient\ of\ s^2}$

12 + 12 = -(-4)4 $\frac{1}{2} + \frac{1}{2} = \frac{-(-4)}{4}$

1 = 1

Product of zeroes = constanttermCoefficientofx $\frac{constant\ term}{Coefficient\ of\ x}$

12 × 12 = 14 $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ 14 = 14 $\frac{1}{4} = \frac{1}{4}$

Hence, the relationship is verified.

(iii) 6s²–3–7x $6s^2 - 3 - 7x$

= 6s²−7x−3 $6s^2 - 7x - 3$ = (3x + 11)(2x – 3)

Zeros of the polynomials are 32 and −13 $\frac{3}{2}$ *and* $\frac{-1}{3}$

Sum of the zeros = −coefficientofxcoefficientofx2 $\frac{-coefficient\ of\ x}{coefficientofx^2}$

-13 + 32 = -(-7)6 $\frac{-1}{3} + \frac{3}{2} = \frac{-(-7)}{6}$ 76 = 76 $\frac{7}{6} = \frac{7}{6}$

Product of the zeroes = constanttermcoefficientofx2 $\frac{constant\ term}{coefficient\ of\ x^2}$

-13 × 32 = -36 $\frac{-1}{3} \times \frac{3}{2} = \frac{-3}{6}$ -36 = -36 $\frac{-3}{6} = \frac{-3}{6}$

Hence, the relationship is verified.

(iv) h(t)=t²–15 $h(t) = t^2 - 15$

We have,

h(t)=t²−15 $h(t) = t^2 - 15$

$= t^2 - \sqrt{15} t^2 - \sqrt{15}$

$= (t+\sqrt{15})(t-\sqrt{15}) \left(t + \sqrt{15}\right) \left(t - \sqrt{15}\right)$

Zeroes of the polynomials are $-\sqrt{15}$ and $\sqrt{15} - \sqrt{15}$ *and* $\sqrt{15}$

Sum of the zeroes = 0

$-\sqrt{15}+\sqrt{15} - \sqrt{15} + \sqrt{15} = 0$

0 = 0

Product of zeroes = constanttermCoefficientofx $\frac{constant\ term}{Coefficient\ of\ x}$ = −151 $\frac{-15}{1}$

$-\sqrt{15}\times\sqrt{15}=-15 - \sqrt{15} \times \sqrt{15} = -15$

-15 = -15

Hence, the relationship verified.

(v) p(x)=x²+2√2x–6 $p(x) = x^2 + 2\sqrt{2}x - 6$

We have,

p(x)=x²+2√2x–6 $p(x) = x^2 + 2\sqrt{2}x - 6$

= x²+3√2x+3√2x–6 $x^2 + 3\sqrt{2}x + 3\sqrt{2}x - 6$

= x(x+3√2)–√2(x+3√2) $x\left(x + 3\sqrt{2}\right) - \sqrt{2}\left(x + 3\sqrt{2}\right)$

= (x+3√2)(x–√2) $\left(x + 3\sqrt{2}\right)\left(x - \sqrt{2}\right)$

Zeroes of the polynomials are 3√2 $3\sqrt{2}$ and –3√2 $-3\sqrt{2}$

Sum of the zeroes = -2√21 $\frac{-2\sqrt{2}}{1}$

√2–3√2=–2√2 $\sqrt{2} - 3\sqrt{2} = -2\sqrt{2}$ –2√2=–2√2 $-2\sqrt{2} = -2\sqrt{2}$

Product of the zeroes = constanttermCoefficientofx $\frac{constant\ term}{Coefficient\ of\ x}$

√2×–3√2=-61 $\sqrt{2} \times -3\sqrt{2} = \frac{-6}{1}$

-6 = -6

Hence, the relationship is verified.

(vi) q(x) = $\sqrt{3}x^2+10x+7\sqrt{3}$ $\sqrt{3}x^2+10x+7\sqrt{3}$

= $\sqrt{3}x^2+7x+3x+7\sqrt{3}$ $\sqrt{3}x^2+7x+3x+7\sqrt{3}$

= $\sqrt{3}x(x+\sqrt{3})7(x+sqrt3)$ $\sqrt{3}x\left(x+\sqrt{3}\right)7\left(x+sqrt3\right)$

= $(x+\sqrt{3})(7+sqrt3)$ $\left(x+\sqrt{3}\right)\left(7+sqrt3\right)$

Zeros of the polynomials are $-\sqrt{3}$and $-7\sqrt{3}$ $-\sqrt{3}and\frac{-7}{\sqrt{3}}$

Sum of zeros = $-10\sqrt{3}$ $\frac{-10}{\sqrt{3}}$

$-\sqrt{3}-7\sqrt{3}=-10\sqrt{3}$ $-\sqrt{3}-\frac{7}{\sqrt{3}}=\frac{-10}{\sqrt{3}}$ $-10\sqrt{3}=-10\sqrt{3}$ $\frac{-10}{\sqrt{3}}=\frac{-10}{\sqrt{3}}$

Product of the polynomials are $-\sqrt{3}, -7\sqrt{3}$ $-\sqrt{3},\frac{-7}{\sqrt{3}}$

7 = 7

Hence, the relationship is verified.

(vii) h(x) = $x^2-(\sqrt{3}+1)x+\sqrt{3}$ $x^2-\left(\sqrt{3}+1\right)x+\sqrt{3}$

= $x^2-\sqrt{3}-x+\sqrt{3}$ $x^2-\sqrt{3}-x+\sqrt{3}$

=$x(x-\sqrt{3})-1(x-\sqrt{3})$ $x(x-\sqrt{3})-1(x-\sqrt{3})$

= $(x-\sqrt{3})(x-1)$ $(x-\sqrt{3})(x-1)$

Zeros of the polynomials are 1 and $\sqrt{3}$ $\sqrt{3}$

Sum of zeros = $-coefficientofxcoefficientofx^2=-[-\sqrt{3}-1]$ $\frac{-co\ efficient\ of\ x}{co\ effi...}=-[-\sqrt{3}-1]$

1+√3=√3+1$1+\sqrt{3}=\sqrt{3}+1$

Product of zeros = −coefficientofxcoefficientofx²=√3$\frac{-co\ efficient\ of\ x}{co\ efficient\ of\ x^2}=\sqrt{3}$

√3=√3$\sqrt{3}=\sqrt{3}$

Hence, the relationship is verified

(viii) g(x) = a[(x²+1)–x(a²+1)]²$a\left[\left(x^2+1\right)-x\left(a^2+1\right)\right]^2$

= ax²+a−a²x−xax^2+a-a^2x-x

= ax²−[(a²x+1)]+a$ax^2-\left[\left(a^2x+1\right)\right]+a$

= ax²−a²x–x+aax^2-a^2x-x+a

= ax(x−a)−1(x–a)$ax(x-a)-1(x-a)$ = (x–a)(ax–1)$(x-a)(ax-1)$

Zeros of the polynomials are 1aand1$\frac{1}{a}\ and\ 1$

Sum of the zeros = a[−a²−1]a $\frac{a[-a^2-1]}{a}$

1a+a=a²+1a a²+1a=a²+1a$\frac{1}{a}+a=\frac{a^2+1}{a}\ \frac{a^2+1}{a}=\frac{a^2+1}{a}$

Product of zeros = a/a

1a×a=aa$\frac{1}{a}\times a=\frac{a}{a}$

1 = 1

Hence, the relationship is verified.

Q.2: If αandβ$\alpha\ and\ \beta$ **are the zeroes of the quadratic polynomial f(x)=x²–5x+4** $f(x)=x^2-5x+4$**, find the value of 1α+1β–2αβ**$\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta$.

Solution: We have,

αandβ $\alpha\ and\ \beta$ are the roots of the quadratic polynomial.

f(x)=x2–5x+4 $f(x) = x^2 - 5x + 4$

Sum of the roots = α+β $\alpha + \beta$ = 5

Product of the roots = αβ $\alpha\beta$ = 4

So,

1α+1β–2αβ $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$ = β+ααβ–2αβ $\frac{\beta+\alpha}{\alpha\beta} - 2\alpha\beta$

= 54–2×4=54–8=-274 $\frac{5}{4} - 2 \times 4 = \frac{5}{4} - 8 = \frac{-27}{4}$

Q.3: If αandβ $\alpha\ and\ \beta$ are the zeroes of the quadratic polynomial f(x)=x2–5x+4 $f(x) = x^2 - 5x + 4$, find the value of 1α+1β–2αβ $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$.

Solution:

Since, αandβ $\alpha\ and\ \beta$ are the zeroes of the quadratic polynomial.

p(y)=x2–5x+4 $p(y) = x^2 - 5x + 4$

Sum of the zeroes = α+β $\alpha + \beta$ = 5

Product of the roots = αβ $\alpha\beta$ = 4

So,

1α+1β–2αβ $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

= β+α–2α2β2αβ $\frac{\beta+\alpha-2\alpha^2\beta^2}{\alpha\beta}$

= (α+β)–2(αβ)2αβ $\frac{(\alpha+\beta)-2(\alpha\beta)^2}{\alpha\beta}$

= (5)–2(4)24 $\frac{(5)-2(4)^2}{4}$

= 5–2×164 $\frac{5-2\times16}{4}$ = 5–324 $\frac{5-32}{4}$ = -274 $\frac{-27}{4}$

Q.4: If αandβα *and* β are the zeroes of the quadratic polynomial p(y)=5y²–7y+1 $p(y) = 5y^2 - 7y + 1$, find the value of 1α+1β $\frac{1}{\alpha} + \frac{1}{\beta}$.

Solution: Since, αandβα *and* β are the zeroes of the quadratic polynomial.

p(y)=5y²–7y+1$p(y) = 5y^2 - 7y + 1$

Sum of the zeroes = α+β$\alpha + \beta$ = 7

Product of the roots = αβ$\alpha\beta$ = 1

So,

1α+1β $\frac{1}{\alpha} + \frac{1}{\beta}$ = α+βαβ $\frac{\alpha+\beta}{\alpha\beta}$ = 71 $\frac{7}{1}$ = 7

Q.5: If αandβα *and* β are the zeroes of the quadratic polynomial f(x)=x²–x–4 $f(x) = x^2 - x - 4$, find the value of 1α+1β–αβ$\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$.

Solution:

Since, αandβα *and* β are the zeroes of the quadratic polynomial.

We have,

f(x)=x²–x–4$f(x) = x^2 - x - 4$

Sum of zeroes = α+β$\alpha + \beta$ = 1

Product of the zeroes = αβ$\alpha\beta$ = -4

So,

1α+1β–αβ$\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$ = α+βαβ–αβ$\frac{\alpha+\beta}{\alpha\beta} - \alpha\beta$

= 1-4–(–4)=-14+4$\frac{1}{-4} - (-4)$ $= \frac{-1}{4} + 4$

= -1+164 $\frac{-1+16}{4}$ = 154 $\frac{15}{4}$

Q.6: If αandβα *and* β are the zeroes of the quadratic polynomial f(x)=x²+x–2 $f(x)=x^2+x-2$, find the value of 1α−1β $\frac{1}{\alpha}-\frac{1}{\beta}$.

Solution:

Since, αandβα *and* β are the zeroes of the quadratic polynomial.

We have,

f(x)=x²+x–2 $f(x)=x^2+x-2$

Sum of zeroes = α+β $\alpha+\beta$ = 1

Product of the zeroes = αβ $\alpha\beta$ = -2

So,

1α+1β−αβ $\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta$ = α+βαβ $\frac{\alpha+\beta}{\alpha\beta}$

= β–ααβ $\frac{\beta-\alpha}{\alpha\beta}$

= β–ααβ × (α–β)αβ = $\frac{\beta-\alpha}{\alpha\beta}\times\frac{(\alpha-\beta)}{\alpha\beta}$ = √(α+β)²–4αβαβ = $\frac{\sqrt{(\alpha+\beta)^2-4\alpha\beta}}{\alpha\beta}$ = √1+82 = √92 = 32

$=\frac{\sqrt{1+8}}{2}=\frac{\sqrt{9}}{2}=\frac{3}{2}$

Q.7: If one of the zero of the quadratic polynomial f(x)=4x²–8kx–9 $f(x)=4x^2-8kx-9$ is negative of the other, then find the value of k.

Solution:

Let, the two zeroes of the polynomial f(x)=4x²–8kx–9 $f(x)=4x^2-8kx-9$ be αand−α α *and* $-\alpha$.

Product of the zeroes = α×−α $\alpha\times-\alpha$ = -9

Sum of the zeroes = α+(−α) $\alpha+(-\alpha)$ = -8k = 0 Since,α–α=0 $Since, \alpha-\alpha=0$

⇒8k=0⇒ $8k = 0$ ⇒k=0⇒ $k = 0$

Q.8: If the sum of the zeroes of the quadratic polynomial f(t)=kt^2+2t+3k $f(t) = kt^2 + 2t + 3k$ is equal to their product, then find the value of k.

Solution: Let the two zeroes of the polynomial f(t)=kt^2+2t+3k$f(t) = kt^2 + 2t + 3k$ be αandβα *and* β.

Sum of the zeroes = α+β$\alpha + \beta$ = 2

Product of the zeroes = α×β$\alpha \times \beta$ = 3k

Now,

−2k=3kk$\frac{-2}{k} = \frac{3k}{k}$ ⇒3k=−2⇒ $3k = -2$ ⇒k=−23⇒ $k = \frac{-2}{3}$

So, k = 0 and ⇒k=−23⇒ $k = \frac{-2}{3}$

Q.9: If αandβα *and* β are the zeroes of the quadratic polynomial p(x)=4x^2–5x–1 $p(x) = 4x^2 - 5x - 1$, find the value of α^{2}β+αβ^2$\alpha^2\beta + \alpha\beta^2$.

Solution:

Since, αandβα *and* β are the zeroes of the quadratic polynomial p(x)=4x^2–5x–1 $p(x) = 4x^2 - 5x - 1$

So, Sum of the zeroes = α+β$\alpha + \beta$ = 54 $\frac{5}{4}$

Product of the zeroes = α×β$\alpha \times \beta$ = −14 $\frac{-1}{4}$

Now,

α^{2}β+αβ^2$\alpha^2\beta + \alpha\beta^2$ = αβ(α+β)$\alpha\beta(\alpha + \beta)$

= 54(−14)$\frac{5}{4}\left(\frac{-1}{4}\right)$

= −516 $\frac{-5}{16}$

Q.10: If αandβα and β are the zeroes of the quadratic polynomial f(t)=t^2–4t+3 $f(t) = t^2 - 4t + 3$, find the value of α^{4}β^3+α^{3}β^4$\alpha^4\beta^3 + \alpha^3\beta^4$.

Solution:

Since, αandβα and β are the zeroes of the quadratic polynomial f(t)=t^2–4t+3 $f(t) = t^2 - 4t + 3$

So, Sum of the zeroes = α+β$\alpha + \beta$ = 4

Product of the zeroes = α×β$\alpha \times \beta$ = 3

Now,

α^{4}β^3+α^{3}β^4$\alpha^4\beta^3 + \alpha^3\beta^4$ = α^{3}β^3(α+β)$\alpha^3\beta^3(\alpha+\beta)$

= (3)^3(4)$(3)^3(4)$ = 108

Q.11: If αandβα and β are the zeroes of the quadratic polynomial f(x)=6x^2+x–2 $f(x) = 6x^2 + x - 2$, find the value of αβ+βα$\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.

Solution:

Since, αandβα and β are the zeroes of the quadratic polynomial f(x)=6x^2+x–2 $f(x) = 6x^2 + x - 2$.

Sum of the zeroes = α+β$\alpha + \beta$ = −16$\frac{-1}{6}$

Product of the zeroes = α×β$\alpha \times \beta$ = −13$\frac{-1}{3}$

Now,

αβ+βα$\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

= (α^2+β^2)–2αβαβ $\frac{(\alpha^2+\beta^2)-2\alpha\beta}{\alpha\beta}$

By substitution the values of the sum of zeroes and product:

= −2512 $\frac{-25}{12}$

Q.12: If αandβ α *and* β are the zeroes of the quadratic polynomial f(x)=6x²+x–2 $f(x) = 6x^2 + x - 2$, find the value of αβ+2[1α+1β]+3αβ $\frac{\alpha}{\beta} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta$.

Solution:

Since, αandβ α *and* β are the zeroes of the quadratic polynomial f(x)=6x²+x–2 $f(x) = 6x^2 + x - 2$.

Sum of the zeroes = α+β $\alpha + \beta$ = 63 $\frac{6}{3}$

Product of the zeroes = α×β $\alpha \times \beta$ = 43 $\frac{4}{3}$

Now,

αβ+2[1α+1β]+3αβ $\frac{\alpha}{\beta} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta$.

= α²+β²αβ+2[1α+1β]+3αβ $\frac{\alpha^2+\beta^2}{\alpha\beta} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta$

= (α+β)²–2αβαβ+2[1α+1β]+3αβ $\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta$

By substituting the values of sum and product of the zeroes, we will get

αβ+2[1α+1β]+3αβ $\frac{\alpha}{\beta} + 2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right] + 3\alpha\beta$ = 8

Q.13: If the squared difference of the zeroes of the quadratic polynomial f(x)=x²+px+45 $f(x) = x^2 + px + 45$ is equal to 144, find the value of p.

Solution:

Let the two zeroes of the polynomial be αandβ α *and* β.

We have,

f(x)=x^2+px+45 $f(x) = x^2 + px + 45$

Now,

Sum of the zeroes = α+β $\alpha + \beta$ =-p

Product of the zeroes = α×β $\alpha \times \beta$ = 45

So,

(α+β)^2–4αβ=144 $(\alpha+\beta)^2 - 4\alpha\beta = 144$ (p)^2–4×45=144 $(p)^2 - 4 \times 45 = 144$
(p)^2=144+180 $(p)^2 = 144 + 180$ (p)^2=324 $(p)^2 = 324$ p=√324 $p = \sqrt{324}$ p=±18
$p = \pm 18$

Thus, in the given equation, p will be either 18 or -18.

Q.14: If αandβ α *and* β are the zeroes of the quadratic polynomial f(x)=x^2–px+q $f(x) = x^2 - px + q$, prove that α²β²+β²α²=p⁴q²−4p²q+2 $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2.$

Solution:

Since, αandβ α *and* β are the roots of the quadratic polynomial given in the question.

f(x)=x^2–px+q $f(x) = x^2 - px + q$

Now,

Sum of the zeroes = p = α+β $\alpha + \beta$

Product of the zeroes = q = α×β $\alpha \times \beta$

LHS = α²β²+β²α² $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}$

= α⁴+β⁴α²β² $\frac{\alpha^4+\beta^4}{\alpha^2\beta^2}$

= (α²+β²)²–2(αβ)²(αβ)² $\frac{(\alpha^2+\beta^2)^2 - 2(\alpha\beta)^2}{(\alpha\beta)^2}$

= [(α+β)²–2αβ]²–2(αβ)²(αβ)² $\frac{\left[(\alpha+\beta)^2 - 2\alpha\beta\right]^2 - 2(\alpha\beta)^2}{(\alpha\beta)^2}$

= [(p)²–2q]^2 –2(q)²(q)² $\frac{\left[(p)^2-2q\right]^2-2(q)^2}{(q)^2}$

= (p⁴+4q²–4p²q)–2q²q² $\frac{(p^4+4q^2-4p^2q)-2q^2}{q^2}$

= p⁴+2q²–4p²qq² $\frac{p^4+2q^2-4p^2q}{q^2}$

= p²q² +2– 4p²q $\frac{p^2}{q^2}+2-\frac{4p^2}{q}$

= p²q² – 4p²q +2 $\frac{p^2}{q^2}-\frac{4p^2}{q}+2$

LHS = RHS

Hence, proved.

Q.15: If αandβ α *and* β are the zeroes of the quadratic polynomial f(x)=x²–p(x+1)–c $f(x)=x^2-p(x+1)-c$, show that (α+1)(β+1)=1–c $(\alpha+1)(\beta+1)=1-c$.

Solution:

Since, αandβ α *and* β are the zeroes of the quadratic polynomial

f(x)=x²–p(x+1)–c $f(x)=x^2-p(x+1)-c$

Now,

Sum of the zeroes = α+β $\alpha+\beta$ = p

Product of the zeroes = α×β $\alpha\times\beta$ = (- p – c)

So,

(α+1)(β+1) $(\alpha+1)(\beta+1)$

= αβ+α+β+1 $\alpha\beta+\alpha+\beta+1$

= αβ+(α+β)+1 $\alpha\beta+(\alpha+\beta)+1$

= (−p–c)+p+1 $(-p-c)+p+1$

= 1 – c = RHS

So, LHS = RHS

Hence, proved.

Q.16: If αandβα *and* β are the zeroes of the quadratic polynomial such that α+β=24andα–β=8$\alpha+\beta=24$ *and* $\alpha-\beta=8$, find a quadratic polynomial having αandβα *and* β as its zeroes.

Solution:

We have,

α+β=24$\alpha+\beta=24$ E-1

α–β=8$\alpha-\beta=8$ E-2

By solving the above two equations accordingly, we will get

2α=32$2\alpha=32$ α=16$\alpha=16$

Substitute the value of αα, in any of the equation. Let we substitute it in E-2, we will get

β=16–8$\beta=16-8$ β=8$\beta=8$

Now,

Sum of the zeroes of the new polynomial = α+β$\alpha+\beta$ = 16 + 8 = 24

Product of the zeroes = αβ$\alpha\beta$ = 16×8$16\times8$ = 128

Then, the quadratic polynomial is-

K x²–(sumofthezeroes)x+(productofthezeroes)
$x^2-(\text{sum of the zeroes})\,x+(\text{product of the zeroes})$ = x²–24x+128
$x^2-24x+128$

Hence, the required quadratic polynomial is f(x)=x²+24x+128$f(x)=x^2+24x+128$

Q.17: If αandβα *and* β are the zeroes of the quadratic polynomial f(x)=x²–1 $f(x)=x^2-1$, find a quadratic polynomial whose zeroes are 2αβand2βα $\frac{2\alpha}{\beta}$ *and* $\frac{2\beta}{\alpha}$.

Solution:

We have,

f(x)=x^2–1 $f(x) = x^2 - 1$

Sum of the zeroes = α+β $\alpha + \beta$ = 0

Product of the zeroes = αβ $\alpha\beta$ = -1

From the question,

Sum of the zeroes of the new polynomial = 2αβ and 2βα $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$

= $2\alpha^2$+$2\beta^2$αβ $\frac{2\alpha^2+2\beta^2}{\alpha\beta}$

= 2(α^2+β^2)αβ $\frac{2(\alpha^2+\beta^2)}{\alpha\beta}$

= 2((α+β)^2–2αβ)αβ $\frac{2\left((\alpha+\beta)^2-2\alpha\beta\right)}{\alpha\beta}$

= 2(2)1−1 $\frac{2(2)1}{-1}$ { By substituting the value of the sum and products of the zeroes }

As given in the question,

Product of the zeroes = (2α)(2β)αβ $\frac{(2\alpha)(2\beta)}{\alpha\beta}$ = 4αβαβ $\frac{4\alpha\beta}{\alpha\beta}$ = 4

Hence, the quadratic polynomial is

x^2–(sumofthezeroes)x+(productofthezeroes)
$x^2 - (sum\ of\ the\ zeroes)\, x + (product\ of\ the\ zeroes)$

= kx^2–(−4)x+4 $x^2 - (-4)\, x + 4$ = x^2+4x+4 $x^2 + 4x + 4$

Hence, the required quadratic polynomial is f(x)=x^2+4x+4 $f(x) = x^2 + 4x + 4$

Q.18: If αandβ α and β are the zeroes of the quadratic polynomial f(x)=x^2–3x–2 $f(x) = x^2 - 3x - 2$, find a quadratic polynomial whose zeroes are 12α+β and 12β+α

$\frac{1}{2\alpha+\beta}$ and $\frac{1}{2\beta+\alpha}$.

Solution:

We have,

f(x)=x²–3x–2 $f(x)=x^2-3x-2$

Sum of the zeroes = α+β $\alpha+\beta$ = 3

Product of the zeroes = αβ $\alpha\beta$ = -2

From the question,

Sum of the zeroes of the new polynomial = 12α+β + 12β+α $\frac{1}{2\alpha+\beta}+\frac{1}{2\beta+\alpha}$

= 2β+α+2α+β(2α+β)(2β+α) $\frac{2\beta+\alpha+2\alpha+\beta}{(2\alpha+\beta)(2\beta+\alpha)}$

= 3α+3β2(α²+β²)+5αβ $\frac{3\alpha+3\beta}{2(\alpha^2+\beta^2)+5\alpha\beta}$

= 3×32[2(α+β)²–2αβ+5×(−2)] $\frac{3\times3}{2\left[2(\alpha+\beta)^2-2\alpha\beta+5\times(-2)\right]}$

= 92[9–(−4)]–10 $\frac{9}{2[9-(-4)]-10}$

= 92[13]–10 $\frac{9}{2[13]-10}$

= 926–10 $\frac{9}{26-10}$ = 916 $\frac{9}{16}$

Product of the zeroes = 12α+β × 12β+α $\frac{1}{2\alpha+\beta}\times\frac{1}{2\beta+\alpha}$

= 1(2α+β)(2β+α) $\frac{1}{(2\alpha+\beta)(2\beta+\alpha)}$

= 14αβ+2α²+2β²+αβ $\frac{1}{4\alpha\beta+2\alpha^2+2\beta^2+\alpha\beta}$

= 15αβ+2(α²+β²) $\frac{1}{5\alpha\beta+2(\alpha^2+\beta^2)}$

= 15αβ+2((α+β)²–2αβ) $\frac{1}{5\alpha\beta+2\left((\alpha+\beta)^2-2\alpha\beta\right)}$

= 15×(−2)+2((3)²–2×(−2)) $\frac{1}{5\times(-2)+2\left((3)^2-2\times(-2)\right)}$

= 1−10+26 $\frac{1}{-10+26}$ = 116 $\frac{1}{16}$

So, the quadratic polynomial is,

x^2–(sumofthezeroes)x+(productofthezeroes)
$x^2 - (sum\ of\ the\ zeroes)\, x + (product\ of\ the\ zeroes)$

= k (x^2+916x+116) $\left(x^2 + \frac{9}{16}x + \frac{1}{16}\right)$

Hence, the required quadratic polynomial is k (x^2+916x+116) $\left(x^2 + \frac{9}{16}x + \frac{1}{16}\right)$.

Q.19: If αandβ $\alpha\ and\ \beta$ are the zeroes of the quadratic polynomial f(x)=x^2+px+q $f(x) = x^2 + px + q$, form a polynomial whose zeroes are $(\alpha+\beta)^2$and$(\alpha-\beta)^2$ $(\alpha+\beta)^2\ and\ (\alpha-\beta)^2$.

Solution:

We have,

f(x)=x^2+px+q $f(x) = x^2 + px + q$

Sum of the zeroes = α+β $\alpha + \beta$ = -p

Product of the zeroes = αβ $\alpha\beta$ = q

From the question,

Sum of the zeroes of new polynomial = $(\alpha+\beta)^2$+$(\alpha-\beta)^2$ $(\alpha+\beta)^2 + (\alpha-\beta)^2$

= $(\alpha+\beta)^2$+α^2+β^2–2αβ

$(\alpha+\beta)^2 + \alpha^2 + \beta^2 - 2\alpha\beta$

= $(\alpha+\beta)^2$+$(\alpha+\beta)^2$–2αβ–2αβ $(\alpha+\beta)^2 + (\alpha+\beta)^2 - 2\alpha\beta - 2\alpha\beta$

= $(-p)^2$+$(-p)^2$–2×q–2×q $(-p)^2 + (-p)^2 - 2 \times q - 2 \times q$

= p^2+p^2–4q $p^2 + p^2 - 4q$

= 2p^2–4q $2p^2 - 4q$

Product of the zeroes of new polynomial = $(\alpha+\beta)^2(\alpha-\beta)^2$(

$= (-p)^2((-p)^2-4q)$

$= p^2(p^2-4q)$

So, the quadratic polynomial is ,

$x^2-(\text{sum of the zeroes})x+(\text{product of the zeroes})$

$= x^2-(2p^2-4q)x+p^2(p^2-4q)$

Hence, the required quadratic polynomial is $f(x)=k(x^2-(2p^2-4q)x+p^2(p^2-4q))$.

Q.20: If α and β are the zeroes of the quadratic polynomial $f(x)=x^2-2x+3$, find a polynomial whose roots are:

(i) $\alpha+2, \beta+2$

(ii) $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$.

Solution:

We have,

$f(x)=x^2-2x+3$

Sum of the zeroes = $\alpha+\beta$ = 2

Product of the zeroes = $\alpha\beta$ = 3

(i) Sum of the zeroes of new polynomial = $(\alpha+2)+(\beta+2)$

= $\alpha+\beta+4$

= 2 + 4 = 6

Product of the zeroes of new polynomial = $(\alpha+1)(\beta+1)$

= αβ+2α+2β+4$\alpha\beta+2\alpha+2\beta+4$

= αβ+2(α+β)+4$\alpha\beta+2\left(\alpha+\beta\right)+4$ = 3+2(2)+43$+2\left(2\right)+4$ = 11

So, quadratic polynomial is:

x2–(sumofthezeroes)x+(productofthezeroes)
$x^2-\left(sum\ of\ the\ zeroes\right)x+\left(product\ of\ the\ zeroes\right)$

= x2–6x+11$x^2-6x+11$

Hence, the required quadratic polynomial is f(x)=k(x2–6x+11)$f\left(x\right)=k\left(x^2-6x+11\right)$

(ii) Sum of the zeroes of new polynomial = α–1α+1 + β–1β+1 $\frac{\alpha-1}{\alpha+1}+\frac{\beta-1}{\beta+1}$

= (α–1)(β+1)+(β–1)(α+1)(α+1)(β+1) $\frac{(\alpha-1)(\beta+1)+(\beta-1)(\alpha+1)}{(\alpha+1)(\beta+1)}$

= αβ+α–β–1+αβ+β–α–1(α+1)(β+1) $\frac{\alpha\beta+\alpha-\beta-1+\alpha\beta+\beta-\alpha-1}{(\alpha+1)(\beta+1)}$

= 3–1+3–13+1+2 $\frac{3-1+3-1}{3+1+2}$ = 46=23 $\frac{4}{6}=\frac{2}{3}$

Product of the zeroes of new polynomial = α–1α+1 + β–1β+1 $\frac{\alpha-1}{\alpha+1}+\frac{\beta-1}{\beta+1}$

= 26=13 $\frac{2}{6}=\frac{1}{3}$

So, the quadratic polynomial is,

x2–(sumofthezeroes)x+(productofthezeroes)
$x^2-\left(sum\ of\ the\ zeroes\right)x+\left(product\ of\ the\ zeroes\right)$

= x2–23x+13$x^2-\frac{2}{3}x+\frac{1}{3}$

Thus, the required quadratic polynomial is f(x)=k(x2–23x+13)$f\left(x\right)=k\left(x^2-\frac{2}{3}x+\frac{1}{3}\right)$.

Q.21: If αandβ$\alpha\ and\ \beta$ are the zeroes of the quadratic polynomial f(x)=ax2+bx+c $f\left(x\right)=ax^2+bx+c$, then evaluate:

(i) α–β $\alpha-\beta$

(ii) 1α−1β $\frac{1}{\alpha}-\frac{1}{\beta}$

(ii) 1α+1β−2αβ $\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta$

(iv) α²β+αβ² $\alpha^2\beta+\alpha\beta^2$

(v) α⁴+β⁴ $\alpha^4+\beta^4$

(vi) 1aα+b+1aβ+b $\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}$

(vii) βaα+b+αaβ+b $\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}$

(viii) a[α²β+β²α]+b[αa+βa] $a\left[\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right]+b\left[\frac{\alpha}{a}+\frac{\beta}{a}\right]$

Solution:

f(x)=ax²+bx+c $f(x)=ax^2+bx+c$

Here,

Sum of the zeroes of polynomial = α+β $\alpha+\beta$ = −ba $\frac{-b}{a}$

Product of zeroes of polynomial = αβ $\alpha\beta$ = ca $\frac{c}{a}$

Since, α+β $\alpha+\beta$ are the roots (or) zeroes of the given polynomial, so

(i) α–β $\alpha-\beta$

The two zeroes of the polynomials are-

−b+√b2−4ac2a−(−b−√b2−4ac2a) $\frac{-b+\sqrt{b^2-4ac}}{2a}-\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$

= −b+√b2−4ac+b+√b2−4ac2a $\frac{-b+\sqrt{b^2-4ac}+b+\sqrt{b^2-4ac}}{2a}$

= 2√b²–4ac2a $\frac{2\sqrt{b^2-4ac}}{2a}$ = √b²–4aca $\frac{\sqrt{b^2-4ac}}{a}$

(ii) 1α−1β $\frac{1}{\alpha}-\frac{1}{\beta}$

= β–ααβ=−(α–β)αβ $\frac{\beta-\alpha}{\alpha\beta}=\frac{-(\alpha-\beta)}{\alpha\beta}$ ….. E.1

From previous question we know that,

α–β$\alpha-\beta$ = √b²–4aca $\frac{\sqrt{b^2-4ac}}{a}$

Also,

αβ$\alpha\beta$ = ca $\frac{c}{a}$

Putting the values in E.1, we will get

−(√b²–4aca ca)− $\left(\frac{\frac{\sqrt{b^2-4ac}}{a}}{\frac{c}{a}}\right)$

= −(√b²–4acc)− $\left(\frac{\sqrt{b^2-4ac}}{c}\right)$

(iii) 1α+1β−2αβ $\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta$

= [1α+1β]−2αβ $\left[\frac{1}{\alpha}+\frac{1}{\beta}\right]-2\alpha\beta$

= [α+βαβ]−2αβ $\left[\frac{\alpha+\beta}{\alpha\beta}\right]-2\alpha\beta$ …….. E-1

Since,

Sum of the zeroes of polynomial = α+β$\alpha+\beta$ = −ba $\frac{-b}{a}$

Product of zeroes of polynomial = αβ$\alpha\beta$ = ca $\frac{c}{a}$

After substituting it in E-1, we will get

−ba × ac−2 ca $\frac{-b}{a}\times\frac{a}{c}-2\frac{c}{a}$

$= -bc - 2ca - \frac{b}{c} - 2\frac{c}{a}$

$= -ab - 2c^2ac \frac{-ab-2c^2}{ac}$

$= -[bc + 2ca] - \left[\frac{b}{c} + \frac{2c}{a}\right]$

(iv) $\alpha^2\beta+\alpha\beta^2 \alpha^2\beta + \alpha\beta^2$

$= \alpha\beta(\alpha+\beta)\alpha\beta(\alpha+\beta)$ …….. E-1.

Since,

Sum of the zeroes of polynomial = $\alpha+\beta\alpha+\beta$ = $-ba \frac{-b}{a}$

Product of zeroes of polynomial = $\alpha\beta\alpha\beta$ = $ca \frac{c}{a}$

After substituting it in E-1, we will get

$ca(-ba)\frac{c}{a}\left(\frac{-b}{a}\right)$

$= -bca^2 \frac{-bc}{a^2}$

(v) $\alpha^4+\beta^4\alpha^4+\beta^4$

$= (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2(\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2$

$= ((\alpha+\beta)^2 - 2\alpha\beta)^2 - (2\alpha\beta)^2\left((\alpha+\beta)^2 - 2\alpha\beta\right)^2 - (2\alpha\beta)^2$ ……. E- 1

Since,

Sum of the zeroes of polynomial = $\alpha+\beta\alpha+\beta$ = $-ba \frac{-b}{a}$

Product of zeroes of polynomial = $\alpha\beta\alpha\beta$ = $ca \frac{c}{a}$

After substituting it in E-1, we will get

$[(-ba) - 2(ca)]^2 - [2(ca)^2]\left[\left(-\frac{b}{a}\right) - 2\left(\frac{c}{a}\right)\right]^2 - \left[2\left(\frac{c}{a}\right)^2\right]$

$= \left[b^2–2aca^2\right]^2 - 2c^2a^2 \left[\frac{b^2-2ac}{a^2}\right]^2 - \frac{2c^2}{a^2}$

$= (b^2–2ac)^2–2a^2c^2a^4 \; \frac{(b^2-2ac)^2-2a^2c^2}{a^4}$

(vi) $1a\alpha+b + 1a\beta+b \; \frac{1}{a\alpha+b} + \frac{1}{a\beta+b}$

$= a\beta+b+a\alpha+b(a\alpha+b)(a\beta+b) \; \frac{a\beta+b+a\alpha+b}{(a\alpha+b)(a\beta+b)}$

$= a(\alpha+\beta)+2ba^2\alpha\beta+ab\alpha+ab\beta+b^2 \; \frac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab\alpha+ab\beta+b^2}$

$= a(\alpha+\beta)+2ba^2\alpha\beta+ab(\alpha+\beta)+b^2 \; \frac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$

Since,

Sum of the zeroes of polynomial = $\alpha+\beta\alpha+\beta = -ba \; \frac{-b}{a}$

Product of zeroes of polynomial = $\alpha\beta\alpha\beta = ca \; \frac{c}{a}$

After substituting it , we will get

$bac–b^2+b^2 \; \frac{b}{ac-b^2+b^2}$

$= bac \; \frac{b}{ac}$

(vii) $\beta a\alpha+b + \alpha a\beta+b \; \frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b}$

$= \beta(a\beta+b)+\alpha(a\alpha+b)(a\alpha+b)(a\beta+b) \; \frac{\beta(a\beta+b)+\alpha(a\alpha+b)}{(a\alpha+b)(a\beta+b)}$

$= a\beta^2+b\beta+\alpha a^2+b\alpha a^2\alpha\beta+ab\alpha+ab\beta+b^2 \; \frac{a\beta^2+b\beta+\alpha a^2+b\alpha}{a^2\alpha\beta+ab\alpha+ab\beta+b^2}$

$= a\alpha^2+b\beta^2+b\alpha+b\beta a^2 \times ca + ab(\alpha+\beta)+b^2 \; \frac{a\alpha^2+b\beta^2+b\alpha+b\beta}{a^2\times\frac{c}{a}+ab(\alpha+\beta)+b^2}$

Since,

Sum of the zeroes of polynomial = $\alpha+\beta\alpha+\beta = -ba \; \frac{-b}{a}$

Product of zeroes of polynomial = αβ$\alpha\beta$ = ca $\frac{c}{a}$

After substituting it , we will get

a[(α+β)²+b(α+β)]ac $\frac{a\left[(\alpha+\beta)^2+b(\alpha+\beta)\right]}{ac}$

= a[(α+β)²–2αβ]–b²aac $\frac{a\left[(\alpha+\beta)^2-2\alpha\beta\right]-\frac{b^2}{a}}{ac}$

= a[b²a–2ca]–b²aac $\frac{a\left[\frac{b^2}{a}-\frac{2c}{a}\right]-\frac{b^2}{a}}{ac}$

= a[b²–2ca]–b²aac $\frac{a\left[\frac{b^2-2c}{a}\right]-\frac{b^2}{a}}{ac}$

= a[b²–2c–b²a]ac $\frac{a\left[\frac{b^2-2c-b^2}{a}\right]}{ac}$

= b²–2c–b²ac $\frac{b^2-2c-b^2}{ac}$

= –2cac $\frac{-2c}{ac}$ = –2a $\frac{-2}{a}$

(viii) a[α²β+β²α]+b[αa+βa]$a\left[\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right]+b\left[\frac{\alpha}{a}+\frac{\beta}{a}\right]$

= a[α²+β²αβ]+b(α²+β²αβ)$a\left[\frac{\alpha^2+\beta^2}{\alpha\beta}\right]+b\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)$

= a[(α+β)²–2αβ]+b((α+β)²–2αβ)αβ $\frac{a\left[(\alpha+\beta)^2-2\alpha\beta\right]+b\left((\alpha+\beta)^2-2\alpha\beta\right)}{\alpha\beta}$

Since,

Sum of the zeroes of polynomial = α+β$\alpha+\beta$ = –ba $\frac{-b}{a}$

Product of zeroes of polynomial = αβ$\alpha\beta$ = ca $\frac{c}{a}$

After substituting it , we will get

a[(–ba)²–3×ca]+b((–ba)²–2ca)ca $\frac{a\left[\left(\frac{-b}{a}\right)^2-3\times\frac{c}{a}\right]+b\left(\left(\frac{-b}{a}\right)^2-2\frac{c}{a}\right)}{\frac{c}{a}}$

$$= a^2c\left[-b^2a^2+3bca^2+b^2a^2-2bca^2\right]\frac{a^2}{c}\left[\frac{-b^2}{a^2}+\frac{3bc}{a^2}+\frac{b^2}{a^2}-\frac{2bc}{a^2}\right]$$

$$=\left[-b^2a^2a^2c+3bca^2a^2c+b^2a^2a^2c-2bca^2a^2c\right]\left[\frac{-b^2a^2}{a^2c}+\frac{3bca^2}{a^2c}+\frac{b^2a^2}{a^2c}-\frac{2bca^2}{a^2c}\right]$$

$$= -b^2ac+3b+b^2ac-2b\frac{-b^2}{ac}+3b+\frac{b^2}{ac}-2b = b$$

Exercise – 2.2: Polynomials

Q.1: Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also, verify the relationship between the zeroes and coefficients in each of the following cases:

(i) $f(x)=2x^3+x^2-5x+2; 12,1,-2$ $f(x) = 2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$

(ii) $g(x)=x^3-4x^2+5x-2;2,1,1$ $g(x) = x^3 - 4x^2 + 5x - 2; 2, 1, 1$

Sol:

(i) $f(x)=2x^3+x^2-5x+2; 12,1,-2$ $f(x) = 2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$

(a) By putting x = 12 $\frac{1}{2}$ in the above equation, we will get

$f(12)=2(12)^3+(12)^2-5(12)+2$ $f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$

$= f(12)=2(18)+14-52+2$ $f\left(\frac{1}{2}\right) = 2\left(\frac{1}{8}\right) + \frac{1}{4} - \frac{5}{2} + 2$

= f(12)=14+14−52+2$f\left(\frac{1}{2}\right) = \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$ = 0

(b) By putting x = 1 in the above equation, we will get

f(1)=2(1)³+(1)²–5(1)+2$f(1) = 2(1)^3 + (1)^2 - 5(1) + 2$

= 2 + 1 – 5 + 2 = 0

(c) By putting x = -2 in the above equation, we will get

f(−2)=2(−2)³+(−2)²–5(−2)+2$f(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$

= -16 + 4 + 10 + 2 = -16 + 16 = 0

Now,

Sum of zeroes = α+β+γ$\alpha + \beta + \gamma$ = − ba $-\frac{b}{a}$

⇒ 12+1−2=−12⇒ $\frac{1}{2} + 1 - 2 = \frac{-1}{2}$ −12=−12 $\frac{-1}{2} = \frac{-1}{2}$

Product of the zeroes = αβ+βγ+αγ$\alpha\beta + \beta\gamma + \alpha\gamma$ = ca $\frac{c}{a}$

12×1+1×(−2)+(−2)×12=−52$\frac{1}{2} \times 1 + 1 \times (-2) + (-2) \times \frac{1}{2} = \frac{-5}{2}$ 12–2–1=−52 $\frac{1}{2} - 2 - 1 = \frac{-5}{2}$ −52=−52 $\frac{-5}{2} = \frac{-5}{2}$

Hence, verified.

(ii) g(x)=x³–4x²+5x–2;2,1,1$g(x) = x^3 - 4x^2 + 5x - 2; 2, 1, 1$

(a) By putting x = 2 in the given equation, we will get

g(2)=(2)³–4(2)²+5(2)–2$g(2) = (2)^3 - 4(2)^2 + 5(2) - 2$

= 8 – 16 + 10 – 2 = 18 – 18 = 0

(b) By putting x = 1 in the given equation, we will get g(1)=$(1)^3$–4$(1)^2$+5(1)–2$g(1) = (1)^3 - 4(1)^2 + 5(1) - 2$

= 1 – 4 + 5 – 2 = 0

Now,

Sum of zeroes = α+β+γ$\alpha + \beta + \gamma$ = −ba$-\frac{b}{a}$

⇒2+1+1=−(−4)$\Rightarrow 2 + 1 + 1 = -(-4)$

4 = 4

Product of the zeroes = αβ+βγ+αγ$\alpha\beta + \beta\gamma + \alpha\gamma$ = ca$\frac{c}{a}$

2×1+1×1+1×2=5$2 \times 1 + 1 \times 1 + 1 \times 2 = 5$

2 + 1 + 2 = 5

5 = 5

αβγ=–(−2)$\alpha\beta\gamma = -(-2)$

2×1×1$2 \times 1 \times 1$ = 2

2 = 2

Hence, verified.

Q.2: Find a cubic polynomial with the sum, sum of the product of its zeroes is taken two at a time, and product of its zeroes as 3, -1 and -3 respectively.

Sol:

Any cubic polynomial is of the form ax^3+bx^2+cx+d$ax^3 + bx^2 + cx + d$:

=x^3= x^3 – (sum of the zeroes) x^2x^2 + (sum of the products of its zeroes) x – (product of the zeroes)

= x^3–3x^2+(−1)x+(−3)$x^3 - 3x^2 + (-1)x + (-3)$

= k[x^3–3x^2–x–3]$k\left[x^3 - 3x^2 - x - 3\right]$

k is any non-zero real numbers.

Q.3: If the zeroes of the polynomial f(x)=2x³–15x²+37x–30 $f(x) = 2x^3 - 15x^2 + 37x - 30$, find them.

Sol:

Let, α=a–d,β=aandγ=a+d $\alpha = a - d,\ \beta = a\ and\ \gamma = a + d$ be the zeroes of the polynomial.

f(x)=2x³–15x²+37x–30 $f(x) = 2x^3 - 15x^2 + 37x - 30$ α+β+γ=–(-152)=152

$\alpha + \beta + \gamma = -\left(\frac{-15}{2}\right) = \frac{15}{2}$ αβγ=–(-302)=15 $\alpha\beta\gamma = -\left(\frac{-30}{2}\right) = 15$

a – d + a + a + d = 152 $\frac{15}{2}$ and a(a–d)(a+d)=15 $a(a - d)(a + d) = 15$

So, 3a = 152 $\frac{15}{2}$

a = 52 $\frac{5}{2}$

And, a(a²+d²)=15 $a(a^2 + d^2) = 15$

d²=14 $d^2 = \frac{1}{4}$ d=12 $d = \frac{1}{2}$

Therefore, α=52–12=42=2 $\alpha = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$

β=52 $\beta = \frac{5}{2}$

γ=52+12 $\gamma = \frac{5}{2} + \frac{1}{2}$ = 3

Q.4: Find the condition that the zeroes of the polynomial f(x)=x³+3px²+3qx+r $f(x) = x^3 + 3px^2 + 3qx + r$ may be in A.P.

Sol:

f(x)=x³+3px²+3qx+r $f(x) = x^3 + 3px^2 + 3qx + r$

Let, a – d, a, a + d be the zeroes of the polynomial.

Then,

The sum of zeroes = −ba $\frac{-b}{a}$

a + a – d + a + d = -3p

3a = -3p

a = -p

Since, a is the zero of the polynomial f(x),

Therefore, f(a) = 0

f(a)=a³+3pa²+3qa+r $f(a) = a^3 + 3pa^2 + 3qa + r$ = 0

Therefore, f(a)=0 $f(a) = 0$

= ⇒a³+3pa²+3qa+r=0 $\Rightarrow a^3 + 3pa^2 + 3qa + r = 0$

= ⇒(−p)³+3p(−p)²+3q(−p)+r=0 $\Rightarrow (-p)^3 + 3p(-p)^2 + 3q(-p) + r = 0$

= −p³+3p³–pq+r=0 $-p^3 + 3p^3 - pq + r = 0$

= 2p³–pq+r=0 $2p^3 - pq + r = 0$

Q.5: If zeroes of the polynomial f(x)=ax³+3bx²+3cx+d $f(x) = ax^3 + 3bx^2 + 3cx + d$ are tin A.P., prove that 2b³–3abc+a²d=0 $2b^3 - 3abc + a^2d = 0$.

Sol:

f(x)=x³+3px²+3qx+r $f(x) = x^3 + 3px^2 + 3qx + r$

Let, a – d, a, a + d be the zeroes of the polynomial.

Then,

The sum of zeroes = −ba $\frac{-b}{a}$

a + a – d + a + d = −3ba $\frac{-3b}{a}$

⇒3a=−3ba⇒ $3a = -\frac{3b}{a}$ ⇒a=-3b3a=-ba⇒ $a = \frac{-3b}{3a} = \frac{-b}{a}$ Since,f(a)=0

$Since, f(a) = 0$ ⇒$a(a^2)+3b(a)^2+3c(a)+d=0$⇒ $a\left(a^2\right) + 3b(a)^2 + 3c(a) + d = 0$

⇒$a(-ba)^3+3b^2a^2-3bca+d=0$⇒ $a\left(\frac{-b}{a}\right)^3 + \frac{3b^2}{a^2} - \frac{3bc}{a} + d = 0$ $2b^3a^2-3bca+d=0$

$\frac{2b^3}{a^2} - \frac{3bc}{a} + d = 0$ $2b^3-3abc+a^2da^2=0$ $\frac{2b^3-3abc+a^2d}{a^2} = 0$ $2b^3-3abc+a^2d=0$

$2b^3 - 3abc + a^2 d = 0$

Q.6: If the zeroes of the polynomial $f(x)=x^3-12x^2+39x+k$
$f(x) = x^3 - 12x^2 + 39x + k$ **are in A.P., find the value of k.**

Sol:

$f(x)=x^3-12x^2+39x+k$ $f(x) = x^3 - 12x^2 + 39x + k$

Let, a-d, a, a+d be the zeroes of the polynomial f(x).

The sum of the zeroes = 12

3a = 12

a = 4

Now,

f(a) = 0

$f(a)=a^3-12a^2+39a+k$ $f(a) = a^3 - 12a^2 + 39a + k$ $f(4)=4^3-12(4)^2+39(4)+k=0$
$f(4) = 4^3 - 12(4)^2 + 39(4) + k = 0$

64 – 192 + 156 + k = 0

k = -28

Exercise – 2.3: Polynomials

Points to note:

Division algorithms fall into two main categories: Slow division and fast division. Slow division algorithms produce one digit of the final quotient per iteration. Fast division methods start with an approximation to the final quotient and produce twice as many digits of the final quotient on each iteration.

Discussion will refer to the form ND=(Q,R)$\frac{N}{D} = (Q, R)$ where,

N = Numerator (dividend) & D = Denominator (divisor) is the input, and Q = Quotient & R = Remainder is the output.

Q.1: Apply division algorithm to find the quotient q(x) and remainder r(x) on dividing f(x) by g(x) in each of the following:

(i) f(x)=x³–6x²+11x–6,g(x)=x²+x+1

$f(x) = x^3 - 6x^2 + 11x - 6,\ g(x) = x^2 + x + 1$

(ii) f(x) = 10x⁴+17x³–62x²+30x–105,g(x)=2x²

$10x^4 + 17x^3 - 62x^2 + 30x - 105,\ g(x) = 2x^2$ **+ 7x + 1**

(iii) $f(x)=4x^3+8x^2+8x+7, g(x)=2x^2-x+1$

$f(x)=4x^3+8x^2+8x+7,\ g(x)=2x^2-x+1$

(iv) $f(x)=15x^3-20x^2+13x-12, g(x)=x^2-2x+2$

$f(x)=15x^3-20x^2+13x-12,\ g(x)=x^2-2x+2$

Solution:

(i) $f(x)=x^3-6x^2+11x-6$ $f(x)=x^3-6x^2+11x-6$ and $g(x)=x^2+x+1$

$g(x)=x^2+x+1$

	$x-7$
x^2+x+1	$x^3-6x^2+11x-6$
	x^3+x^2+x
	$-7x^2+10x-6$
	$-7x^2-7x-7$
	$17x-1$

(ii) $f(x)=10x^4+17x^3-62x^2+30x-105, g(x)=2x^2+7x+1$

$f(x)=10x^4+17x^3-62x^2+30x-105,\ g(x)=2x^2+7x+1$

$f(x)=10x^4+17x^3-62x^2+30x-105$ $f(x)=10x^4+17x^3-62x^2+30x-105$

$g(x)=2x^2+7x+1$ $g(x)=2x^2+7x+1$

Quotient: $5x^2 - 9x - 2$

Divisor: $2x^2 + 7x + 1$

$$
\begin{array}{l}
10x^4 + 17x^3 - 62x^2 + 30x - 3 \\
10x^4 + 35x^3 + 5x^2 \\
\hline
-18x^3 - 67x^2 + 30x - 3 \\
-18x^3 - 63x^2 - 9x \\
\hline
-4x^2 + 39x - 3 \\
4x^2 - 14x - 2 \\
\hline
53x - 1
\end{array}
$$

(iii) f(x)=4x³+8x²+8x+7,g(x)=2x²–x+1

$f(x) = 4x^3 + 8x^2 + 8x + 7,\ g(x) = 2x^2 - x + 1$

Quotient: $2x + 5$

Divisor: $2x^2 - x + 1$

$$
\begin{array}{l}
4x^3 + 8x^2 + 8x + 7 \\
4x^3 - 2x^2 + 2x \\
\hline
10x^2 + 6x + 7 \\
10x^2 - 5x + 5 \\
\hline
11x + 2
\end{array}
$$

f(x)=4x³+8x²+8x+7 $f(x) = 4x^3 + 8x^2 + 8x + 7$ g(x)=2x²–x+1 $g(x) = 2x^2 - x + 1$

(iv) f(x)=15x³–20x²+13x–12,g(x)=x²–2x+2

$f(x) = 15x^3 - 20x^2 + 13x - 12,\ g(x) = x^2 - 2x + 2$

4

f(x)=15x³–20x²+13x–12 $f(x) = 15x^3 - 20x^2 + 13x - 12$ g(x)=x²–2x+2

$g(x) = x^2 - 2x + 2$

Q.2: Check whether the first polynomial is a factor of the second polynomial by applying the division algorithm:

(i) g(t)=t²–3;f(t)=2t⁴+3t³–2t²–9t–12 $g\left(t\right)=t^2-3\ ;f\left(t\right)=2t^4+3t^3-2t^2-9t-12$

(ii) g(x)=x²–3x+1;f(x)=x⁵–4x³+x²+3x+1
$g\left(x\right)=x^2-3x+1\ ;f\left(x\right)=x^5-4x^3+x^2+3x+1$

(iii) g(x)=2x²–x+3;f(x)=6x⁵–x⁴+4x³–5x²–x–15
$g\left(x\right)=2x^2-x+3\ ;f\left(x\right)=6x^5-x^4+4x^3-5x^2-x-15$

Solution:

(i) g(t)=t²–3;f(t)=2t⁴+3t³–2t²–9t–12 $g\left(t\right)=t^2-3\ ;f\left(t\right)=2t^4+3t^3-2t^2-9t-12$

	$2t^2+3t+4$
t^2-3	$2t^4+3t^3-2t^2-9t-12$
	$2t^4 \quad -6t^2$
	$3t^3+4t^2-9t$
	$3t^3 \quad -9t$
	$4t^2-12$
	$4t^2-12$
	0

g(t)=t²–3 $g\left(t\right)=t^2-3$ f(t)=2t⁴+3t³–2t²–9t $f\left(t\right)=2t^4+3t^3-2t^2-9t$

Therefore, g(t) is the factor of f(t).

(ii) g(x)=x²–3x+1;f(x)=x⁵–4x³+x²+3x+1
$g\left(x\right)=x^2-3x+1\ ;f\left(x\right)=x^5-4x^3+x^2+3x+1$

	$x^2 - 1$
$x^3 - 3x + 1$	$x^5 - 4x^3 + x^2 + 3x + 1$
	$x^5 - 3x^3 + x^2$
	$- x^3 \quad + 3x + 1$
	$- x^3 \quad + 3x - 1$
	2

g(x)=x^2–3x+1 $g(x) = x^2 - 3x + 1$ f(x)=x^5–4x^3+x^2+3x+1
$f(x) = x^5 - 4x^3 + x^2 + 3x + 1$

Therefore, g(x) is not the factor of f(x).

(iii) g(x)=2x^2–x+3;f(x)=6x^5–x^4+4x^3–5x^2–x–15
$g(x) = 2x^2 - x + 3\ ;\ f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

	$3x^3 + x^2 - 2x - 5$
$2x^2 - x + 3$	$6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$
	$6x^5 - 3x^4 + 9x^3$
	$2x^4 - 5x^3 - 5x^2$
	$2x^4 - x^3 + 3x^2$
	$- 4x^3 - 8x^2 - x$
	$- 4x^3 + 2x^2 - 6x$
	$- 10x^2 + 5x - 15$
	$- 10x^2 + 5x - 15$
	0

g(x)=2x^2–x+3 $g(x) = 2x^2 - x + 3$ f(x)=6x^5–x^4+4x^3–5x^2–x–15
$f(x) = 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

Q.3: Obtain all zeroes of the polynomial f(x) = f(x)=2x^4+x^3–14x^2–19x–6
$f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$**, if two of its zeroes are -2 and -1.**

Solution:

f(x)=$2x^4+x^3–14x^2–19x–6$ $f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$

If the two zeroes of the polynomial are -2 and -1, then its factors are (x + 2) and (x + 1)

$(x+2)(x+1)=x^2+x+2x+2=x^2+3x+2$
$(x+2)(x+1) = x^2 + x + 2x + 2 = x^2 + 3x + 2$

	$2x^2 - 5x - 3$
$x^2 + 3x + 2$	$2x^4 + x^3 - 14x^2 - 19x - 6$
	$2x^4 + 6x^3 + 4x^2$
	$-5x^3 - 18x^2 - 19x$
	$-5x^3 - 15x^2 - 10x$
	$-3x^2 - 9x - 6$
	$-3x^2 - 9x - 6$
	0

f(x)=$2x^4+x^3–14x^2–19x–6$ $f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$ = $(2x^2–5x–3)(x^2+3x+2)$
$(2x^2 - 5x - 3)(x^2 + 3x + 2)$

= (2x + 1)(x – 3)(x + 2)(x + 1)

Therefore, zeroes of the polynomial = −12 $\frac{-1}{2}$, 3, -2 , -1

Q-4: Obtain all zeroes of f(x) = f(x)=$x^3+13x^2+32x+20$
$f(x) = x^3 + 13x^2 + 32x + 20$**, if one of its zeroes is -2.**

Solution:

f(x) = f(x)=$x^3+13x^2+32x+20$ $f(x) = x^3 + 13x^2 + 32x + 20$

Since, the zero of the polynomial is -2 so, it means its factor is (x + 2).

	$x^2 + 11x + 10$
$x + 2$	$x^3 + 13x^2 + 32x + 20$
	$x^3 + 2x^2$
	$11x^2 + 32x + 20$
	$11x^2 + 22x$
	$10x + 20$
	$10x + 20$
	0

So, $f(x)=x^3+13x^2+32x+20$ $f(x) = x^3 + 13x^2 + 32x + 20$ = $(x^2+11x+10)(x+2)$ $\left(x^2 + 11x + 10\right)(x + 2)$

= $(x^2+10x+x+10)(x+2)$ $\left(x^2 + 10x + x + 10\right)(x + 2)$

= $(x+10)(x+1)(x+2)$ $(x + 10)(x + 1)(x + 2)$

Therefore, the zeroes of the polynomial are -1, -10, -2.

Q-5: Obtain all zeroes of the polynomial $f(x)=x^4-3x^3-x^2+9x-6$ $f(x) = x^4 - 3x^3 - x^2 + 9x - 6$, if the two of its zeroes are $-\sqrt{3}$ and $\sqrt{3}$ $-\sqrt{3}$ *and* $\sqrt{3}$.

Solution:

$f(x)=x^4-3x^3-x^2+9x-6$ $f(x) = x^4 - 3x^3 - x^2 + 9x - 6$

Since, two of the zeroes of polynomial are $-\sqrt{3}$ and $\sqrt{3}$ $-\sqrt{3}$ *and* $\sqrt{3}$ so, $(x+\sqrt{3})(x-\sqrt{3})$ $\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$ = x^2-3 $x^2 - 3$

	$x^2 - 3x + 2$
$x^2 - 3$	$x^4 - 3x^3 - x^2 + 9x - 6$
	$x^4 \qquad - 3x^2$
	$- 3x^3 + 2x^2 + 9x$
	$- 3x^3 \qquad + 9x$
	$2x^2 - 6$
	$2x^2 - 6$
	0

So, f(x)=x^4–$3x^2$–x^2+9x–6 $f(x) = x^4 - 3x^2 - x^2 + 9x - 6$ = (x^2–3)(x^2–3x+2) $(x^2 - 3)(x^2 - 3x + 2)$

= (x+√3)(x–√3) $(x + \sqrt{3})(x - \sqrt{3})$ (x^2–2x–2+2) $(x^2 - 2x - 2 + 2)$

= (x+√3)(x–√3) $(x + \sqrt{3})(x - \sqrt{3})$ (x–1)(x–2) $(x - 1)(x - 2)$

Therefore, the zeroes of the polynomial are −√3, √3 $-\sqrt{3}, \sqrt{3}$, 1, 2.

Q-6: Obtain all zeroes of the polynomial f(x)=$2x^4$–$2x^3$–$7x^2$+x–1 $f(x) = 2x^4 - 2x^3 - 7x^2 + x - 1$, if the two of its zeroes are –√32 and √32 $-\sqrt{\frac{3}{2}}$ *and* $\sqrt{\frac{3}{2}}$.

Solution:

f(x)=$2x^4$–$2x^3$–$7x^2$+x–1 $f(x) = 2x^4 - 2x^3 - 7x^2 + x - 1$

Since, –√32 and √32 $-\sqrt{\frac{3}{2}}$ *and* $\sqrt{\frac{3}{2}}$ are the zeroes of the polynomial, so the factors are

(x–√32)and(x+√32) $\left(x - \sqrt{\frac{3}{2}}\right)$ *and* $\left(x + \sqrt{\frac{3}{2}}\right)$

	$2x^2 - 2x - 4$
$x^2 - 3/2$	$2x^4 - 2x^3 - 7x^2 + 3x + 6$
	$2x^4 \quad - 3x^2$
	$- 2x^3 - 4x^2 + 3x + 6$
	$- 2x^3 \quad + 3x$
	$- 4x^2 + 6$
	$- 4x^2 + 6$
	0

So, f(x)=$2x^4–2x^3–7x^2$+x–1 $f(x) = 2x^4 - 2x^3 - 7x^2 + x - 1$ = (x–$\sqrt{32}$)(x+$\sqrt{32}$) $\left(x - \sqrt{\frac{3}{2}}\right)\left(x + \sqrt{\frac{3}{2}}\right)$ ($2x^2$–2x–4) $(2x^2 - 2x - 4)$

= (x–$\sqrt{32}$)(x+$\sqrt{32}$) $\left(x - \sqrt{\frac{3}{2}}\right)\left(x + \sqrt{\frac{3}{2}}\right)$ ($2x^2$–4x+2x–4) $(2x^2 - 4x + 2x - 4)$

= (x–$\sqrt{32}$)(x+$\sqrt{32}$) $\left(x - \sqrt{\frac{3}{2}}\right)\left(x + \sqrt{\frac{3}{2}}\right)$ (x+2)(x–2) $(x+2)(x-2)$

Therefore, the zeroes of the polynomial = x = -1, 2, –$\sqrt{32}$ and $\sqrt{32}$ $-\sqrt{\frac{3}{2}}$ *and* $\sqrt{\frac{3}{2}}$.

Q.7: Find all the zeroes of the polynomial $x^4+x^3–34x^2–4x+120$ $x^4 + x^3 - 34x^2 - 4x + 120$, if the two of its zeroes are 2 and -2.

Solution:

$x^4+x^3–34x^2–4x+120$ $x^4 + x^3 - 34x^2 - 4x + 120$

Since, the two zeroes of the polynomial given is 2 and -2

So, factors are (x + 2)(x – 2) = x^2+2x–2x–4 $x^2 + 2x - 2x - 4$ = x^2–4 $x^2 - 4$

	$x^2 + x - 30$
$x^2 - 4$	$x^4 + x^3 - 34x^2 - 4x + 120$
	$x^4 \quad - 4x^2$
	$x^3 - 30x^2 - 4x + 120$
	$x^3 \quad - 4x$
	$-30x^2 + 120$
	$-30x^2 + 120$
	0

So, $x^4+x^3–34x^2–4x+120$ $x^4 + x^3 - 34x^2 - 4x + 120$ = $(x^2–4)(x^2+x–30)$ $(x^2 - 4)(x^2 + x - 30)$

$=(x–2)(x+2)(x^2+6x–5x–30)$ $(x-2)(x+2)(x^2+6x-5x-30)$

$= (x–2)(x+2)(x+6)(x–5)$ $(x-2)(x+2)(x+6)(x-5)$

Therefore, the zeroes of the polynomial = x = 2, -2, -6, 5

Q-8: Find all the zeroes of the polynomial $2x^4+7x^3–19x^2$ $2x^4 + 7x^3 - 19x^2$ – 14x + 30, if the two of its zeroes are $\sqrt{2}$and$-\sqrt{2}$ $\sqrt{2}$ and $-\sqrt{2}$.

Solution:

$2x^4+7x^3–19x^2–14x+30$ $2x^4 + 7x^3 - 19x^2 - 14x + 30$

Since, $\sqrt{2}$and$-\sqrt{2}$ $\sqrt{2}$ *and* $-\sqrt{2}$ are the zeroes of the polynomial given.

So, factors are $(x+\sqrt{2})$and$(x-\sqrt{2})$ $(x+\sqrt{2})$ *and* $(x-\sqrt{2})$ = $x^2+\sqrt{2}x-\sqrt{2}x-2$ $x^2 + \sqrt{2}x - \sqrt{2}x - 2$ = $x^2–2$ $x^2 - 2$

	$2x^2 + 7x - 15$
$x^2 - 2$	$2x^4 + 7x^3 - 19x^2 - 14x + 30$
	$2x^4 \quad - 4x^2$
	$7x^3 - 15x^2 - 14x + 30$
	$7x^3 \quad - 14x$
	$-15x^2 + 30$
	$-15x^2 + 30$
	0

So, $2x^4+7x^3-19x^2-14x+30$ $2x^4 + 7x^3 - 19x^2 - 14x + 30$ = $(x^2-2)(2x^2+7x-15)$ $(x^2-2)\left(2x^2+7x-15\right)$

= $(2x^2+10x-3x-15)(x+\sqrt{2})(x-\sqrt{2})$ $\left(2x^2+10x-3x-15\right)\left(x+\sqrt{2}\right)\left(x-\sqrt{2}\right)$

= $(2x-3)(x+5)(x+\sqrt{2})(x-\sqrt{2})$ $(2x-3)(x+5)\left(x+\sqrt{2}\right)\left(x-\sqrt{2}\right)$

Therefore, the zeroes of the polynomial is $\sqrt{2}, -\sqrt{2}, -5$, 32 $\sqrt{2}, -\sqrt{2}, -5, \frac{3}{2}$.

Q-9: Find all the zeroes of the polynomial $f(x)=2x^3+x^2-6x-3$ $f(x) = 2x^3 + x^2 - 6x - 3$, if two of its zeroes are $-\sqrt{3}$and$\sqrt{3}$ $-\sqrt{3}$ and $\sqrt{3}$.

Solution:

$f(x)=2x^3+x^2-6x-3$ $f(x) = 2x^3 + x^2 - 6x - 3$

Since, $-\sqrt{3}$and$\sqrt{3}$ $-\sqrt{3}$ *and* $\sqrt{3}$ are the zeroes of the given polynomial

So, factors are $(x-\sqrt{3})$and$(x+\sqrt{3})$ $\left(x-\sqrt{3}\right)$ *and* $\left(x+\sqrt{3}\right)$ = $(x^2-\sqrt{3}x+\sqrt{3}x-3)$ $\left(x^2-\sqrt{3}x+\sqrt{3}x-3\right)$ = (x^2-3) $\left(x^2-3\right)$

	$2x+1$
x^2-3	$2x^3+x^2-6x-3$
	$2x^3-6x$
	x^2-3
	x^2-3
	0

So, f(x)=$2x^3+x^2$–6x–3$f(x)=2x^3+x^2-6x-3$ = (x^2–3)(2x+1)$(x^2-3)(2x+1)$

= (x–√3)(x+√3)(2x+1)$(x-\sqrt{3})(x+\sqrt{3})(2x+1)$

Therefore, set of zeroes for the given polynomial = √3,–√3,–12$\sqrt{3},-\sqrt{3},\frac{-1}{2}$

Q-10: Find all the zeroes of the polynomial f(x)=x^3+3x^2–2x–6 $f(x)=x^3+3x^2-2x-6$, if the two of its zeroes are √2and–√2$\sqrt{2}$ and $-\sqrt{2}$.

Solution:

f(x)=x^3+3x^2–2x–6$f(x)=x^3+3x^2-2x-6$

Since, √2and−√2$\sqrt{2}$ *and* $-\sqrt{2}$ are the two zeroes of the given polynomial.

So, factors are (x+√2)and(x−√2)$(x+\sqrt{2})$ *and* $(x-\sqrt{2})$ = x^2+√2x–√2x–2 $x^2+\sqrt{2}x-\sqrt{2}x-2$ = x^2–2x^2-2

	$x+3$
x^2-2	x^3+3x^2-2x-6
	$x^3 \quad -2x$
	$3x^2 \quad -6$
	$3x^2 \quad -6$
	0

By division algorithm, we have:

f(x)=$x^3+3x^2–2x–6$ $f(x) = x^3 + 3x^2 - 2x - 6$ = $(x^2–2)(x+3)$ $(x^2 - 2)(x+3)$

= $(x–\sqrt{2})(x+\sqrt{2})(x+3)$ $(x - \sqrt{2})(x + \sqrt{2})(x+3)$

Therefore, the zeroes of the given polynomial is $-\sqrt{2},\sqrt{2}$and-3 $-\sqrt{2}, \sqrt{2}\ and\ -3$.

Q-11: What must be added to the polynomial f(x)=$x^4+2x^3–2x^2+x–1$ $f(x) = x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by g(x)=$x^2+2x–3$ $g(x) = x^2 + 2x - 3$.

Sol:

f(x)=$x^4+2x^3–2x^2+x–1$ $f(x) = x^4 + 2x^3 - 2x^2 + x - 1$

$x^2 - 1$

$x^2 + 2x - 3$) $x^4 + 2x^3 - 2x^2 + x - 1$

$x^4 + 2x^3 - 3x^2$

$x^2 + x - 1$

$x^2 + 2x - 3$

$-x + 2$

We must add (x – 2) in order to get the resulting polynomial exactly divisible by g(x)=$x^2+2x–3$ $g(x) = x^2 + 2x - 3$.

Q-12: What must be subtracted from the polynomial f(x)=$x^4+2x^3–13x^2–12x+21$ $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$ so that the resulting polynomial is exactly divisible by g(x)=$x^2–4x+3$ $g(x) = x^2 - 4x + 3$.

Sol:

f(x)=$x^4+2x^3–13x^2–12x+21$ $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$

$x^2 - 6x + 8$

$x^2 - 4x + 3$) $x^4 + 2x^3 - 13x^2 - 12x + 21$

$x^4 - 4x^3 + 3x^2$

$+ 6x^3 - 16x^2 - 12x + 21$

$2x^3 - 24x^2 - 18x$

$8x^2 - 30x + 21$

$8x^2 - 32x + 24$

$2x - 3$

We must subtract (2x – 3) in order to get the resulting polynomial exactly divisible by g(x)=$x^2–4x+3$ $g(x) = x^2 - 4x + 3$.

Q-13: Given that √2 $\sqrt{2}$ is a zero of the cubic polynomial f(x)=$6x^3+\sqrt{2}x^2–10x–4\sqrt{2}$ $f(x) = 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$, find its other two zeroes.

Solution:

f(x)=$6x^3+\sqrt{2}x^2–10x–4\sqrt{2}$ $f(x) = 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$

Since, √2 $\sqrt{2}$ is a zero of the cubic polynomial

So, factor is (x–√2) $(x - \sqrt{2})$

$$6x^2+7\sqrt{2}x+4$$

$x-\sqrt{2}$) $6x^3+\sqrt{2}x^2-10x-4\sqrt{2}$

$6x^3-6\sqrt{2}x^2$

$7\sqrt{2}x^2-10x-4\sqrt{2}$

$7\sqrt{2}x^2-14x$

$4x-4\sqrt{2}$

$4x-4\sqrt{2}$

0

Since, $f(x)=6x^3+\sqrt{2}x^2-10x-4\sqrt{2}$ $f(x)=6x^3+\sqrt{2}x^2-10x-4\sqrt{2}$ = $(x-\sqrt{2})(6x^2+7\sqrt{2}x+4)$ $(x-\sqrt{2})\ (6x^2+7\sqrt{2}x+4)$

= $(x-\sqrt{2})(6x^2+4\sqrt{2}x+3\sqrt{2}x+4)$ $(x-\sqrt{2})\ (6x^2+4\sqrt{2}x+3\sqrt{2}x+4)$

= $(x-\sqrt{2})(3x+2\sqrt{2})(2x+\sqrt{2})$ $(x-\sqrt{2})\ (3x+2\sqrt{2})\ (2x+\sqrt{2})$

So, the zeroes of the polynomial is $-2\sqrt{2}3, -\sqrt{2}2, \sqrt{2}$ $-\frac{2\sqrt{2}}{3}, -\frac{\sqrt{2}}{2}, \sqrt{2}$

Q-14: Given that $x-\sqrt{5}$ $x-\sqrt{5}$ is a factor of the cubic polynomial $x^3-3\sqrt{5}x^2+13x-3\sqrt{5}$ $x^3-3\sqrt{5}x^2+13x-3\sqrt{5}$, find all the zeroes of the polynomial.

Solution:

$x^3-3\sqrt{5}x^2+13x-3\sqrt{5}$ $x^3-3\sqrt{5}x^2+13x-3\sqrt{5}$

In the question, it's given that $x-\sqrt{5}$ $x-\sqrt{5}$ is a factor of the cubic polynomial.

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Since, $x^3-3\sqrt{5}x^2+13x-3\sqrt{5}$ $x^3-3\sqrt{5}x^2+13x-3\sqrt{5}$ = $(x-\sqrt{5})(x^2-2\sqrt{5}x+3)$ $(x-\sqrt{5})\ (x^2-2\sqrt{5}x+3)$

$= (x-\sqrt{5})(x-(\sqrt{5}+\sqrt{2}))(x-(\sqrt{5}-\sqrt{2}))$ $(x-\sqrt{5})\,(x-(\sqrt{5}+\sqrt{2}))\,(x-(\sqrt{5}-\sqrt{2}))$

So, the zeroes of the polynomial = $\sqrt{5}, (\sqrt{5}-\sqrt{2}), (\sqrt{5}+\sqrt{2})$ $\sqrt{5},\ (\sqrt{5}-\sqrt{2}),\ (\sqrt{5}+\sqrt{2})$.