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Solutions

Class 9 Maths

Chapter 6

Ex *6.4*

In each of the following, use factor theorem to find whether polynomial g(x) is a factor of polynomial f(x) , or not : (1 – 7)

Q1. $f(x) = x^3 - 6x^2 + 11x - 6, g(x) = x - 3$

Sol :

Here , $f(x) = x^3 - 6x^2 + 11x - 6$

g(x) = x – 3

To prove that g(x) is the factor of f(x) ,

we should show => f(3) = 0

here , x – 3 = 0

=> x = 3

Substitute the value of x in f(x)

$f(3) = 3^3 - 6 * (3)^2 + 11(3) - 6$

= 27 – (6*9) + 33 – 6

= 27 – 54 + 33 – 6

= 60 – 60

= 0

Since, the result is 0 g(x) is the factor of f(x)

Q2. $f(x) = 3x^4 + 17x^3 + 9x^2 - 7x - 10, g(x) = x + 5$

Sol :

Here , $f(x) = 3x^4 + 17x^3 + 9x^2 - 7x - 10$

g(x) = x + 5

To prove that g(x) is the factor of f(x) ,

we should show => f(-5) = 0

here , x + 5 = 0

=> x = -5

Substitute the value of x in f(x)

$f(-5) = 3(-5)^4 + 17(-5)^3 + 9(-5)^2 - 7(-5) - 10$

= (3 * 625) + (12 * (-125)) +(9*25) + 35 – 10

= 1875 – 2125 + 225 + 35 – 10

= 2135 – 2135

= 0

Since, the result is 0 g(x) is the factor of f(x)

Q3. $f(x) = x^5 + 3x^4 - x^3 - 3x^2 + 5x + 15, g(x) = x + 3$

Sol :

Here , $f(x) = x^5 + 3x^4 – x^3 – 3x^2 + 5x + 15$

g(x) = x + 3

To prove that g(x) is the factor of f(x) ,

we should show => f(-3) = 0

here , x + 3 = 0

=> x = -3

Substitute the value of x in f(x)

$f(-3) = (-3)^5 + 3(-3)^4 – (-3)^3 – 3(-3)^2 + 5(-3) + 15$

= -243 + 243 + 27 − 27 − 15 + 15

= 0

Since, the result is 0 g(x) is the factor of f(x)

Q4. $f(x) = x^3 – 6x^2 – 19x + 84, g(x) = x – 7$

Sol :

Here , $f(x) = x^3 – 6x^2 – 19x + 84$

g(x) = x − 7

To prove that g(x) is the factor of f(x) ,

we should show => f(7) = 0

here , x − 7 = 0

=> x = 7

Substitute the value of x in f(x)

$f(7) = 7^3 – 6(7)^2 – 19(7) + 84$

= 343 − (6*49) − (19*7) + 84

= 342 − 294 − 133 + 84

= 427 − 427

= 0

Since, the result is 0 g(x) is the factor of f(x)

Q5. $f(x) = 3x^3 + x^2 – 20x + 12, g(x) = 3x – 2$

Sol :

Here , $f(x) = 3x^3 + x^2 – 20x + 12$

g(x) = 3x − 2

To prove that g(x) is the factor of f(x) , we should show => $f(\frac{2}{3}) = 0$

here , $3x - 2 = 0$

=> $3x = 2$

=> $x = \frac{2}{3}$

Substitute the value of x in f(x)

$f(\frac{2}{3}) = 3(\frac{2}{3})$^{3} $+ (\frac{2}{3})^2 - 20(\frac{2}{3}) + 12$

$= 3(\frac{8}{27}) + \frac{4}{9} - \frac{40}{3} + 12$

$= \frac{8}{9} + \frac{4}{9} - \frac{40}{3} + 12$

$= \frac{12}{9} - \frac{40}{3} + 12$

Taking L.C.M

$= \frac{12-120+108}{9}$

$= \frac{120-120}{9}$

= 0

Since, the result is 0 g(x) is the factor of f(x)

Q6. $f(x) = 2x^3 - 9x^2 + x + 13,\ g(x) = 3 - 2x$

Sol :

Here , $f(x) = 2x^3 - 9x^2 + x + 13$

$g(x) = 3 - 2x$

To prove that g(x) is the factor of f(x) ,

To prove that g(x) is the factor of f(x) ,

we should show => $f(\frac{3}{2}) = 0$

here , $3 - 2x = 0$

=> $-2x = -3$

=> $2x = 3$

=> $x = \frac{3}{2}$

Substitute the value of x in f(x)

$f(\frac{3}{2}) = 2(\frac{3}{2})^3 - 9(\frac{3}{2})^2 + (\frac{3}{2}) + 13$

$= 2(\frac{27}{8}) - 9(\frac{9}{4}) + \frac{3}{2} + 12$

$= (\frac{27}{4}) - (\frac{81}{4}) + \frac{3}{2} + 12$

Taking L.C.M

= $\frac{21-81+6+48}{4}$

= $\frac{81-81}{4}$

= 0

Since, the result is 0 g(x) is the factor of f(x)

Q7. $f(x) = x^3 - 6x^2 + 11x - 6, g(x) = x^2 - 3x + 2$

Sol :

Here , $f(x) = x^3 - 6x^2 + 11x - 6$

$g(x) = x^2 - 3x + 2$

First we need to find the factors of $x^2 - 3x + 2$

=>$x^2 - 2x - x + 2$

=> x(x − 2) -1(x − 2)

=> (x − 1) and (x − 2) are the factors

To prove that g(x) is the factor of f(x) ,

The results of f(1) and f(2) should be zero

Let , x − 1 = 0

x = 1

substitute the value of x in f(x)

$f(1) = 1^3 - 6(1)^2 + 11(1) - 6$

= 1 − 6 + 11 − 6

= 12 − 12

= 0

Let , x − 2 = 0

x = 2

substitute the value of x in f(x)

$f(2) = 2^3 - 6(2)^2 + 11(2) - 6$

= 8 − (6 * 4) + 22 − 6

= 8 − 24 + 22 − 6

= 30 − 30

= 0

Since, the results are 0 g(x) is the factor of f(x)

Q8. Show that (x − 2) , (x + 3) and (x − 4) are the factors of $x^3 - 3x^2 - 10x + 24$

Sol :

Here , f(x) = $x^3 - 3x^2 - 10x + 24$

The factors given are (x − 2) , (x + 3) and (x − 4)

To prove that g(x) is the factor of f(x) ,

The results of f(2) , f(-3) and f(4) should be zero

Let , x − 2 = 0

=> x = 2

Substitute the value of x in f(x)

f(2) = $2^3 - 3(2)^2 - 10(2) + 24$

= 8 − (3 * 4) − 20 + 24

= 8 − 12 − 20 + 24

= 32 − 32

= 0

Let , x + 3 = 0

=> x = -3

Substitute the value of x in f(x)

f(-3) = $(-3)^3 - 3(-3)^2 - 10(-3) + 24$

= -27 − 3(9) + 30 + 24

= -27 − 27 + 30 + 24

= 54 − 54

= 0

Let , x − 4 = 0

=> x = 4

Substitute the value of x in f(x)

f(4) = $(4)^3 - 3(4)^2 - 10(4) + 24$

= 64 − (3*16) − 40 + 24

= 64 − 48 − 40 + 24

= 84 − 84

= 0

Since, the results are 0 g(x) is the factor of f(x)

Q9. Show that (x + 4) , (x − 3) and (x − 7) are the factors of $x^3 - 6x^2 - 19x + 84$

Sol :

Here , f(x) = $x^3 - 6x^2 - 19x + 84$

The factors given are (x + 4) , (x − 3) and (x − 7)

To prove that g(x) is the factor of f(x) ,

The results of f(-4) , f(3) and f(7) should be zero

Let, x + 4 = 0

=> x = -4

Substitute the value of x in f(x)

f(-4) = $(-4)^3 - 6(-4)^2 - 19(-4) + 84$

= -64 − (6 * 16) − (19 * (-4)) + 84

= -64 − 96 + 76 + 84

= 160 − 160

= 0

Let, x − 3 = 0

=> x = 3

Substitute the value of x in f(x)

f(3) = $(3)^3 - 6(3)^2 - 19(3) + 84$

= 27 − (6 * 9) − (19 * 3) + 84

= 27 − 54 − 57 + 84

= 111 − 111

= 0

Let, x − 7 = 0

=> x = 7

Substitute the value of x in f(x)

f(7) = $(7)^3 - 6(7)^2 - 19(7) + 84$

= 343 − (6 * 49) − (19 * 7) + 84

= 343 − 294 − 133 + 84

= 427 − 427

= 0

Since, the results are 0 g(x) is the factor of f(x)

Q10. For what value of a is (x − 5) a factor of $x^3 - 3x^2 + ax - 10$

Sol :

Here, f(x) = $x^3 - 3x^2 + ax - 10$

By factor theorem

If (x − 5) is the factor of f(x) then , f(5) = 0

=> x – 5 = 0

=> x = 5

Substitute the value of x in f(x)

f(5) = $5^3 - 3(5)^2 + a(5) - 10$

= 125 – (3 * 25) + 5a – 10

= 125 – 75 + 5a – 10

= 5a + 40

Equate f(5) to zero

f(5) = 0

=> 5a + 40 = 0

=> 5a = -40

=> a = $\frac{-40}{5}$

= -8

When a= -8 , (x – 5) will be factor of f(x)

Q11. Find the value of a such that (x – 4) is a factor of $5x^3 - 7x^2 - ax - 28$

Sol :

Here, f(x) = $5x^3 - 7x^2 - ax - 28$

By factor theorem

If (x – 4) is the factor of f(x) then , f(4) = 0

=> x – 4 = 0

=> x = 4

Substitute the value of x in f(x)

f(4) = $5(4)^3 - 7(4)^2 - a(4) - 28$

= 5(64) – 7(16) – 4a – 28

= 320 – 112 – 4a – 28

= 180 – 4

Equate f(4) to zero, to find a

f(4) = 0

=> 180 – 4a = 0

=> -4a = -180

=> 4a = 180

=> a = $\frac{180}{4}$

=> a = 45

When a = 45 , (x − 4) will be factor of f(x)

Q12. Find the value of a, if (x + 2) is a factor of $4x^4 + 2x^3 - 3x^2 + 8x + 5a$

Sol :

Here, f(x) = $4x^4 + 2x^3 - 3x^2 + 8x + 5a$

By factor theorem

If (x + 2) is the factor of f(x) then , f(-2) = 0

=> x + 2 = 0

=> x = -2

Substitute the value of x in f(x)

f(-2) = $4(-2)^4 + 2(-2)^3 - 3(-2)^2 + 8(-2) + 5a$

= 4(16) + 2(-8) − 3(4) − 16 + 5a

= 64 − 16 − 12 − 16 + 5a

= 5a + 20

equate f(-2) to zero

f(-2) = 0

=> 5a + 20 = 0

=> 5a = -20

=> a = $\frac{-20}{5}$

=> a = -4

When a = -4 , (x + 2) will be factor of f(x)

Q13. Find the value of k if x − 3 is a factor of $k^2x^3 - kx^2 + 3kx - k$

Sol :

Let f(x) = $k^2x^3 - kx^2 + 3kx - k$

From factor theorem if x − 3 is the factor of f(x) then f(3) = 0

=> x − 3 = 0

=> x = 3

Substitute the value of x in f(x)

f(3) = $k^2(3)^3 - k(3)^2 + 3k(3) - k$

= $27k^2 - 9k + 9k$ − k

= $27k^2 - k$

= k(27k − 1)

Equate f(3) to zero, to find k

=> f(3) = 0

=> k(27k − 1) = 0

=> k = 0 and 27k − 1 = 0

=> k = 0 and 27k = 1

=> k = 0 and k = $\frac{1}{27}$

When k = 0 and $\frac{1}{27}$, (x − 3) will be the factor of f(x)

Q14. Find the values of a and b, if x^2 − 4 is a factor of $ax^4 + 2x^3 – 3x^2 + bx – 4$

Sol :

Given , f(x) = $ax^4 + 2x^3 – 3x^2 + bx – 4$

g(x) = x^2 − 4

first we need to find the factors of g(x)

=> x^2 − 4

=> x^2 = 4

=> x = $\sqrt{4}$

=> x = ±2

(x − 2) and (x + 2) are the factors

By factor therorem if (x − 2) and (x + 2) are the factors of f(x) the result of f(2) and f(-2) should be zero

Let , x − 2 = 0

=> x = 2

Substitute the value of x in f(x)

f(2) = $a(2)^4 + 2(2)^3 – 3(2)^2 + b(2) – 4$

= 16a + 2(8) − 3(4) + 2b − 4

= 16a + 2b + 16 − 12 − 4

= 16a + 2b

Equate f(2) to zero

=> 16a + 2b = 0

=> 2(8a + b) = 0

=> 8a + b = 0 ———- 1

Let , x + 2 = 0

=> x = -2

Substitute the value of x in f(x)

f(-2) = $a(-2)^4 + 2(-2)^3 – 3(-2)^2 + b(-2) – 4$

= 16a + 2(-8) − 3(4) − 2b − 4

= 16a − 2b − 16 − 12 − 4

= 16a − 2b − 32

= 16a − 2b − 32

Equate f(2) to zero

=> 16a − 2b − 32 = 0

=> 2(8a − b) = 32

=> 8a − b = 16 −−−− 2

Solve equation 1 and 2

8a + b = 0

8a − b = 16

16a = 16

a = 1

substitute a value in eq 1

8(1) + b = 0

=> b = -8

The values are a = 1 and b = -8

Q15. Find α , β if (x + 1) and (x + 2) are the factors of $x^3 + 3x^2 - 2\alpha x + \beta$

Sol:

Given, f(x) = $x^3 + 3x^2 - 2\alpha x + \beta$ and the factors are (x + 1) and (x + 2)

From factor theorem, if they are tha factors of f(x) then results of f(-2) and f(-1) should be zero

Let , x + 1 = 0

=> x = -1

Substitute value of x in f(x)

f(-1) = $(-1)^3 + 3(-1)^2 - 2\alpha(-1) + \beta$

$= -1 + 3 + 2\alpha + \beta$

$= 2\alpha + \beta$ + 2 −−−− 1

Let , x + 2 = 0

=> x = -2

Substitute value of x in f(x)

f(-2) = $(-2)^3 + 3(-2)^2 - 2\alpha(-2) + \beta$

$= -8 + 12 + 4\alpha + \beta$

$= 4\alpha + \beta$ + 4 −−−−− 2

Solving 1 and 2 i.e (1 − 2)

=> $2\alpha + \beta + 2 - (4\alpha + \beta + 4) = 0$

=> $-2\alpha - 2 = 0$

=> $2\alpha = -2$

=> $\alpha = -1$

Substitute α= -1 in equation 1

=> $2(-1) + \beta$ = -2

=> β= -2 + 2

=> β= 0

The values are $\alpha = -1 \text{and} \beta = 0$

Q16. Find the values of p and q so that $x^4 + px^3 + 2x^2 - 3x + q$ is divisible by (x^2 – 1)

Sol :

Here , f(x) = $x^4 + px^3 + 2x^2 - 3x + q$

g(x) = $x^2 - 1$

first, we need to find the factors of $x^2 - 1$

=> $x^2 - 1$ = 0

=> $x^2 = 1$

=> x = ±1

=> (x + 1) and (x – 1)

From factor theorem , if x = 1, -1 are the factors of f(x) then f(1) = 0 and f(-1) = 0

Let us take , x + 1

=> x + 1 = 0

=> x = -1

Substitute the value of x in f(x)

f(-1) = $(-1)^4 + p(-1)^3 + 2(-1)^2 - 3(-1) + q$

= 1 – p + 2 + 3 + q

= -p + q + 6 ——- 1

Let us take , x – 1

=> x – 1 = 0

=> x = 1

Substitute the value of x in f(x)

f(1) = $(1)^4 + p(1)^3 + 2(1)^2 - 3(1) + q$

= 1 + p + 2 – 3 + q

= p + q ——- 2

Solve equations 1 and 2

-p + q = -6

p + q = 0

2q = -6

q = -3

substitute q value in equation 2

p + q = 0

p − 3 = 0

p = 3

the values of are p = 3 and q = -3

Q17. Find the values of a and b so that (x + 1) and (x − 1) are the factors of $x^4 + ax^3 - 3x^2 + 2x + b$

Sol :

Here, f(x) = $x^4 + ax^3 - 3x^2 + 2x + b$

The factors are (x + 1) and (x − 1)

From factor theorem , if x = 1, -1 are the factors of f(x) then f(1) = 0 and f(-1) = 0

Let , us take x + 1

=> x + 1 = 0

=> x = -1

Substitute value of x in f(x)

f(-1) = $(-1)^4 + a(-1)^3 - 3(-1)^2 + 2(-1) + b$

= 1 − a − 3 − 2 + b

= -a + b − 4 ——- 1

Let , us take x − 1

=> x − 1 = 0

=> x = 1

Substitute value of x in f(x)

f(1) = $(1)^4 + a(1)^3 - 3(1)^2 + 2(1) + b$

= 1 + a − 3 + 2 + b

= a + b ——- 2

Solve equations 1 and 2

-a + b = 4

a + b = 0

2b = 4

b = 2

substitute value of b in eq 2

a + 2 = 0

a = -2

the values are a = -2 and b = 2

Q18. If $x^3 + ax^2 - bx + 10$ is divisible by $x^3 - 3x + 2$, find the values of a and b

Sol :

Here , f(x) = $x^3 + ax^2 - bx + 10$

g(x) = $x^3 - 3x + 2$

first, we need to find the factors of g(x)

g(x) = $x^3 - 3x + 2$

= $x^3 - 2x - x + 2$

= x(x − 2) -1(x − 2)

= (x − 1) and (x − 2) are the factors

From factor theorem , if x = 1, 2 are the factors of f(x) then f(1) = 0 and f(2) = 0

Let, us take x − 1

=> x − 1 = 0

=> x = 1

Substitute the value of x in f(x)

f(1) = $1^3 + a(1)^2 - b(1) + 10$

= 1 + a − b + 10

= a − b + 11 —–- 1

Let, us take x − 2

=> x − 2 = 0

=> x = 2

Substitute the value of x in f(x)

f(2) = $2^3 + a(2)^2 - b(2) + 10$

= 8 + 4a − 2b + 10

= 4a − 2b + 18

Equate f(2) to zero

=> 4a − 2b + 18 = 0

=> 2(2a − b + 9) = 0

=> 2a − b + 9 ——- 2

Solve 1 and 2

a − b = -11

2a − b = -9

(-) (+) (+)

-a = -2

a = 2

substitute a value in eq 1

=> 2 − b = -11

=> − b = -11 − 2

=> -b = -13

=> b = 13

The values are a = 2 and b = 13

Q19. If both (x + 1) and (x − 1) are the factors of $ax^3 + x^2 - 2x + b$, Find the values of a and b

Sol:

Here, f(x) = $ax^3 + x^2 - 2x + b$

(x + 1) and (x − 1) are the factors

From factor theorem , if x = 1, -1 are the factors of f(x) then f(1) = 0 and f(-1) = 0

Let, x − 1= 0

=> x = -1

Substitute x value in f(x)

f(1) = $a(1)^3 + (1)^2 - 2(1) + b$

= a + 1 − 2 + b

= a + b − 1 ———- 1

Let, x + 1= 0

=> x = -1

Substitute x value in f(x)

f(-1) = $a(-1)^3 + (-1)^2 - 2(-1) + b$

= -a + 1 + 2 + b

= -a + b + 3 ———- 2

Solve equations 1 and 2

a + b = 1

-a + b = -3

2b = -2

=> b = -1

substitute b value in eq 1

=> a − 1 = 1

=> a = 1 + 1

=> a = 2

The values are a= 2 and b = -1

Q20. What must be added to $x^3 - 3x^2 - 12x + 19$ so that the result is exactly divisible by $x^2 + x - 6$

Sol :

Here , p(x) = $x^3 - 3x^2 - 12x + 19$

g(x) = $x^2 + x - 6$

by division algorithm, when p(x) is divided by g(x) , the remainder wiil be a linear expression in x

let, r(x) = ax + b is added to p(x)

=> f(x) = p(x) + r(x)

= $x^3 - 3x^2 - 12x + 19$ + ax + b

f(x) = $x^3 - 3x^2 + x(a - 12) + 19$ + b

We know that , g(x) = $x^2 + x - 6$

First, find the factors for g(x)

g(x) = $x^2 + 3x - 2x - 6$

= x(x + 3) -2(x + 3)

= (x + 3) (x − 2) are the factors

From, factor theorem when (x + 3) and (x − 2) are the factors of f(x) the f(-3) = 0 and f(2) = 0

Let, x + 3 = 0

=> x = -3

Substitute the value of x in f(x)

f(-3) = $(-3)^3 - 3(-3)^2 + (-3)(a - 12) + 19$ + b

= -27 − 27 − 3a + 24 + 19 + b

= -3a + b + 1 −−− 1

Let, x − 2 = 0

=> x = 2

Substitute the value of x in f(x)

f(2) = $(2)^3 - 3(2)^2 + (2)(a - 12) + 19$ + b

= 8 − 12 + 2a − 24 + b

= 2a + b − 9 −−− 2

Solve equations 1 and 2

-3a + b = -1

2a + b = 9

(-) (-) (-)

-5a = − 10

a = 2

substitute the value of a in eq 1

=> -3(2) + b = -1

=> -6 + b = -1

=> b = -1 + 6

=> b = 5

∴ r(x) = ax + b

= 2x + 5

∴ $x^3 - 3x^2 - 12x + 19$ is divided by $x^2 + x - 6$ when it is added by 2x + 5

Q21. What must be added to $x^3 - 6x^2 - 15x + 80$ so that the result is exactly divisible by $x^2 + x - 12$

Sol :

Let, p(x) = $x^3 - 6x^2 - 15x + 80$

q(x) = $x^2 + x - 12$

by division algorithm, when p(x) is divided by q(x) the remainder is a linear expression in x.

so, let r(x) = ax + b is subtracted from p(x), so that p(x) − q(x) is divisible by q(x)

let f(x) = p(x) − q(x)

q(x) = $x^2 + x - 12$

= $x^2 + 4x - 3x - 12$

= x(x + 4) (-3)(x + 4)

= (x+4) , (x − 3)

clearly, (x − 3) and (x + 4) are factors of q(x)

so, f(x) wiil be divisible by q(x) if (x − 3) and (x + 4) are factors of q(x)

from , factor theorem

f(-4) = 0 and f(3) = 0

=> f(3) = $3^3 - 6(3)^2 - 3(a + 15) + 80$ − b = 0

= 27 − 54 -3a -45 + 80 −b

= -3a −b + 8 −−− 1

Similarly,

f(-4) = 0

=> f(-4) => $(-4)^3 - 6(-4)^2 - (-4)(a + 15) + 80$ – b = 0

=> -64 – 96 -4a + 60 + 80 –b = 0

=> 4a – b – 20 = 0 ——- 2

Substract eq 1 and 2

=> 4a – b – 20 – 8 + 3a + b = 0

=> 7a – 28 = 0

=> a = $\frac{28}{7}$

=> a= 4

Put a = 4 in eq 1

=> -3(4) – b = -8

=> -b – 12 = -8

=> -b = -8 + 12

=> b = -4

Substitute a and b values in r(x)

=> r(x) = ax + b

= 4x – 4

Hence, p(x) is divisible by q(x) , if r(x) = 4x – 4 is subtracted from it

Q22. What must be added to $3x^3 + x^2 - 22x + 9$ so that the result is exactly divisible by $3x^2 + 7x - 6$

Sol :

Let, p(x) = $3x^3 + x^2 - 22x + 9$ and q(x) = $3x^2 + 7x - 6$

By division theorem, when p(x) is divided by q(x) , the remainder is a linear equation in x.

Let, r(x) = ax + b is added to p(x) , so that p(x) + r(x) is divisible by q(x)

f(x) = p(x) + r(x)

=> f(x) = $3x^3 + x^2 - 22x + 9(ax + b)$

=> = $3x^3 + x^2 + x(a - 22) + b + 9$

We know that,

q(x) = $3x^2 + 7x - 6$

= $3x^2 + 9x - 2x - 6$

= 3x(x+3) – 2(x+3)

= (3x-2) (x+3)

So, f(x) is divided by q(x) if (3x-2) and (x+3) are the factors of f(x)

From, factor theorem

$f(\frac{2}{3}) = 0$ and $f(-3) = 0$

let , $3x - 2 = 0$

$3x = 2$

$x = \frac{2}{3}$

=> $f(\frac{2}{3}) = 3(\frac{2}{3})^3 + (\frac{2}{3})$^{2} + $(\frac{2}{3})(a - 22) + b + 9$

= $3(\frac{8}{27}) + \frac{4}{9} + \frac{2}{3}a - \frac{44}{3} + b + 9$

= $\frac{12}{9} + \frac{2}{3}a - \frac{44}{3} + b + 9$

= $\frac{12+6a-132+9b+81}{9}$

Equate to zero

=> $\frac{12+6a-132+9b+81}{9} = 0$

=> $6a + 9b - 39 = 0$

=> $3(2a + 3b - 13) = 0$

=> $2a + 3b - 13 = 0$ ———- 1

Similarly,

Let, $x + 3 = 0$

=> $x = -3$

=> $f(-3) = 3(-3)^3 + (-3)^2 + (-3)(a-22) + b + 9$

= $-81 + 9 - 3a + 66 + b + 9$

= $-3a + b + 3$

Equate to zero

$-3a + b + 3 = 0$

Multiply by 3

$-9a + 3b + 9 = 0$ ——– 2

Substact eq 1 from 2

=> $-9a + 3b + 9 - 2a - 3b + 13 = 0$

=> $-11a + 22 = 0$

=> $-11a = -22$

=> $a = \frac{22}{11}$

=> $a = 2$

Substitute a value in eq 1

=> $-3(2) + b = -3$

=> -6 + b = -3

=> b = -3 + 6

=> b = 3

Put the values in r(x)

r(x) = ax + b

= 2x + 3

Hence, p(x) is divisible by q(x) , if r(x) = 2x + 3 is added to it

Q23. If x – 2 is a factor of each of the following two polynomials , find the value of a in each case :

1. $x^3 - 2ax^2 + ax - 1$
2. $x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4$

Sol :

(1) let f(x) = $x^3 - 2ax^2 + ax - 1$

from factor theorem

if (x – 2) is the factor of f(x) the f(2) = 0

let , x – 2 = 0

=> x = 2

Substitute x value in f(x)

f(2) = $2^3 - 2a(2)^2 + a(2) - 1$

= 8 – 8a + 2a – 1

= -6a + 7

Equate f(2) to zero

=> -6a + 7 = 0

=> -6a = -7

=> a= $\frac{7}{6}$

When , (x – 2) is the factor of f(x) then a= $\frac{7}{6}$

(2) Let, f(x) = $x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4$

from factor theorem

if (x – 2) is the factor of f(x) the f(2) = 0

let , x – 2 = 0

=> x = 2

Substitute x value in f(x)

f(2) = $2^5 - 3(2)^4 - a(2)^3 + 3a(2)^2 + 2a(2) + 4$

= 32 – 48 – 8a + 12 + 4a + 4

= 8a − 12

Equate f(2) to zero

=> 8a − 12 = 0

=> 8a = 12

=> a = $\frac{12}{8}$

= $\frac{3}{2}$

So, when (x − 2) is a factor of f(x) then a= $\frac{3}{2}$

Q24. In each of the following two polynomials , find the value of a, if (x − a) is a factor :

1. $x^6 - ax^5 + x^4 - ax^3 + 3x - a + 2$
2. $x^5 - a^2x^3 + 2x + a + 1$

Sol :

(1) $x^6 - ax^5 + x^4 - ax^3 + 3x - a + 2$

let , f(x) = $x^6 - ax^5 + x^4 - ax^3 + 3x - a + 2$

here , x − a = 0

=> x = a

Substitute the value of x in f(x)

f(a) = $a^6 - a(a)^5 + (a)^4 - a(a)^3 + 3(a) - a + 2$

= $a^6 - a^6 + (a)^4 - a^4 + 3(a) - a + 2$

= 2a + 2

Equate to zero

=> 2a + 2 = 0

=> 2(a + 1) = 0

=> a = -1

So, when (x − a) is a factor of f(x) then a = -1

(2) $x^5 - a^2x^3 + 2x + a + 1$

let, $f(x) = x^5 - a^2x^3 + 2x + a + 1$

here , x − a = 0

=> x = a

Substitute the value of x in f(x)

f(a) = $a^5 - a^2a^3 + 2(a) + a + 1$

= $a^5 - a^5 + 2a + a + 1$

= 3a + 1

Equate to zero

=> 3a + 1 = 0

=> 3a = -1

=> a= $\frac{-1}{3}$

So, when (x − a) is a factor of f(x) then a = $\frac{-1}{3}$

Q25. In each of the following two polynomials , find the value of a, if (x + a) is a factor :

1. $x^3 + ax^2 - 2x + a + 4$
2. $x^4 - a^2x^2 + 3x - a$

Sol :

(1) $x^3 + ax^2 - 2x + a + 4$

let, f(x) = $x^3 + ax^2 - 2x + a + 4$

here , x + a = 0

=> x = -a

Substitute the value of x in f(x)

f(-a) = $(-a)^3 + a(-a)^2 - 2(-a) + a + 4$

= $(-a)^3 + a^3 - 2(-a) + a + 4$

= 3a + 4

Equate to zero

=> 3a + 4 = 0

=> 3a = -4

=> a = $\frac{-4}{3}$

So, when (x + a) is a factor of f(x) then a = $\frac{-4}{3}$

(2) $x^4 - a^2x^2 + 3x - a$

let, $f(x) = x^4 - a^2x^2 + 3x - a$

here , x + a = 0

=> x = -a

Substitute the value of x in f(x)

f(-a) = $(-a)^4 - a^2(-a)^2 + 3(-a) - a$

= $a^4 - a^4 - 3(a) - a$

= -4a

Equate to zero

=> -4a = 0

=> a = 0

So, when (x + a) is a factor of f(x) then a = 0