RD SHARMA

Solutions

Class 9 Maths

Chapter 6

Ex 6.3

In each of the following , using the remainder theorem, find the remainder when f(x) is divided by g(x) and verify the by actual division : (1 – 8)

Q1. f(x) = $x^3 + 4x^2 - 3x + 10$, g(x) = x + 4

Sol :

Here, f(x) = $x^3 + 4x^2 - 3x + 10$

g(x) = x + 4

from, the remainder theorem when f(x) is divided by g(x) = x – (-4) the remainder will be equal to f(-4)

Let , g(x) = 0

=> x + 4 = 0

=> x = -4

Substitute the value of x in f(x)

f(-4) = $(-4)^3 + 4(-4)^2 - 3(-4) + 10$

= -64 + (4 * 16) + 12 + 10

= -64 + 64 + 12 + 10

= 12 + 10

= 22

Therefore, the remainder is 22

Q2. f(x) = $4x^4 - 3x^3 - 2x^2 + x - 7$, g(x) = x – 1

Sol :

Here, f(x) = $4x^4 - 3x^3 - 2x^2 + x - 7$

g(x) = x – 1

from, the remainder theorem when f(x) is divided by g(x) = x – (-1) the remainder will be equal to f(1)

Let , g(x) = 0

=> x – 1 = 0

=> x = 1

Substitute the value of x in f(x)

f(1) = $4(1)^4 - 3(1)^3 - 2(1)^2 + 1 - 7$

= 4 – 3 – 2 + 1 – 7

= 5 – 12

= -7

Therefore, the remainder is 7

Q3. f(x) = $2x^4 - 6x^3 + 2x^2 - x + 2$, g(x) = x + 2

Sol :

Here, f(x) = $2x^4 - 6x^3 + 2x^2 - x + 2$

g(x) = x + 2

from, the remainder theorem when f(x) is divided by g(x) = x − (-2) the remainder will be equal to f(-2)

Let , g(x) = 0

=> x + 2 = 0

=> x = -2

Substitute the value of x in f(x)

f(-2) = $2(-2)^4 - 6(-2)^3 + 2(-2)^2 - (-2) + 2$

= (2 * 16) − (6 * (-8)) + (2 * 4) + 2 + 2

= 32 + 48 + 8 + 2 + 2

= 92

Therefore, the remainder is 92

Q4. f(x) = $4x^3 - 12x^2 + 14x - 3$, g(x) = 2x − 1

Sol:

Here, f(x) = $4x^3 - 12x^2 + 14x - 3$

g(x) = 2x − 1

from, the remainder theorem when f(x) is divided by g(x) = $2(x - \frac{1}{2})$, the remainder is equal to f($\frac{1}{2}$)

Let , g(x) = 0

=> 2x − 1 = 0

=> 2x = 1

=> x = $\frac{1}{2}$

Substitute the value of x in f(x)

f($\frac{1}{2}$) = $4(\frac{1}{2})^3 - 12(\frac{1}{2})^2 + 14(\frac{1}{2} - 3$

= $4(\frac{1}{8}) - 12(\frac{1}{4}) + 4(\frac{1}{2}) - 3$

= $(\frac{1}{2})$ − 3 + 7 − 3

= $(\frac{1}{2})$ + 1

Taking L.C.M

= $(\frac{2+1}{2})$

= $(\frac{3}{2})$

Therefore, the remainder is $(\frac{3}{2})$

Q5. f(x) = $x^3 - 6x^2 + 2x - 4$, g(x) = 1 − 2x

Sol :

Here, f(x) = $x^3 - 6x^2 + 2x - 4$

g(x) = 1 − 2x

from, the remainder theorem when f(x) is divided by g(x) = -2(x − $\frac{1}{2}$) , the remainder is equal to f($\frac{1}{2}$)

Let , g(x) = 0

=> 1 − 2x = 0

=> -2x = -1

=> 2x = 1

=> x = $\frac{1}{2}$

Substitute the value of x in f(x)

f($\frac{1}{2}$) = ($\frac{1}{2}$)³ − 6($\frac{1}{2}$)² + 2($\frac{1}{2}$) − 4

= $\frac{1}{8} - 8(\frac{1}{4}) + 2(\frac{1}{2}) - 4$

= $\frac{1}{8} - (\frac{1}{2}) + 1 - 4$

= $\frac{1}{8} - (\frac{1}{2}) - 3$

Taking L.C.M

= $\frac{1-4+8-32}{8}$

= $\frac{1-36}{8}$

= $\frac{1-36}{8}$

= $\frac{-35}{8}$

Therefore, the remainder is $\frac{-35}{8}$

Q6. f(x) = $x^4 - 3x^2 + 4$, g(x) = x − 2

Sol :

Here, f(x) = $x^4 - 3x^2 + 4$

g(x) = x − 2

from, the remainder theorem when f(x) is divided by g(x) = x − 2 the remainder will be equal to f(2)

let , g(x) = 0

=> x − 2 = 0

=> x = 2

Substitute the value of x in f(x)

f(2) = $2^4 - 3(2)^2 + 4$

= 16 − (3* 4) + 4

= 16 − 12 + 4

= 20 − 12

= 8

Therefore, the remainder is 8

Q7. f(x) = $9x^3 - 3x^2 + x - 5$, g(x) = $x - \frac{2}{3}$

Sol :

Here, f(x) = $9x^3 - 3x^2 + x - 5$

g(x) = $x - \frac{2}{3}$

from, the remainder theorem when f(x) is divided by g(x) = x − $\frac{2}{3}$ the remainder will be equal to f($\frac{2}{3}$)

substitute the value of x in f(x)

f($\frac{2}{3}$) = 9($\frac{2}{3}$) − 3($\frac{2}{3}$)^2 + ($\frac{2}{3}$) − 5

= $9(\frac{8}{27}) - 3(\frac{4}{9}) + \frac{2}{3} - 5$

= $(\frac{8}{3}) - (\frac{4}{3}) + \frac{2}{3} - 5$

= $\frac{8-4+2-15}{3}$

= $\frac{10-19}{3}$

= $\frac{-9}{3}$

= -3

Therefore, the remainder is -3

Q8. f(x) = $3x^4 + 2x^3 - \frac{x^3}{3} - \frac{x}{9} + \frac{2}{27}$, g(x) = $x + \frac{2}{3}$

Sol :

Here, f(x) = $3x^4 + 2x^3 - \frac{x^3}{3} - \frac{x}{9} + \frac{2}{27}$

g(x) = $x + \frac{2}{3}$

from remainder theorem when f(x) is divided by g(x) = $x - (-\frac{2}{3})$, the remainder is equal to f($-\frac{2}{3}$)

substitute the value of x in f(x)

f($-\frac{2}{3}$) = 3($-\frac{2}{3}$)^4 + 2($-\frac{2}{3}$)^3 − $\frac{(-\frac{2}{3})^3}{3}$ − \(\frac{(-\frac{2}{3})}{9} + [latex]\frac{2}{27}\)\(\frac{2}{27}\]”>

= [latex]3(\frac{16}{81}) + 2(\frac{-8}{27}) − \frac{4}{(9 * 3)} − (\frac{-2}{(9 * 3)}) + \frac{2}{27}\)

= $(\frac{16}{27}) - (\frac{16}{27}) - \frac{4}{27} + (\frac{2}{27}) + \frac{2}{27}$

$= (\frac{4}{27}) - (\frac{4}{27})$

= 0

Therefore, the remainder is 0

Q9. If the polynomial $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 4x + a$ leave the same remainder when divided by x − 2 , Find the value of a

Sol :

Given , the polymials are

f(x) = $2x^3 + ax^2 + 3x - 5$

p(x) = $x^3 + x^2 - 4x + a$

The remainders are f(2) and p(2) when f(x) and p(x) are divided by x − 2

We know that,

f(2) = p(2)   (given in problem)

we need to calculate f(2) and p(2)

for, f(2)

substitute (x = 2) in f(x)

f(2) = $2(2)^3 + a(2)^2 + 3(2) - 5$

= (2 * 8) + a4 + 6 − 5

= 16 + 4a + 1

= 4a + 17 ——- 1

for, p(2)

substitute (x = 2) in p(x)

p(2) = $2^3 + 2^2 - 4(2) + a$

= 8 + 4 − 8 + a

= 4 + a ——— 2

Since, f(2) = p(2)

Equate eqn 1 and 2

=> 4a + 17 = 4 + a

=> 4a − a = 4 − 17

=> 3a = -13

=> a = $\frac{-13}{3}$

The value of a = $\frac{-13}{3}$

Q10. If polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$ when divided by (x − 4) leave the remainders as R_1 and R_2 respectively. Find the values of a in each of the following cases, if

1. $R_1 = R_2$
2. $R_1 + R_2 = 0$
3. $2R_1 - R_2 = 0$

Sol :

Here, the polynomials are

f(x) = $ax^3 + 3x^2 - 3$

p(x) = $2x^3 - 5x + a$

let,

R_1 is the remainder when f(x) is divided by x − 4

=> R_1 = f(4)

=> R_1 = $a(4)^3 + 3(4)^2 - 3$

= 64a + 48 − 3

= 64a + 45 ——— 1

Now , let

R_2 is the remainder when p(x) is divided by x − 4

=> R_2 = p(4)

=> R_2 = $2(4)^3 - 5(4) + a$

= 128 − 20 + a

= 108 + a ——— 2

1. Given , $R_1 = R_2$

=> 64a + 45 = 108 + a

=> 63a = 63

=> a= 1

2. Given, $R_1 + R_2$ = 0

=> 64a + 45 + 108 + a = 0

=> 65a + 153 = 0

=> a = $\frac{-153}{65}$

3.Given, $2R_1 - R_2$ = 0

=> 2(64a + 45) − 108 − a = 0

=> 128a + 90 − 108 − a = 0

=> 127a − 18 = 0

=> a = $\frac{18}{127}$

Q11. If the polynomials $ax^3 + 3x^2 – 13$ and $2x^3 – 5x + a$ when divided by (x − 2) leave the same remainder, Find the value of a

Sol :

Here , the polynomials are

f(x) = $ax^3 + 3x^2 – 13$

p(x) = $2x^3 – 5x + a$

equate , x − 2 = 0

x = 2

substitute the value of x in f(x) and p(x)

f(2) = $(2)^3 + 3(2)^2 – 13$

= 8a + 12 − 13

= 8a − 1 −−−- 1

p(2) = $2(2)^3 – 5(2) + a$

= 16 − 10 + a

= 6 + a −−−− 2

f(2) = p(2)

=> 8a − 1 = 6 + a

=> 8a − a = 6 + 1

=> 7a = 7

=> a = 1

The value of a = 1

Q12. Find the remainder when $x^3 + 3x^3 + 3x + 1$ is divided by,

1. x + 1
2. x − $\frac{1}{2}$
3. x
4. x + π
5. 5 + 2x

Sol :

Here, f(x) = $x^3 + 3x^2 + 3x + 1$

by remainder theorem

1. => x + 1 = 0

=> x = -1

substitute the value of x in f(x)

f(-1) = $(-1)^3 + 3(-1)^2 + 3(-1) + 1$

= -1 + 3 − 3 + 1

= 0

2. x − $\frac{1}{2}$

Sol :

Here, f(x) = $x^3 + 3x^2 + 3x + 1$

By remainder theorem

=> x − $\frac{1}{2}$ = 0

=> x = $\frac{1}{2}$

substitute the value of x in f(x)

f($\frac{1}{2}$) = ($\frac{1}{2}$)³ + 3($\frac{1}{2}$)² + 3($\frac{1}{2}$) + 1

= $(\frac{1}{2})^3 + 3(\frac{1}{2})^2 + 3(\frac{1}{2}) + 1$

= $\frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1$

= $\frac{1+6+12+8}{8}$

= $\frac{27}{8}$

3. x

Sol :

Here, f(x) = $x^3 + 3x^2 + 3x + 1$

by remainder theorem

=> x = 0

substitute the value of x in f(x)

f(0) = $0^3 + 3(0)^2 + 3(0) + 1$

= 0 + 0 + 0 + 1

= 1

4. x + π

Sol :

Here, f(x) = $x^3 + 3x^2 + 3x + 1$

by remainder theorem

=> x + π = 0

=> x = -π

Substitute the value of x in f(x)

f(-π) = $(-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$

= $-(\pi)^3 + 3(\pi)^2 - 3(\pi) + 1$

5. 5 + 2x

Sol :

Here, f(x) = $x^3 + 3x^2 + 3x + 1$

by remainder theorem

5 + 2x = 0

2x = -5

x = $\frac{-5}{2}$

substitute the value of x in f(x)

$f(\frac{-5}{2}) = (\frac{-5}{2})^3 + 3(\frac{-5}{2})^2 + 3(\frac{-5}{2}) + 1$

= $\frac{-125}{8} + 3(\frac{25}{4}) + 3(\frac{-5}{2}) + 1$

= $\frac{-125}{8} + \frac{75}{4} - \frac{15}{2} + 1$

Taking L.C.M

= $\frac{-125+150-50+8}{8}$

= $\frac{-27}{8}$