RD SHARMA
Solutions
Class 9 Maths
Chapter 5
Ex 5.1

Q1 . $x^3 + x - 3x^2 - 3$

SOLUTION :

Taking x common in $x^3 + x$

=$x\,(x^2 + 1) - 3x^2 - 3$

Taking − 3 common in $-3x^2 - 3$

=$x\,(x^2 + 1) - 3\,(x^2 + 1)$

Now , we take $(x^2 + 1)$ common

=$(x^2 + 1)$ (x − 3)

$\therefore$ $x^3 + x - 3y^2 - 3$ =$(x^2 + 1)$ (x − 3)

Q2 . $a(a + b)^3 - 3a^2b\,(a + b)$

SOLUTION :

Taking (a + b) common in the two terms

= (a + b) {a(a + b) ² − 3a ²b}

Now, using $(a + b)^2 = a^2 + b^2 + 2ab$

=$(a + b)\,\{a\,(a^2 + b^2 + 2ab) - 3a^2b\}$

=$(a + b)\,\{a^3 + ab^2 + 2a^2b - 3a^2b\}$

=$(a + b)\,\{a^3 + ab^2 - a^2b\}$

=$(a + b)\,p\,\{a^2 + b^2 - ab\}$

=$p\,(a + b)\,(a^2 + b^2 - ab)$

$\therefore$ $a(a + b)^3 - 3a^2b\,(a + b) = a\,(a + b)\,(a^2 + b^2 - ab)$

Q3 . $x\,(x^3 - y^3) + 3xy\,(x - y)$

SOLUTION :

Elaborating $x^3 - y^3$ using the identity $x^3 - y^3 = (x - y)\,(x^2 + xy + y^2)$

=$x\,(x - y)\,(x^2 + xy + y^2) + 3xy\,(x - y)$

Taking common x(x-y) in both the terms

=$x\,(x - y)\,(x^2 + xy + y^2 + 3y)$

$\therefore$ $x\,(x^3 - y^3) + 3xy\,(x - y)$= $x\,(x - y)\,(x^2 + xy + y^2 + 3y)$

Q4 . $a^2x^2 + (ax^2 + 1)x + a$

SOLUTION :

We multiply $x(ax^2 + 1) = ax^3 + x$

=$a^2x^2 + ax^3 + x + a$

Taking common ax^2 in $(a^2x^2 + ax^3)$ and 1in (x + a)

=$ax^2(a + x) + 1(x + a)$

=$ax^2(a + x) + 1(a + x)$

Taking (a + x) common in both the terms

=$(a + x)(ax^2 + 1)$

$\therefore a^2x^2 + (ax^2 + 1)x + a = (a + x)(ax^2 + 1)$

Q5 . $x^2 + y - xy - x$

SOLUTION :

On rearranging

$x^2 - xy - x + y$

Taking x common in the $(x^2 - xy)$ and -1 in(-x+y)

=x(x − y) − 1 (x − y)

Taking (x − y) common in the terms

=(x − y)(x − 1)

latex]$\therefore$[/latex]$x^2 + y - xy - x$ =(x − y)(x − 1)

Q6 . $x^3 - 2x^2b + 3xy^2 - 6y^3$

SOLUTION :

Taking x^2 common in $(x^3 - 2x^2y)$ and +$3y^2$ common in $(3xy^2 - 6y^3)$

= $x^2(x - 2y) + 3y^2(x - 2y)$

Taking (x − 2y) common in the terms

= $(x - 2y)(x^2 + 3y^2)$

latex]$\therefore$[/latex] $x^3 - 2x^2y + 3xy^2 - 6y^3$ = $(x - 2y)(x^2 + 3y^2)$

Q7 . $6ab - b^2 + 12ac - 2bc$

SOLUTION :

Taking b common in $(6ab - b^2)$ and 2c in (12ac – 2bc)

=b(6a – b) + 2c (6a – b)

Taking (6a – b) common in the terms

=(6a – b)(b + 2c)

latex]∴[/latex]$6ab - b^2 + 12ac - 2bc$ =(6a – b)(b + 2c)

Q8 . $[x^2 + \frac{1}{x^2}] - 4\,[x + \frac{1}{x}] + 6$

SOLUTION :

=$x^2 + \frac{1}{x^2} - 4x - \frac{4}{x} + 4 + 2$

=$x^2 + \frac{1}{x^2} + 4 + 2 - \frac{4}{x} - 4x$

=$(x^2) + (\frac{1}{x})^2 + (-2)^2 + 2 \times x \times \frac{1}{x} + 2 \times \frac{1}{x} \times (-2) + 2\,(-2)\,x$

Using identity

$x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

We get,

=$[x + \frac{1}{x} + (-2)]^2$

=$[x + \frac{1}{x} - 2]^2$

=$[x + \frac{1}{x} - 2]\,[x + \frac{1}{x} - 2]$

$\therefore\ [x^2 + \frac{1}{x^2}] - 4\,[x + \frac{1}{x}] + 6$ =$[x + \frac{1}{x} - 2]\,[x + \frac{1}{x} - 2]$

Q9 . x(x- 2)(x – 4) + 4x – 8

SOLUTION :

=x(x – 2)(x – 4) + 4(x – 2)

Taking (x – 2) common in both the terms

=(x – 2){x(x – 4) + 4}

=(x – 2) $\{x^2 - 4x + 4\}$

Now splitting the middle term of $x^2 - 4x + 4$

=(x – 2){ $x^2 - 2x - 2x + 4$}

=(x – 2){ x(x – 2) -2(x -2)}

$=(x-2)\{(x-2)(x-2)\}$

$=(x-2)(x-2)(x-2)$

$=(x-2)^3$

$\therefore x(x-2)(x-4)+4x-8=(x-2)^3$

Q10 .(x + 2) $(x^2+25)-10x^2-20x$

SOLUTION :

$(x+2)(x^2+25)-10x(x+2)$

Taking $(x+2)$ common in both the terms

$=(x+2)(x^2+25-10x)$

$=(x+2)(x^2-10x+25)$

Splitting the middle term of $(x^2-10x+25)$

$=(x+2)(x^2-5x-5x+25)$

$=(x+2)\{x(x-5)-5(x-5)\}$

$=(x+2)(x-5)(x-5)$

$\therefore (x+2)(x^2+25)-10x^2-20x=(x+2)(x-5)(x-5)$

Q11 . $2a^2+2\sqrt{6}ab+3b^2$

SOLUTION :

$=(\sqrt{2}a)^2+2\times\sqrt{2}a\times\sqrt{3}b+(\sqrt{3}b)^2$

Using the identity $(p+q)^2=p^2+q^2+2pq$

$=(\sqrt{2}a+\sqrt{3}b)^2$

$=(\sqrt{2}a+\sqrt{3}b)(\sqrt{2}a+\sqrt{3}b)$

$\therefore 2a^2+2\sqrt{6}ab+3b^2=(\sqrt{2}a+\sqrt{3}b)(\sqrt{2}a+\sqrt{3}b)$

Q 12 . $(a-b+c)^2+(b-c+a)^2+2(a-b+c)\times(b-c+a)$

SOLUTION :

Let $(a-b+c)=x$ and $(b-c+a)=y$

$= x^2 + y^2 + 2xy$

Using the identity $(a+b)^2 = a^2 + b^2 + 2ab$

$= (x+y)^2$

Now , substituting x and y

$(a-b+c+b-c+a)^2$

Cancelling −b , +b & +c , -c

$= (2a)^2$

$= 4a^2$

$\therefore (a-b+c)^2 + (b-c+a)^2 + 2(a-b+c) \times (b-c+a) = 4a^2$

Q13 . $a^2 + b^2 + 2(ab + bc + ca)$

SOLUTION :

$= a^2 + b^2 + 2ab + 2bc + 2ca$

Using the identity $(p+q)^2 = p^2 + q^2 + 2pq$

We get,

$= (a+b)^2 + 2bc + 2ca$

$= (a+b)^2 + 2c(b + a)$

Or $(a+b)^2 + 2c(a + b)$

Taking (a + b) common

= (a + b)(a + b + 2c)

$\therefore a^2 + b^2 + 2(ab + bc + ca)$ = (a + b)(a + b + 2c)

Q14 . $4(x-y)^2 - 12(x-y)(x+y) + 9(x+y)^2$

SOLUTION :

Let (x − y) = x,(x + y) = y

$= 4x^2 - 12xy + 9y^2$

Splitting the middle term − 12 = -6 -6 also $4 \times 9 = -6 \times -6$

$=4x^2 - 6xy - 6xy + 9y^2$

=2x(2x − 3y) -3y(2x − 3y)

=(2x − 3y) (2x − 3y)

$=(2x - 3y)^2$

Substituting x = x − y & y = x + y

=$[2(x-y)-3(x+y)]^2$=[2x − 2y − 3x − 3y $]^2$

=(2x-3x-2y-3y $)^2$

=$[-x-5y]^2$

=$[(-1)(x+5y)]^2$

=$(x+5y)^2$ [∵ $(-1)^2 = 1$]

∴ $4(x-y)^2-12(x-y)(x+y)+9(x+y)^2$=$(x+5y)^2$

Q 15 . $a^2-b^2+2bc-c^2$

SOLUTION :

$a^2-(b^2-2bc+c^2)$

Using the identity $(a-b)^2 = a^2+b^2-2ab$

= $a^2-(b-c)^2$

Using the identity $a^2-b^2 = (a+b)(a-b)$

=(a + b − c)(a − (b − c))

=(a + b − c)(a − b + c)

∴ $a^2-b^2+2bc-c^2$=(a + b − c)(a − b + c)

Q16 . $a^2+2ab+b^2-c^2$

SOLUTION :

Using the identity $(p+q)^2 = p^2+q^2+2pq$

=$(a+b)^2-c^2$

Using the identity $p^2-q^2 = (p+q)(p-q)$

=(a + b + c)(a + b − c)

∴ $a^2+2ab+b^2-c^2$ =(a + b + c)(a + b − c)

Q 17 . $a^2+4b^2-4ab-4c^2$

SOLUTION :

On rearranging

= $a^2-4ab+4b^2-4c^2$

= $(a)^2 - 2 \times a \times 2b + (2b)^2 - 4c^2$

Using the identity $(a-b)^2 = a^2 + b^2 - 2ab$

=$(a-2b)^2 - 4c^2$

=$(a-2b)^2 - (2c)^2$

Using the identity $a^2 - b^2 = (a+b)(a-b)$

=(a – 2b – 2c) (a – 2b + 2c)

$\therefore a^2 + 4b^2 - 4ab - 4c^2$ =(a – 2b – 2c) (a – 2b + 2c)

Q18 . $xy^9 - yx^9$

SOLUTION :

=$xy\,(y^8 - x^8)$

=$xy\,((y^4)^2 - (x^4)^2)$

Using the identity $p^2 - q^2$= (p + q)(p – q)

=$xy\,(y^4 + x^4)(y^4 - x^4)$

=$xy\,(y^4 + x^4)((y^2)^2 - (x^2)^2)$

Using the identity $p^2 - q^2$= (p + q)(p – q)

=$xy\,(y^4 + x^4)(y^2 + x^2)(y^2 - x^2)$

= $xy\,(y^4 + x^4)(y^2 + x^2)(y + x)(y - x)$

= $xy\,(x^4 + y^4)(x^2 + y^2)(x + y)(-1)(x - y)$

$\because (y - x) = -1\,(x - y)$

=$-xy\,(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$

$\therefore xy^9 - yx^9$ = $-xy\,(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$

Q 19 . $x^4 + x^2y^2 + y^4$

SOLUTION :

Adding x^2y^2 and subtracting x^2y^2 to the given equation

= $x^4 + x^2y^2 + y^4 + x^2y^2 - x^2y^2$

= $x^4 + 2x^2y^2 + y^4 - x^2y^2$

= $(x^2)^2 + 2 \times x^2 \times y^2 + (y^2)^2 - (xy)^2$

Using the identity $(p+q)^2 = p^2 + q^2 + 2pq$

= $(x^2 + y^2)^2 - (xy)^2$

Using the identity $p^2 - q^2 = (p+q)(p-q)$

= $(x^2 + y^2 + xy)(x^2 + y^2 - xy)$

$\therefore x^4 + x^2y^2 + y^4 = (x^2 + y^2 + xy)(x^2 + y^2 - xy)$

Q20 . $x^2 - y^2 - 4xz + 4z^2$

SOLUTION :

On rearranging the terms

= $x^2 - 4xz + 4z^2 - y^2$

= $(x)^2 - 2 \times x \times 2z + (2z)^2 - y^2$

Using the identity $x^2 - 2xy + y^2 = (x-y)^2$

= $(x - 2z)^2 - y^2$

Using the identity $p^2 - q^2 = (p+q)(p-q)$

= $(x - 2z + y)(x - 2z - y)$

$\therefore x^2 - y^2 - 4xz + 4z^2 = (x - 2z + y)(x - 2z - y)$

Q21 . $x^2 + 6\sqrt{2}x + 10$

SOLUTION :

Splitting the middle term ,

= $x^2 + 5\sqrt{2}x + \sqrt{2}x + 10$ $[\because 6\sqrt{2} = 5\sqrt{2} + \sqrt{2} \text{ and } 5\sqrt{2} \times \sqrt{2} = 10]$

= $x(x + 5\sqrt{2}) + \sqrt{2}(x + 5\sqrt{2})$

= $(x + 5\sqrt{2})(x + \sqrt{2})$

$\therefore x^2 + 6\sqrt{2}x + 10 = (x + 5\sqrt{2})(x + \sqrt{2})$

Q22 . $x^2 - 2\sqrt{2}x - 30$

SOLUTION :

Splitting the middle term,

= $x^2 - 5\sqrt{2}x + 3\sqrt{2}x - 30$

$[\because -2\sqrt{2} = -5\sqrt{2} + 3\sqrt{2}$ also $-5\sqrt{2} \times 3\sqrt{2} = -30]$

= $x(x - 5\sqrt{2}) + 3\sqrt{2}(x - 5\sqrt{2})$

= $(x - 5\sqrt{2})(x + 3\sqrt{2})$

$\therefore x^2 - 2\sqrt{2}x - 30 = (x - 5\sqrt{2})(x + 3\sqrt{2})$

Q23 . $x^2 - \sqrt{3}x - 6$

SOLUTION :

Splitting the middle term,

=$x^2 - 2\sqrt{3}x + \sqrt{3}x - 6$ $[\because -\sqrt{3} = -2\sqrt{3} + \sqrt{3}$ also $-2\sqrt{3} \times \sqrt{3} = -6]$

= $x(x - 2\sqrt{3}) + \sqrt{3}(x - 2\sqrt{3})$

= $(x - 2\sqrt{3})(x + \sqrt{3})$

$\therefore x^2 - \sqrt{3}x - 6 = (x - 2\sqrt{3})(x + \sqrt{3})$

Q24 . $x^2 + 5\sqrt{5}x + 30$

SOLUTION :

Splitting the middle term,

= $x^2 + 2\sqrt{5}x + 3\sqrt{5}x + 30$ $[\because 5\sqrt{5} = 2\sqrt{5} + 3\sqrt{5}$ also $2\sqrt{5} \times 3\sqrt{5} = 30]$

= $x(x + 2\sqrt{5}) + 3\sqrt{5}(x + 2\sqrt{5})$

= $(x + 2\sqrt{5})(x + 3\sqrt{5})$

$\therefore x^2 + 5\sqrt{5}x + 30 = (x + 2\sqrt{5})(x + 3\sqrt{5})$

Q25 . $x^2 + 2\sqrt{3}x - 24$

SOLUTION :

Splitting the middle term,

$=x^2 + 4\sqrt{3}x - 2\sqrt{3}x - 24$ $[\because 2\sqrt{3} = 4\sqrt{3} - 2\sqrt{3}$ also $4\sqrt{3}(-2\sqrt{3}) = -24]$

$= x(x + 4\sqrt{3}) - 2\sqrt{3}(x + 4\sqrt{3})$

$= (x + 4\sqrt{3})(x - 2\sqrt{3})$

$\therefore x^2 + 2\sqrt{3}x - 24 = (x + 4\sqrt{3})(x - 2\sqrt{3})$

Q26 . $2x^2 - \frac{5}{6}x + \frac{1}{12}$

SOLUTION :

Splitting the middle term,

$= 2x^2 - \frac{x}{2} - \frac{x}{3} + \frac{1}{12}$ $[\because -\frac{5}{6} = -\frac{1}{2} - \frac{1}{3}$ also $-\frac{1}{2} \times -\frac{1}{3} = 2 \times \frac{1}{12}]$

$= x(2x - \frac{1}{2}) - \frac{1}{6}(2x - \frac{1}{2})$

$= (2x - \frac{1}{2})(x - \frac{1}{6})$

$\therefore 2x^2 - \frac{5}{6}x + \frac{1}{12} = (2x - \frac{1}{2})(x - \frac{1}{6})$

Q27 . $x^2 + \frac{12}{35}x + \frac{1}{35}$

SOLUTION :

Splitting the middle term,

$=x^2 + \frac{5}{35}x + \frac{7}{35}x + \frac{1}{35}$ $[\because \frac{12}{35} = \frac{5}{35} + \frac{7}{35}$ and $\frac{5}{35} \times \frac{7}{35} = \frac{1}{35}]$

$= x^2 + \frac{x}{7} + \frac{x}{5} + \frac{1}{35}$

$= x(x + \frac{1}{7}) + \frac{1}{5}(x + \frac{1}{7})$

$= (x + \frac{1}{7})(x + \frac{1}{5})$

$\therefore x^2 + \frac{12}{35}x + \frac{1}{35} = (x + \frac{1}{7})(x + \frac{1}{5})$

Q28 . $21x^2 - 2x + \frac{1}{21}$

SOLUTION :

$= (\sqrt{21}x)^2 - 2\sqrt{21}x \times \frac{1}{\sqrt{21}} + (\frac{1}{\sqrt{21}})^2$

Using the identity $(x-y)^2 = x^2 + y^2 - 2xy$

$= (\sqrt{21}x - \frac{1}{\sqrt{21}})^2$

$\therefore 21x^2 - 2x + \frac{1}{21} = (\sqrt{21}x - \frac{1}{\sqrt{21}})^2$

Q29 . $5\sqrt{5}x^2 + 20x + 3\sqrt{5}$

SOLUTION :

Splitting the middle term,

$= 5\sqrt{5}x^2 + 15x + 5x + 3\sqrt{5}$ $[\because 20 = 15 + 5 \text{ and } 15 \times 5 = 5\sqrt{5} \times 3\sqrt{5}]$

$= 5x(\sqrt{5}x + 3) + \sqrt{5}(\sqrt{5}x + 3)$

$= (\sqrt{5}x + 3)(5x + \sqrt{5})$

$\therefore 5\sqrt{5}x^2 + 20x + 3\sqrt{5} = (\sqrt{5}x + 3)(5x + \sqrt{5})$

Q30 . $2x^2 + 3\sqrt{5}x + 5$

SOLUTION :

Splitting the middle term,

$= 2x^2 + 2\sqrt{5}x + \sqrt{5}x + 5$

$= 2x(x + \sqrt{5}) + \sqrt{5}(x + \sqrt{5})$

$= (x + \sqrt{5})(2x + \sqrt{5})$

$\therefore 2x^2 + 3\sqrt{5}x + 5 = (x + \sqrt{5})(2x + \sqrt{5})$

Q31 . $9(2a-b)^2 - 4(2a-b) - 13$

SOLUTION :

Let $2a - b = x$

$= 9x^2 - 4x - 13$

Splitting the middle term,

$= 9x^2 - 13x + 9x - 13$

$= x(9x - 13) + 1(9x - 13)$

$= (9x - 13)(x + 1)$

Substituting $x = 2a - b$

$= [9(2a - b) - 13](2a - b + 1)$

$= (18a - 9b - 13)(2a - b + 1)$

$\therefore 9(2a - b)^2 - 4(2a - b) - 13 = (18a - 9b - 13)(2a - b + 1)$

Q 32 . $7(x - 2y)^2 - 25(x - 2y) + 12$

SOLUTION :

Let x-2y = P

$= 7P^2 - 25P + 12$

Splitting the middle term,

$= 7P^2 - 21P - 4P + 12$

$= 7P(P - 3) - 4(P - 3)$

$= (P - 3)(7P - 4)$

Substituting $P = x - 2y$

$= (x - 2y - 3)(7(x - 2y) - 4)$

$= (x - 2y - 3)(7x - 14y - 4)$

$\therefore 7(x - 2y)^2 - 25(x - 2y) + 12 = (x - 2y - 3)(7x - 14y - 4)$

Q33 . $2(x + y)^2 - 9(x + y) - 5$

SOLUTION :

Let x+y = z

$= 2z^2 - 9z - 5$

Splitting the middle term,

$= 2z^2 - 10z + z - 5$

$= 2z(z - 5) + 1(z - 5)$

$= (z - 5)(2z + 1)$

Substituting $z = x + y$

$= (x + y - 5)(2(x + y) + 1)$

= $(x + y - 5)(2x + 2y + 1)$

$\therefore 2(x + y)^2 - 9(x + y) - 5 = (x + y - 5)(2x + 2y + 1)$

Q34 . Give the possible expression for the length & breadth of the rectangle having $35y^2 - 13y - 12$ *as its area.*

SOLUTION :

Area is given as $35y^2 - 13y - 12$

Splitting the middle term,

Area = $35y^2 + 218y - 15y - 12$

= $7y(5y + 4) - 3(5y + 4)$

= $(5y + 4)(7y - 3)$

We also know that area of rectangle = length $\times$breadth

$\therefore$ Possible length = $(5y + 4)$ and breadth=$(7y - 3)$

Or possible length = $(7y - 3)$ and breadth= $(5y + 4)$

Q35 . What are the possible expression for the cuboid having volume $3x^2 - 12x$.

SOLUTION :

Volume = $3x^2 - 12x$

= $3x(x - 4)$

= $3 \times x(x - 4)$

Also volume = Length $\times$Breadth$\times$Height

$\therefore$ Possible expression for dimensions of cuboid are = 3 , x , $(x - 4)$