# RD SHARMA

# Solutions

# Class 9 Maths

# Chapter 4

# Ex 4.4

Q1. Find the following products

(a) $(3x + 2y)(9x^2 - 6xy + 4y^2)$

(b) $(4x - 5y)(16x^2 + 20xy + 25y^2)$

(c) $(7p^4 + q)(49p^8 - 7p^4q + q^2)$

(d) $(\frac{x}{2} + 2y)($

$\frac{x^2}{4} - xy + 4y^2)$

(e) $(\frac{3}{x} - \frac{5}{y})(\frac{9}{x^2} + \frac{25}{y^2} + \frac{15}{xy})$

(f) $(3 + \frac{5}{x})(9 - \frac{15}{x} + \frac{25}{x^2})$

(g) $(\frac{2}{x} + 3x)(\frac{4}{x^2} + 9x^2 - 6)$

(h) $(\frac{3}{x} - 2x^2)(\frac{9}{x^2} + 4x^4 - 6x)$

(i) $(1 - x)(1 + x + x^{2})$

(j) $(1 + x)(1 - x + x^{2})$

(k) $(x^2 - 1)(x^4 + x^2 + 1)$

(l) $(x^2 + 1)(x^6 - x^3 + 1)$

Sol :

(a) $(3x + 2y)(9x^2 - 6xy + 4y^2)$

Given, $(3x + 2y)(9x^2 - 6xy + 4y^2)$

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$(3x + 2y)(9x^2 - 6xy + 4y^2)$ can we written as

=> $(3x + 2y)[(3x)^2 - (3x)(2y) + (2y)^2)]$

=> $(3x)^3 + (2y)^3$

=> $27x^3 + 8y^3$

Hence, the value of $(3x + 2y)(9x^2 - 6xy + 4y^2) = 27x^3 + 8y^3$

(b) $(4x - 5y)(16x^2 + 20xy + 25y^2)$

Given, $(4x - 5y)(16x^2 + 20xy + 25y^2)$

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$(4x - 5y)(16x^2 + 20xy + 25y^2)$ can we written as

=> $(4x - 5y)[(4x)^2 + (4x)(5y) + (5y)^2)]$

=> $(4x)^3 - (5y)^3$

=> $16x^3 - 25y^3$

Hence, the value of $(4x - 5y)(16x^2 + 20xy + 25y^2) = 16x^3 - 25y^3$

(c) $(7p^4 + q)(49p^8 - 7p^4q + q^2)$

Given, $(7p^4 + q)(49p^8 - 7p^4q + q^2)$

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$(7p^4 + q)(49p^8 - 7p^4q + q^2)$ can be written as

=> $(7p^4 + q)[(7p^4)^2 - (7p^4)(q) + (q)^2)]$

=> $(7p^4)^3 + (q)^3$

=> $343p^{12} + q^3$

Hence, the value of $(7p^4 + q)(49p^8 - 7p^4q + q^2) = 343p^{12} + q^3$

(d) $(\frac{x}{2} + 2y)(\frac{x^2}{4} - xy + 4y^2)$

Sol :

Given, $(\frac{x}{2} + 2y)(\frac{x^2}{4} - xy + 4y^2)$

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$(\frac{x}{2} + 2y)(\frac{x^2}{4} - xy + 4y^2)$ can be written as

=> $(\frac{x}{2} + 2y)[(\frac{x}{2})^2 - \frac{x}{2}(2y) + (2y)^2]$

=> $(\frac{x}{2})^3 + (2y)^3$

=> $\frac{x^3}{8} + 8y^3$

(e) $(\frac{3}{x} - \frac{5}{y})(\frac{9}{x^2} + \frac{25}{y^2} + \frac{15}{xy})$

Sol:

Given, $(\frac{3}{x} - \frac{5}{y})(\frac{9}{x^2} + \frac{25}{y^2} + \frac{15}{xy})$

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$(\frac{3}{x} - \frac{5}{y})(\frac{9}{x^2} + \frac{25}{y^2} + \frac{15}{xy})$

Can be written as,

=> $(\frac{3}{x} - \frac{5}{y})(\frac{3}{x})^2 + (\frac{5}{y})^2 + (\frac{3}{x})(\frac{5}{y})$

=> $(\frac{3}{x})^3 - (\frac{5}{y})^3$

=> $(\frac{27}{x^3}) - (\frac{125}{y^3})$

Hence, the value of $(\frac{3}{x} - \frac{5}{y})(\frac{9}{x^2} + \frac{25}{y^2} + \frac{15}{xy}) = (\frac{27}{x^3}) - (\frac{125}{y^3})$

(f) $(3 + \frac{5}{x})(9 - \frac{15}{x} + \frac{25}{x^2})$

Sol:

Given, $(3 + \frac{5}{x})(9 - \frac{15}{x} + \frac{25}{x^2})$

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$(3 + \frac{5}{x})(9 - \frac{15}{x} + \frac{25}{x^2})$ can be written as,

=> $(3 + \frac{5}{x})[(3^2) - 3(\frac{5}{x}) + (\frac{5}{x})^2]$

=> $(3)^3 + (\frac{5}{x})^3$

=> 27 + $\frac{125}{x^3}$

Hence, the value of $(3 + \frac{5}{x})(9 - \frac{15}{x} + \frac{25}{x^2})$ is 27 + $\frac{125}{x^3}$

(g) $(\frac{2}{x} + 3x)(\frac{4}{x^2} + 9x^2 - 6)$

Sol:

Given, $(\frac{2}{x} + 3x)(\frac{4}{x^2} + 9x^2 - 6)$

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$(\frac{2}{x} + 3x)(\frac{4}{x^2} + 9x^2 - 6)$ can be written as,

=> $(\frac{2}{x} + 3x)[(\frac{2}{x})^2 + (3x)^2 - (\frac{2}{x})(3x)]$

=> $(\frac{2}{x})^3 + (3x)^3$

=> $\frac{8}{x^3} + 9x^3$

Hence, the value of $(\frac{2}{x} + 3x)(\frac{4}{x^2} + 9x^2 - 6)$ is $\frac{8}{x^3} + 9x^3$

(h) $(\frac{3}{x} - 2x^2)(\frac{9}{x^2} + 4x^4 - 6x)$

Sol:

Given, $(\frac{3}{x} - 2x^2)(\frac{9}{x^2} + 4x^4 - 6x)$

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$(\frac{3}{x} - 2x^2)(\frac{9}{x^2} + 4x^4 - 6x)$ can be written as,

=> $(\frac{3}{x} - 2x^2)[(\frac{3}{x})^2 + (2x^2)^2 - (\frac{3}{x})(2x^2)]$

=> $(\frac{3}{x} - 2x^2)[(\frac{9}{x^2}) + 4x^4 - (\frac{3}{x})(2x^2)]$

=> $(\frac{3}{x})^3 - (2x^2)^3$

=> $\frac{27}{x^3} - 8x^6$

Hence, $(\frac{3}{x} - 2x^2)(\frac{9}{x^2} + 4x^4 - 6x)$ is $\frac{27}{x^3} - 8x^6$

(i) (1 − x)(1 + x + x^{2})

Sol:

Given, $(1 - x)(1 + x + x^{2})$

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$(1 - x)(1 + x + x^{2})$ can be written as,

=> $(1 - x)[(1^2 + (1)(x) + x^2)]$

=> $(1)^3 - (x)^3$

=> $1 - x^3$

Hence, the value of $(1 - x)(1 + x + x^{2})$ is $1 - x^3$

(j) $(1 + x)(1 - x + x^{2})$

Sol:

Given, $(1 + x)(1 - x + x^{2})$

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$(1 + x)(1 - x + x^{2})$ can be written as,

=> $(1 + x)[(1^2 - (1)(x) + x^2)]$

=> $(1)^3 + (x)^3$

=> $1 + x^3$

Hence, the value of $(1 + x)(1 + x - x^{2})$ is $1 + x^3$

(k) $(x^2 - 1)(x^4 + x^2 + 1)$

Sol:

Given, $(x^2 - 1)(x^4 + x^2 + 1)$

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$(x^2 - 1)(x^4 + x^2 + 1)$ can be written as,

=> $(x^2 - 1)[(x^2)^2 - 1^2 + (x^2)(1)]$

=> $(x^2)^3 - 1^3$

=> $x^6 - 1$

Hence, $(x^2 - 1)(x^4 + x^2 + 1)$ is $x^6 - 1$

(l) $(x^2 + 1)(x^6 - x^3 + 1)$

Sol:

Given, $(x^2 + 1)(x^6 - x^3 + 1)$

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$(x^2 + 1)(x^6 - x^3 + 1)$ can be written as,

=> $(x^3 + 1)[(x^3)^2 - (x^3)(1) + 1^2]$

=> $(x^3)^3 + 1^3$

=> $x^9 + 1$

Hence, the value of $(x^2 + 1)(x^6 - x^3 + 1)$ is $x^9 + 1$

Q2. Find x= 3 and y = -1, Find the values of each of the following using in identity:

(a) $(9x^2 - 4x^2)(81y^4 + 36x^2y^2 + 16x^4)$

(b) $(\frac{3}{x} - \frac{5}{y})(\frac{9}{x^2} + \frac{25}{y^2} + \frac{15}{xy})$

(c) $(\frac{x}{7} + \frac{y}{3})(\frac{x^2}{49} + \frac{y^2}{9} - \frac{xy}{21})$

(d) $(\frac{x}{4} - \frac{y}{3})(\frac{x^2}{16} + \frac{y^2}{9} + \frac{xy}{21})$

(e) $(\frac{5}{x} + 5x)(\frac{25}{x^2} - 25 + 25x^2)$

Sol:

(a) $(9x^2 - 4x^2)(81y^4 + 36x^2y^2 + 16x^4)$

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$(9x^2 - 4x^2)(81y^4 + 36x^2y^2 + 16x^4)$ can be written as,

=> $(9x^2 - 4x^2)[(9y^2)^2 + (9)(4)x^2y^2 + (4x^2)^2]$

=> $(9y^2)^3 - (4x^2)^3$

=> $729y^6 - 64x^6$

Substitute the value x = 3, y = -1 in $729y^6 - 64x^6$ we get,

=> $729y^6 - 64x^6$

=> $729(-1)^6 - 64(3)^6$

=> $729(1) - 64(729)$

=> $729 - 46656$

=> $-45927$

Hence, the product value of $(9x^2 - 4x^2)(81y^4 + 36x^2y^2 + 16x^4) = -45927$

(b) $(\frac{3}{x} - \frac{5}{y})(\frac{9}{x^2} + \frac{25}{y^2} + \frac{15}{xy})$

Sol:

Given,

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$(\frac{3}{x} - \frac{5}{y})(\frac{9}{x^2} + \frac{25}{y^2} + \frac{15}{xy})$

Can be written as,

=> $(\frac{3}{x} - \frac{x}{3})[(\frac{3}{x})^2 + (\frac{x}{3})^2 + (\frac{3}{x})(\frac{x}{3})]$

=> $(\frac{3}{x})^3 - (\frac{x}{3})^3$

=> $(\frac{27}{x^3}) - (\frac{x^3}{27})$ -- 1

Substitute x = 3 in eq 1

=> $(\frac{27}{3^3}) - (\frac{3^3}{27})$

=> $(\frac{27}{27}) - (\frac{27}{27})$

=> 0

Hence, the value of $(\frac{3}{x} - \frac{5}{y})(\frac{9}{x^2} + \frac{25}{y^2} + \frac{15}{xy})$ is 0

(c) $(\frac{x}{7} + \frac{y}{3})(\frac{x^2}{49} + \frac{y^2}{9} - \frac{xy}{21})$

Sol:

Given,

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$(\frac{x}{7} + \frac{y}{3})(\frac{x^2}{49} + \frac{y^2}{9} - \frac{xy}{21})$

Can be written as,

=> $(\frac{x}{7} + \frac{y}{3})[(\frac{x}{7})^2 + (\frac{y}{3})^2 - (\frac{x}{7})(\frac{y}{3})]$

=> $(\frac{x}{7})^3 + (\frac{y}{3})^3$

=> $(\frac{x^3}{343}) + (\frac{y^3}{27})$ -- 1

Substitute x = 3, y = -1 in eq 1

=> $(\frac{3^3}{343}) + (\frac{(-1)^3}{27})$

=> $(\frac{27}{343}) - (\frac{1}{27})$

Taking least common multiple, we get

=> $\frac{27*27}{343*27} - \frac{1*343}{27*343}$

=> $\frac{729}{9261} - \frac{343}{9261}$

=> $\frac{729-343}{9261}$

=> $\frac{386}{9261}$

Hence, the value of $(\frac{x}{7} + \frac{y}{3})(\frac{x^2}{49} + \frac{y^2}{9} - \frac{xy}{21})$ = $\frac{386}{9261}$

(d) $(\frac{x}{4} - \frac{y}{3})(\frac{x^2}{16} + \frac{y^2}{9} - + \frac{xy}{21})$

Sol:

Given,

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$(\frac{x}{4} - \frac{y}{3})(\frac{x^2}{16} + \frac{y^2}{9} + \frac{xy}{21})$

Can be written as,

=> $(\frac{x}{4} - \frac{y}{3})[(\frac{x}{4})^2 + (\frac{y}{3})^2 + (\frac{x}{4})(\frac{y}{3})]$

=> $(\frac{x}{4})^3 - (\frac{y}{3})^3$

=> $(\frac{x^3}{64}) - (\frac{y^3}{27})$ –– 1

Substitute x = 3, y = -1 in eq 1

=> $(\frac{3^3}{343}) - (\frac{(-1)^3}{27})$

=> $(\frac{27}{64}) + (\frac{1}{27})$

Taking least common multiple, we get

=> $\frac{27*27}{64*27} + \frac{1*64}{27*64}$

=> $\frac{729}{1728} + \frac{64}{1728}$

=> $\frac{729+64}{1728}$

=> $\frac{793}{9261}$

Hence, the value of $(\frac{x}{4} - \frac{y}{3})(\frac{x^2}{16} + \frac{y^2}{9} + \frac{xy}{21}) = \frac{793}{1728}$

(e) $(\frac{5}{x} + 5x)(\frac{25}{x^2} - 25 + 25x^2)$

Sol:

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$(\frac{5}{x} + 5x)(\frac{25}{x^2} - 25 + 25x^2)$ can be written as,

=> $(\frac{5}{x} + 5x)[(\frac{5}{x})^2 + (5x)^2 - (\frac{5}{x})(5x)]$

=> $(\frac{5}{x})^3 + (5x)^3$

=> $\frac{125}{x^3} + 125x^3$ ––- 1

Substitute x = 3, in eq 1

=> $\frac{125}{3^3} + 125(3)^3$

=> $\frac{125}{27} + 125 * 27$

=> $\frac{125}{27} + 3375$

Taking least common multiple, we get

=> $\frac{125}{27} + \frac{3375*27}{27*1}$

=> $\frac{125}{27} + \frac{91125}{27}$

=> $\frac{125+91125}{27}$

=> $\frac{91250}{25}$

Hence, the value of $(\frac{5}{x} + 5x)(\frac{25}{x^2} - 25 + 25x^2)$ is $\frac{91250}{25}$

Q3. If a + b = 10 and ab = 16, find the value of $a^2 - ab + b^2$ and $a^2 + ab + b^2$

Sol :

Given, a + b = 10, ab = 16

We know that, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

=> $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

=> $a^3 + b^3 = (10)^3 - 3(16)(10)$

=> $a^3 + b^3 = 1000 - 480$

=> $a^3 + b^3 = 520$

Substitute , $a^3 + b^3 = 520$, a + b = 10 in $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$520 = 10(a^2 + b^2 - ab)$

$\frac{520}{10} = (a^2 + b^2 - ab)$

=> $(a^2 + b^2 - ab) = 52$

Now, we need to find $(a^2 + b^2 + ab)$

Add and subtract 2ab in $a^2 + b^2 + ab$

=> $a^2 + b^2 + ab - 2ab + 2ab$

=> $(a + b)^2 - ab$

Substitute a + b = 10, ab

=> $a^2 + b^2 + ab = 10^2 - 16$

= 100 − 16

= 84

Hence, the values of $(a^2 + b^2 - ab) = 52$ and $(a^2 + b^2 + ab) = 84$

Q4. If a + b = 8 and ab = 6, find the value of $a^3 + b^3$

Sol:

Given, a + b = 8 and ab = 6

We know that, $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

=> $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

=> $a^3 + b^3 = (8)^3 - 3(6)(8)$

=> $a^3 + b^3 = 512 - 144$

=> $a^3 + b^3 = 368$

Hence, the value of $a^3 + b^3$ is 368

Q5. If a − b = 6 and ab = 20, find the value of $a^3 - b^3$

Sol:

Given, a − b = 6 and ab = 20

We know that, $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$

=> $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$

=> $a^3 - b^3 = (6)^3 + 3(20)(6)$

=> $a^3 - b^3 = 216 + 360$

=> $a^3 - b^3 = 576$

Hence, the value of $a^3 - b^3$ is 576

Q6. If x = -2 and y = 1, by using an identity find the value of the following:

(a) $(4y^2 - 9x^2)(16y^4 + 36x^2y^2 + 81x^4)$

(b) $(\frac{2}{x} - \frac{x}{2})(\frac{4}{x^2} + \frac{x^2}{4} + 1)$

(c) $(5y + \frac{15}{y})(25y^2 - 75 + \frac{225}{y^2})$

Sol:

Given,

(a) $(4y^2 - 9x^2)(16y^4 + 36x^2y^2 + 81x^4)$

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$(4y^2 - 9x^2)(16y^4 + 36x^2y^2 + 81x^4)$ can be written as,

=> $(4y^2 - 9x^2)[(4x)^2 + 4y^2*9x^2 + (9x^2)^2)$

=> $(4y^2)^3 - (9x^2)^3$

=> $64y^6 - 729x^6$ ———- 1

Substitute x = -2 and y = 1 in eq 1

=> $64y^6 - 729x^6$

=> $64(1)^6 - 729(-2)^6$

=> 64 − 729(64)

=> 64(1 − 729)

=> 64(-728)

=> -46592

Hence, the value of $(4y^2 - 9x^2)(16y^4 + 36x^2y^2 + 81x^4)$ is -46592

(b) $(\frac{2}{x} - \frac{x}{2})(\frac{4}{x^2} + \frac{x^2}{4} + 1)$

here x = -2

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$(\frac{2}{x} - \frac{x}{2})(\frac{4}{x^2} + \frac{x^2}{4} + 1)$ can be witten as,

=> $(\frac{2}{x} - \frac{x}{2})[(\frac{2}{x})^2 + (\frac{x}{2})^2 + (\frac{2}{x})(\frac{x}{2})]$

=> $(\frac{2}{x})^3 - (\frac{x}{2})^3$

=> $(\frac{8}{x^3}) - (\frac{x^3}{8})$ ---- 1

Substitute x = -2 in eq 1

=> $(\frac{8}{(-2)^3}) - (\frac{(-2)^3}{8})$

=> $(\frac{8}{-8}) - (\frac{-8}{8})$

=> -1 + 1

=> 0

Hence, the value of $(\frac{2}{x} - \frac{x}{2})(\frac{4}{x^2} + \frac{x^2}{4} + 1)$ is 0

(c) $(5y + \frac{15}{y})(25y^2 - 75 + \frac{225}{y^2})$

Sol:

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$(5y + \frac{15}{y})(25y^2 - 75 + \frac{225}{y^2})$ can be written as,

=> $(5y + \frac{15}{y})[(5y)^2 + (\frac{15}{y})^2 - (5y)(\frac{15}{y})]$

=> $(5y)^3 + (\frac{15}{y})^3$

=> $125y^3 + (\frac{3375}{y^3})$ ---- 1

Substitute y = 1 in eq 1

=> $125(1)^3 + (\frac{3375}{(1)^2})$

=> 125 + 3375

=> 3500

Hence, the value of $(5y + \frac{15}{y})(25y^2 - 75 + \frac{225}{y^2})$ is 3500.

EXERCISE 4.5

Q1. Find the following products:

(a) $(3x + 2y + 2z)(9x^2 + 4y^2 + 4z^2 - 6xy - 4yz - 6zx)$

(b) $(4x - 3y + 2z)(16x^2 + 9y^2 + 4z^2 + 12xy + 6yz - 8zx)$

(c) $(2a - 3b - 2c)(4a^2 + 9b^2 + 4c^2 + 6ab - 6bc + 4ca)$

(d) $(3x - 4y + 5z)(9x^2 + 16y^2 + 25z^2 + 12xy - 15zx + 20yz)$

Sol:

Given,

(a) $(3x + 2y + 2z)(9x^2 + 4y^2 + 4z^2 - 6xy - 4yz - 6zx)$

we know that,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

so,

$(3x + 2y + 2z)(9x^2 + 4y^2 + 4z^2 - 6xy - 4yz - 6zx) = (3x)^3 + (2y)^3 + (2z)^3 - 3(3x)(2y)(2z)$

$= 27x^3 + 8y^3 + 8z^3 - 36xyz$

Hence, the value of $(3x + 2y + 2z)(9x^2 + 4y^2 + 4z^2 - 6xy - 4yz - 6zx)$ is $27x^3 + 8y^3 + 8z^3 - 36xyz$

(b) $(4x - 3y + 2z)(16x^2 + 9y^2 + 4z^2 + 12xy + 6yz - 8zx)$

we know that,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

so,

$(4x - 3y + 2z)(16x^2 + 9y^2 + 4z^2 + 12xy + 6yz - 8zx) = (4x)^3 + (-3y)^3 + (2z)^3 - 3(4x)(-3y)(2z)$

$= 64x^3 - 27y^3 + 8z^3 + 72xyz$

Hence, the value of $(4x - 3y + 2z)(16x^2 + 9y^2 + 4z^2 + 12xy + 6yz - 8zx)$ is $64x^3 - 27y^3 + 8z^3 + 72xyz$

(c) $(2a - 3b - 2c)(4a^2 + 9b^2 + 4c^2 + 6ab - 6bc + 4ca)$

we know that,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

so,

$(2a - 3b - 2c)(4a^2 + 9b^2 + 4c^2 + 6ab - 6bc + 4ca) = (2a)^3 + (-3b)^3 + (-2c)^3 - 3(2a)(-3b)(-2c)$

$= 8a^3 - 27b^3 - 8c^3 - 36abc$

Hence, the value of (c) $(2a - 3b - 2c)(4a^2 + 9b^2 + 4c^2 + 6ab - 6bc + 4ca)$ is $8a^3 - 27b^3 - 8c^3 - 36abc$

(d) $(3x - 4y + 5z)(9x^2 + 16y^2 + 25z^2 + 12xy - 15zx + 20yz)$

we know that,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

so,

$(3x - 4y + 5z)(9x^2 + 16y^2 + 25z^2 + 12xy - 15zx + 20yz) = (3x)^3 + (-4y)^3 + (5z)^3 - 3(3x)(-4y)(5z)$

$= 27x^3 - 64y^3 + 125z^3 + 180xyz$

Hence, the value of $(3x - 4y + 5z)(9x^2 + 16y^2 + 25z^2 + 12xy - 15zx + 20yz)$ is $27x^3 - 64y^3 + 125z^3 + 180xyz$

Q2. If $x + y + z = 8$ and $xy + yz + zx = 20$, Find the value of $x^3 + y^3 + z^3 - 3xyz$

Sol:
given, $x + y + z = 8$ and $xy + yz + zx = 20$

We know that,

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(20)$

$(x + y + z)^2 = x^2 + y^2 + z^2 + 40$

$8^2 = x^2 + y^2 + z^2 + 40$

$64 - 40 = x^2 + y^2 + z^2$

$x^2 + y^2 + z^2 = 24$

we know that,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)[(x^2 + y^2 + z^2) - (xy + yz + zx)]$

here, $x + y + z = 8$, $xy + yz + zx = 20$, $x^2 + y^2 + z^2 = 24$

$x^3 + y^3 + z^3 - 3xyz = 8[(24 - 20)]$

$= 8 * 4$

$= 32$

Hence, the value of $x^3 + y^3 + z^3 - 3xyz$ is 32

Q3. If $a + b + c = 9$ and $ab + bc + ca = 26$, Find the value of $a^3 + b^3 + c^3 - 3abc$

Sol :

Given, $a + b + c = 9$ and $ab + bc + ca = 26$

We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(26)$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 52$

$9^2 = a^2 + b^2 + c^2 + 52$

$81 - 52 = a^2 + b^2 + c^2$

$a^2 + b^2 + c^2 = 29$

we know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$a^3 + b^3 + c^3 - 3abc = (a + b + c)[(a^2 + b^2 + c^2) - (ab + bc + ca)]$

here, $a + b + c = 9$, $ab + bc + ca = 26$, $a^2 + b^2 + c^2 = 29$

$a^3 + b^3 + c^3 - 3abc = 9[(29 - 26)]$

$= 9 * 3$

= 27

Hence, the value of $a^3 + b^3 + c^3 - 3abc$ is 27

Q4. If $a + b + c = 9$ and $a^2 + b^2 + c^2 = 35$, Find the value of $a^3 + b^3 + c^3 - 3abc$

Sol:

Given, $a + b + c = 9$ and $a^2 + b^2 + c^2 = 35$

We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$9^2 = 35 + 2(ab + bc + ca)$

$81 = 35 + 2(ab + bc + ca)$

$81 - 35 = 2(ab + bc + ca)$

$\frac{46}{2} = ab + bc + ca$

$ab + bc + ca = 23$

we know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$a^3 + b^3 + c^3 - 3abc = (a + b + c)[(a^2 + b^2 + c^2) - (ab + bc + ca)]$

here, $a + b + c = 9$, $ab + bc + ca = 23$, $a^2 + b^2 + c^2 = 35$

$a^3 + b^3 + c^3 - 3abc = 9[(35 - 23)]$

$= 9 * 12$

$= 108$

Hence, the value of $a^3 + b^3 + c^3 - 3abc$ is 108

Q5. Evaluate:

(a) $25^3 - 75^3 + 50^3$

(b) $48^3 - 30^3 - 18^3$

(c) $(\frac{1}{2})^3 + (\frac{1}{3})^3 - (\frac{5}{6})^3$

(d) $(0.2)^3 - (0.3)^3 + (0.1)^3$

Sol:

Given,

(a) $25^3 - 75^3 + 50^3$

we know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

here, $a = 25$, $b = -75$, $c = 50$

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$a^3 + b^3 + c^3 = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$

$a^3 + b^3 + c^3 = (25 - 75 + 50)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$

$a^3 + b^3 + c^3 = (0)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$

$a^3 + b^3 + c^3 = 3abc$

$25^3 + (-75)^3 + 50^3 = 3abc$

$= 3(25)(-75)(50)$

$= -281250$

Hence, the value $25^3 + (-75)^3 + 50^3 = -281250$

(b) $48^3 - 30^3 - 18^3$

we know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

here, a = 48, b = -30, c = -18

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$a^3 + b^3 + c^3 = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$

$a^3 + b^3 + c^3 = (48 - 30 - 18)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$

$a^3 + b^3 + c^3 = (0)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$

$a^3 + b^3 + c^3 = 3abc$

$48^3 + (-30)^3 + (-18)^3 = 3abc$

$= 3(48)(-30)(-18)$

$= 77760$

Hence, the value $48^3 + (-30)^3 + (-18)^3 = 77760$

(c) $(\frac{1}{2})^3 + (\frac{1}{3})^3 - (\frac{5}{6})^3$

we know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

here, a= $\frac{1}{2}$, b= $\frac{1}{3}$, c= $\frac{-5}{6}$

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$a^3 + b^3 + c^3 = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$

$a^3 + b^3 + c^3 = (\frac{1}{2} + \frac{1}{3} - \frac{5}{6})( a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$

by using least common multiple

$a^3 + b^3 + c^3 = (\frac{1*6}{2*6} + \frac{1*4}{3*4} - \frac{5*2}{6*2})( a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$

$a^3 + b^3 + c^3 = (\frac{6}{12} + \frac{4}{12} - \frac{10}{12})( a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$

$a^3 + b^3 + c^3 = 0( a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$

$a^3 + b^3 + c^3 = 3abc$

$(\frac{1}{2})^3 + (\frac{1}{3})^3 - (\frac{-5}{6})^3 = 3 * \frac{1}{2} * \frac{1}{3} * \frac{-5}{6}$

$= \frac{1}{2} * \frac{-5}{6}$

$= \frac{-5}{12}$

Hence, the value of $(\frac{1}{2})^3 + (\frac{1}{3})^3 - (\frac{5}{6})^3$ is $\frac{-5}{12}$

(d) $(0.2)^3 - (0.3)^3 + (0.1)^3$

we know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

here, a = 0.2, b = 0.3, c = 0.1

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$a^3 + b^3 + c^3 = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$

$a^3 + b^3 + c^3 = (0.2 - 0.3 + 0.1)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$

$a^3 + b^3 + c^3 = (0)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$

$a^3 + b^3 + c^3 = 3abc$

$(0.2)^3 - (0.3)^3 + (0.1)^3 = 3abc$

$= 3(0.2)(-0.3)(0.1)$

$= -0.018$

Hence, the value $(0.2)^3 - (0.3)^3 + (0.1)^3$ is 0.018