

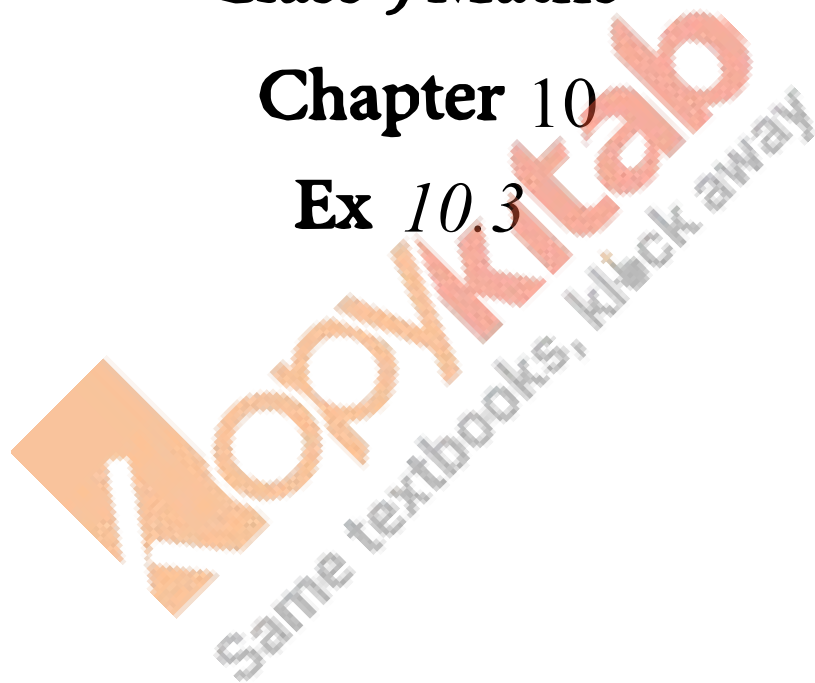
RD SHARMA

Solutions

Class 9 Maths

Chapter 10

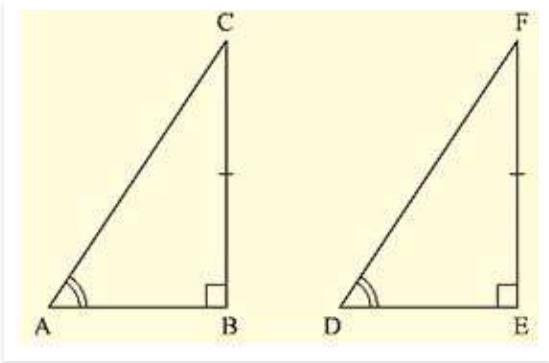
Ex 10.3



(1) In two right triangles one side and an acute angle of one are equal to the corresponding side and angle of the other. Prove that the triangles are congruent.

Solution:

Given that, in two right triangles one side and acute angle of one are equal to the corresponding side and angles of the other.



We have to prove that the triangles are congruent.

Let us consider two right triangles such that

$$\angle B = \angle E = 90^\circ \quad \dots\dots(i)$$

$$AB = DE \quad \dots\dots(ii)$$

$$\angle C = \angle F \quad \text{From(iii)}$$

Now observe the two triangles ABC and DEF

$$\angle C = \angle F \quad [\text{From (iii)}]$$

$$\angle B = \angle E \quad [\text{From (i)}]$$

$$\text{and } AB = DE \quad [\text{From (ii)}]$$

So, by AAS congruence criterion, we have $\triangle ABC \cong \triangle DEF$

Therefore, the two triangles are congruent

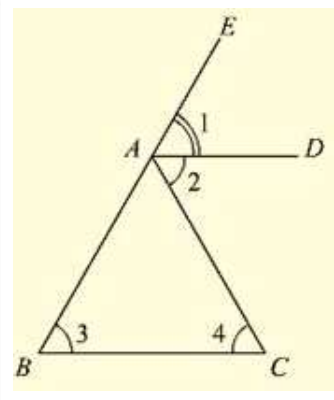
Hence proved

(2) If the bisector of the exterior vertical angle of a triangle be parallel to the base. Show that the triangle is isosceles.

Solution:

Given that the bisector of the exterior vertical angle of a triangle is parallel to the base and we have to prove that the triangle is isosceles. Let ABC be a triangle such that AD is the angular bisector of exterior vertical angle EAC and $AD \parallel BC$

$$\text{Let } \angle EAD = (i), \angle DAC = (ii), \angle ABC = (iii) \text{ and } \angle ACB = (iv)$$



We have,

$$(i) = (ii) \quad [\text{AD is a bisector of } \angle \text{EAC}]$$

$$(i) = (iii) \quad [\text{Corresponding angles}]$$

$$\text{and } (ii) = (iv) \quad [\text{alternative angle}]$$

$$(iii) = (iv)$$

$$AB = AC$$

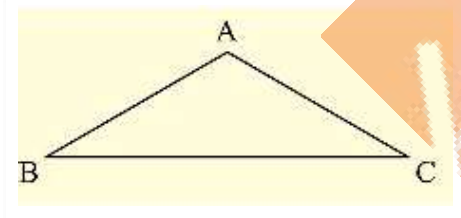
Since, in $\triangle ABC$, two sides AB and AC are equal we can say that $\triangle ABC$ is isosceles triangle.

(3) In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

Solution:

Let $\triangle ABC$ be isosceles such that $AB = AC$.

$$\angle B = \angle C$$



Given that vertex angle A is twice the sum of the base angles B and C. i.e., $\angle A = 2(\angle B + \angle C)$

$$\angle A = 2(\angle B + \angle B)$$

$$\angle A = 2(2 \angle B)$$

$$\angle A = 4(\angle B)$$

Now, We know that sum of angles in a triangle $= 180^\circ$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$4 \angle B + \angle B + \angle B = 180^\circ$$

$$6 \angle B = 180^\circ$$

$$\angle B = 30^\circ$$

Since, $\angle B = \angle C$

$$\angle B = \angle C = 30^\circ$$

And $\angle A = 4 \angle B$

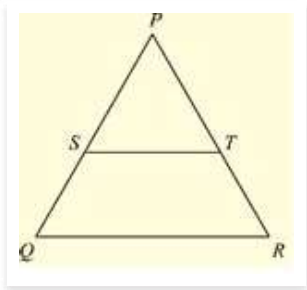
$$\angle A = 4 \times 30^\circ = 120^\circ$$

Therefore, angles of the given triangle are 120° , 30° and 30° .

= 428 and $LB = LC$]

(4) PQR is a triangle in which $PQ = PR$ and is any point on the side PQ. Through S, a line is drawn parallel to QR and intersecting PR at T. Prove that $PS = PT$.

Solution : Given that PQR is a triangle such that $PQ = PR$ and S is any point on the side PQ and $ST \parallel QR$.



To prove,

$$PS = PT$$

Since, $PQ = PR$

PQR is an isosceles triangle.

$$\angle Q = \angle R \text{ (or) } \angle PQR = \angle PRQ$$

Now, $\angle PST = \angle PQR$ and $\angle PTS = \angle PRQ$ [Corresponding angles as $ST \parallel QR$]

Since, $\angle PQR = \angle PRQ$

$$\angle PST = \angle PTS$$

Now, In $\triangle PST$, $\angle PST = \angle PTS$

$\triangle PST$ is an isosceles triangle

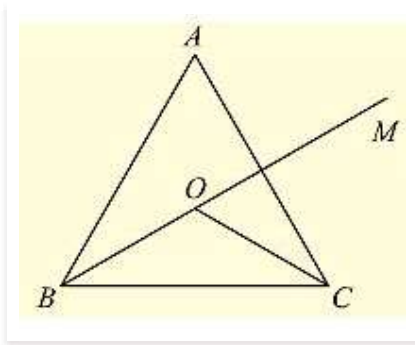
Therefore, $PS = PT$

(5) In a $\triangle ABC$, it is given that $AB = AC$ and the bisectors of B and C intersect at O. If M is a point on BO produced, prove that $\angle MOC = \angle ABC$.

Solution:

Given that in $\triangle ABC$,

$AB = AC$ and the bisector of $\angle B$ and $\angle C$ intersect at O. If M is a point on BO produced



We have to prove $\angle MOC = \angle ABC$

Since,

$AB = AC$

$\triangle ABC$ is isosceles

$\angle B = \angle C$ (or)

$\angle ABC = \angle ACB$

Now,

BO and CO are bisectors of $\angle ABC$ and $\angle ACB$ respectively

$$\Rightarrow \angle ABO = \angle OBC = \angle ACO = \angle OCB = \frac{1}{2} \angle ABC = \frac{1}{2} \angle ACB \quad \dots(i)$$

We have, in $\triangle OBC$

$$\angle OBC + \angle OCB + \angle BOC = 180^\circ \quad \dots(ii)$$

And also

$$\angle BOC + \angle COM = 180^\circ \quad \dots(iii) \text{ [Straight angle]}$$

Equating (ii) and (iii)

$$\angle OBC + \angle OCB + \angle BOC = \angle BOC + \angle MOC$$

$$\angle OBC + \angle OCB = \angle MOC \text{ [From (i)]}$$

$$2 \angle OBC = \angle MOC \text{ [From (i)]}$$

$$2 \left(\frac{1}{2} \angle ABC \right) = \angle MOC \text{ [From (i)]}$$

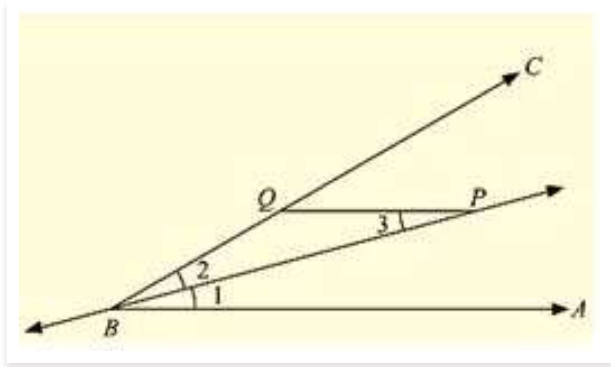
$$\angle ABC = \angle MOC$$

Therefore, $\angle MOC = \angle ABC$

(6) P is a point on the bisector of an angle ABC. If the line through P parallel to AB meets BC at Q, prove that triangle BPQ is isosceles.

Sol:

Given that P is a point on the bisector of an angle ABC, and $PQ \parallel AB$.



We have to prove that $\triangle BPQ$ is isosceles.

Since,

BP is bisector of $\angle ABC = \angle ABP = \angle PBC$ (i)

Now,

$PQ \parallel AB$

$\angle BPQ = \angle ABP$ (ii) [alternative angles]

From (i) and (ii), we get

$\angle BPQ = \angle PBC$ (or) $\angle BPQ = \angle PBQ$

Now,

In BPQ ,

$\angle BPQ = \angle PBQ$

$\triangle BPQ$ is an isosceles triangle.

Hence proved

(7) Prove that each angle of an equilateral triangle is 60° .

Sol:

Given to prove each angle of an equilateral triangle is 60° .

Let us consider an equilateral triangle ABC .

Such that $AB = BC = CA$

Now, $AB = BC$

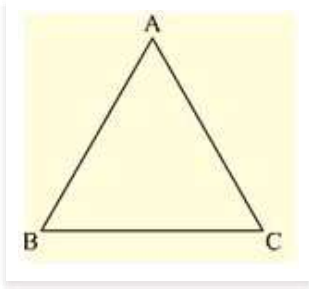
$\angle A = \angle C$ (i) [Opposite angles to equal sides are equal]

And $BC = AC$

$\angle B = \angle A$ (ii)

From (i) and (ii), we get

$\angle A = \angle B = \angle C$ (iii)



We know that

Sum of angles in a triangle = 180

$$\angle A + \angle B + \angle C = 180$$

$$\angle A + \angle A + \angle A = 180$$

$$3 \angle A = 180$$

$$\angle A = 60$$

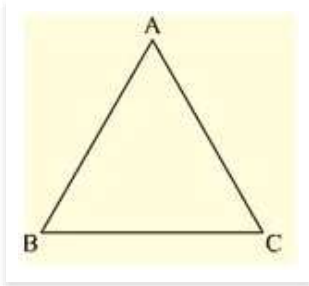
$$\angle A = \angle B = \angle C = 60$$

Hence, each angle of an equilateral triangle is 60° .

(8) Angles $\angle A, B, C$ of a triangle ABC are equal to each other. Prove that ABC is equilateral.

Sol:

Given that angles A, B, C of a triangle ABC equal to each other.



We have to prove that $\triangle ABC$ is equilateral

We have, $\angle A = \angle B = \angle C$

Now,

$$\angle A = \angle B$$

$$BC = AC \text{ [opposite sides to equal angles are equal]}$$

And $\angle B = \angle C$

$$AC = AB$$

From the above we get

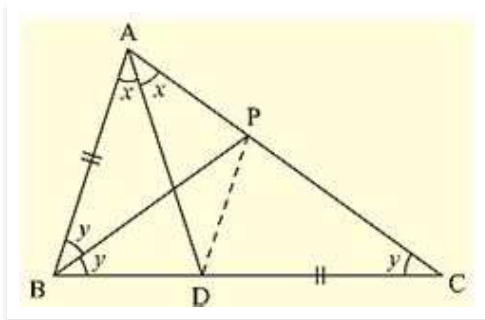
$$AB = BC = AC$$

$\triangle ABC$ is equilateral.

(9) ABC is a triangle in which $\angle B = 2 \angle C$. D is a point on BC such that AD bisects $\angle BAC$ and $AB = CD$. Prove that $\angle BAC = 72^\circ$.

Solution: Given that in ABC, $\angle B = 2 \angle C$ and D is a point on BC such that AD bisectors $\angle BAC$ and

$AB = CD$.



We have to prove that $\angle BAC = 72^\circ$

Now, draw the angular bisector of $\angle ABC$, which meets AC in P.

Join PD

Let $C = \angle ACB = y$

$\angle B = \angle ABC = 2 \angle C = 2y$ and also

Let $\angle BAD = \angle DAC$

$\angle BAC = 2x$ [AD is the bisector of $\angle BAC$]

Now, in $\triangle BPC$,

$\angle CBP = y$ [BP is the bisector of $\angle ABC$]

$\angle PCB = y$

$\angle CBP = \angle PCB = y$ [PC = BP]

Consider, $\triangle ABP$ and $\triangle DCP$, we have

$\triangle ABP = \triangle DCP = y$

$AB = DC$ [Given]

And $PC = BP$ [From above]

So, by SAS congruence criterion, we have $\triangle ABP \cong \triangle DCP$

Now,

$\angle BAP = \angle CDP$ and $AP = DP$ [Corresponding parts of congruent triangles are equal]

$\angle BAP = \angle CDP = 2$

Consider, $\triangle APD$,

We have $AP = DP$

$\angle ADP = \angle DAP$

But $\angle DAP = x$

$\angle ADP = \angle DAP = x$

Now

In $\triangle ABD$.

$\angle ABD + \angle BAD + \angle ADB = 180$

And also $\angle ADB + \angle ADC = 180^\circ$ [Straight angle]

From the above two equations, we get

$$\angle ABD + \angle BAD + \angle ADB = \angle ADB + \angle ADC$$

$$2y + x = \angle ADP + \angle PDC$$

$$2y + x = x + 2x$$

$$2y = 2x$$

$$y = x \text{ (or) } x = y$$

We know,

Sum of angles in a triangle = 180°

So, In $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$2x + 2y + y = 180^\circ [\angle A = 2x, \angle B = 2y, \angle C = y]$$

$$2(y) + 3y = 180^\circ [x = y]$$

$$5y = 180^\circ$$

$$y = 36^\circ$$

Now, $\angle A = \angle BAC = 2 \times 36^\circ = 72^\circ$

(10) ABC is a right angled triangle in which $\angle A = 90^\circ$ and $AB = AC$. Find $\angle B$ and $\angle C$.

Solution: Given that ABC is a right angled triangle such that $\angle A = 90^\circ$ and $AB = AC$

Since, $AB = AC$

$\triangle ABC$ is also isosceles.

Therefore, we can say that $\triangle ABC$ is right angled isosceles triangle.

$$\angle C = \angle B \text{ and } \angle A = 90^\circ \dots\dots(i)$$

Now, we have sum of angles in a triangle = 180°

$$\angle A + \angle B + \angle C = 180^\circ$$

$$90^\circ + \angle B + \angle B = 180^\circ \text{ [From(i)]}$$

$$2\angle B = 180^\circ - 90^\circ$$

$$\angle B = 45^\circ$$

Therefore, $\angle B = \angle C = 45^\circ$

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