1. Triangles ABC and DEF are similar.

(i) If area of (ΔABC ΔABC) = 16 cm^2 , area (ΔDEF ΔDEF) = 25 cm^2 and BC = 2.3 cm, find EF.

(ii) If area (ΔABC ΔABC) = 9 cm^2 , area (ΔDEF ΔDEF) = 64 cm^2 and DE = 5.1 cm, find AB.

(iii) If AC = 19 cm and DF = 8 cm, find the ratio of the area of two triangles.

(iv) If area of (ΔABC ΔABC) = 36 cm^2 , area (ΔDEF ΔDEF) = 64 cm^2 and DE = 6.2 cm, find AB.

(v) If AB = 1.2 cm and DE = 1.4 cm, find the ratio of the area of two triangles.

Answer:

(i) We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

arΔABCarΔDEF =(BCEF $)^2$ $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{BC}{EF})^2$ 1625 =(2.3EF $)^2$ $\frac{16}{25} = (\frac{2.3}{EF})^2$ 45 = 2.3EF $\frac{4}{5} = \frac{2.3}{EF}$

EF = 2.875 cm

(ii) arΔABCarΔDEF =(ABDE $)^2$ $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{AB}{DE})^2$

964 =(ABDE $)^2$ $\frac{9}{64} = (\frac{AB}{DE})^2$ 38 = AB5.1 $\frac{3}{8} = \frac{AB}{5.1}$

AB = 1.9125 cm

(iii) arΔABCarΔDEF =(ACDF $)^2$ $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{AC}{DF})^2$

arΔABCarΔDEF =(198 $)^2$ $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{19}{8})^2$ arΔABCarΔDEF =(36164) $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{361}{64})$

(iv) arΔABCarΔDEF =(ABDE $)^2$ $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{AB}{DE})^2$

3664 =(ABDE $)^2$ $\frac{36}{64} = (\frac{AB}{DE})^2$ 68 = AB6.2 $\frac{6}{8} = \frac{AB}{6.2}$

AB = 4.65 cm

(v) arΔABCarΔDEF =(ABDE $)^2$ $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{AB}{DE})^2$

arΔABCarΔDEF =(1.21.4 $)^2$ $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{1.2}{1.4})^2$ arΔABCarΔDEF =(3649) $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{36}{49})$

2. In the fig 4.178, ΔACB ΔACB is similar to ΔAPQ ΔAPQ . If BC = 10 cm, PQ = 5 cm, BA = 6.5 cm, AP = 2.8 cm, find CA and AQ. Also, find the Area of ΔACB ΔACB : Area of ΔAPQ ΔAPQ .

Answer:

Given: ΔACB ΔACB is similar to ΔAPQ ΔAPQ

BC = 10 cm

PQ = 5 cm

BA = 6.5 cm

AP = 2.8 cm

Find:

(1) CA and AQ

(2) Area of ΔACB ΔACB : Area of ΔAPQ ΔAPQ

(1) It is given that ΔACB ΔACB – ΔAPQ ΔAPQ

We know that for any two similar triangles the sides are proportional. Hence

ABAQ=BCPQ=ACAP $\frac{AB}{AQ} = \frac{BC}{PQ} = \frac{AC}{AP}$

ABAQ=BCPQ $\frac{AB}{AQ} = \frac{BC}{PQ}$ 6.5AQ=105 $\frac{6.5}{AQ} = \frac{10}{5}$

AQ = 3.25 cm

Similarly,

BCPQ=CAAP $\frac{BC}{PQ} = \frac{CA}{AP}$ CA2.8=105 $\frac{CA}{2.8} = \frac{10}{5}$

CA = 5.6 cm

(2) We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

arΔACQarΔAPQ=(BCPQ)^2 $\frac{ar\Delta ACQ}{ar\Delta APQ} = (\frac{BC}{PQ})^2$

= (105)^2 $(\frac{10}{5})^2$

$= (21)^2(\frac{2}{1})^2$

$= 41\frac{4}{1}$

3. The areas of two similar triangles are 81 cm² and 49 cm² respectively. Find the ration of their corresponding heights. What is the ratio of their corresponding medians?

Answer:

Given: The area of two similar triangles is $81cm^2$ and $49cm^2$respectively.

To find:

(1) The ratio of their corresponding heights.

(2) The ratio of their corresponding medians.

(1) We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

ar(triangle1)ar(triangle2)=(altitude1altitude2)^2 $\frac{\text{ar (triangle 1)}}{\text{ar (triangle 2)}} = (\frac{\text{altitude 1}}{\text{altitude 2}})^2$ 8149=(altitude1altitude2)^2

$\frac{81}{49} = (\frac{\text{altitude 1}}{\text{altitude 2}})^2$

Taking square root on both sides, we get

97=altitude1altitude2 $\frac{9}{7} = \frac{\text{altitude 1}}{\text{altitude 2}}$

Altitude 1: altitude 2 = 9: 7

(2) We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their medians.

ar(triangle1)ar(triangle2)=(median1median2)^2 $\frac{\text{ar (triangle 1)}}{\text{ar (triangle 2)}} = (\frac{\text{median 1}}{\text{median 2}})^2$ 8149=(median1median2)^2

$\frac{81}{49} = (\frac{\text{median 1}}{\text{median 2}})^2$

Taking square root on both sides, we get

97=median1median2 $\frac{9}{7} = \frac{\text{median 1}}{\text{median 2}}$

Median 1: median 2 = 9: 7

4. The areas of two similar triangles are 169 cm^2 and 121 cm^2 respectively. If the longest side of the larger triangle is 26 cm, find the longest side of the smaller triangle.

Answer:

Given:

The area of two similar triangles is 169cm2 and 121cm2 respectively. The longest side of the larger triangle is 26cm.

To find:

Longest side of the smaller triangle

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

ar(largertriangle)ar(smallertriangle)=(sideofthelargertrianglesideofthesmallertriangle)^2

$$\frac{\text{ar (larger triangle)}}{\text{ar (smaller triangle)}} = \left(\frac{\text{side of the larger triangle}}{\text{side of the smaller triangle}}\right)^2$$

169121=(sideofthelargertrianglesideofthesmallertriangle)^2

$$\frac{169}{121} = \left(\frac{\text{side of the larger triangle}}{\text{side of the smaller triangle}}\right)^2$$

Taking square root on both sides, we get

1311=sideofthelargertrianglesideofthesmallertriangle

$$\frac{13}{11} = \frac{\text{side of the larger triangle}}{\text{side of the smaller triangle}}$$

1311=26sideofthesmallertriangle

$$\frac{13}{11} = \frac{26}{\text{side of the smaller triangle}}$$

Side of the smaller triangle = 11×2613 $\frac{11 \times 26}{13}$ = 22 cm

Hence, the longest side of the smaller triangle is 22 cm.

5. The corresponding altitudes of two similar triangles are 6 cm and 9 cm respectively. Find the ratio of their areas.

Answer:

Given:

The area of two similar triangles is $25cm^2$ and $36cm^2$ respectively. If the altitude of first triangle 2.4cm.

To find:

The altitude of the other triangle

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

ar(triangle1)ar(triangle2) = (altitude1altitude2)^2 $\frac{\text{ar (triangle 1)}}{\text{ar (triangle 2)}} = (\frac{\text{altitude 1}}{\text{altitude 2}})^2$ 2536 = (2.4altitude2)^2

$\frac{25}{36} = (\frac{2.4}{\text{altitude 2}})^2$

Taking square root on both sides, we get

56 = 2.4altitude2 $\frac{5}{6} = \frac{2.4}{\text{altitude 2}}$

Altitude 2 = 2.88 cm

Hence, the corresponding altitude of the other is 2.88 cm.

6. ABC is a triangle in which ∠ A = 90°, AN ⊥ BC, BC = 12 cm and AC = 5 cm. Find the ratio of the areas of ΔANCΔANC and ΔABCΔABC .

Answer:

Given:

The corresponding altitudes of two similar triangles are 6 cm and 9 cm respectively.

To find:

Ratio of areas of triangle

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

ar(triangle1)ar(triangle2) = (altitude1altitude2)^2 $\frac{\text{ar (triangle 1)}}{\text{ar (triangle 2)}} = (\frac{\text{altitude 1}}{\text{altitude 2}})^2$ ar(triangle1)ar(triangle2) = (69)^2

$\frac{\text{ar (triangle 1)}}{\text{ar (triangle 2)}} = (\frac{6}{9})^2$ ar(triangle1)ar(triangle2) = 3681 $\frac{\text{ar (triangle 1)}}{\text{ar (triangle 2)}} = \frac{36}{81}$ ar(triangle1)ar(triangle2) = 49

$\frac{\text{ar (triangle 1)}}{\text{ar (triangle 2)}} = \frac{4}{9}$

ar (triangle 1): ar (triangle 2) = 4: 9

Hence, the ratio of the areas of two triangles is 4: 9.

7. ABC is a triangle in which ∠A°,AN⊥BC$\angle A°, AN \perp BC$, BC = 12 cm and AC = 5 cm. Find the ratio of the areas of ΔANCandΔABC$\Delta ANC \text{ and } \Delta ABC$.

Answer:

Given:

In ΔABC, $\angle A = 90^\circ$, AN $\perp$ BC, BC= 12 cm and AC = 5 cm.

To find:
Ratio of the triangles ΔANC and ΔABC.

In ΔANC and ΔABC,

$\angle ACN = \angle ACB$ (Common)

$\angle A = \angle ANC$ (90°)

Therefore, $\Delta ANC - \Delta ABC$ (AA similarity)

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

Therefore,

$$\frac{Ar(\Delta ANC)}{Ar(\Delta ABC)} = \left(\frac{AC}{BC}\right)^2$$

$$\frac{Ar(\Delta ANC)}{Ar(\Delta ABC)} = \left(\frac{5cm}{12cm}\right)^2$$

$$\frac{Ar(\Delta ANC)}{Ar(\Delta ABC)} = \frac{25}{144}$$

8. In Fig, DE || BC

(i) If DE = 4m, BC = 6 cm and Area $(\Delta ADE) = 16\,cm^2$, find the area of ΔABC.

(ii) If DE = 4cm, BC = 8 cm and Area $(\Delta ADE) = 25\,cm^2$, find the area of ΔABC.

(iii) If DE : BC = 3 : 5. Calculate the ratio of the areas of ΔADE and the trapezium BCED.

Answer:

In the given figure, we have DE $\parallel$ BC.

In ΔADE and ΔABC

$\angle ADE = \angle B$ (Corresponding angles)

∠DAE=∠BAC $\angle DAE = \angle BAC$ (Common)

So, ΔADEandΔABC $\Delta ADE \text{ and } \Delta ABC$ (AA Similarity)

(i) We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

Hence,

Ar(ΔADE)Ar(ΔABC)=DE²BC² $\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{DE^2}{BC^2}$ 16Ar(ΔABC)=4²6² $\frac{16}{Ar(\Delta ABC)} = \frac{4^2}{6^2}$ Ar(ΔABC)=6²×164²

$$Ar(\Delta ABC) = \frac{6^2 \times 16}{4^2}$$

Ar(ΔABC) $Ar(\Delta ABC)$ = 36 cm^2

(ii) We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

Hence,

Ar(ΔADE)Ar(ΔABC)=DE²BC² $\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{DE^2}{BC^2}$ 25Ar(ΔABC)=4²8² $\frac{25}{Ar(\Delta ABC)} = \frac{4^2}{8^2}$ Ar(ΔABC)=8²×254²

$$Ar(\Delta ABC) = \frac{8^2 \times 25}{4^2}$$

Ar(ΔABC) $Ar(\Delta ABC)$ = 100 cm^2

(iii) We know that

Ar(ΔADE)Ar(ΔABC)=DE²BC² $\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{DE^2}{BC^2}$ Ar(ΔADE)Ar(ΔABC)=3²5²

$\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{3^2}{5^2}$ Ar(ΔADE)Ar(ΔABC)=925 $\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{9}{25}$

Let the area of ΔADE ΔADE = 9x sq units

area of ΔABC ΔABC = 25x sq units

Now, Ar(ΔADE)Ar(trapBCED)=9x16x $\frac{Ar(\Delta ADE)}{Ar(trapBCED)} = \frac{9x}{16x}$

Ar(ΔADE)Ar(trapBCED)=916 $\frac{Ar(\Delta ADE)}{Ar(trapBCED)} = \frac{9}{16}$

9. In ΔABC ΔABC, D and E are the mid- points of AB and AC respectively. Find the ratio of the areas ΔADE ΔADE and ΔABC ΔABC.

Answer:

Given:

In $\Delta ABC \Delta ABC$, D and E are the midpoints of AB and AC respectively.

To find:

Ratio of the areas of $\Delta ADE and \Delta ABC \Delta ADE$ and ΔABC

It is given that D and E are the midpoints of AB and AC respectively.

Therefore, DE II BC (Converse of mid-point theorem)

Also, DE = $12 \frac{1}{2}$BC

In $\Delta ADE and \Delta ABC \Delta ADE$ and ΔABC

$\angle ADE=\angle B \angle ADE = \angle B$ (Corresponding angles)

$\angle DAE=\angle BAC \angle DAE = \angle BAC$ (common)

So, $\Delta ADE – \Delta ABC \Delta ADE - \Delta ABC$ (AA Similarity)

We know that the ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides.

$Ar(\Delta ADE)Ar(\Delta ABC) = AD^2AB^2 \frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{AD^2}{AB^2}$ $Ar(\Delta ADE)Ar(\Delta ABC) = 1^22^2$

$\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{1^2}{2^2}$ $Ar(\Delta ADE)Ar(\Delta ABC) = 14 \frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{1}{4}$

10. The areas of two similar triangles are 100 cm2 and 49 cm2 respectively. If the altitude of the bigger triangles is 5 cm, find the corresponding altitude of the other.

Answer:

Given: the area of the two similar triangles is $100cm^2$ $100cm^2$ and $49cm^2$ $49cm^2$ respectively. If the altitude of the bigger triangle is 5cm

To find: their corresponding altitude of the other triangle

We know that the ratio of the areas of the two similar triangles is equal to the ratio of squares of their corresponding altitudes.

ar(biggertriangle1)Ar(triangle2) $\frac{\text{ar(bigger triangle1)}}{\text{Ar(triangle2)}} = ($ altitudeofthebiggertriangle1altitude2 $)^2$

$(\frac{\text{altitude of the bigger triangle1}}{\text{altitude2}})^2$

$(10049)(\frac{100}{49})=(5altitude2)^2(\frac{5}{altitude2})^2$

Taking squares on both the sides

$(107(\frac{10}{7}=(5altitude2)(\frac{5}{altitude2})$

Altitude 2=3.5cm

11. The areas of two similar triangles are 121 cm2 and 64 cm2 respectively. If the median of the first triangle is 12.1 cm, find the corresponding median of the other.

Answer:

Given : the area of the two triangles is $121cm^2$ $121cm^2$ and $64cm^2$ $64cm^2$respectively. If the merdian of the first triangle is 12.1cm

To find the corresponding medians of the other triangle

We know that ratio of the areas of the two similar triangles are equal to the ratio of the squares of their merdians

$(ar(triangle1)ar(triangle2)(\frac{ar(triangle1)}{ar(triangle2)}=(median1median2)^2(\frac{median1}{median2})^2$

$(12164(\frac{121}{64}=(12.1median2)^2(\frac{12.1}{median2})^2$

Taking the squareroot on the both sides

$(118(\frac{11}{8}=(12.1median2)(\frac{12.1}{median2})$

Median2=8.8cm

12. If ΔABC~ΔDEF $\Delta ABC \sim \Delta DEF$ such that AB = 5cm, area (ΔABCΔABC) = 20 cm2 and area (ΔDEFΔDEF) = 45 cm2, determine DE.

Answer:

Given : the area of the two similar ΔABCΔABC =$20cm^2$$20cm^2$and ΔDEF$\Delta DEF$ =$45cm^2$ $45cm^2$ and AB=5cm

To measure of DE

We know that the ratio of the areas of the two similar triangles is equal to the ratio of the squares of their corresponding sides.

ArΔABCArΔDEF =(ABDE)^2 $\frac{Ar\Delta ABC}{Ar\Delta DEF} = \left(\frac{AB}{DE}\right)^2$

2045 $\frac{20}{45}$ = 5DE ^{2} $\left(\frac{5}{DE}\right)^2$

2045 $\frac{20}{45}$ = 5DE $\frac{5}{DE}$

DE^2 DE^2 = 25×4520 $\frac{25\times45}{20}$

DE^2 DE^2 = 2254 $\frac{225}{4}$

DE=7.5cm

13. In ΔABC ΔABC , PQ is a line segment intersecting AB at P and AC at Q such that PQ || BC and PQ divides ΔABC ΔABC into two parts equal in area. Find BPAB $\frac{BP}{AB}$.

Answer:

Given: in ΔABC ΔABC , PQ is a line segment intersecting AB at P and AC at such that PQ∥BC $PQ \parallel BC$ and PQ divides ΔABC ΔABC in two parts equal in area

To find : BPAB $\frac{BP}{AB}$

We have PQ∥BC $PQ \parallel BC$ and

Ar(ΔAPQ ΔAPQ) = Ar(quad BPQC)

Ar(ΔAPQ ΔAPQ) +Ar(ΔAPQ ΔAPQ) =Ar(quad BPQC) +Ar(ΔAPQ ΔAPQ)

2(Ar(ΔAPQ ΔAPQ) =Ar(ΔABC ΔABC)

Now PQ∥BC $PQ \parallel BC$ and BA is a transversal

In ΔABC ΔABC and ΔAPQ ΔAPQ)

∠APQ=∠B $\angle APQ = \angle B$ (corresponding angles)

∠PAQ=∠BAC $\angle PAQ = \angle BAC$ (common)

In ΔABC~ΔAPQ $\Delta ABC \sim \Delta APQ$) (AA similarity)

We know that the ratio of the areas of the two similar triangles is used and is equal to the ratio of their squares of the corresponding sides.

Hence

ArΔAPQArΔABC=(APAB)^2 $\frac{Ar\Delta APQ}{Ar\Delta ABC} = (\frac{AP}{AB})^2$ ArΔAPQ2ArΔABC=(APAB)^2 $\frac{Ar\Delta APQ}{2Ar\Delta ABC} = (\frac{AP}{AB})^2$ 12=

(AP AB)^2 $\frac{1}{2} = (\frac{AP}{AB})^2$ √12=(AP AB)√ $\frac{1}{2} = (\frac{AP}{AB})$

AB=√2AP $\sqrt{2AP}$

AB=√2(AB−BP $\sqrt{2(AB-BP}$

√2BP $\sqrt{2BP}$ =√2AB−AB $\sqrt{2AB-AB}$

BPAB $\frac{BP}{AB}$ = √2−1√2 $\frac{\sqrt{2}-1}{\sqrt{2}}$

14. The areas of two similar triangles ABC and PQR are in the ratio 9 : 16. If BC = 4.5 cm, find the length of QR.

Answer:

Given: the areas of the two similar triangles ABC and PQR are in the ratio 9:16. BC=4.5cm

To find: Length of QR

We know that the ratio of the areas of the two similar triangles is equal to the ratio of the squares of their corresponding sides.

ArΔABCArΔPQR=(BCQR)^2 $\frac{Ar\Delta ABC}{Ar\Delta PQR} = (\frac{BC}{QR})^2$

916 $\frac{9}{16}$ =(4.5QR)^2$(\frac{4.5}{QR})^2$

34 $\frac{3}{4}$=4.5QR $\frac{4.5}{QR}$

QR=183 $\frac{18}{3}$ = 6cm

15. ABC is a triangle and PQ is a straight line meeting AB and P and AC in Q. If AP = 1 cm, PB = 3 cm, AQ = 1.5 cm, QC = 4.5 m, prove that area of ΔAPQ ΔAPQ is one – sixteenth of the area of ΔABC ΔABC .

Answer:

Given : in ΔABC , PQ is a line segment intersecting AB at P and AC at Q. AP = 1cm , PB = 3cm, AQ= 1.5 cm and QC= 4.5cm

To find Ar(ΔAPQ)= $\frac{1}{16} \times \Delta ABC$)

In ΔABC

$\frac{AP}{PB} = \frac{AQ}{QC}$

$\frac{1}{3} = \frac{1}{3}$

According to converse of basic proportional theorem if a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Hence,

$PQ \parallel BC$

Hence in ΔABC and ΔAPQ

$\angle APQ = \angle B$ (corresponding angles)

$\angle PAQ = \angle BAC$ (common)

$\Delta ABC \sim \Delta APQ$ $\frac{Ar\Delta APQ}{Ar\Delta ABC} = (\frac{AP}{AB})^2$

$\frac{Ar\Delta APQ}{Ar\Delta ABC} = (\frac{AP}{(AB+BP)})^2$

$\frac{Ar\Delta APQ}{Ar\Delta ABC} = (\frac{1}{4})^2$ (given)

$\frac{Ar\Delta APQ}{Ar\Delta ABC} = (\frac{1}{16})$

16. If D is a point on the side AB of ΔABC such that AD : DB = 3 : 2 and E is a point on BC such that DE || AC. Find the ratio of areas of ΔABC and ΔBDE .

Answer:

Given In ΔABC , D is appoint on the side AB such that AD:DB=3:2. E is a point on side BC such that $DE \parallel AC$

To find

ΔABCΔBDE $\frac{\Delta ABC}{\Delta BDE}$

In ΔABCΔABC ,ΔBDEΔBDE ,

∠BDE$\angle BDE$ =∠A$\angle A$ (corresponding angles)

∠DBE$\angle DBE$ =∠ABC$\angle ABC$

ΔABC~ΔBDE$\Delta ABC \sim \Delta BDE$

We know that the ratio of the two similar triangles is equal to the ratio of the squares of their corresponding sides

Let AD=2x and BD =3x

Hence

ArΔABCArΔBDE=(ABBD)^2 $\frac{Ar\Delta ABC}{Ar\Delta BDE} = (\frac{AB}{BD})^2$ (AB+DABD)^2$(\frac{AB+DA}{BD})^2$ (3x+2x2x)^2 $(\frac{3x+2x}{2x})^2$ ArΔABCArΔBDE=(254)$\frac{Ar\Delta ABC}{Ar\Delta BDE} = (\frac{25}{4})$

17. If ΔABCΔABC and ΔBDEΔBDE are equilateral triangles, where D is the midpoint of BC, find the ratio of areas of ΔABCΔABC and ΔBDEΔBDE .

Answer:

Given In ΔABCΔABC , ΔBDEΔBDE are equilateral triangles. D is the point of BC.

To find ArΔABCArΔBDE $\frac{Ar\Delta ABC}{Ar\Delta BDE}$

In ΔABCΔABC , ΔBDEΔBDE

ΔABC~ΔBDE$\Delta ABC \sim \Delta BDE$ (AAA criteria of similarity all angles of the equilateral triangles are equal)

Since D is the mid point of BC, BD : DC=1

We know that the ratio of the areas of the two similar triangles is equal to the ratio of squares of their corresponding sides.

Let DC=x, and BD= x

Hence

ArΔABCArΔBDE=(BCBD)^2 $\frac{Ar\Delta ABC}{Ar\Delta BDE} = (\frac{BC}{BD})^2$

=(BD+DADC)^2$(\frac{BD+DA}{DC})^2$

$= \left(\frac{1x+1x}{1x}\right)^2$

$\frac{Ar\Delta ABC}{Ar\Delta BDE} = 4:1$

18.Two isosceles triangles have equal vertical angles and their areas are in the ratio 36 : 25. Find the ratio of their corresponding heights.

Answer:

Given:

Two isosceles triangles have equal vertical angles and their areas are in the ratio of 36: 25.

To find:

Ratio of their corresponding heights

Suppose ΔABC and ΔPQR are two isosceles triangles with $\angle A = \angle P$.

Therefore,

$\frac{AB}{AC} = \frac{PQ}{PR}$

In ΔABC and ΔPQR ,

$\angle A = \angle P$ $\frac{AB}{AC} = \frac{PQ}{PR}$

$\therefore \Delta ABC \sim \Delta PQR$ (SAS similarity)

Let AD and PS be the altitudes of ΔABC and ΔPQR , respectively.

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

$\frac{ar\ \Delta ABC}{ar\ \Delta PQR} = \left(\frac{AD}{PS}\right)^2$ $\frac{36}{25} = \left(\frac{AD}{PS}\right)^2$ $\frac{AD}{PS} = \frac{6}{5}$

Hence, the ratio of their corresponding heights is 6: 5.

19. In the given figure. ΔABC and ΔDBC are on the same base BC. If AD and BC intersect at O, Prove that

$\frac{\text{Area of } (\Delta ABC)}{\text{Area of } (\Delta DBC)} = \frac{AO}{DO}$

Answer:

Given ΔABC and ΔDBC are on the same BC. AD and BC intersect at O.

Prove that : $\frac{Ar\Delta ABC}{Ar\Delta DBC} = \frac{AO}{DO}$

$AL \perp BC and DM \perp BC$

Now, in ΔALO and ΔALO we have

$\angle ALO = \angle DMO = 90°$

$\angle AOL = \angle DOM$ (vertically opposite angles)

Therefore $\Delta ALO \sim \Delta DMO$

$\therefore \frac{AL}{DM} = \frac{AO}{DO}$

$\frac{Ar\Delta ABC}{Ar\Delta BCD} = \frac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times BC \times DM}$

$= \frac{AL}{DM}$

$= \frac{AO}{DO}$

20. ABCD is a trapezium in which AB || CD. The Diagonal AC and BC intersect at O. Prove that :

(i) $\Delta AOB\ \Delta COD$

(ii) If OA = 6 cm, OC = 8 cm,

Find:

(a) $\frac{\text{Area of } (\Delta AOB)}{\text{Area of } (\Delta COD)}$

(b) $\frac{\text{Area of } (\Delta AOD)}{\text{Area of } (\Delta COD)}$

Answer: Given ABCD is the trapezium which $AB \parallel CD$

The diagonals AC and BD intersect at o.

To prove:

(i) $\Delta AOB \sim \Delta COD$

(ii) If OA = 6 cm, OC = 8 cm

To find :

(a) ArΔAOBArΔCOD $\frac{Ar\Delta AOB}{Ar\Delta COD}$

(b) ArΔAODArΔCOD $\frac{Ar\Delta AOD}{Ar\Delta COD}$

Construction : Draw a line MN passing through O and parallel to AB and CD

Now in ΔAOBandΔCOD $\Delta AOB and \Delta COD$

(i) Now in ∠OAB $\angle OAB$ =∠OCD $\angle OCD$ (Alternate angles)

(ii) ∠OBA $\angle OBA$ =∠ODC $\angle ODC$ (Alternate angles)

∠AOB $\angle AOB$ =∠COD $\angle COD$ (vertically opposite angle)

ΔAOB~ΔCOD $\Delta AOB \sim \Delta COD$ (A.A.Acrieteria)

a) We know that the ratio of areas of two triangles is equal to the ratio of squares of their corresponding sides.

ArΔAOBArΔCOD =(AOCO)^2 $\frac{Ar\Delta AOB}{Ar\Delta COD} = (\frac{AO}{CO})^2$

=(68)^2 $(\frac{6}{8})^2$

ArΔAOBArΔCOD $\frac{Ar\Delta AOB}{Ar\Delta COD}$ =(68)^2 $(\frac{6}{8})^2$

b) We know that the ratio of two similar triangles is equal to the artio of their corresponding

sides.

ArΔAOBArΔCOD =(AOCO)^2 $\frac{Ar\Delta AOB}{Ar\Delta COD} = (\frac{AO}{CO})^2$

=(6cm8cm)^2 $(\frac{6cm}{8cm})^2$ =(68)^2 $(\frac{6}{8})^2$

21. In ΔABC ΔABC , P divides the side AB such that AP : PB = 1 : 2. Q is a point in AC such that PQ || BC. Find the ratio of the areas of ΔAPQ ΔAPQ and trapezium BPQC.

Answer: Given : In $\Delta ABC \Delta ABC$, P divides the side AB such that AP: PB =1:2, Q is a point on AC on such that PQ ∥∥ BC

To find : The ratio of the areas of $\Delta APQ \Delta APQ$ and the trapezium BPQC.

In $\Delta APQ \Delta APQ$ and $\Delta ABC \Delta ABC$

$\angle APQ = \angle B \angle APQ = \angle B$ (corresponding angles)

$\angle PAQ = \angle BAC \angle PAQ = \angle BAC$ (common)

So, $\Delta APQ \sim \Delta ABC \Delta APQ \sim \Delta ABC$ (AA Similarity)

We know that the ratio of areas of the twosimilar triangles is equal to the ratio of the squares of their corresponding sides.

$Ar\Delta APQ Ar\Delta ABC = (AP AB)^2 \frac{Ar\Delta APQ}{Ar\Delta ABC} = \left(\frac{AP}{AB}\right)^2$ $Ar\Delta APQ Ar\Delta ABC = 1x^2 (1x+2x)^2$
$\frac{Ar\Delta APQ}{Ar\Delta ABC} = \frac{1x^2}{(1x+2x)^2}$ $Ar\Delta APQ Ar\Delta ABC = 19 \frac{Ar\Delta APQ}{Ar\Delta ABC} = \frac{1}{9}$

Let Area of $\Delta APQ \Delta APQ$ =1 sq. units and Area of $\Delta ABC \Delta ABC$ =9x sq.units

Ar[trapBCED]=Ar($\Delta ABC \Delta ABC$) – Ar($\Delta APQ \Delta APQ$)

=9x-1x

=8x sq units

Now,

$Ar\Delta APQ Ar(trapBCED) \frac{Ar\Delta APQ}{Ar(trapBCED)} = xsqunits 8xsqunits \frac{xsqunits}{8xsqunits} = 18 \frac{1}{8}$