Mark the correct alternative in each of the following :
Question 1.
If sec θ + tan θ = x, then sec θ =

(a) $\frac{x^2+1}{x}$

(b) $\frac{x^2+1}{2x}$

(c) $\frac{x^2-1}{2x}$

(d) $\frac{x^2-1}{x}$

Solution:

$\sec\theta + \tan\theta = x$

We know that

$\Rightarrow \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow x(\sec\theta - \tan\theta) = 1$

$\sec\theta - \tan\theta = \frac{1}{x}$

Now $\sec\theta + \tan\theta = x$

Adding we get,

$2\sec\theta = \frac{1}{x} + x = \frac{1+x^2}{x}$

$\sec\theta = \frac{1+x^2}{2x}$ (b)

Question 2.

(a) $\frac{x^2+1}{x}$

(b) $\frac{x^2-1}{x}$

(c) $\frac{x^2+1}{2x}$

(d) $\frac{x^2-1}{2x}$

Solution:

$\sec\theta + \tan\theta = x$ ....(*i*)

We know that

$\sec^2\theta + \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow x(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{x}$ ....(*ii*)

Subtracting (*ii*) from (*i*)

$$2\tan\theta = x - \frac{1}{x} = \frac{x^2-1}{x}$$

$$\tan\theta = \frac{x^2-1}{2x}$$ (d)

Question 3.

$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ **is equal to**

(a) $\sec\theta + \tan\theta$
(b) $\sec\theta - \tan\theta$
(c) $\sec^2\theta + \tan^2\theta$
(d) $\sec^2\theta - \tan^2\theta$

Solution:

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} = \frac{1+\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$= \sec\theta + \tan\theta$ (a)

Question 4.

The value of $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$ is

(a) $\cot\theta - \text{cosec}\,\theta$ (b) $\text{cosec}\,\theta + \cot\theta$
(c) $\text{cosec}^2\theta + \cot^2\theta$ (d) $(\cot\theta + \text{cosec}\,\theta)^2$

Solution:

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} = \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}$$

$$= \frac{1+\cos\theta}{\sin\theta} = \text{cosec}\,\theta + \cot\theta \qquad \text{(b)}$$

Question 5.
$\sec^4 A - \sec^2 A$ is equal to
(a) $\tan^2 A - \tan^4 A$
(b) $\tan^4 A - \tan^2 A$
(c) $\tan^4 A + \tan^2 A$
(d) $\tan^2 A + \tan^4 A$
Solution:

$\sec^4 A - \sec^2 A = \sec^2 A (\sec^2 A - 1)$
$= (1 + \tan^2 A) \tan^2 A$

$$\begin{cases} \sec^2 A = 1 + \tan^2 A \\ \sec^2 A - 1 = \tan^2 A \end{cases}$$

$= \tan^2 A + \tan^4 A$
$= \tan^4 A + \tan^2 A$ (c)

Question 6.
$\cos^4 A - \sin^4 A$ is equal to
(a) $2 \cos^2 A + 1$
(b) $2 \cos^2 A - 1$
(c) $2 \sin^2 A - 1$
(d) $2 \sin^2 A + 1$
Solution:
$\cos^4 A - \sin^4 A = (\cos^2 A + \sin^2 A) (\cos^2 A - \sin^2 A)$
$= 1 (\cos^2 A - \sin^2 A) = \cos^2 A - (1 - \cos^2 A)$
$= \cos^2 A - 1 + \cos^2 A$
$= 2 \cos^2 A - 1$ **(b)**

Question 7.

$$\frac{\sin \theta}{1 + \cos \theta} \text{ is equal to}$$

(a) $\frac{1 + \cos \theta}{\sin \theta}$ (b) $\frac{1 - \cos \theta}{\cos \theta}$

(c) $\frac{1 - \cos \theta}{\sin \theta}$ (d) $\frac{1 - \sin \theta}{\cos \theta}$

Solution:

$$\frac{\sin \theta}{1 + \cos \theta} = \frac{\sin \theta (1 - \cos \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$$

$$= \frac{\sin \theta (1 - \cos \theta)}{1 - \cos^2 \theta} = \frac{\sin \theta (1 - \cos \theta)}{\sin^2 \theta}$$

$$= \frac{1 - \cos \theta}{\sin \theta} \quad \text{(c)}$$

Question 8.

$$\frac{\sin\theta}{1-\cot\theta}+\frac{\cos\theta}{1-\tan\theta} \text{ is equal to}$$

(a) 0 (b) 1

(c) $\sin\theta+\cos\theta$ (d) $\sin\theta-\cos\theta$

Solution:

$$\frac{\sin\theta}{1-\cot\theta}+\frac{\cos\theta}{1-\tan\theta}=\frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}$$

$$=\frac{\sin\theta\times\sin\theta}{\sin\theta-\cos\theta}+\frac{\cos\theta\times\cos\theta}{\cos\theta-\sin\theta}$$

$$=\frac{\sin^2\theta}{\sin\theta-\cos\theta}-\frac{\cos^2\theta}{\sin\theta-\cos\theta}$$

$$=\frac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}$$

$$=\frac{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}{\sin\theta-\cos\theta}$$

$=\sin\theta+\cos\theta$ (c)

Question 9.

The value of $(1+\cot\theta-\text{coscc}\,\theta)(1+\tan\theta+\sec\theta)$ is

(a) 1

(b) 2

(c) 4

(d) 0

Solution:

$(1 + \cot\theta - \text{cosec}\,\theta)(1 + \tan\theta + \sec\theta)$

$$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta}\right)$$

$$= \frac{(\sin\theta + \cos\theta - 1)(\cos\theta + \sin\theta + 1)}{\sin\theta \times \cos\theta}$$

$$= \frac{\{(\cos\theta + \sin\theta) - 1\}\{(\cos\theta + \sin\theta) + 1\}}{\sin\theta\cos\theta}$$

$$= \frac{(\cos\theta + \sin\theta)^2 - 1}{\sin\theta\cos\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta} = \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$= 2$ (b)

Question 10.

$\frac{\tan\theta}{\sec\theta - 1} + \frac{\tan\theta}{\sec\theta + 1}$ **is equal to**

(a) $2\tan\theta$ **(b)** $2\sec\theta$

(c) $2\,\text{cosec}\,\theta$ **(d)** $2\tan\theta\sec\theta$

Solution:

$$\frac{\tan\theta}{\sec\theta-1}+\frac{\tan\theta}{\sec\theta+1}$$

$$=\tan\theta\left(\frac{1}{\sec\theta-1}+\frac{1}{\sec\theta+1}\right)$$

$$=\frac{\tan\theta(\sec\theta+1+\sec\theta-1)}{(\sec\theta-1)(\sec\theta+1)}$$

$$=\frac{\tan\theta\times 2\sec\theta}{\sec^2\theta-1}=\frac{2\tan\theta\sec\theta}{\tan^2\theta}$$

$$=\frac{2\sec\theta}{\tan\theta}=\frac{2\times\cos\theta}{\cos\theta\times\sin\theta}=\frac{2}{\sin\theta}$$

$$=2\operatorname{cosec}\theta$$ (c)

Question 11.
(cosec θ – sin θ) (sec θ – cos θ) (tan θ + cot θ) is equal
(a) 0
(b) 1
(c) -1
(d) None of these
Solution:

$(\operatorname{cosec}\theta-\sin\theta)(\sec\theta-\cos\theta)(\tan\theta+\cot\theta)$

$$=\left(\frac{1}{\sin\theta}-\sin\theta\right)\left(\frac{1}{\cos\theta}-\cos\theta\right)\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)$$

$$=\frac{1-\sin^2\theta}{\sin\theta}\times\frac{1-\cos^2\theta}{\cos\theta}\times\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$$

$$=\frac{\cos^2\theta}{\sin\theta}\times\frac{\sin^2\theta}{\cos\theta}\times\frac{1}{\sin\theta\cos\theta}$$

$$=\frac{\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}=1$$ (b)

Question 12.
If x = a cos θ and y = b sin θ, then $b^2x^2 + a^2y^2$ =
(a) a^2b^2
(b) ab
(c) a^4b^4
(d) $a^2 + b^2$
Solution:
x = a cos θ, y = b sin θ …(i)
bx = ab cos θ, ay = ab sin θ ….(ii)
Adding (i) and (ii) we get,
$b^2x^2 + a^2y^2 = a^2b^2 \cos^2\theta + a^2b^2 \sin^2\theta$
$= a^2b^2 (\cos^2\theta + \sin^2\theta)$
$= a^2b^2 \times 1$
$= a^2b^2$ **(a)**

Question 13.
If x = a sec θ and y-b tan θ, then $b^2x^2 – a^2y^2$
(a) ab
(b) $a^2 – b^2$
(c) $a^2 + b^2$
(d) a^2b^2
Solution:
x = a sec θ and y = b tan θ
$b^2x^2 – a^2y^2 = b^2 (a \sec\theta)^2 – a^2 (b \tan\theta)^2$
$= a^2b^2 \sec^2\theta – a^2b^2 \tan^2\theta$
$= a^2b^2 (\sec^2\theta – \tan^2\theta)$
$= a^2b^2 \times 1$
$= a^2b^2$ **(d)**

Question 14.
$$\frac{\cot\theta}{\cot\theta - \cot 3\theta} + \frac{\tan\theta}{\tan\theta - \tan 3\theta}$$ is equal to
(a) 0 (b) 1
(c) –1 (d) 2

Solution:

$$\frac{\cot\theta}{\cot\theta-\cot 3\theta}+\frac{\tan\theta}{\tan\theta-\tan 3\theta}$$

$$\frac{\cot\theta\tan\theta-\cot\theta\tan 3\theta+\cot\theta\tan\theta-\tan\theta\cot 3\theta}{(\cot\theta-\cot 3\theta)(\tan\theta-\tan 3\theta)}$$

$\{\tan\theta\cot\theta=1\}$

$$\Rightarrow\frac{1-\cot\theta\tan\theta+1-\tan\theta\cot 3\theta}{\cot\theta\tan\theta-\cot\theta\tan 3\theta-\tan\theta\cot 3\theta+\cot 3\theta\tan 3\theta}$$

$$=\frac{2-\cot\theta\tan 3\theta-\tan\theta\cot 3\theta}{1-\cot\theta\tan 3\theta-\tan\theta\cot 3\theta+1}$$

$$=\frac{2-\cot\theta\tan 3\theta-\tan\theta\cot 3\theta}{2-\cot\theta\tan 3\theta-\tan\theta\cot 3\theta}=1\quad\text{(b)}$$

Question 15.
$2(\sin^6\theta+\cos^6\theta)-3(\sin^4\theta+\cos^4\theta)$ is equal to
(a) 0
(b) 1
(c) -1
(d) None of these
Solution:

$2(\sin^6\theta+\cos^6\theta)-3(\sin^4\theta+\cos^4\theta)$

$=2[(\sin^2\theta)^3+(\cos^2\theta)^3]-3[(\sin^2\theta)^2+(\cos^2\theta)^2]$

$=2[(\sin^2\theta+\cos^2\theta)(\sin^4\theta+\cos^4\theta-\sin^2\theta\cos^2\theta)]-3[(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\cos^2\theta]$

$\{\because a^3+b^3=(a+b)^3-3ab(a+b)\}$

$=2[1(\sin^2\theta)^2+(\cos^2\theta)^2+2\sin^2\theta\cos^2\theta-3\sin^2\theta+\cos^2\theta]-3[(1)^2-2\sin^2\theta\cos^2\theta]$

$=2[(\sin^2\theta+\cos^2\theta)^2-3\sin^2\theta\cos^2\theta]-3[1-2\sin^2\theta\cos^2\theta]$

$=2[1-3\sin^2\theta\cos^2\theta)-3(1-2\sin^2\theta\cos^2\theta]$

$=2-6\sin^2\theta\cos^2\theta-3+6\sin^2\theta\cos^2\theta$

$=-1$ (c)

Question 16.
If $a\cos\theta+b\sin\theta=4$ and $a\sin\theta-b\cos\theta=3$, then $a^2+b^2=$

(a) 7
(b) 12
(c) 25
(d) None of these
Solution:
$a \cos \theta + b \sin \theta = 4$
$a \sin \theta – b \cos \theta = 3$
Squaring and adding
$a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \sin \theta \cos \theta=16$
$a^2 \sin^2 \theta + b^2 \cos^2 \theta – 2ab \sin \theta \cos \theta = 9$
$a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta) = 25$ ($\because \sin^2 \theta + \cos^2 \theta=1$)
$\Rightarrow a^2 \text{ x } 1 + b^2 \text{ x } 1 = 25$
$\Rightarrow a^2 + b^2 = 25$ **(c)**

Question 17.
If $a \cot \theta + b \operatorname{cosec} \theta = p$ and $b \cot \theta + a \operatorname{cosec} \theta = q$, then $p^2 – q^2 =$
(a) $a^2 – b^2$
(b) $b^2 – a^2$
(c) $a^2 + b^2$
(d) $b – a$
Solution:
$a \cot \theta + b \operatorname{cosec} \theta = p$
$b \cot \theta + a \operatorname{cosec} \theta = q$
Squaring and subtracting,
$p^2 – q^2 = (a \cot \theta + b \operatorname{cosec} \theta)^2 – (b \cot \theta + a \operatorname{cosec} \theta)^2$
$= a^2 \cot^2 \theta + b^2 \operatorname{cosec}^2 \theta + 2ab \cot \theta \operatorname{cosec} \theta – (b^2 \cot^2 \theta + a^2 \operatorname{cosec}^2 \theta + 2ab \cot \theta \operatorname{cosec} \theta)$
$= a^2 \cot^2 \theta + b^2 \operatorname{cosec}^2 \theta + lab \cot \theta \operatorname{cosec} \theta – b^2 \cot^2 \theta – a^2 \operatorname{cosec}^2 \theta – lab \cot \theta \operatorname{cosec} \theta$
$= a^2 (\cot^2 \theta – \operatorname{cosec}^2 \theta) + b^2 (\operatorname{cosec}^2 \theta – \cot^2 \theta)$
$= -a^2 (\operatorname{cosec}^2 \theta – \cot^2 \theta) + b^2 (\operatorname{cosec}^2 \theta – \cot^2 \theta)$
$= -a^2 \text{ x } 1 + b^2 \text{ x } 1 = b^2 – a^2$ **(b)**

Question 18.
The value of $\sin^2 29° + \sin^2 61°$ is
(a) 1
(b) 0
(c) $2\sin^2 29$“
(d) $2\cos^2 61°$
Solution:
$\sin^2 29° + \sin^2 61° = \sin^2 29° + \sin^2 (99° – 29°)$
$= \sin^2 29 + \cos^2 29°$ **(a)**
$(\sin^2 \theta + \cos^2 \theta=1)$

Question 19.
If $x = r \sin \theta \cos \varphi$, $y = r \sin \theta \sin \varphi$ and $z – r \cos \theta$, then
(a) $x^2 +y^2 + z^2 = r^2$
(b) $x^2 +y^2 – z^2 = r^2$
(c) $x^2– y^2+ z^2 = r^2$
(d) $z^2 + y^2 – x^2 = r^2$

Solution:

$$x = r \sin\theta \cos\phi \Rightarrow \frac{x}{r} = \sin\theta\cos\phi \quad ....(i)$$

$$y = r \sin\theta \sin\phi \Rightarrow \frac{y}{r} = \sin\theta\sin\phi \quad ....(ii)$$

$$z = r \cos\theta \Rightarrow \frac{z}{r} = \cos\theta \quad ....(iii)$$

Squaring and adding (*i*) and (*ii*)

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \sin^2\theta\cos^2\phi + \sin^2\sin^2\phi$$

$= \sin^2\theta(\cos^2\phi + \sin^2\phi)$

$= \sin^2\theta \times 1 \quad \{\sin^2\theta + \cos^2\theta = 1\}$

$= \sin^2\theta$

Now adding (*iii*) in it

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} + \frac{z^2}{r^2} = \sin^2\theta + \cos^2\theta = 1$$

Hence $\frac{x^2}{r^2} + \frac{y^2}{r^2} + \frac{z^2}{r^2} = 1$

$$\Rightarrow \frac{x^2 + y^2 + z^2}{r^2} = 1 \Rightarrow x^2 + y^2 + z^2 = r^2 \quad \text{(a)}$$

Question 20.
If $\sin\theta + \sin^2\theta = 1$, then $\cos^2\theta + \cos^4\theta$
(a) -1
(b) 1
(c) 0
(d) None of these
Solution:
$\sin\theta + \sin^2\theta = 1$
$\Rightarrow \sin\theta = 1 - \sin^2\theta$
$\Rightarrow \sin\theta = \cos^2\theta$
$\cos^2\theta + \cos^4\theta = \sin\theta + \sin^2\theta$ $\{\because \cos^2\theta = \sin\theta\}$
$\Rightarrow \cos^2\theta + \cos^4\theta = 1$ **(b)**
$\{\because \sin\theta + \sin^2\theta = 1$ (given)$\}$

Question 21.
If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = it$, then $a^2 + b^2 =$
(a) $m^2 - it^2$

(b) m^2n^2
(c) $n^2 – m^2$
(d) $m^2 + n^2$
Solution:
a cos θ + b sin θ = m
a sin θ – b cos θ = n
Squaring and adding
$a^2 \cos^2 \theta + b^2 \sin^2 \theta + lab \sin \theta \cos \theta = m^2$
$a^2 \sin^2 \theta + b^2 \cos^2 \theta – 2ab \sin \theta \cos \theta = n^2$
$a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta) = m^2 + n^2$ $\{\sin^2 \theta + \cos^2 \theta=1\}$
$\Rightarrow a^2 + 1 + b^2 x 1 = m^2 – n^2$
$\Rightarrow a^2 + b^2 = m^2 + n^2$
Hence $a^2 + b^2 = m^2 + n^2$ **(d)**

Question 22.
If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A =$
(a) -1
(b) 0
(c) 1
(d) None of these
Solution:
$\cos A + \cos^2 A = 1$
$\Rightarrow \cos A = 1 – \cos^2 A$
$\Rightarrow \cos A = \sin^2 A$
Now, $\sin^2 A + \sin^4 A = \cos A + \cos^2 A = 1$ ($\because \cos A + \cos^2 A = 1$) (given)
$\therefore \sin^2 A + \sin^4 A = 1$ **(c)**

Question 23.
If $x = a \sec \theta \cos \phi$, $y = b \sec \theta \sin \phi$ and $z = c \tan \theta$, then $\frac{x^2}{a^2} + \frac{y^2}{b^2} =$

(a) $\frac{z^2}{c^2}$
(b) $1 - \frac{z^2}{c^2}$
(c) $\frac{z^2}{c^2} - 1$
(d) $1 + \frac{z^2}{c^2}$

Solution:

$x = a \sec \theta \cos \phi$

$y = b \sec \theta \sin \phi$

$z = c \tan \theta$

$$\frac{x}{a} = \sec \theta \cos \phi \quad ....(i)$$

$$\frac{y}{b} = \sec \theta \sin \phi \quad ....(ii)$$

$$\frac{z}{c} = \tan \theta \quad ....(iii)$$

Squaring and adding (*i*) and (*ii*)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \sec^2 \theta \cos^2 \phi + \sec^2 \theta \sin^2 \phi$$

$= \sec^2 \theta (\cos^2 \phi + \sin^2 \phi)$

$= \sec^2 \theta \times 1 = \sec^2 \theta$

Squaring (*iii*) and subtracting from (*iv*)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = \sec^2 \theta - \tan^2 \theta = 1$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 + \frac{z^2}{c^2} \quad \text{(d)}$$

Question 24.

If $a \cos \theta - b \sin \theta = c$, then $a \sin \theta + b \cos \theta =$

(a) $\pm\sqrt{a^2 + b^2 + c^2}$ **(b)** $\pm\sqrt{a^2 + b^2 - c^2}$

(c) $\pm\sqrt{c^2 - a^2 - b^2}$ **(d) None of these**

Solution:

$a \cos\theta - b \sin\theta = c$

Squaring,

$a^2 \cos^2\theta + b^2 \sin\theta - 2ab \sin\theta \cos\theta = c^2$

$\Rightarrow a^2 (1 - \sin^2\theta) + b^2 (1 - \cos^2\theta) - 2ab \sin\theta \cos\theta = c^2$

$\Rightarrow a^2 - a^2 \sin^2\theta + b^2 - b^2 \cos^2\theta - 2ab \sin\theta \cos\theta = c^2$

$\Rightarrow -a^2 \sin^2\theta - b^2 \cos^2\theta - 2ab \sin\theta \cos\theta = c^2 - a^2 - b^2$

$\Rightarrow a^2 \sin^2\theta + b^2 \cos^2\theta + 2ab \sin\theta \cos\theta = a^2 + b^2 - c^2$ (Dividing by –1)

$(a \sin\theta + b \cos\theta)^2 = a^2 + b^2 - c^2$

$\therefore a \sin\theta + b \cos\theta = \pm\sqrt{a^2 + b^2 - c^2}$ (b)

Question 25.
$9\sec^2 A - 9\tan^2 A$ is equal to
(a) 1
(b) 9
(c) 8
(d) 0
Solution:
$9\sec^2 A - 9\tan^2 A = 9 (\sec^2 A - \tan^2 A)$
$= 9 \times 1$ ($\because \sec^2 A - \tan^2 A = 1$)
$= 9$ **(b)**

Question 26.
$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) =$
(a) 0
(b) 1
(c) 1
(d) -1

Solution:

(1 + tan θ + sec θ) (1 + cot θ – cosec θ)
= 1 + cot θ – cosec θ + tan θ + cot θ tan θ
– tan θ cosec θ + sec θ + sec θ cot θ – sec
θ cosec θ

$$= 1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta} + \frac{\sin\theta}{\cos\theta} + 1 - \frac{\sin\theta}{\cos\theta}$$

$$\times \frac{1}{\sin\theta} + \frac{1}{\cos\theta} + \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} -$$

$$\frac{1}{\cos\theta} \times \frac{1}{\sin\theta}$$

$$= 2 + \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} - \frac{1}{\sin\theta} - \frac{1}{\cos\theta} +$$

$$\frac{1}{\cos\theta} + \frac{1}{\sin\theta} - \frac{1}{\sin\theta\cos\theta}$$

$$= 2 + \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta\cos\theta}$$

$$= 2 + \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} - \frac{1}{\sin\theta\cos\theta}$$

$$= 2 + \frac{1}{\sin\theta\cos\theta} - \frac{1}{\sin\theta\cos\theta} = 2 \quad \text{(c)}$$

Question 27.
(sec A + tan A) (1 – sin A) =
(a) sec A
(b) sin A
(c) cosec A
(d) cos A

Solution:

$$(\sec A + \tan A)(1 - \sin A)$$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{1+\sin A}{\cos A} \times (1 - \sin A) = \frac{(1+\sin A)(1-\sin A)}{\cos A}$$

$$= \frac{1-\sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A . \qquad \text{(d)}$$

Question 28.

$\dfrac{1+\tan^2 A}{1+\cot^2 A}$ **is equal to**

(a) $\sec^2 A$ **(b)** -1
(c) $\cot^2 A$ **(d)** $\tan^2 A$

$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{\sin^2 A}{\cos^2 A}$$

$= \tan^2 A$ (d)

Question 29.
If $\sin\theta \cos\theta = 0$, then the value of $\sin^4\theta + \cos^4\theta$ is

(a) 1 **(b)** $\frac{3}{4}$

(c) $\frac{1}{2}$ **(d)** $\frac{1}{4}$

Solution:

$\sin\theta - \cos\theta = 0$

$\Rightarrow \sin\theta = \cos\theta$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \theta = 45°$

Now, put the value of θ in the given equation

$\sin^4\theta + \cos^4\theta$

$= \sin^4 45° + \cos^4 45°$

$= \left(\dfrac{1}{\sqrt{2}}\right)^4 + \left(\dfrac{1}{\sqrt{2}}\right)^4$

$= \dfrac{1}{4} + \dfrac{1}{4} = \dfrac{2}{4}$

$= \dfrac{1}{2}$ (c)

Question 30.
The value of sin (45° + θ) – cos (45° – θ) is equal to
(a) 2 cos θ
(b) 0
(c) 2 sin θ
(d) 1
Solution:
sin (45° + θ) – cos (45° – θ)
= sin (45° + θ) – sin (90° – 45° + θ)
= sin (45° + θ) – sift (45° + θ)
= 0 **(b)**

Question 31.
If ΔABC is right-angled at C, then the value of cos (A + B) is

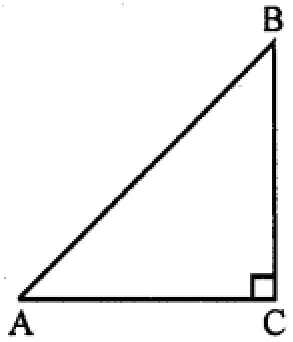
(a) 0 **(b)** 1

(c) $\dfrac{1}{2}$ **(d)** $\dfrac{\sqrt{3}}{2}$

Solution:

$\angle A + \angle B + \angle C = 180°$

$\angle A + \angle B + 90° = 180°$



$\Rightarrow \angle A + \angle B = 90°$

$\therefore \cos(A + B) = \cos 90° = 0$ **(a)**

Question 32.

If cos 9θ = sin θ and 9θ < 90°, then value of tan 6θ is

(a) $\frac{1}{\sqrt{3}}$ **(b) $\sqrt{3}$**

(c) 1 **(d) 0**

Solution:

cos (9θ) = sin θ

⇒ sin (90° – 9θ) = sin θ

⇒ 90° – 90 = θ

⇒ 9θ = 90°

⇒ θ= 10

tan 6θ = tan 6

= tan 60° = 3–√ **(b)**

Question 33.

If cos (α + β) =0, then sin (α – β) can be reduced to

(a) cos β

(b) cos 2β

(c) sin α

(d) sin 2α

Solution:

cos (α + β) = 0

⇒ α + β = 90° [∵ cos 90° = 0]

⇒ θ = 90° – β …(i)

sin (α – β) = sin (90° – β – β) [using (i)]

= sin (90° – 2β)

= cos 2β [∵ sin (90° – θ) = cos θ] **(b)**