Answer each of the following questions either in one word or one sentence or as per the requirement of the question :

Question 1.

Write the value of k for which the quadratic equation $x^2 - kx + 4 = 0$ has equal roots.

Solution:

$x^2 - kx + 4 = 0$

Here a = 1, b = – k, c = 4

Discriminant (D) = $b^2 - 4ac$

= $(-k)^2 - 4 \times 1 \times 4 = k^2 - 16$

The roots are equal

D = 0 => $k^2 - 16 = 0$

=> $(k + 4)(k - 4) = 0$.

Either k + 4 = 0, then k = – 4

or k – 4 = 0, then k = 4

k = 4, -4

Question 2.

What is the nature of roots of the quadratic equation $4x^2 - 12x - 9 = 0$?

Solution:

$4x^2 - 12x - 9 = 0$

Here a = 4, b = -12, c = – 9

Discriminant (D) = $b^2 - 4ac = (-12)^2 - 4 \times 4 \times (-9)$

= 144 + 144 = 288

D > 0

Roots are real and distinct

Question 3.

If $1 + \sqrt{2}$ is a root of a quadratic equation with rational coefficients, write its other root.

Solution:

The roots of the quadratic equation with rational coefficients are conjugate

The other root will be $1 - \sqrt{2}$

Question 4.

Write the number of real roots of the equation $x^2 + 3|x| + 2 = 0$.

Solution:

$x^2 + 3|x| + 2 + 0 \Rightarrow x^2 + 3x + 2 = 0$

Here $a = 1, b = 3, c = 2$ $\quad (\because |x| = x)$

$D = b^2 - 4ac = (3)^2 - 4 \times 1 \times 2$

$= 9 - 8 = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{1}}{2 \times 1} = \frac{-3 \pm 1}{2}$$

$$x_1 = \frac{-3+1}{2} = \frac{-2}{2} = -1$$

and $x_2 = \frac{-3-2}{2} = \frac{-4}{2} = -2$

$\therefore$ Real roots are $-1, -2$

Question 5.

Write the sum of the real roots of the equation $x^2 + |x| - 6 = 0$.

Solution:

$x^2 + |x| - 6 = 0 \Rightarrow x^2 + x - 6 = 0 \quad (\because |x| = x)$

Here $a = 1, b = 1, c = -6$

$\therefore$ Sum of roots $= \frac{-b}{a} = \frac{-1}{1} = -1$

Question 6.

Write the set of values of 'a' for which the equation $x^2 + ax - 1 = 0$, has real roots.

Solution:

$x^2 + ax - 1 = 0$

Here $a = 1, b = a, c = -1$

$D = b^2 - 4ac = (a)^2 - 4 \times 1 \times (-1) = a^2 + 4$

Roots are real

$D \geq 0 \Rightarrow a^2 + 4 \geq 0$

For all real values of a the equation has real roots.

Question 7.

In there any real value of 'a' for which the equation $x^2 + 2x + (a^2 + 1) = 0$ has real roots ?

Solution:

$x^2 + 2x + (a^2 + 1) = 0$

$D = (-b)^2 - 4ac = (2)^2 - 4 \times 1 (a^2 + 1) = 4 - 4a^2 - 4 = -4a^2$

For real value of x, $D \geq 0$

But $-4a^2 \leq 0$

So it is not possible

There is no real value of a

Question 8.

Write the value of λ, for which $x^2 + 4x + \lambda$ is a perfect square.

Solution:
In $x^2 + 4x + \lambda$
$a = 1, b = 4, c = \lambda$
$x^2 + 4x + \lambda$ will be a perfect square if $x^2 + 4x + \lambda = 0$ has equal roots
$D = b^2 - 4ac = (4)^2 - 4 \times 1 \times \lambda = 16 - 4\lambda$
$D = 0$
$\Rightarrow 16 - 4\lambda = 0$
$\Rightarrow 16 = 4A$
$\Rightarrow \lambda = 4$
Hence $\lambda = 4$

Question 9.
Write the condition to be satisfied for which equations $ax^2 + 2bx + c = 0$ and $bx^2 - 2\sqrt{ac}\, x + b = 0$ have equal roots.
Solution:
In $ax^2 + 2bx + c = 0$

$D_1 = (-2b)^2 - 4 \times a \times c$
$= 4b^2 - 4ac$
$\because$ Roots are equal
$\therefore D_1 = 0$
$\therefore 4b^2 - 4ac = 0 \Rightarrow 4b^2 = 4ac$
$\Rightarrow b^2 = ac$

and in $bx^2 - 2\sqrt{ac}\, x + b = 0$

$D_2 = \left(-2\sqrt{ac}\right)^2 - 4 \times b \times b$
$= 4ac - 4b^2$
$\because$ Roots are equal
$\therefore 4ac = 4b^2 \Rightarrow b^2 = ac$
$\therefore$ The required condition is $b^2 = ac$

Question 10.
Write the set of values of k for which the quadratic equation has $2x^2 + kx - 8 = 0$ has real roots.
Solution:
In $2x^2 + kx - 8 = 0$
$D = b^2 - 4ac = (k)^2 - 4 \times 2 \times (-8) = k^2 + 64$
The roots are real
$D \geq 0$
$k^2 + 64 \geq 0$
For all real values of k, the equation has real roots.

Question 11.
Write a quadratic polynomial, a sum of whose zeros is $2\sqrt{3}$ and their product is 2.
Solution:
Sum of zeros = $2\sqrt{3}$

and product of zeros = 2
The required polynomial will be

$$k\left(x^2-\frac{b}{a}x+\frac{c}{a}\right)$$ where k is any real number

Whose $\frac{b}{a}=2\sqrt{3}$ and $\frac{c}{a}=2$

$\therefore$ $k\,(x^2-2\sqrt{3}\,x+2)$ whose k is any real number

Question 12.
Show that x = – 3 is a solution of $x^2 + 6x + 9 = 0$ **(C.B.S.E. 2008)**
Solution:
The given equation is $x^2 + 6x + 9 = 0$
If x = -3 is its solution then it will satisfy it
L.H.S. = $(-3)^2 + 6(-3) + 9 = 9 - 18 + 9 = 18 - 18 = 0$ = R.H.S.
Hence x = – 3 is its one root (solution)

Question 13.
Show that x = – 2 is a solution of $3x^2 + 13x + 14 = 0$. **(C.B.S.E. 2008)**
Solution:
The given equation is $3x^2 + 13x + 14 = 0$
If x = – 2 is its solution, then it will satisfy it
L.H.S. = $3(-2)^2 + 13(-2) + 14 = 3 \times 4 - 26 + 14$
$= 12 - 26 + 14 = 26 - 26 = 0$ = R.H.S.
Hence x = – 2 is its solution

Question 14.
Find the discriminant of the quadratic equation $3\sqrt{3}\,x^2 + 10x + \sqrt{3} = 0$. **(C.B.S.E. 2009)**
Solution:

$$3\sqrt{3}\,x^2 + 10x + \sqrt{3} = 0$$

Here $a = 3\sqrt{3}$, $b = 10$, $c = \sqrt{3}$

$\therefore$ Discriminant $D = b^2 - 4ac$

$= (10)^2 - 4 \times 3\sqrt{3} \times \sqrt{3}$

$= 100 - 12 \times 3 = 100 - 36 = 64$

Question 15.
If x = −12, is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of k. **[CBSE 2015]**

Solution:

$x = \frac{-1}{2}$ is the solution of $3x^2 + 2kx - 3 = 0$

Substituting the value of x in the given equation

$$3\left(\frac{-1}{2}\right)^2 - 2k\left(\frac{-1}{2}\right) - 3 = 0$$

$$3 \times \frac{1}{4} + k - 3 = 0 \Rightarrow k = 3 - \frac{3}{4} = \frac{9}{4}$$

Hence $k = \frac{9}{4}$