Q1) In a ΔABC, if $\angle A = 55^0$, $\angle B = 40^0$, Find $\angle C$.

Solution:

Given Data:

$\angle B = 55^0$, $\angle B = 40^0$, then $\angle C$ =?

We know that

In a ΔABC sum of all angles of a triangle is 180^0

i.e., $\angle A + \angle B + \angle C = 180^0$

$\Rightarrow 55^0 + 40^0 + \angle C = 180^0$

$\Rightarrow 95^0 + \angle C = 180^0$

$\Rightarrow \angle C = 180^0 - 95^0$

$\Rightarrow \angle C = 85^0$

Q2) If the angles of a triangle are in the ratio 1:2:3, determine three angles.

Solution:

Given that,

Angles of a triangle are in the ratio 1:2:3

Let the angles be x, 2x, 3x

∴ We know that,

Sum of all angles of triangles is 180^0

x+2x+3x = 180^0

=>6x = 180^0

=>$x = \frac{180^0}{6}$

=>x=30^0

Since x=30^0

2x = $2(30)^0$ = 60^0

3x = $3(30)^0$ = 90^0

Therefore, angles are $30^0, 60^0, 90^0$

Q3) The angles of a triangle are $(x - 40^0), (x - 20^0)$ and $(\frac{1}{2}x - 10^0)$. *Find the value of x.*

Solution:

Given that,

The angles of a triangle are

$(x - 40^0), (x - 20^0)$ and $(\frac{1}{2}x - 10^0)$

We know that,

Sum of all angles of triangle is 180^0

$\therefore (x-40^0)+(x-20^0)+(\frac{1}{2}x-10^0)=180^0$

$2x+\frac{1}{2}x-70^0=180^0$

$\frac{5}{2}x=180^0+70^0$

$5x=2(250)^0$

$x=\frac{500^0}{5}$

$\therefore x=100^0$

Q4) The angles of a triangle are arranged in ascending order of magnitude. If the difference between two consecutive angles is 10^0, find the three angles.

Solution:

Given that,

The difference between two consecutive angles is 10^0

Let x, x+10^0, x+20^0 be the consecutive angles that differ by 10^0

We know that,

Sum of all angles in a triangle is 180^0

x+x+10^0+x+20^0 = 180^0

3x+30^0 = 180^0

=>3x = $180^0 - 30^0$

=>3x = 150^0

=>x = 50^0

Therefore, the required angles are

x = 50^0

x+10^0 = 50^0 + 10^0 = 60^0

x+20^0 = 50^0 + 20^0 = 70^0

As the difference between two consecutive angles is 10^0, the three angles are $50^0,\ 60^0,\ 70^0$.

Q5) Two angles of a triangle are equal and the third angle is greater than each of those angles by 30^0. Determine all the angles of the triangle.

Solution:

Given that,

Two angles of a triangle are equal and the third angle is greater than each of those angles by 30^0.

Let x, x, x+30^0 be the angles of a triangle

We know that,

Sum of all angles in a triangle is 180^0

x + x + x + 30^0 = 180^0

3x + 30^0 = 180^0

3x = $180^0 - 30^0$

3x = 150^0

x = 50^0

Therefore, the three angles are $50^0,\ 50^0,\ 80^0$.

Q6) If one angle of a triangle is equal to the sum of the other two, show that the triangle is a right angle triangle.

Solution:

If one angle of a triangle is equal to the sum of the other two angles

=>$\angle B = \angle A + \angle C$

In ΔABC,

Sum of all angles of a triangle is 180^0

=>$\angle A + \angle B + \angle C = 180^0$

=>$\angle B + \angle B = 180^0$[∠ B=∠ A+∠C]

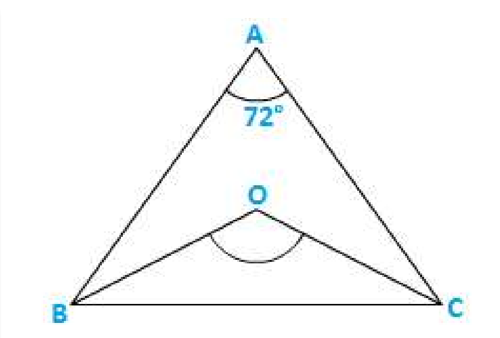
=>$2\angle B = 180^0$

=>$\angle B = \frac{180^0}{2}$

=>$\angle B = 90^0$

Therefore, ABC is a right angled triangle.

Q7) ABC is a triangle in which $\angle A = 72^0$, the internal bisectors of angles B and C meet in O. Find the magnitude of $\angle BOC$.



Solution:

Given,

ABC is a triangle where $\angle A = 72^0$ and the internal bisector of angles B and C meeting O.

In ΔABC,

$\angle A + \angle B + \angle C = 180^0$

=>$72^0 + \angle B + \angle C = 180^0$

=>$\angle B + \angle C = 180^0 - 72^0$

Dividing both sides by ‘2’

=>$\frac{\angle B}{2} + \frac{\angle C}{2} = \frac{108^0}{2}$

=>$\angle OBC + \angle OCB = 54^0$

Now, $\text{In } \Delta BOC \Rightarrow \angle OBC + \angle OCB + \angle BOC = 180^0$

=>$54^0 + \angle BOC = 180^0$

=>$\angle BOC = 180^0 - 54^0 = 126^0$

$\therefore \angle BOC = 126^0$

Q8) The bisectors of base angles of a triangle cannot enclose a right angle in any case.

Solution:

In ΔXYZ,

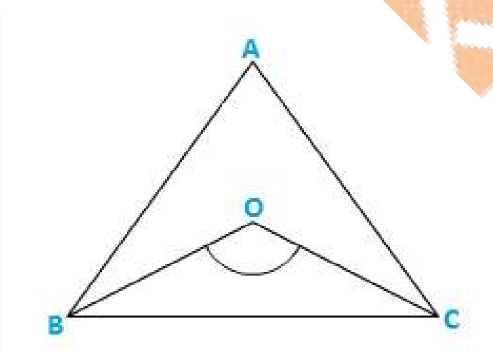
Sum of all angles of a triangle is 180^0

i.e., $\angle X + \angle Y + \angle Z = 180^0$

Dividing both sides by ‘2’

=>$\frac{1}{2}\angle X + \frac{1}{2}\angle Y + \frac{1}{2}\angle Z = 180^0$

=>$\frac{1}{2}\angle X + \angle OYZ + \angle OYZ = 90^0$ [∵ OY, OZ,∠ Y and ∠ Z]



$\Rightarrow \angle OYZ + \angle OZY = 90^0 - \frac{1}{2}\angle X$

Now in ΔYOZ

$\therefore \angle YOZ + \angle OYZ + \angle OZY = 180^0$

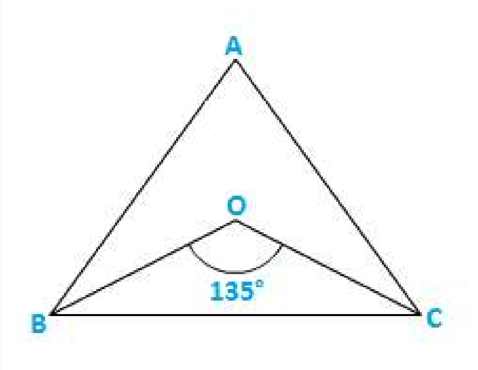
$\Rightarrow \angle YOZ + 90^0 - \frac{1}{2}\angle X = 180^0$

$\Rightarrow \angle YOZ = 90^0 - \frac{1}{2}\angle X$

Therefore, the bisectors of a base angle cannot enclosure right angle.

Q9) If the bisectors of the base angles of a triangle enclose an angle of 135^0*, prove that the triangle is a right angle.*

Solution:



Given the bisectors of the base angles of a triangle enclose an angle of 135^0

i.e., $\angle BOC = 135^0$

But, We know that

$\angle BOC = 90^0 + \frac{1}{2}\angle A$

=>$135^0 = 90^0 + \frac{1}{2}\angle A$

=>$\frac{1}{2}\angle A = 135^0 - 90^0$

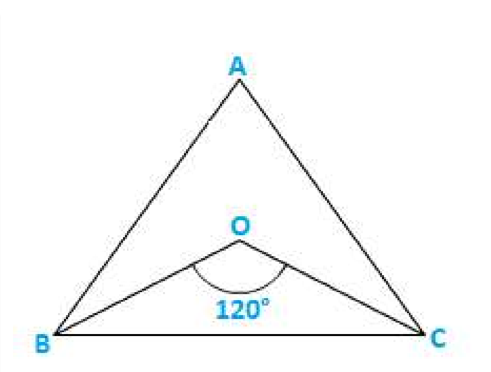
=>$\angle A = 45^0(2)$

=>$\angle A = 90^0$

Therefore, ΔABC is a right angle triangle that is right angled at A.

Q10) In a ΔABC, $\angle ABC = \angle ACB$ *and the bisectors of* $\angle ABC$ and $\angle ACB$ *intersect at O such that* $\angle BOC = 120^0$*. Show that* $\angle A = \angle B = \angle C = 60^0$.

Solution:



Given,

In ΔABC,

$\angle ABC = \angle ACB$

Dividing both sides by '2'

$\frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

=>$\angle OBC = \angle OCB$ [$\therefore$ OB, OC bisects $\angle B$ and $\angle C$]

Now,

$\angle BOC = 90^0 + \frac{1}{2}\angle A$

=>$120^0 - 90^0 = \frac{1}{2}\angle A$

=>$30^0 * (2) = \angle A$

=>$\angle A = 60^0$

Now in ΔABC

$\angle A + \angle ABC + \angle ACB = 180^0$ (Sum of all angles of a triangle)

=>$60^0 + 2\angle ABC = 180^0$ [$\therefore \angle ABC = \angle ACB$]

=>$2\angle ABC = 180^0 - 60^0$

=>$\angle ABC = \frac{120^0}{2} = 60^0$

=>$\angle ABC = \angle ACB$

$\therefore \angle ACB = 60^0$

Hence Proved.

Q11) Can a triangle have:

(i) Two right angles?

(ii) Two obtuse angles?

(iii) Two acute angles?

(iv) All angles more than 60°?

(v) All angles less than 60°?

(vi) All angles equal to 60"?

Justify your answer in each case.

Sol:

(i) No,

Two right angles would up to 180°. So the third angle becomes zero. This is not possible, so a triangle cannot have two right angles. [Since sum of angles in a triangle is 180^0]

(ii) No,

A triangle can't have 2 obtuse angles. Obtuse angle means more than 90° So that the sum of the two sides will exceed 180° which is not possible. As the sum of all three angles of a triangle is 180°.

(iii) Yes

A triangle can have 2 acute angles. Acute angle means less the 90" angle.

(iv) No

Having angles more than 60^0 make that sum more than 180^0. This is not possible. [Since the sum of all the internal angles of a triangle is 180^0]

(v) No

Having all angles less than 60^0 will make that sum less than 180^0 which is not possible.[Therefore, the sum of all the internal angles of a triangle is 180^0]

(vi) Yes

A triangle can have three angles equal to 60^0 . Then the sum of three angles equal to the 180^0. Such triangles are called as equilateral triangle. [Since, the sum of all the internal angles of a triangle is180^0]

Q12) If each angle of a triangle is less than the sum of the other two, show that the triangle is acute angled.

Solution

Given each angle of a triangle less than the sum of the other two

$\therefore \angle X + \angle Y + \angle Z$

=>$\angle X + \angle X < \angle X + \angle Y + \angle Z$

=>$2\angle X < 180^0$ [Sum of all the angles of a triangle]

=>$\angle X < 90^0$

Similarly $\angle Y < 90^0$ and $\angle Z < 90^0$

Hence, the triangles are acute angled.