# Exercise 6.1

Prove the following trigonometric identities:

**1.**

**Sol:**

We know $\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$\Rightarrow \sin^2 A \cdot \cos ec^2 A$

$\Rightarrow \sin^2 A \cdot \frac{1}{\sin^2 A} = 1$ $\quad\quad\quad \therefore L.H.S = R.H.S$

**2.**

**Sol:**

We know that $\cos ec^2 A - a^2 - A = 1$

$\cos ec^2 A = 1 + \cot^2 A$

$\Rightarrow \cos ec^2 A \cdot \sin^2 A = 1$

$\frac{1}{\sin A} \cdot n^2 A \cdot 1$

$1 = 1 \quad L.H.S = R.H.S$

**3.**

**Sol:**

$L.H.S \Rightarrow \frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^2\theta = \sin^2\theta$

$R.H.S \Rightarrow 1 - \cos^2\theta \quad\quad \left[1 = \sin\theta + \cos^2\theta\right]$

$\Rightarrow \sin^2\theta \quad\quad\quad\quad \left[\therefore \sin^2\theta = 1 - \cos^2\theta\right]$

$L.H.S = R.H.S$

**4.**

**Sol:**

$LHS = \cos ec\theta\sqrt{\sin^2\theta} \quad\quad\quad \left[\because 1 - \cos^2\theta = \sin^2\theta\right]$

$= \cos ec\theta \cdot \sin\theta$

$= 1$

$\therefore L.H.S = R.H.S$

**5.**

**Sol:**

We know that $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \sec^2\theta = 1, \tan\theta$

$\cos ec^2\theta - \cos^2\theta = 1$

$\cos ec^2\theta - \cot^2\theta$

$\tan^2\theta \cdot \cot^2\theta = \tan^2\theta \dfrac{1}{\tan^2\theta}$

**6.**

**Sol:**

$LHS = \tan\theta + \dfrac{1}{\tan\theta} \Rightarrow \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\dfrac{\sin\theta}{\cos\theta}}$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$\Rightarrow \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \dfrac{1}{\sin\theta\cos\theta}$

$\Rightarrow \sec\theta \cos ec\theta$

Hence L.H.S = R.H.S

**7.**

**Sol:**

$\cos\theta - \cos 2 \cdot \dfrac{\theta}{2} = \cos^2\dfrac{\theta}{2} - \sin^2\dfrac{\theta}{2}$

$\sin\theta = \sin\dfrac{\theta}{2} \cdot 2 \Rightarrow 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$

$\Rightarrow LHS = \dfrac{\cos^2\dfrac{\theta}{2} - \sin^2\dfrac{\theta}{2}}{\left[\cos^2\dfrac{\theta}{2} + \sin^2\dfrac{\theta}{2}\right] - 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}} \quad \left[\because 1 = \cos^2\dfrac{\theta}{2} + \sin^2\dfrac{\theta}{2}\right]$

$\Rightarrow \dfrac{\left[\cos\dfrac{\theta}{2} - \sin\dfrac{\theta}{2}\right]\left[\cos\dfrac{\theta}{2} + \sin\dfrac{\theta}{2}\right]}{\left[\cos\dfrac{\theta}{2} - \sin\dfrac{\theta}{2}\right]} \quad \left[\because a^2 - b^2 = (a-b)(a+b)(a-b)^2 = a^2 + b^2 - 2ab\right]$

$\Rightarrow \dfrac{\cos\dfrac{\theta}{2} - \sin\dfrac{\theta}{2}}{\cos\dfrac{\theta}{2} - \sin\dfrac{\theta}{2}}.$

$$R.H.S\ \frac{1+\sin\theta}{\cos\theta} \Rightarrow \frac{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}$$

$$\Rightarrow \frac{\left[\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right]}{\left[\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right]}$$

$$\Rightarrow \frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}$$

$$\therefore L.H.S = R.H.S$$

**8.**

**Sol:**

$$\cos\theta = \cos 2\frac{\theta}{2} = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$$

$$\sin\theta = \sin 2\cdot\frac{\theta}{2} = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$1 = \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}$$

$$LHS = \frac{\cos\theta}{1+\sin\theta} = \frac{\cos^2\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cot\frac{\theta}{2}}$$

$$\Rightarrow \frac{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}{\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)^2}$$

$$\Rightarrow \frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}$$

$$RHS = \frac{1-\sin\theta}{\cos\theta} = \frac{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}$$

$$\Rightarrow \frac{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)^2}{\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)}$$

$$\Rightarrow \frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}$$

$\therefore$ LHS = RHS

**9.**

**Sol:**

$1+\cot^2 A=\operatorname{cosec}^2 A \qquad \left[\because \operatorname{cosec}^2 A-\cot^2 A=1\right.$

$\qquad \operatorname{cosec}^2 A=1+\cot^2 A.$

$$\Rightarrow \cot^2 A+\frac{1}{\operatorname{cosec}^2 A}$$

$\Rightarrow \cos^2 A+\sin^2 A=1 \qquad \therefore LHS = RHS$

**10.**

**Sol:**

$1+\tan^2 A=\sec^2 \qquad \left[\because \sec^2 A-\tan^2 A=1\right]$

$\Rightarrow \sin^2 A+\frac{1}{\sec^2} \qquad \left[1+\tan^2 A-\sec^2 A\right]$

$\Rightarrow \sin^2 A+\cos^2 A=1$

$\therefore LHS = RHS$

**11.**

**Sol:**

$\text{L.H.S}=\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$ Rationalize numerator with $\sqrt{1-\cos\theta}$

$$\Rightarrow \frac{\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}}\times\frac{\sqrt{1-\cos\theta}}{1-\cos\theta}$$

$$=\frac{\left(\sqrt{1-\cos\theta}\right)^2}{\sqrt{(1-\cos\theta)(1+\cos\theta)}}$$

$$=\frac{1-\cos\theta}{\sqrt{1-\cos^2\theta}}=\frac{1-\cos\theta}{\sqrt{\sin^2\theta}}=\frac{1-\cos\theta}{\sin\theta}$$

$= \cos ec\theta - \cot\theta$

**12.**

**Sol:**

$1 = \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}$

$\cos\theta = \cos 2\cdot\frac{\theta}{2} = \cos^2\frac{\theta}{2} - 8\sin^2\frac{\theta}{2}$

$\sin\theta = \sin 2\cdot\frac{\theta}{2} = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$$LHS = \frac{1-\cos\theta}{\sin\theta} = \frac{\cos^2\frac{\theta}{2} = \sin^2\frac{\theta}{2} - \left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$\frac{\cos\frac{\theta}{2} + \sin^2\frac{\theta}{2} - \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \tan\frac{\theta}{2}.$$

$$RHS = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \tan\frac{\theta}{2}$$

$\therefore L.H.S = R.H.S$

**13.**

**Sol:**

$$LHS = \frac{\sin\theta}{1-\cos\theta}$$

Rationalizer both Nr and Or with $1+\cos\theta$

$$\Rightarrow \frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$$

$$\Rightarrow \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta} \qquad \left[\because (a-b)(a+b)=a^2-b^2\right]$$

$$\Rightarrow \frac{\sin\theta+\sin\theta}{\sin^2\theta} \qquad \left[\because 1-\cos^2\theta-\sin^2\theta\right]$$

$$\Rightarrow \frac{\sin\theta}{\sin^2\theta}+\frac{\sin\theta\cos\theta}{\sin^2\theta}$$

$$\Rightarrow \frac{1}{\sin\theta}\frac{\cos\theta}{\sin\theta} \Rightarrow \cos ec\theta+\cot\theta$$

$\therefore LHS = RHS$

**14.**

**Sol:**

$$LHS = \frac{1-\sin\theta}{1+\sin\theta}$$

Rationalize both Nr and Or with $(1-\sin\theta)$ multiply

$$\Rightarrow \quad \frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}$$

$$\Rightarrow \quad \frac{(1-\sin\theta)^2}{\cos^2\theta} \qquad \left[\because (1-\sin\theta)(1+\sin\theta)=\cos^2\theta\right]$$

$$\Rightarrow \left[\frac{1-\sin\theta}{\cos\theta}\right]^2 \Rightarrow \left[\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right]^2$$

$$\Rightarrow [\sec\theta-\tan\theta]^2$$

$= LHS = RHS$ Hence proved



**15.**

**Sol:**

$$LHS \Rightarrow \cos ec^2\theta-\sin^2\theta \qquad \left[(a+b)(a-b)=a^2-b^2\right]$$

$$\Rightarrow 1+\cot^2\theta-(1-\cos^2\theta) \qquad \left[\because \cos ec^2\theta=1+\cot^2\theta \sin^2\theta=1-\cos^2\theta\right]$$

$$\Rightarrow 1+\cot^2-1+\cos^2\theta$$

$$\Rightarrow \cot^2\theta+\cos^2\theta$$

$= LHS = RHS$ Hence proved

**16.**

**Sol:**

$$LHS = \frac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta} \qquad \left[\because \cos ec^2\theta=1+\cot^2\theta\right]$$

$$\Rightarrow \frac{\cos ec^2\theta \cdot \tan\theta}{\sec^2} \Rightarrow \frac{1}{\sin^2\theta} \cdot \frac{\cos^2\theta}{1} \cdot \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \frac{\cos\theta}{\sin\theta} = \cot\theta$$

$= LHS = RHS$ Hence proved

**17.**

**Sol:**

$LHS = \sec^2\theta - \cos^2\theta \qquad \left[\because (\sec\theta + \cos\theta)(\sec\theta - \cos\theta) - \sec^2\theta - \cos^2\theta\right]$

$\Rightarrow 1 + \tan^2\theta - \left(1 - \sin^2\theta\right) \qquad \left[\because \sec^2\theta = 1 + \tan^2\theta \cos^2\theta = 1 - \tan^2\theta\right]$

$\Rightarrow 1 + \tan^2\theta - l + \sin^2\theta$

$\tan^2\theta + \sin^2\theta$

$= LHS = RHS$ Hence proved

**18.**

**Sol:**

$$LHS = \frac{1}{\cos + 1} = (1 - \sin A) \times \left[\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right] \qquad \left[\because \sec A = \frac{1}{\cos A} \text{ and } \tan A = \frac{\sin A}{\cos A}\right]$$

$$\Rightarrow \frac{1}{\cos A} \times (1 - \sin A)\frac{(1 + \sin A)}{\cos A}$$

$$= \frac{\cos^2 A}{\cos^2 A} = 1 \qquad \left[\because (1 - \sin A)(1 + \sin A) \cdot \cos^2 A = 1 - \sin^2 A\right]$$

$= LHS = RHS$ Hence proved

**19.**

**Sol:**

$$LHS = \left[\frac{1}{\sin A} - \sin A\right]\left[\frac{1}{\cos A} - \cos A\right]\left[\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right]$$

$$\Rightarrow \frac{1 - \sin^2 A}{\sin A} \times \frac{1 - \cos^2 A}{\cos A} \times \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$$

$$\Rightarrow \frac{\cos^2 A \cdot \sin^2 A \cdot 1}{\sin^2 A \cos^2 A} \qquad \left[\begin{aligned} &\because \operatorname{cosec} A = \frac{1}{\sin} A \\ &\sec A = \frac{1}{\cos} A \\ &\tan A = \frac{\sin A}{\cos A} \\ &\cot A = \frac{\cos A}{\sin A} \end{aligned}\right]$$

$$= 1 \qquad \left[\begin{aligned} &\because 1 - \sin^2 A = \cos^2 A \\ &1 - \cos^2 A = \sin^2 A \\ &\sin^2 A + \cos^2 A = 1 \end{aligned}\right]$$

$= LHS = RHS$ Hence proved

**20.**

**Sol:**

$$LHS = \tan^2\theta - \sin^2\theta = \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta \qquad \left[\because \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}\right]$$

$$\Rightarrow \sin^2\theta\left[\frac{1}{\cos^2\theta} - 1\right]$$

$$\sin^2\theta\left[\frac{1-\cos^2\theta}{\cos^2\theta}\right]$$

$$\Rightarrow \sin^2\theta \cdot \frac{\sin^2\theta}{\cos^2\theta} = \sin^2\theta \tan^2\theta$$

$= LHS = RHS$ Hence proved

**21.**

**Sol:**

$$LHS = \left(1 + \tan^2\theta\right)\left(1 - \sin^2\theta\right) \qquad \left[\because (a-b)(a+b) = a^2 - b^2\right]$$

$$\Rightarrow \quad \sec^2\theta \cdot \cos^2\theta \qquad \left[\because \sec^2\theta = 1 + \tan^2\theta\right]$$

$= 1$

$= LHS = RHS$ Hence proved

**22.**

**Sol:**

$$LHS = \sin^2 A \cdot \frac{\cos^2 A}{\sin^2 A} + \cos^2 A \cdot \frac{\sin^2 A}{\cos^2 A}$$

$= \cos^2 A + \sin^2 A \qquad \left[\because \cot^2 A = \cos^2 \dfrac{A}{\sin^2 A} \tan^2 A = \dfrac{\sin^2 A}{\cos^2 A}\right]$

$= LHS = RHS$ Hence proved

**23.**

**Sol:**

$L.H.S = \dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} = \dfrac{\cos^2\theta}{\sin\theta\cos\theta} \qquad \left[\because \cos^2\theta - \sin^2\theta = \cos\theta\right]$

$\left[\because \cos^2 = 2\cos^2\theta - 1\right]$

$= \dfrac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$

$= LHS = RHS$ Hence proved

(ii) $\tan\theta - \cot\theta = \dfrac{2\sin^2\theta - 1}{\sin\theta\cos\theta}$

**Sol:**

$LHS = \dfrac{\sin\theta}{\cos\theta} - \dfrac{\cos\theta}{\sin\theta}$

$\Rightarrow \dfrac{\sin^2\theta - \cos^2}{\cos\theta\sin\theta}$

$\Rightarrow \dfrac{\sin^2\theta - \left(1 - \sin^2\theta\right)}{\cos\theta\sin\theta} \qquad \left[\because \cos^2\theta = 1\sin^2\theta\right]$

$\Rightarrow \dfrac{2\sin^2\theta - 1}{\sin\theta\cos\theta}$

$\therefore LHS = RHS$ Hence proved



**24.**

**Sol:**

$LHS = \dfrac{\left(15^2\right) - \sin\theta\cos ec\theta + \sin^2\theta}{\sin\theta}$

$\Rightarrow \dfrac{\cos^2\theta + \sin^2\theta - 1}{\sin\theta} \qquad \left[\because \sin\theta\cos ec\theta = 1\right]$

$= 0$

$\therefore LHS = RHS$ Hence proved

**25.**

**Sol:**

$$LHS = \frac{1-\sin A + 1 + \sin A}{(1+\sin A)(1-\sin A)}$$

$$\Rightarrow \frac{2}{1-\sin^2 A} \qquad \left[\because (1+\sin A)(1-\sin A) = 1-\sin^2 A\right]$$

$$\Rightarrow \frac{2}{\cos^2 A} \Rightarrow 2\sec^2 A \qquad \left[\because 1-\sin A = \cos A\right]$$

$\therefore LHS = RHS$ Hence proved

**26.**

**Sol:**

$$LHS = \frac{(1+\sin\theta)^2 + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{1+\sin^2\theta + 2\sin\theta + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$\Rightarrow \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)} = 2\sec\theta$$

$\therefore LHS = RHS$ Hence proved

**27.**

**Sol:**

$$LHS = \frac{1+\sin^2\theta + 2\sin\theta + 1 + \sin^2\theta - 2\sin\theta}{2\cos\theta}$$

$$\Rightarrow \frac{2(1+\sin^2\theta)}{2\cos^2\theta} \Rightarrow \frac{1+\sin^2\theta}{1-\sin^2\theta} \qquad \left[\because \cos^2\theta = 1-\sin^2\theta\right]$$

$\therefore LHS = RHS$ Hence proved

**28.**

**Sol:**

$$LHS \Rightarrow \frac{1+\tan^2\theta}{1+\cot^2\theta} = \frac{\sec^2\theta}{\cos ec^2\theta} \qquad \left[\because \tan^2\theta + 1 = \sec^2\theta \;\; 1+\cot^2\theta = \cos ec^2\theta\right]$$

$$= \frac{1}{\cos^2\theta \cdot 1}\sin^2 = \tan^2\theta$$

$$\Rightarrow \left[\frac{1-\tan\theta}{1-\cot\theta}\right]^2 \Rightarrow \left[\frac{1-\tan\theta}{1-\frac{1}{\tan\theta}}\right]^2$$

$$\Rightarrow \left[\frac{1-\tan\theta}{(1-\tan\theta)}\cdot\tan\theta\right]^2 = \tan^2\theta$$

$\therefore LHS = RHS$ Hence proved

**29.**

**Sol:**

$$LHS = \frac{1+\sec\theta}{\sec\theta} = \frac{1+\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$$= \frac{\cos\theta+1}{\cos\theta}\cdot\cos\theta$$

$$= 1+\cos\theta$$

$$RHS = \frac{\sin^2\theta}{1-\cos\theta} \Rightarrow \frac{1-\cos^2\theta}{1-\cos\theta}$$

$$\Rightarrow \frac{\left(1-2\sqrt{5}b\right)+\left(\cos\theta\right)}{1-48} = 1+\cos\theta$$

$\therefore LHS = RHS$ Hence proved

**30.**

**Sol:**

$$LHS = \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\cot\theta}{1-\tan\theta}$$

$$\Rightarrow -\frac{\tan^2\theta}{(1-\tan\theta)} + \frac{\cot\theta}{1-\tan\theta}$$

$$\frac{1}{1-\tan\theta}\left[\frac{1}{\tan\theta} - \tan^2\theta\right]$$

$$\frac{1}{1-\tan\theta}\left[\frac{1-\tan^3\theta}{\tan\theta}\right]$$

$\Rightarrow \frac{1}{1-\tan\theta}\frac{(1-\tan\theta)(1+\tan\theta+\tan^2\theta)}{\tan\theta}$ $\left[\because a^3-b^3=(a-b)(a^2+ab+b^2)\right]$

$\Rightarrow \frac{1+\tan\theta+\tan^2\theta}{\tan\theta}$

$\Rightarrow \cot\theta+1+\tan\theta$

$\therefore LHS = RHS$ Hence proved

**31.**

**Sol:**

We know that $\sec^2\theta-\tan^2\theta$:

Cubing on both sides

$\left(\sec\theta-\tan^2\theta\right)^3=1$

$\sec^6\theta\cdot\tan^6\theta-3\sec^2\theta\left(\sec^2\theta-\tan^2\theta\right)=1$

$\tan^2\theta$ $\left[\because (a-b)^3=a^3-b^3-3ab(a-b)\right]$

$\Rightarrow \sec\theta-\tan^6\theta=3\sec^2\theta\tan^2\theta=1$

$\Rightarrow \sec^6\theta=\tan^6\theta+1+3\tan^2\theta\sec^2\theta$

Hence proved

**32.**

**Sol:**

We know that $\cos ec^2\theta-\cot^2\theta=1$

Cubing on both sides

$\left(\cos ec^2\theta-\cot^2\theta\right)^3=(1)^3$

$\Rightarrow \cos ec^6\theta-\cot^6\theta-3\cos ec^2\theta\cot^2\theta\left(\cos ec^2\theta-\cot^2\theta\right)=1$

$\left[\because (a-b)^3-a^3-b^3-3ab(a-b)\right]$

$\Rightarrow \cos ec^6=1+3\cos ec^2\theta\cot^2\theta+\cot^6\theta$

Hence proved

**33.**

**Sol:**

$\sec^2\theta=\tan^2\theta=1$

$\therefore \sec^2\theta=1+\tan^2\theta$

$LHS=\frac{\sec^2\theta\cdot\cot\theta}{\cos ec^2\theta}\Rightarrow\frac{1\cdot\sin^6\theta}{\cos^2\theta}\cdot\frac{\cos\theta}{\sin\theta}$

$$\left[\because \sec\theta = \frac{1}{\cos\theta}, \cos ec\theta = \frac{1}{\sin\theta} \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$\therefore LHS = RHS$ Hence proved

**34.**

**Sol:**

We know that $\sin^2 A + \cos^2 A = 1$

$$\sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin^2 A = (1-\cos A)(1+\cos A)$$

$$\Rightarrow LHS = \frac{(1+\cos A)}{(1-\cos A)(1+\cos A)} = \frac{1}{1-\cos A}$$

$\therefore L.H.S = R.H.S$ Hence proved

**35.**

**Sol:**

$$LHS = \frac{\sec\theta - \tan\theta}{\sec A + \tan A}$$

Rationalizing the denominator y multiply and diving with $\sec A + \tan A$ we get

$$\frac{(\sec A - \tan A)}{(\sec A + \tan A)} \times \frac{(\sec A + \tan A)}{(\sec A + \tan A)} = \frac{\sec^2 A - \tan^2 A}{(\sec A + \tan A)^2} = \frac{1}{(\sec A + \tan A)^2}$$

$$\left[\because \sec^2 A - \tan^2 A = 1\right]$$

$$= \frac{1}{\sec^2 A + \tan^2 A + 2\sec A \tan A} = \frac{1}{\frac{1}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A} + \frac{2\sin A}{\cos^2 A}}$$

$$\Rightarrow \frac{\cos^2}{1 + \sin^2 A + 2\sin A} = \frac{\cos^2 A}{(1+\sin A)^2}$$

$\therefore L.H.S = R.H.S$ Hence proved

**36.**

**Sol:**

$$LHS = \frac{1+\cos A}{\sin A} \qquad ...(1)$$

Multiply both Nr and Dr with $(1-\cos A)$ we get

$$\frac{(1+\cos A)(1-\cos A)}{\sin A(1-\cos A)}=\frac{1-\cos^2 A}{\sin A(1-\cos A)}$$

$$=\frac{\sin^2 A}{\sin A(1-\cos A)}=\frac{1-\cos^2 A}{\sin A(1-\cos A)}$$

$$=\frac{\sin^2}{\sin A(1-\cos A)} \qquad [\because \cos^2 A=\sin^2 A]$$

$$=\frac{\sin A}{1-\cos A}$$

$\therefore L.H.S = R.H.S$ Hence proved

**37.**

**Sol:**

$$LHS=\sqrt{\frac{1+\sin A}{1-\sin A}}.$$

Rationalize the Nr. By multiplying both Nr and Dr with $\sqrt{1+\sin A}$.

$$\Rightarrow \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1+\sin A)(1-\sin A)}}=\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} \qquad [\because (1+\sin A)(1-\sin A)=\cos^2 A]$$

$$=\frac{1+\sin A}{\cos A}=\frac{1}{\cos A}+\frac{\sin A}{\cos A}$$

$\sec A+\tan A$

$\therefore L.H.S = R.H.S$ Hence proved

**38.**

**Sol:**

Rationalizing both Nr and Or by multiplying both with $\sqrt{1-\cos A}$ we get

$$\Rightarrow \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}}. \qquad [\because (1+\cos A)(1-\cos A)=1-\cos^2 A=\sin^2 A]$$

$$\sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}$$

$$=\frac{1-\cos A}{\sin A}$$

$= \cos ecA-\cot A.$

$\therefore L.H.S = R.H.S$ Hence proved

**39.**

**Sol:**

$LHS = (\sec A - \tan A)^2$

$\Rightarrow \left[\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right]^2 \Rightarrow \frac{(1-\sin A)^2}{\cos^2 A.}$

$\Rightarrow \frac{(1-\sin A)^2}{1-\sin^2 A}$ $\qquad [\because 1-\sin^2 A = \cos^2 A]$

$\Rightarrow \frac{(1-\sin A)^2}{(1-\sin A)(1+\sin A)}$ $\qquad [\because a^2 - b^2 = (a-b)(a+b)]$

$= \frac{1-\sin A}{1+\sin A}$

$\therefore L.H.S = R.H.S$ Hence proved

**40.**

**Sol:**

$LHS = \frac{1-\cos A}{1+\cos A}$

Rationalizing Nr by multiplying and dividing with $1-\cos A.$

$= \frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}$

$\Rightarrow \frac{(1-\cos A)^2}{1-\cos^2 A}$

$\Rightarrow \frac{(1-\cos A)^2}{\sin^2 A}$ $\qquad [\because (a+b)(a-b) = a^2 - b^2 \quad 1-\cos^2 A = \sin^2 A]$

$= \left[\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right]^2$ $\qquad (\cos ecA - \cot A)^2$

$= (\cot A - \cos ecA)^2$

$\therefore L.H.S = R.H.S$ Hence proved

**41.**

**Sol:**

$$LHS=\frac{\sec A+1+\sec A-1}{(\sec A+1)(\sec A-1)}=\frac{2\sec A}{(\sec^2 A-1)}$$

$$\left[\because (a+b)(a-b)=a^2-b^2 \sec^2 A-1=\tan^2 A\right]$$

$$\Rightarrow \frac{2\sec A}{\tan^2 A}=\frac{2\cdot 1\cos^2 A}{\cos A\cdot\sin^2 A} \qquad \left[\because \sec A=\frac{1}{\cos A}\tan^2 A=\frac{\sin^2 A}{\cos^2 A}\right]$$

$$\Rightarrow 2\cos ecA\cot A$$

$\therefore L.H.S=R.H.S$ Hence proved

**42.**

**Sol:**

$$LHS=\frac{\cos A}{1-\tan A}+\frac{\sin A}{\left(1-\frac{1}{\tan A}\right)}$$

$$=\frac{\cos A}{1-\tan A}-\frac{\sin A\cdot\tan A}{1-\tan A}$$

$$\Rightarrow \frac{\cos A-\sin A\tan A}{(1-\tan A)}$$

$$\Rightarrow \frac{\cos A-\sin A\cdot\frac{\sin A}{\cos A}}{1-\frac{\sin A}{\cos A}}$$

$$\Rightarrow \frac{\cos^2 A-\sin^2 A\cos A}{(\cos A-\sin A)\cos A}=\frac{(\cos A-\sin A)(\cos A+\sin A)}{\cos A-\sin A}$$

$$\Rightarrow \cos A+\sin A.$$

$\therefore L.H.S=R.H.S$ Hence proved

**43.**

**Sol:**

$$LHS\cos ecA\left[\frac{\cos ec+1+\cos ec-1}{\cos ec^2 A-1}\right] \qquad \left[\because \cos ec^2 A-1=\cot^2 A\right]$$

$$\Rightarrow \cos ecA\left[\frac{2\cos ecA}{\cot^2 A}\right]$$

$$\Rightarrow \frac{2}{\sin^2 A}\frac{\sin^2 A}{\cos^2 A}=2\sec^2 A.$$

$\therefore LHS=RHS$ Hence proved.

**44.**

**Sol:**

$$LHS=\left[1+\frac{\sin^2 A}{\cos^2 A}\right]+\left[1+\frac{\cos^2 A}{\sin^2 A^2}\right]$$

$$\Rightarrow \frac{\cos^2 A+\sin^2 A}{\cos^2 A}+\frac{\sin^2 A+\cos^2 A}{\sin^2 A}$$

$$\Rightarrow \frac{1}{\cos^2 A}+\frac{1}{\sin^2 A} \qquad \left[\because \sin^2 A+\cos^2 A=1\right]$$

$$\Rightarrow \frac{\sin^2 A+\cos^2 A}{\sin^2 A\cos^2 A}=\frac{1}{\sin^2 A(1-\sin A)} \qquad \left[\cos^2 A=1-\sin^2 A\right]$$

$$\Rightarrow \frac{1}{\sin^2 A-\sin^2 A}$$

$\therefore LHS = RHS$ Hence proved.

**45.**

**Sol:**

We know that

$$\sec^2 A=1+\tan^2 A$$

$$\cos ec^2 A=1+\cot^2 A.$$

$$\therefore LHS=\frac{\tan^2}{\sec^2}+\frac{\cot^2}{\cos ec^2 A}$$

$$\Rightarrow \frac{\sin^2 A}{\cos^2 A}\times\frac{\cos^2 A}{1}+\frac{\cos^2 A}{\sin^2 A}=\frac{\sin^2 A}{1}$$

$$\left[\because \tan A=\sin\frac{A}{\cos A}\sec A=\frac{1}{\cos A}\cot A=\frac{\cos A}{\sin A}\cos ec=\frac{1}{\sin A}\right]$$

$$\Rightarrow \sin^2 A+\cos^2 A$$

$$=1$$

$\therefore LHS = RHS$ Hence proved.

**46.**

**Sol:**

$$=\frac{\frac{\cos A}{\sin A}-\cos A}{\frac{\cos A}{\sin A}+\cos A} \qquad \left[\because \cot A=\frac{\cos A}{\sin A}\right]$$

$$= \frac{\cos A\left[\frac{1}{\sin A} - 1\right]}{\cos A\left[\frac{1}{\sin A} + 1\right]}$$

$$= \frac{\cos ecA - 1}{\cos ecA + 1}$$

**47.**

**Sol:**

(i) $\Rightarrow \dfrac{1 + \cos\theta + \sin\theta}{1 + \cos\theta - \sin\theta}$

Dividing the equation with $\cos\theta$ we get or both Nr and Dr

$$\frac{\frac{1 + \cos\theta + \sin\theta}{\cos\theta}}{\frac{1 + \cos\theta - \sin\theta}{\cos\theta}} = \frac{\frac{1}{\cos\theta} + \frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta} - \frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\sec\theta + 1 + ta\theta}{\sec\theta + 1 - \tan\theta}$$

$$= \frac{\sec\theta + \tan\theta + \sec^2\theta - \tan^2\theta}{\sec^2\theta - \tan\theta + 1} \qquad \left[\because \sec^2\theta - \tan^2\theta = 1\right]$$

Or

$$\frac{\sec\theta + \tan\theta + 1}{\sec\theta - \tan\theta + 1}$$

$$\frac{\frac{1}{\sec\theta - \tan\theta} + 1}{\sec\theta - \tan\theta + 1} \qquad \left[\because \sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}\right]$$

Or

$$\frac{\sec\theta+\tan\theta+1}{\sec\theta-\tan\theta+1}$$

$$\frac{\frac{1}{\sec\theta-\tan\theta}+1}{\sec\theta-\tan\theta+1} \qquad \left[\because \sec\theta+\tan\theta=\frac{1}{\sec\theta-\tan\theta}\right]$$

$$=\frac{1+\sec\theta-\tan\theta}{1+\sec\theta-\tan\theta}\times\frac{1}{\sec\theta-\tan\theta}$$

$$=\frac{1}{\sec\theta-\tan\theta}=\sec\theta+\tan\theta$$

$$=\sec\theta+\tan\theta$$

$$=\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}$$

$$=\frac{1+\cos\theta}{\cos\theta}$$

(ii) $\dfrac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}$

Divide Nr and Dr with $\cos\theta$, we get

$$\frac{\frac{\sin\theta-\cos\theta+1}{\cos\theta}}{\frac{\sin\theta+\cos\theta-1}{\cos\theta}}=\frac{\tan\theta-1+\sec\theta}{\tan\theta+1-\sec\theta}.$$

$$=\frac{\frac{1}{\sec\theta-\tan\theta}-1}{\tan\theta-\sec\theta+1}$$

$$=\frac{1-\sec\theta+\tan\theta}{1-\sec\theta+\tan\theta}\times\frac{1}{\sec\theta-\tan\theta}$$

$$=\frac{1}{\sec\theta-\tan\theta}$$

(iii) $\dfrac{\cos\theta-\sin\theta+1}{\cos\theta+\sin\theta-1}=\cos ec+\cot\theta$

Divide both Nr and Dr with $\sin\theta$

$$\frac{\frac{\cos\theta-\sin\theta+1}{\sin\theta}}{\frac{\cos\theta+\sin\theta-1}{\sin\theta}}$$

$$=\frac{\cot\theta-1+\cos ec}{\cot\theta+1-\cos ec}$$

$$=\frac{\cot\theta+\cos ec\theta-\left(\cos ec^2\theta-\cot^2\theta\right)}{\cot\theta-\cos ec\theta+1}$$

$$=\frac{\cot\theta+\cos ec\theta-\left(\cos ec^2\theta+\cot^2\theta\right)}{\cot\theta-\cos ec\theta+1}$$

$$=\frac{\cot\theta+\cos ec\left(1-\left(\cos ec-\cot\theta\right)\right)}{\cot\theta-\cos ec\theta+1}$$

$$=\cot\theta+\cos ec\theta$$

**48.**

**Sol:**

$$LHS:\sec A-\tan A\left[\therefore\frac{1}{\sec A+\tan A}=\sec A-\tan A\right]$$

$$=-\tan A$$

$$RHS\ \frac{1}{\cos A}-\frac{1}{\sec A-\tan A}$$

$$\sec A-\left(\sec A+\tan A\right)$$

$$\left[\because\frac{1}{\sec A-\tan A}=\sec A+\tan A\right]$$

$$=-\tan A$$

$$LHS=RHS$$

**49.**

**Sol:**

$$\tan^2 A+\cot^2 A=\frac{\sin^2 A}{\cos^2 A}+\frac{\cos^2 A}{\sin^2 A}$$

$$=\frac{\sin^4 A+\cos^4 A}{\cos^2 A\sin^2 A}$$

$$=\frac{1-2\sin^2 A\cos^2 A}{\sin^2 A\cos^2 A}\qquad\left[\because\sin^4 A+\cos^4 A=1-2\sin^2 A\cos^2 A\right]$$

$$=\sec^2 A\cos ec^2 A-2$$

$\sin^4 A+\cos^4 A$ is in the form of $a^4+b^4$

$$a^4+b^4=\left(a^2+b^2\right)^2-2a^2b^2$$

Here $a=\sin A, b=\cos A$

$$=\left(\sin^2 A+\cos^2 A\right)^2-2\sin^2 A\cos^2 A$$

$$=1-2\sin^2 A\cos^2 14$$

**50.**

**Sol:**

$$\frac{1-\frac{\sin^2 A}{\cos^2 A}}{\frac{\cos^2}{\sin^2}-1}=\frac{\frac{\cos^2 A-\sin^2 A}{\cos^2 A}}{\frac{\cos^2 A-\sin^2 A}{\sin^2 A}}$$

$$=\frac{\sin^2 A}{\cos^2 A}$$

$$=\tan^2 A.$$

**51.**

**Sol:**

$$1+\frac{\cos ec^2\theta-1}{1+\cos ec\theta} \qquad \left[\because \cos ec^2\theta-\cot^2\theta=1, \cot^2\theta=\cos ec^2\theta-1\right]$$

$$1+\frac{(\cos ec\theta-1)(\cos ec\theta+1)}{1+\cos ec\theta}$$

$$=1+\cos ec\theta-1 \qquad \left[\because (a+b)(a-b)=a^2-b^2 a=\cos ec\theta, b=1.\right]$$

$$=\cos ec\theta$$

**52.**

**Sol:**

$$\frac{\cos\theta}{\frac{1}{\sin\theta}+1}+\frac{\cos\theta}{\frac{1}{\sin\theta}-1}$$

$$\frac{\frac{\cos\theta}{1+\sin\theta}}{\sin\theta}+\frac{\frac{\cos\theta}{1-\sin\theta}}{\sin\theta}$$

$$\frac{(\cos\theta)(\sin\theta)}{1+\sin\theta}+\frac{(\cos\theta)(\sin\theta)}{1-\sin\theta}$$

$$\frac{(1-\sin\theta)(\sin\theta\cos\theta)+(\sin\theta\cos\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$\frac{\sin\theta\cos\theta-\sin\theta\cos\theta+\sin\theta\cos\theta+\sin^2\theta\cos^2}{1-\sin^2\theta}$$

$$= \frac{\sin\theta\cos\theta}{\cos^2\theta}$$

$$= \frac{2\sin\theta}{\cos\theta}$$

$$= 2\tan\theta$$

**53.**

**Sol:**

$$\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1-\sin^2\theta+\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\cos^2\theta+\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$$= \cot\theta.$$

**54.**

**Sol:**

$$\frac{\tan^3\theta}{\sec^2\theta}+\frac{\cos^2\theta}{\cos ec^2\theta} \qquad \left[\because \sec^2\theta-\tan^2\theta=1 \cos ec^2\theta-\cot^2\theta=1\right]$$

$$\cos ec^2\theta = 1+\cot^2\theta.$$

$$\tan\theta+\cos^2\theta+\cot^3\theta\times\sin^3\theta \qquad \left[\because \frac{1}{\sec^2\theta}=\cos^2\theta, \frac{1}{\cos ec^2\theta}=1+\cot^2\theta\right]$$

$$\frac{\sin^3\theta}{\cos^3\theta}\times\cos^2\theta+\frac{\cos^3\theta}{\sin^3\theta}\times\sin^2\theta$$

$$\frac{\sin^3\theta}{\cos\theta}+\frac{\cos^3\theta}{\sin\theta}$$

$$= \frac{\sin^4\theta+\cos^4\theta}{\sin\theta\cos\theta}$$

$$\frac{1-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$\frac{1}{\sin\theta\cos\theta}-\frac{2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$\sec\theta\cos ec\theta-2\sin\theta\cos\theta.$$

**55.**

**Sol:**

$$LHS=\frac{\left(\sin^3\theta+\cos^3\theta\right)-\left(\sin^3\theta+\cos^5\theta\right)}{\sin\theta+\cos\theta}$$

$$=\frac{\sin^3\theta\left(1-\sin^2\theta\right)+\cos^3\theta+1-\cos^2\theta}{\sin\theta+\cos\theta}$$

$$=\frac{\sin^3\theta\times\cos^3\theta+\cos^3\theta\times\sin^2\theta}{\sin\theta+\cos\theta}$$

$$=\frac{\sin^2\theta+\cos\theta\left(\sin\theta+\cos\theta\right)}{\sin\theta+\cos\theta}$$

$$=\sin^3\theta\cos^2\theta$$

$$\frac{T_5-T_9}{T_3}=\frac{\left(\sin^5\theta+\cos^5\theta\right)-\left(\sin^7\theta+\cos^7\theta\right)}{\sin^3\theta+\cos^3\theta}$$

$$=\frac{\sin^5\theta\left(1-\sin^2\theta\right)+\cos^5\theta\left(\sin^2\theta\right)}{\sin^3\theta+\cos^3\theta}$$

$$=\frac{\sin^5\theta+\cos^2+\cos^5\theta\left(\sin^2\theta\right)}{\sin^3\theta+\cos^3\theta}$$

$$=\frac{\sin^2\theta\cos^2\theta\left(\sin^3\theta+\cos^3\theta\right)}{\sin^3\theta+\cos^3\theta}$$

$$=\sin^2\theta\cos^2\theta$$

$L.H.S=R.H.S$ Hence Proval.

$$=\frac{\sin^2\theta\cos^2\theta\left(\sin\theta+\cos\theta\right)}{\sin^2\theta+\cos\theta}$$

$$=\sin^2\theta\cos^2\theta$$

$L.H.S=R.H.S$

**56.**

**Sol:**

$$\Rightarrow\left(\tan\theta+\sec\theta\right)^2+\left(\tan\theta-\sec\theta\right)^2$$

$$=\tan^2\theta+\sec^2\theta+2\tan\theta\sec\theta+\tan^2\theta+\sec^2\theta+2\tan\theta\sec\theta.$$

$$= 2\tan^2\theta + 2\sec^2\theta$$

$$= 2\left[\tan^2\theta + \sec^2\right]$$

$$= 2\left[\frac{\sin^2\theta}{\cos^2\theta} + \frac{1}{\cos^2\theta}\right]$$

$$= 2\left(\frac{\sin + \sin^2\theta}{\cos^2\theta}\right)$$

**57.**

**Sol:**

$$\Rightarrow \left[\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\cos ec^2\theta - \sin^2\theta}\right]\sin^2\theta\cos^2\theta.$$

$$= \left[\frac{1}{\frac{1-\cos^4\theta}{\cos^2\theta}} + \frac{1}{\frac{1-\sin^4\theta}{\sin^2\theta}}\right]\sin^2\theta\cos^2\theta.$$

$$= \left[\frac{\cos^2\theta}{1-\cos^4\theta} + \frac{\sin^2\theta}{1-\sin^4\theta}\right]\sin^2\theta\cos^2\theta$$

$$= \left[\frac{\cos^2\theta}{\cos^2\theta + \sin^2\theta - \cos^4\theta} + \frac{\sin^2}{\cos^2\theta + \sin^2\theta - \sin^4\theta}\right]\sin^2\theta\cos^2\theta.$$

$$= \left[\frac{\cos^2\theta}{\cos^2\theta\left(1-\cos^2\theta\right) + \sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta\left(1-\sin^2\theta\right) + \cos^2\theta}\right]\sin^2\theta\cos^2\theta.$$

$$= \left[\frac{\cos^2\theta}{\cos^2\theta\sin^2\theta + \sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta\cos^2\theta + \cos^2\theta}\right]\sin^2\theta\cos^2\theta$$

$$= \left[\frac{\cos^2\theta}{\sin^2\theta\left(\cos^2\theta + 1\right)} + \frac{\sin^2\theta}{\cos^2\theta\left(\sin^2\theta + 1\right)}\right]\sin^2\theta\cos^2\theta.$$

$$= \left[\frac{\cos^4\theta\left(1+\sin^2\theta\right) + \sin^4\theta\left(1+\cos^2\theta\right)}{\sin^2\theta\cos^2\theta\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}\right]\sin^2\theta\cos^2\theta$$

$$= \frac{\cos^4\theta\left(1+\sin^2\theta\right) + \sin^4\theta\left(1+\cos^2\theta\right)}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}$$

$$= \frac{\cos^4\theta + \cos^4\theta\sin^2\theta + \sin^4\theta + \sin^4\theta\cos^2\theta}{1+\sin^2\theta + \cos^2\theta + \cos^2\theta\sin^2\theta}.$$

$$=\frac{1-2\sin^2\theta\cos^2\theta+\sin^2\theta\cos^2\theta\left(\cos^2\theta+\sin^2\theta\right)}{1+1+\cos^2\theta\sin^2\theta}\quad\left(\because\cos^2\theta+\sin^2\theta=1\right)$$

$$=\frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}.$$

**58.**

**Sol:**

$$\Rightarrow\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\times\frac{1+\sin\theta-\cos\theta}{1+\sin\theta-\cos\theta}\right)^2$$

$$\Rightarrow\left[\frac{\left(1+\sin\theta-\cos\theta\right)^2}{\left(1+\sin\theta\right)^2-\cos^2\theta}\right]$$

$$=\left[\frac{(1)^2+\sin^2\theta+\cos^2\theta+2\times1\times\sin\theta+2\times\sin\theta\left(-\cos\theta\right)-2\cos\theta}{1-\cos^2\theta+\sin^2\theta+2\sin\theta}\right]$$

$(\text{Since}, \sin^2\theta+\cos^2\theta=1]$

$$=\left[\frac{1+1+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{\sin^2\theta-\sin^2\theta+2\sin\theta}\right]^2$$

$$=\left[\frac{2\times2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{2\sin^2\theta+2\sin\theta}\right]^2$$

$$=\left[\frac{2\left(1+\sin\theta\right)-2\cos\theta\left(\sin\theta+1\right)}{2\sin\theta\left(\sin\theta+1\right)}\right]^2$$

$$=\left[\frac{\left(1+\sin\theta\right)\left(2-2\cos\theta\right)}{2\sin\theta\left(\sin\theta+1\right)}\right]^2$$

$$=\left[\frac{2-2\cos\theta}{2\sin\theta}\right]^2$$

$$=\left[\frac{2}{2}-\left(\frac{1-\cos\theta}{\sin\theta}\right)\right]^2$$

$$=\left[\frac{1-\cos\theta}{\sin\theta}\right]^2$$

$$=\frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$=\frac{(1-\cos\theta)\times(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$=\frac{1-\cos\theta}{1+\cos\theta}.$$

**59.**

**Sol:**

$$=\left(\sec A+\tan A-\left\{\sec^2 A-\tan^2 A\right\}\right)\left[\sec A-\tan A+\left(\sec^2 A-\tan^2 A\right)\right]$$

$$=(\sec A+\tan A-\sec A+\tan A)(\sec A-\tan A)(\sec A-\tan A+(\sec A+\tan A)(\sec A-\tan A))$$

$$=(\sec A+\tan A)(1-(\sec A-\tan A))(\sec A-\tan A)(1+\sec A\tan A)$$

$$=(\sec A+\tan A)(1-\sec A+\tan A)(\sec A-\tan A)(1+\sec A\tan A)$$

$$=(\sec A+\tan A)(\sec A-\tan A)(1-\sec A+\tan A)(1+\sec A\tan A)$$

$$=\left(\sec^2 A-\tan^2 A\right)(1-\sec A+\tan A)(1-\sec A\tan A)$$

$$=\left[1-\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right]\left[1+\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right]$$

$$=\left(\frac{\cos A-1+\sin A}{\cos A}\right)\left(\frac{\cos A+1+\sin A}{\cos A}\right)$$

$$=\left(\frac{\cos A+\sin A^2-1}{\cos^2 A}\right)$$

$$=\frac{\cos^2 A+\sin^2 A+2\sin A\cos A-1}{\cos^2 A}$$

$$=\frac{1+2\sin A\cos A}{\cos^2 A}-1$$

$$=\frac{2\sin A\cos A}{\cos^2 A}\qquad\left[\because \sin^2 A+\cos^2 A=1\right]$$

$$=2\tan A$$

**60.**

**Sol:**

$$LHS = (1+\cot A - \cos ecA)(1+\tan A+\sec A)$$

$$= \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$$

$$= \left(\frac{\sin A+\cos A-1}{\sin}\right)\left(\frac{\cos A+\sin A+1}{\cos A}\right)$$

$$= \frac{(\sin A+\cos A)^2-1}{\sin A\cos A}$$

$$= \frac{1+2\sin A\cos A-1}{\sin A\cos A} \qquad \left[\because \sin^2 A+\cos^2 A=1\right]$$

$$= 2.$$

**61.**

**Sol:**

$LHS$

$$(\cos ec\theta - \sec\theta)(\cot\theta - \tan\theta)$$

$$\left[\frac{1}{\sin\theta}-\frac{1}{\cos\theta}\right]\left[\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}\right]$$

$$\left[\frac{\cos\theta-\sin\theta}{\sin\theta\cos\theta}\right]\left[\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}\right]$$

$$\left[\frac{(\cos\theta-\sin\theta)^2(\cos\theta+\sin\theta)}{\cos^2\theta\sin^2\theta}\right]$$

$RHS$

$$(\cos ec\theta+\sec\theta)(\sec\theta\cos ec\theta-2)$$

$$= \left[\frac{1}{\sin\theta}+\frac{1}{\cos\theta}\right]\left[\frac{1}{\cos\theta}-\frac{1}{\sin\theta}-2\right]$$

$$= \left[\frac{\sin\theta-\cos\theta}{\sin\theta\cos\theta}\right]\left[\frac{1-2\sin\theta\cos\theta}{\sin\theta\cos\theta}\right]$$

$$= \left[\frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}\right]\left[\frac{\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta}{\sin\theta\cos\theta}\right]$$

$$= \frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)^2}{\sin^2\theta\cos^2\theta} \qquad \left[\because \cos^2\theta+\sin^2\theta=1\right]$$

$L.H.S = R.H.S$ Hence proved

**62.**

**Sol:**

$LHS = (\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A)$

$= \left[\frac{1}{\cos A} - \frac{1}{\sin A}\right]\left[1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right]$

$= \left[\frac{\sin A - \cos A}{\sin A \cos A}\right]\left[\frac{\cos A \sin A + \sin^2 A + \cos^2 A}{\sin A \cos A}\right]$

$= \frac{(\sin A - \cos A)(\sin^2 A + \cos A \sin A + \cos^2 A)}{\sin^2 A \cos^2 A}$

$= \frac{(\sin^3 A - \cos^3 A)}{\sin^2 A \cos^2 A}$ $\quad\left[\therefore (a-b)(a^2 + ab) + b = (a^3 - b^3)\right]$

$RHS = \tan A \sec A - \cot A \operatorname{cosec} A$

$= \frac{\sin A}{\cos A} \times \frac{1}{\cos A} - \frac{\cos A}{\sin A} \times \frac{1}{\cos A}$

$= \frac{\sin A}{\cos^2 A} - \frac{\cos A}{\sin A}$

$= \frac{\sin^3 A - \cos^3 A}{\sin^2 A \cos^2 A}$

$L.H.S = R.H.S$ Hence proved.

**63.**

**Sol:**

$LHS \frac{\cos A \operatorname{cosec} A - \sin A \sec A}{\cos A + \sin A}$

$= \frac{\cos A \times \frac{1}{\sin A} - \sin A \times \frac{1}{\cos A}}{\cos A + \sin A}$

$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\cos A}}{\cos A + \sin A}$

$= \frac{\frac{\cos^2 A - \sin^2 A}{\sin A \cos A}}{\cos A + \sin A}$

$= \frac{\cos^2 A - \sin^2 A}{\sin A \cos A} \times \frac{1}{\cos A + \sin A}$

$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\sin A \cos A \times (\cos A + \sin A)}$

$$= \frac{\cos A - \sin A}{\sin A \cos A}$$

$$= \frac{\cos A}{\sin A \cos A} - \frac{\sin A}{\sin A \cos A}$$

$$= \frac{1}{\sin A} - \frac{1}{\cos A}$$

$$= \cos ecA - \sec A$$

$$= R.H.S$$

Hence proved.

**64.**

**Sol:**

$$LHS = \frac{\sin A}{\frac{1}{\cos A} + \frac{\sin A}{\cos A} - 1} + \frac{\cos A}{\frac{1}{\sin A} + \frac{\cos A}{\sin A} - 1}$$

$$= \frac{\sin A}{\frac{1 + \sin A - \cos A}{\cos A}} + \frac{\cos A}{\frac{1 + \cos A - \sin A}{\sin A}}$$

$$= \frac{\sin A \cos A}{1 + \sin A - \cos A} + \frac{\sin A \cos A}{1 + \cos A - \sin A}$$

$$= \sin A \cos A \left[ \frac{1}{1 + \sin A - \cos A} + \frac{1}{1 + \cos A - \sin A} \right]$$

$$= \sin A \cos A \left[ \frac{1 + \cos A - \sin A + \cot A \sin A - \cos A}{(1 + \sin \theta - \cos \theta)(1 + \cos A - \sin A)} \right]$$

$$= \sin A \cos A \left[ \frac{2}{\cos A - \sin A + \sin A + \sin A \cos A - \sin^2 A - \cos A - \cos^2 A + \cos A \sin A} \right]$$

$$= \sin A \cos A \left[ \frac{2}{1 - \sin^2 A - \cos^2 A + 2 \sin A \cos A} \right]$$

$$= \sin A \cos A \left[ \frac{2}{1 - \left( \sin^2 A + \cos^2 A \right) + 2 \sin A \cos A} \right]$$

$$= \sin A \cos A \left[ \frac{2}{1 - 1 + 2 \sin A \cos A} \right] \left( \because \sin^2 A + \cos^2 A = 1 \right)$$

$$= \sin A \times \cos A \times \frac{2}{2 \sin A \cos A}$$

$$= 1$$

$$L.H.S = R.H.S$$

**65.**

**Sol:**

$$= \frac{\tan A}{(\sec^2 A)^2} + \frac{\cos A}{(\operatorname{cosec}^2 A)^2} \qquad \left[\begin{array}{l} \because 1+\tan^2 A = \sec^2 A \\ 1+\cot^2 A = \operatorname{cosec}^2 A \end{array}\right]$$

$$= \frac{\frac{\sin A}{\cos A}}{\sec^4 A} + \frac{\frac{\cot A}{\sin A}}{\operatorname{cosec}^4 A}$$

$$= \frac{\frac{\sin A}{\cos A}}{\frac{1}{\cos^4 A}} + \frac{\frac{\cos A}{\sin A}}{\frac{1}{\sin^4 A}}$$

$$= \frac{\sin A}{\cos A} \times \frac{\cos^4 A}{1} + \frac{\cos A}{\sin A} \times \frac{\sin^4}{1}$$

$$= \sin A \times \cos^3 A + \cos A - \sin^3$$

$$= \sin A \cos A (\cos^2 A + \sin^2 A)$$

$$= \sin A \cos A$$

L.H.S = R.H.S

Hence proved.

**66.**

**Sol:**

$$LHS = \sec^4 A(1-\sin^4 A) - 2\tan^4 A$$

$$= \sec^4 A - \sec^4 A \times \sin^4 A - 2\tan^2 A$$

$$= \sec^4 A - \frac{1}{\cos^4 A} \times \sin^4 - 2\tan^2 A$$

$$= \sec^4 A - \tan^4 A - 2\tan^2 A$$

$$= (\sec^2 A)^2 = \tan^4 A - 2\tan^2 A$$

$$= (1+\tan^2 A)^2 - \tan^4 A - 2\tan^2 A \qquad [\because \sec^2 A - \tan^2 A = 1]$$

$$= 1 + \tan^4 A + 2\tan^2 A - \tan^4 A - 2\tan^2 A$$

$$= 1 = RHS$$

Hence proved.

**67.**

**Sol:**

$$=\frac{\frac{\cos^2 A}{\sin^2 A}\left(\frac{1}{\cos A}-1\right)}{1+\sin A}$$

$$=\frac{\frac{\cos^2 A}{\sin^2 A}\left(\frac{1-\cos A}{\cos A}\right)}{1+\sin A} \qquad \left[\because \sin^2 A+\cos^2 A=1\right]$$

$$=\frac{\frac{(\cos A\times\cos A)}{(1-\cos^2 A)}\left[\frac{1-\cos A}{\cos A}\right]}{1+\sin A}$$

$$=\frac{(\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}=\frac{1}{1+\sin A}$$

$$=\frac{\cos A}{(1+\cos A)(1+\sin A)}$$

Solving

$$RHS=\sec^2\left[\frac{1-\sin A}{1+\sec A}\right]$$

$$=\frac{1}{\cos^2 A}\left[\frac{1-\sec A}{1+\sec A}\right]$$

$$=\frac{1}{\cos^2 A}\left[\frac{1-\sec A}{\cos A+1}\right](\cos A)$$

$$=\frac{(1-\sin A)}{(\cos A)(\cos A+1)}$$

By multiplying Nr and Dr with $(1+\sin A)$

$$=\frac{(1-\sin A)}{(\cos A)(1+\cos A)}\times\frac{1+\sin A}{1+\sin A}$$

$$=\frac{(1)^2-\sin^2 A}{\cos A(1+\cos A)(1+\sin A)}$$

$$=\frac{\cos^2 A}{\cos A(1+\cos A)(1+\sin A)}$$

$$=\frac{\cos^2 A}{(1+\cos A)(1+\sin A)}$$

$L.H.S=R.H.S$ hence proved.

**68.**

**Sol:**

$(1+\cot A+\tan A)(\sin A-\cos A)$

$\sin A-\cos A+\cot A\sin A-\cot A\cos A+\sin A\tan A-\tan A\cos A$

$\sin A-\cos A+\frac{\cos A}{\sin A}\times\sin A-\cot A\cos A+\sin A\tan A-\frac{\sin A}{\cos A}\times\cos A$

$\sin A-\cos+\cos A-\cot A\cos A+\sin A\tan A-\sin A$

$=\sin A\cos A\ \cos A\cot A$

Solving:

$\frac{\sec A}{\cos ec^2 A}-\frac{\cos ecA}{\sec^2 A}$

$\frac{\frac{1}{\cos A}}{\frac{1}{\sin^2 A}}-\frac{\frac{1}{\sin A}}{\frac{1}{\cos^2 A}}$

$\frac{\sin^2 A}{\cos A}-\frac{\cos^2 A}{\sin A}$

$\frac{\sin^3 A-\cos^3 A}{\sin A\cos A}$

$=\sin A\times\frac{\sin A}{\cos A}-\cos A\times\frac{\cos A}{\sin A}$

$=\sin A\tan A-\cos A\cot A$

$L.H.S=R.H.S$

**69.**

**Sol:**

$LHS=\sin^2 A\cos^2 B-\cos^2 A\sin^2 B.$

$=\sin^2 A\left(1-\sin^2 B\right)-\left(1-\sin^2 A\right)\left(\sin^2 A\right)\qquad\left(\because \cos^2 A=1-\sin^2 A\right)$

$=\sin^2 A-\sin^2 A\sin^2 B-\sin^2 B+\sin^2 A\sin^2 B$

$=\sin^2 A-\sin^2 B$

$R.H.S$ Hence Proved.

**70.**

**Sol:**

$LHS=\frac{\cot A+\tan B}{\cot B+\tan A}$

$$= \frac{\frac{\cos A}{\sin A} + \frac{\sin B}{\cos B}}{\frac{\cos B}{\sin B} + \frac{\cos ecA}{\cos A}}$$

$$= \frac{\frac{\cos A \cos B - \sin A \sin B}{\sin A \cos B}}{\frac{\cos A \cos B + \sin A \sin B}{\cos A \sin B}}$$

$$= \frac{\cos A \cos B + \sin A \sin B}{\sin A \cos B} \times \frac{\cos A \sin B}{\cos A \cos B + \sin A \sin B}$$

$$= \frac{\cos A \sin B}{\sin A \cos B}$$

$$= \cot A \tan B$$

$$= RHS$$

Hence proved

**71.**

**Sol:**

$$LHSS = \frac{\tan A + \tan B}{\cot A + \cot B}$$

$$= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B}}$$

$$= \frac{\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\frac{\cos A \sin B + \cos B \sin A}{\sin A \sin B}}$$

$$= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B} \times \frac{\sin A \sin B}{\cos A \sin B + \cos B \sin A}$$

$$= \frac{\sin A \sin B}{\cos A \cos B}$$

$$= \tan A + \tan B = RHS$$

Hence proved

**72.**

**Sol:**

$$LHS = \cot^2 A \cos ec^2 B - \cot^2 B \cos ec^2 A$$

$$= \cot^2 A \left(1 + \cot^2 B\right) - \cot^2 B \left(1 + \cot^2 B\right) \qquad \left[\because \cos ec^2\theta = 1 + \cot^2 \theta\right]$$

$$= \cot^2 A + \cot^2 A \cot^2 B - \cot^2 B - \cot^2 B \cot^2 A$$

$= \cot^2 A - \cot^2 B.$

Hence proved

**73.**

**Sol:**

$LHS = \tan^2 A \sec^2 B - \sec^2 A \tan^2 B$

$= \tan^2 A + \left(1 + \tan^2 B\right) - \sec^2 A\left(\tan^2 A\right)$

$= \tan^2 A + \tan^2 A \tan^2 B - \tan^2 B\left(1 + \tan^2 A\right) \qquad \left(\because \sec^2 A = 4\tan^2 A\right)$

$= \tan^2 + \tan^2 A \tan^2 B - \tan^2 B - \tan^2 B \tan^2 A$

$= \tan^2 A - \tan^2 B$

$= RHS$

**74.**

**Sol:**

$L.H.S = x^2 - y^2$

$= \left(a\sec\theta + b\tan\theta\right)^2 - \left(a\tan\theta + b\sec\theta\right)^2$

$= a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta\tan\theta - a^2\tan^2\theta - b^2\sec\theta - 2ab\sec\theta\tan\theta.$

$= a^2 - \sec^2\theta - b^2\sec^2\theta + b^2\tan^2\theta - a^2\tan^2\theta$

$= \sec^2\theta\left(a^2 - b^2\right) + \tan^2\theta\left(b^2 - a^2\right)$

$= \sec^2\theta\left(a^2 - b^2\right) - \tan^2\theta\left(a^2 - b^2\right)$

$= \left(a^2 - b^2\right)\left(\sec^2\theta - \tan^2\theta\right) \qquad \left[\because \sec^2\theta - \tan^2\theta = 1\right]$

$= a^2 - b^2$

**75.**

**Sol:**

$\left[\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right]^2 + \left[\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right]^2 = (1)^2 + (1)^2$

$\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta \frac{2xy}{ab}\cos\theta\sin\theta + \frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta$

$-\frac{2xy}{ab}\sin\theta\cos\theta = 1 + 1$

$\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + \frac{x^2}{a^2}\sin^2\theta = 2$

$\cos^2\theta\left[\frac{x^2}{a^2} + \frac{y^2}{b^2}\right] + \sin^2\theta\left(\frac{x^2}{a^2} + \frac{y^2}{a^2}\right) = 2$

$$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)\left(\cos^2\theta+\sin^2\theta\right)=2$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}= \qquad \left(\therefore \cos^2\theta+\sin^2\theta=1\right)$$

**76.**

**Sol:**

$$\cos ec\theta-\sin\theta=a^3$$

$$\frac{1}{\sin\theta}-\sin\theta=a^3$$

$$\frac{1-\sin^2}{\sin\theta}=a^3$$

$$\frac{\cos^2\theta}{\sin\theta}=a^3$$

$$a=\frac{\cos^{\frac{1}{3}}\theta}{\sin^{\frac{1}{3}}\theta}$$

$$\Rightarrow a^2=\frac{\cos\frac{4}{3}\theta}{\sin\frac{2}{3}\theta}$$

$$\sec\theta-\cos\theta=b^3$$

$$\frac{1}{\cos\theta}\cos\theta=b^3$$

$$\frac{1-\cos^2\theta}{\cos\theta}=b^3$$

$$\frac{\sin^2\theta}{\cos\theta}=b^3$$

$$b=\frac{\sin^{\frac{2}{3}}\theta}{\cos^{\frac{1}{3}}\theta}$$

Now, $a^2b^2\left(a^2+b^2\right)$

$$=\frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta}\times\frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta}\left(\frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta}=\frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta}\right)$$

$$=\cos^{\frac{4}{3}-\frac{2}{3}}\theta\times\sin^{\frac{4-2}{3}}\left(\frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta}+\frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta}\right)$$

$$=\cos^{\frac{2}{3}}\theta\sin^{\frac{2}{3}}\left(\frac{1}{\sin^{\frac{2}{3}}\theta\cos^{\frac{2}{3}}\theta}\right)\quad\left(\because\cos^2\theta+\sin^2\theta=1\right)$$

$$=1$$

$$L.H.S=R.H.S$$

**77.**

**Sol:**

$$=\left(a\cos^3\theta+3a\cos\theta\sin^2\theta+a\sin^3\theta+3a\cos^2\theta\sin\theta\right)^{\frac{2}{3}}$$

$$+\left(a\cos^3\theta+3a\cos\theta\sin^2\theta-a\sin^3\theta-3a\cos^2\theta\sin\theta\right)^{\frac{2}{3}}$$

$$=a^{\frac{1}{3}}\left(\cos^3\theta+3\cos\theta\sin^2\theta+\sin^3\theta+3\cos^2\theta\sin\theta\right)^{\frac{2}{3}}$$

$$+a^{\frac{2}{3}}\left(\cos^3\theta+3\cos\theta\sin^2\theta+\sin^3\theta-3\cos^2\theta\sin\theta\right)^{\frac{2}{3}}$$

$$=a^{\frac{1}{3}}\left[\left(\cos\theta+\sin\theta\right)^3\right]^{\frac{2}{3}}+a^{\frac{2}{3}}\left(\cos\theta-\sin\theta\right)^3\Big]^{\frac{2}{3}}$$

$$=a^{\frac{2}{3}}\left[\left(\cos\theta+\sin\theta\right)^2\right]+a^{\frac{2}{3}}\left(\cos\theta-\sin\theta\right)^2$$

$$=a^{\frac{2}{3}}\left[\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta\right]$$

$$=a^{\frac{2}{3}}\left[\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta\right]+a^{\frac{2}{3}}\left[\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta\right]$$

$$=a^{\frac{2}{3}}\left[1+2\sin\theta\cos\theta\right]+a^{\frac{2}{3}}\left[1-2\sin\theta\cos\theta\right]$$

$$=a^{\frac{2}{3}}\left[1+2\sin\theta\cos\theta+1-2\sin\theta\cos\theta\right]$$

$$=a^{\frac{1}{3}}\left(1+1\right)=2a^{\frac{2}{3}}$$

$$=R.H.S$$

Hence proved.

**78.**

**Sol:**

$x = a\cos^3\theta : y = b\sin^3\theta$

$\frac{x}{a} = \cos^3\theta : \frac{y}{b} = \sin^3\theta$

$L.H.S = \left[\frac{x}{a}\right]^{\frac{2}{3}} + \left[\frac{y}{b}\right]^{\frac{2}{3}}$

$= \left(\cos^3\theta\right)^{\frac{2}{3}} + \left(\sin^3\theta\right)^{\frac{2}{3}}$

$= \cos^2\theta + \sin^2\theta \qquad \left(\therefore \cos^2\theta + \sin^2\theta = 1\right)$

$= 1$

Hence proved

**79.**

**Sol:**

Given $3\sin\theta + 5\cos\theta = 5$

$3\sin\theta = 5 - 5\cos\theta$

$3\sin\theta = 5\left(1-\cos\theta\right)$

$3\sin\theta = \frac{5\left(1-\cos\theta\right)\left(1-\cos\theta\right)}{1+\cos\theta}$

$3\sin\theta = \frac{5\left(1-\cos^2\theta\right)}{\left(1+\cos\theta\right)}$

$3\sin\theta = \frac{5\sin^2\theta}{1+\cos\theta}$

$3 + 3\cos\theta = 5\sin\theta$

$3 = 5\sin - 3\cos\theta$

$= RHS$

Hence proved.

**80.**

**Sol:**

$R.H.S = m^2\sin^2$

$= \left(a\cos\theta + b\sin\theta\right)^2 + \left(a\sin\theta - b\cos\theta\right)^2$

$= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta$

$+a^2\sin^2\theta+b^2\cos^2\theta-2ab\sin\theta\cos\theta$

$=a^2\cos^2\theta+b^2\cos^2\theta+b^2\sin^2\theta+a^2\sin^2\theta$

$=a^2\left(\sin^2\theta+\cos^2\theta\right)+b^2\left(\sin^2\theta+\cos^2\theta\right)$

$=a^2+b^2 \qquad \left(\because \sin^2\theta+\cos^2\theta=1\right)$

**81.**

**Sol:**

$LHS=mn$

$=\left(\cos ec\theta+\cot\theta\right)\left(\cos ec\theta-\cot\theta\right)$

$=\cos ec^2\theta-\cot^2\theta$

$=1 \qquad \left[\because \left(a+b\right)\left(a-b\right)=a^2-b^2\cos ec^2\theta-\cot^2\theta=1\right]$

$=R.H.S$

**82.**

**Sol:**

$\cos A+\cos^2 A=1$

$\cos A=1-\cos^2 A$

$\cos A=\sin^2 A$

$LHS=\sin^2 A+\sin^4 A$

$=\sin^2 A+\left(\sin^2 A\right)$

$=\sin^2 A+\left(\cos A\right)^2$

$=\sin^2 A+\cos A$

$=1$

**83.**

**Sol:**

$$LHS=\sqrt{\frac{\frac{1}{\cos\theta}-1}{\frac{1}{\cos\theta}+1}}+\sqrt{\frac{\frac{1}{\cos\theta}+1}{\frac{1}{\cos\theta}-1}}$$

$$=\sqrt{\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}}+\sqrt{\frac{\frac{1+\cos\theta}{\cos\theta}}{\frac{1-\cos\theta}{\cos\theta}}}$$

$$=\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}+\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$

$$=\sqrt{\frac{(1-\cos\theta)}{(1+\cos\theta)}\times\frac{(1-\cos\theta)}{1-\cos\theta}}+\sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}}$$

$$=\sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}+\sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}}$$

$$=\frac{1-\cos\theta}{\sin\theta}+\frac{1+\cos\theta}{\sin\theta}$$

$$=\frac{1-\cos\theta+1+\cos\theta}{\sin\theta}$$

$$=\frac{2}{\sin\theta}$$

$$=2\cos ec$$

(2) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}\times\frac{(1+\sin\theta)}{1+\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}\times\frac{1-\sin\theta}{1-\sin\theta}}$$

$$\sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}=\sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$=\sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}+\sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$$

$$=\frac{1+\cos\theta}{\sin\theta}+\frac{1-\cos\theta}{\sin\theta}$$

$$=\frac{2}{\sin\theta}=2\cos ec\theta$$

(3) Not given

(4) $\frac{\sec\theta-1}{\sec\theta+1}$

$$= \frac{\frac{1}{\cos\theta} - 1}{\frac{1}{\cos\theta} + 1}$$

$$= \frac{1 - \cos\theta}{1 + \cos\theta}$$

$$= \frac{1 - \cos\theta}{1 + \cos\theta} \times \frac{1 + \cos\theta}{1 + \cos\theta}$$

$$= \frac{1 - \cos^2\theta}{(1 + \cos\theta)^2}$$

$$= \frac{\sin^3}{(1 + \cos\theta)^2}$$

$$= \left[\frac{\sin\theta}{1 + \cos\theta}\right]^2$$

$= RHS$

Hence proved.

**84.**

**Sol:**

$\cos\theta + \cos^2\theta = 1$

$\cos\theta = 1 - \cos^2\theta$

$\cos\theta = \sin^2\theta$ .....(1)

Now, $\sin^{12}\theta + 3\sin^{10}\theta + 3\sin^8\theta + \sin^6\theta + 2\sin^4\theta + 2\sin^2\theta - 2$

$= \left(\sin^4\theta\right)^3 + 3\sin^4\theta \cdot \sin^2\theta\left(\sin^4\theta + \sin^2\theta\right)$

$+ \left(\sin^2\theta\right)^3 + 2\left(\sin^2\theta\right)^2 + 2\sin^2\theta 2$

Using $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$ and also from

(1) $\sin^2\theta\cos\theta$

$\left(\sin^4\theta + \sin^2\theta\right)^3 + 2\cos\theta^2 + 2\cos\theta - 2.$

$\left(\left(\sin^2\theta\right)^2 + \sin^2\theta\right) + 2\cos^2\theta + 2\cos\theta - 2$

$\left(\cos^2 + \sin^2\theta\right)^3 + 2\cos^2\theta + 2\cos\theta - 2$

$\left(\cos^2 + \sin\right)^3 + 2\cos^2\theta + 2\sin^2\theta - 2 \quad \left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$1+2\left(\sin^2\theta+\cos^2\theta\right)-2$

$1+2(1)-2$

$=1$

$L.H.S = R.H.S$

Hence proved.

**85.**

**Sol:**

L.H.S

We know that $1+\cos\theta = 1+\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2} = 2\cos^2\frac{\theta}{2}$

$\therefore \Rightarrow 2\cos^2\frac{\alpha}{2}\cdot 2\cos^2\frac{\beta}{2}\cdot 2\cos^2\frac{\gamma}{2} \qquad ....(1)$

Multiply (1) with $\sin\alpha\sin\beta\sin\gamma$ and divide it with same we get

$$\frac{8\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2}\cos^2\frac{\gamma}{2}}{\sin\alpha\sin\beta\sin\gamma}\times\sin\alpha\sin\beta\sin\gamma$$

$$\Rightarrow \frac{2\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2}\cos^2\frac{\gamma}{2}\times\sin\alpha\sin\beta\sin\gamma}{\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}}$$

$$\Rightarrow \sin\alpha\sin\beta\sin\gamma\times\cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$$

$RHS\ (1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)$

We know that $1-\cos\theta = 1-\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2} = 2\sin^2\frac{\theta}{2}$

$\Rightarrow 2\sin^2\frac{\alpha}{2}2\cdot\sin^2\frac{\beta}{2}\cdot 2\sin^2\frac{\gamma}{2}$

Multiply and divide by $\sin\alpha\sin\beta\sin\gamma$ we get

$$\frac{2\sin^2\frac{\alpha}{2}2\sin^2\frac{\beta}{2}2\sin^2\frac{\gamma}{2}\cdot\sin\alpha\sin\beta\sin\gamma}{\sin\alpha\sin\beta\sin\gamma}$$

$$\Rightarrow \frac{2\sin^2\frac{\alpha}{2}\cdot 2\sin^2\frac{\beta}{2}\cdot 2\sin^2\frac{\gamma}{2}\cdot\sin\alpha\sin\beta\sin\gamma}{2\sin\frac{\alpha}{2}\cos\frac{\beta}{2}2\sin\frac{\beta}{2}\cos\frac{\beta}{2}2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2}}$$

$$\Rightarrow \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}\sin\alpha\sin\beta\sin\gamma$$

Hence $\sin\alpha \sin\beta \sin\gamma$ is the member of equality.

**86.**

**Sol:**

$\sin\theta + \cos\theta = x$

Squaring on both sides

$(\sin\theta + \cos\theta)^2 = x^2$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = x^2$

$\therefore \sin\theta\cos\theta = \dfrac{x^2 - 1}{2}$ ......(1)

We know $\sin^2\theta + \cos^2\theta = 1$

Cubing on both sides

$(\sin^2\theta + \cos^2\theta)^3 = (1)^3$

$\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta) = 1$

$\Rightarrow \sin^6\theta + \cos^6 7 = 1 - 3\sin^2\theta\cos^2\theta$

$= 1 - 3\dfrac{(x^2-1)^2}{4}$ from (1)

$\therefore \sin^6\theta + \cos^6\theta = \dfrac{4 - 3(x^2-1)^2}{4}$

Hence proved

**87.**

**Sol:**

$x^2 = a^2\sec^2\theta\cos^2\theta$ .....$(i)$

$y^2 = b^2\sec^2\theta\sin^2\theta$ ....$(ii)$

$z^2 = c^2\tan^2\theta$ ....$(iii)$

## Exercise 6.2

**1.**

**Sol:**

We have $\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\dfrac{4}{5}\right)^2}$

$$= \sqrt{1 - \frac{16}{25}}$$

$$= \sqrt{25 - \frac{16}{25}}$$

$$= \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\therefore \sin\theta = \frac{3}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3/5}{4/5} = \frac{3}{4} \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$$

$$\cos ec = \frac{1}{\sec\theta} = \frac{1}{\frac{3}{5}} = \frac{5}{3} \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{3}{4}} = \frac{4}{3}.$$

**2.**

**Sol:**

We have $\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(\frac{1}{2}\right)^2}$

$$= \sqrt{\frac{1-1}{2}} = \sqrt{\frac{1}{2}}$$

$$\therefore \cos\theta = \frac{1}{\sqrt{2}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$$

$$\cos ec\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{1} = 1$$

**3.**

**Sol:**

We know that $\sec\theta = \sqrt{1+\tan^2\theta}$

$$= \sqrt{1+\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \sqrt{1+\frac{1}{2}} = \sqrt{\frac{3}{2}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

$$\cos ec\theta = \sqrt{1+\cot^2\theta} \Rightarrow \sqrt{1+2} = \sqrt{3}$$

Substituting it in (1) we get

$$\Rightarrow \frac{\left(\sqrt{3}\right)^2 - \left(\sqrt{3}\right)^2}{\left(\sqrt{3}\right)^2 + \left(\sqrt{3}\right)^2} = \frac{3-\frac{3}{2}}{3+2} = \frac{\frac{3}{2}}{5}$$

$$= \frac{3}{10}$$

**4.**

**Sol:**

$$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+\left(\frac{3}{4}\right)^2} = \sqrt{1+\frac{9}{16}}$$

$$\Rightarrow \sqrt{\frac{16-9}{16}} = \frac{5}{4}$$

$$\therefore \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{5}{4}} = \frac{4}{5} = \cos\theta$$

$$\therefore \text{We get } \frac{1-\frac{4}{5}}{1+\frac{4}{5}} = \frac{\frac{1}{5}}{\frac{9}{5}} = \frac{1}{9}.$$

**5.**

**Sol:**

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{12}{5}} = \frac{5}{12}$$

$$\cos ec = \sqrt{1+\cot^2\theta} = \sqrt{1+\left[\frac{5}{12}\right]^2} = \sqrt{\frac{144+25}{(12)^2}} = \sqrt{\frac{169}{144}} = \frac{13}{12}.$$

$$\sin\theta = \frac{1}{\cos ec\theta} = \frac{1}{\frac{13}{12}} = \frac{12}{13}.$$

We get $\dfrac{1+\frac{12}{13}}{1-\frac{12}{13}} = \dfrac{\frac{13+12}{18}}{\frac{13-12}{18}} = \dfrac{25}{1} = 25$

**6.**

**Sol:**

$$\cos ec\theta = \sqrt{1+\cot^2\theta} = \sqrt{1+\frac{1}{3}} = \sqrt{\frac{4}{3}}$$

$$\therefore \cos ec\theta = \frac{2}{\sqrt{3}}$$

$$\sin\theta = \frac{1}{\cos ec\theta} = \frac{1}{\frac{2}{\sqrt{3}}} = \frac{\sqrt{3}}{2}$$

$$and\ \frac{1}{\cot\theta} = \frac{\sin\theta}{\cos\theta} = \cos = \frac{\sin\theta}{\frac{1}{\cos\theta}} \Rightarrow \frac{\frac{\sqrt{3}}{2}}{\sqrt{3}} = \frac{1}{2}.$$

$\therefore$ on substituting we get

$$\frac{1-\frac{1}{4}}{2-\frac{3}{4}} = \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{5}.$$

**7.**

**Sol:**

We know that $\cot A = \sqrt{\cos ec^2 A - 1}$

$= \sqrt{(2)^2 - 1} = \sqrt{2-1}$

$-1.$

$\tan A = \frac{1}{\cot A} = \frac{1}{1} = 1$

$\sin A = \frac{1}{\cos ec} A = \frac{1}{\sqrt{2}} \therefore \sin A = \frac{1}{\sqrt{2}}$

$\cos A \sqrt{1 - \sin^2 A} = \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} - \sqrt{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}}.$

On substituting we get

$$\frac{2\left[\frac{1}{\sqrt{2}}\right]^3 + 3[1]^2}{4\left[(1) - \left(\frac{1}{\sqrt{2}}\right)^2\right]} = \frac{2 = \frac{1}{2} + 3}{4\left[1 - \frac{1}{2}\right]}$$

$$\Rightarrow \frac{1+3}{4 \cdot \frac{1}{2}} = \frac{4}{2} = 2.$$

**8.**

**Sol:**

$\cos ec\theta = \sqrt{1 + \cot^2 \theta} = \sqrt{1 + \left(\sqrt{3}\right)^2} = \sqrt{1+3} = 2$

$\sin \theta = \frac{1}{\cos ec} \theta = \frac{1}{2} \cot \theta = \frac{\cos \theta}{\sin \theta} \quad \therefore \cos \theta = \cot \theta \cdot \sin \theta$

$\Rightarrow \cos \theta = \frac{\sqrt{3}}{2}$

$\sec \theta = \frac{1}{\cos \theta} = \frac{2}{\sqrt{3}}$

On substituting we get

$$\frac{(2)^2 + \left(\sqrt{3}\right)^2}{(2)^2 - \left(\frac{2}{\sqrt{3}}\right)^2} = \frac{4+3}{\frac{12-4}{3}} = \frac{7}{\frac{8}{3}}$$

$$= \frac{21}{8}.$$

**9.**

**Sol:**

$\cos\theta = \frac{1}{3}$ $\quad\quad$ $\sin = \sqrt{1+\cos^2\theta}$

$= \sqrt{1-\frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2\sqrt{2}}{3\cdot\frac{1}{3}} = 2\sqrt{2}$

On substituting in (1) we get

$$\frac{6\left[\frac{2\sqrt{2}}{3}\right]^2 + \left(2\sqrt{2}\right)^2}{4\cdot\frac{1}{3}} = \frac{6\cdot\frac{3}{5}}{\frac{4}{5}} = \frac{\frac{16+24}{3}}{\frac{4}{3}}$$

$= \frac{40}{4} = 10$

**10.**

**Sol:**

$\sqrt{3}\cdot\frac{\sin\theta}{\cos\theta} = \sin\theta$

$\cos\theta = \frac{\sqrt{3}}{3} \Rightarrow \frac{1}{\sqrt{3}}$

$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{1}{\sqrt{3}}\right)^2}$

$\therefore \sin^2\theta - \cos^2\theta = \left(\sqrt{\frac{2}{3}}\right)^2 - \left[\frac{1}{\sqrt{3}}\right]^2$

$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

**11.**

**Sol:**

$$\sin\theta = \frac{1}{\cos ec\theta} = \frac{1}{\frac{13}{12}} = \frac{12}{13}$$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left[\frac{12}{13}\right]^2} = \sqrt{1-\frac{144}{169}}$$

$$= \sqrt{\frac{25}{169}} = \frac{5}{13}$$

$$\Rightarrow \frac{2\cdot\frac{12}{13}-3\cdot\frac{5}{13}}{4\cdot\frac{12}{13}-9\cdot\frac{5}{13}} = \frac{\frac{24-15}{13}}{\frac{48-15}{13}} = \frac{9}{3} = 3$$

**12.**

**Sol:**

$$L.H.S \Rightarrow \sin\theta + \cos\theta = \sqrt{2}\sin\theta \qquad \left[\because \cos(90-\theta) = \sin\theta\right]$$

$$\Rightarrow \cos\theta - \sin\theta\left(\sqrt{2}\right) - \sin\theta$$

$$\cos\theta - \sin\theta\left(\sqrt{2}-1\right)$$

Divide both sides with $\sin\theta$ we get

$$\frac{\cos\theta}{\sin\theta} = \frac{\sin\theta}{\sin\theta}\left(\sqrt{2}-1\right)$$

$$= \cot\theta = \sqrt{2}-1$$