Surface Area and volume of A Right Circular cone – 20.2

1.

Sol:

(i) Given that,

Radius of cone $(r) = 6m$

Height of cone $(h) = 7cm$

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (6)^2 \times 7$

$= 264cm^3$

(ii) Given,

Radius of cone $(r) = 3\cdot 5cm$

Height of cone $(h) = 12cm$

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (3\cdot 5)^2 \times 12$

$= 154cm^3$

(iii) From the relation

$l^2 = r^2 + h^2$, we have

$r = \sqrt{l^2 - h^2} = \sqrt{(28)^2 - (21)^2} = 7\sqrt{7}cm$

So, volume of cone $= \frac{1}{3} \times \pi r^2 \times h$

$= \frac{1}{3} \times \frac{22}{7} \times (2)^2 \times (7\sqrt{7})^2$

$= 7546cm^3$

2.

Sol:

(i) Radius of cone $(r) = 7cm$

Slant height (l) of cone $= 25cm$

Height (h) of cone $= \sqrt{l^2 - r^2}$

$= \sqrt{(25)^2 - b^2} = \sqrt{25^2 - 7^2} = 24cm.$

Volume of cone $= \frac{1}{3}\pi r^2 h = \left[\frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 24\right] cm^3$

$= 1232\ cm^3.$

(ii) Height (h) of cone = 12cm.

Slant height of cone $(l) = 13cm.$

Radius (r) of cone $= \sqrt{l^2 - r^2} = \sqrt{13^2 - 12^2}\, cm$

$= 5cm.$

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 12 \Big]\, cm^3$

$= \frac{2200}{7} cm^3$

Capacity of the conical vessel $= \left(\frac{2200}{7000}\right) liters$

$= \frac{11}{35} liters$

3.

Sol:

Given that, let height $\rightarrow h$ say

Height of 1st cone = h

Height of 2nd cone = 3h

Let the ratio of radii be r

$\therefore$ Radius of 1st cone = 3r

Radius of 2nd cone = r

$\therefore$ ratio of volume $= \frac{V_1}{V_2}$

$$\Rightarrow \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{r_1^2 h_1}{r_2^2 h_2}$$

$$= \frac{(3r)^2 \times h}{r^2 \times 3h}$$

$$= \frac{9r^2 h}{3r^2 h}$$

$$= \frac{3}{1}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{3}{1}.$$

4.

Sol:

Let thee ratio be x

$\therefore$ Radius $'r' = 5x$

Height 'h' = $12x$

WKT,

$\therefore$ Slant height $= \sqrt{r^2 + h^2} = \sqrt{(5x)^2 + (12x)^2} = 13x$

Now volume $= 314m^3$ [given data]

$\Rightarrow \frac{1}{3}\pi r^2 h = 314m^3$

$\Rightarrow \frac{1}{3} \times 3 \cdot 14 \times 25x^2 \times 12x = 314$

$\Rightarrow x^3 = \frac{314 \times 3}{3 \cdot 14 \times 25 \times 12}$

$\Rightarrow x^3 = 1 \Rightarrow x = 1$

$\therefore$ Slant height $= 13x = 13m$

Radius $= 5x = 5m.$

5.

Sol:

Let the ratio be x

Radius 'r' = $5x$

Height 'h' = $12x$

$\therefore$ Slant height $'l' = \sqrt{r^2 + h^2} = \sqrt{(5x)^2 + (12x)^2} = 13x.$

Now volume $= 2512cm^3$

$\Rightarrow \frac{1}{3} \times \pi \times (5x)^2 \times 12x = 2512$

$\Rightarrow \frac{1}{3} \times 3 \cdot 14 \times 2 \cdot 5x^2 \times 128x = 2512$

$\Rightarrow x^3 = \frac{2512 \times 3}{314 \times 25 \times 2}$

$\Rightarrow x = 2.$

$\therefore$ Slant height $= 13x = 13 \times 2 = 26cm$

And, Radius of cone $= 5x = 5 \times 2 = 10cm.$

6.

Sol:

Let ratio of radius be $'r'$

Radius of 1st cone = 2r

Radius of 2nd cone = 3r

Similarly

Let volume ratio be 'v'

Volume of 1st cone $\to 4v$

Similarly volume of 2nd cone $\to 5v$

$$\therefore \frac{V_1}{V_2} = \frac{4v}{5v} = \frac{4}{5}$$

$$\Rightarrow \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{4}{5}$$

$$\Rightarrow \frac{h_1(2r)^2}{h_2(3r)^2} = \frac{4}{5}$$

$$\Rightarrow \frac{h_1}{h_2} \times \frac{4r^2}{9r^2} = \frac{4}{5}$$

$$\Rightarrow \frac{h_1}{h_2} \times \frac{36}{20} = \frac{18}{20} = \frac{9}{5}$$

$\therefore$ Ratio of the inner height is $9:5$

7.

Sol:

Given that,

A cylinder and a cone have equal radii of their equal bases and heights

Let t_1 radius of cone = radius of cylinder = r

Let t_1 height of cone = height of cylinder = h

Let $V_1 =$ volume of cone

$V_2 =$ volume of cylinder

$$\Rightarrow \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r^2 h}{\pi r^2 h} = \frac{1}{3}$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{3}{1}$$

Hence their volumes are in the ratio $3:4$.

8.

Sol:

Let radius of cone is r and height is h

Volume $V_1 = \frac{1}{3}\pi r^2 h.$

In another case,

Radius of cone = half of radius $= \frac{r}{2}$

Height $= h$

$\therefore$ Volume $= (V_2) = \frac{1}{3}\pi\left(\frac{1}{2}r\right)^2 h$

$= \frac{1}{3}\pi \times \frac{r^2}{4} \times h$

$= \frac{1}{12}\pi r^2 h.$

$\therefore \frac{V_1}{V_2} = \frac{\frac{1}{12}\pi r^2 h}{\frac{1}{3}\pi r^2 h} = \frac{3}{12} = \frac{1}{4}.$

$\therefore$ Ratio will be $(1:4)$.

9.

Sol:

Diameter of heap $d = 9m$

Radius $= \frac{9}{2}m = 4\cdot 5m.$

Height $(h) = 3\cdot 5m.$

Volume of heap $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\left[3\cdot 14 \times (4\cdot 5)^2 \times 3\cdot 4\right] m^3$

$= 74\cdot 18 m^3$

Slant height $l = \sqrt{r^2 + h^2} = \sqrt{(4\cdot 5)^2 + (3\cdot 5)^2}$

$= 5\cdot 70m.$

Area of canvas required = CSA of cone

$= \pi rl$

$= 3\cdot 14 \times 4\cdot 5 \times 5\cdot 7 m^2$

$= 80 \cdot 54 m^2$

10.

Sol:

Given diameter of cone 14cm

$\therefore$ Radius of cone $= 7cm$

Height of cone $= 51cm.$

$\therefore$ Volume of cone $= \frac{1}{3} \times \pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 5 \times 51$

$= 2618 cm^3$

No with is given that $1cm^3$ weight 105m

$\therefore$ $2618 cm^3$ weight $(261 \times 10)\, gm$

i.e., $26 \cdot 180 kg.$

11.

Sol:

Given, radius of cone $(r) = 6 \cdot 3cm$

Height of cone $(h) = 10cm$

$\therefore$ WKT, Slant height $+(1) = \sqrt{(6 \cdot 3)^2 + (10)^2}$

$= 11 \cdot 819 cm \left[l = \sqrt{r^2 + h^2} \right]$

$\therefore$ Volume of cone $= \frac{1}{3} \pi r^2 h = \frac{1}{3} \times 3 \cdot 14 \times (6 \cdot 3)^2 \times 10 = 4158 cm^3$

And CSA of cone $= \pi rl$

$= \frac{22}{7} \times 6 \cdot 3 \times 11 \cdot 819 = 234 \cdot 01 cm^2$

12.

Sol:

For largest circular cone radius of the base of the cone $= \frac{1}{2}$ edge of cube

$= \frac{1}{2} \times 14 = 7cm$

And height of the cone $= \frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \times 3 \cdot 4 \times 7 \times 7 \times 14$$

$$= 718 \cdot 666\, cm^3.$$

13.

Sol:

(i) Radius of cone $= \left(\frac{28}{2}\right) cm = 14cm$

Let height of cone is h

Volume of cone $= 9856cm^3$

$$\Rightarrow \frac{1}{3}\pi r^2 h = 9856cm^2$$

$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times (14)^2 \times h \Big] cm^2 = 9856cm^3$$

$h = 48cm.$

Thus the height of the cone is $48cm.$

(ii) Slant height (l) of cone $= \sqrt{r^2 + h^2}$

$$= \left(\sqrt{(14)^2 + (48)^2} cm \right.$$

$$= \sqrt{196 + 2304} = \sqrt{2500} cm$$

$$= 50cm$$

Thus, the slant height of cone is 50cm.

(iii) CSA of cone $= \pi r l = \left(\frac{22}{7} \times 14 \times 50 \right) cm^2$

$$= 2200cm^2.$$

14.

Sol:

Radius (r) of pit $= \frac{3 \cdot 5}{2} m = 1 \cdot 75m.$

Depth (h) of pit = 12m.

Volume of pit $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (1 \cdot 75)^2 \times 12$

$$= 38 \cdot 5m^3$$

$\therefore$ Capacity of the pit $= (38 \cdot 5 \times 1)$ Kilometers

$= 38 \cdot 5$ Kilo liters

15.

Sol:

Given that,

Area of canvas $= 551m^2$ and area of the canvas 10st in wastage is $1m^2$

$\therefore$ area of canvas available for making the tent is $(551-1)m^2 = 550m^2$.

SA of tent $= 550m^2$ required $\cdot$ base radius of conical tent $= 7m$.

CSA of tent $= 550m^2$

$\pi rl = 550m^2$

$\Rightarrow \frac{22}{7} \times 7 \times l = 550$

$\Rightarrow l = \frac{550}{22} = 25m$

Now, WKT

$l^2 = r^2 + h^2$

$\Rightarrow (25)^2 - (7)^2 = h^2$

$\Rightarrow h = \sqrt{625-49}$

$= \sqrt{576} = 24m$

So, the volume of the conical tent $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3 \cdot 14 \times (7 \times 7)(24)m^3 = 1232m^3$.