Exercise -1.5

1.

Sol:

(i) Every point on the number line corresponds to a Real number which may be either rational or irrational.

(ii) The decimal form of an irrational number is neither terminating nor repeating

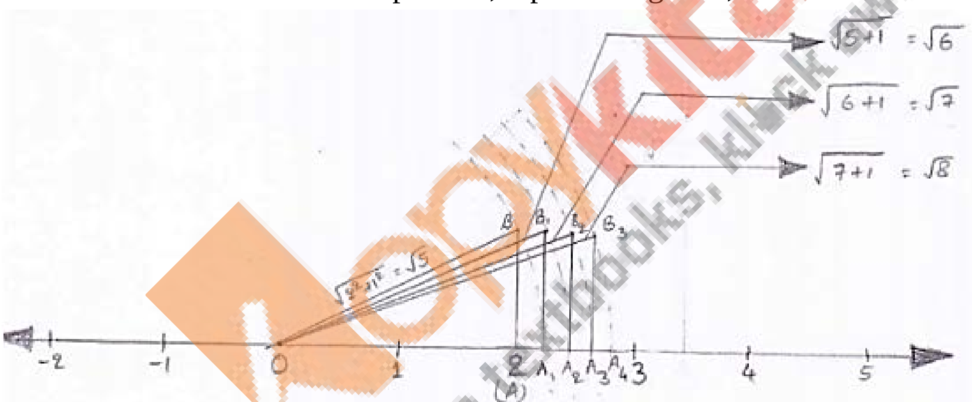
(iii) The decimal representation of a rational number is either terminating, non-terminating or recurring.

(iv) Every real number is either a rational number or an irrational number.

2.

Sol:

Draw a number line and mark point O, representing zero, on it



Suppose point A represents 2 as shown in the figure

Then $OA = 2.$ Now, draw a right triangle OAB such that $AB = 1.$

By Pythagoras theorem, we have

$OB^2 = OA^2 + AB^2$

$\Rightarrow OB^2 = 2^2 + 1^2$

$\Rightarrow OB^2 = 4 + 1 = 5 \Rightarrow OB = \sqrt{5}$

Now, draw a circle with center O and radius OB.

We fine that the circle cuts the number line at A

Clearly, $OA_1 = OB =$ radius of circle $= \sqrt{5}$

Thus, A_1 represents $\sqrt{5}$ on the number line.

But, we have seen that $\sqrt{5}$ is not a rational number. Thus we find that there is a point on the number which is not a rational number

Now, draw a right triangle OA_1B_1, Such that $A_1B_1 = AB = 1$

Again, by Pythagoras theorem, we have

$$(OB_1)^2 = (OA_1)^2 + (A_1B_1)^2$$

$$\Rightarrow (OB_1)^2 = \left(\sqrt{5}\right)^2 + (1)^2$$

$$\Rightarrow \left(OB_1^2\right) = 5 + 1 = 6 \Rightarrow OB_1 = \sqrt{6}$$

Draw a circle with center O and radius $OB_1 = \sqrt{6}$. This circle cuts the number line at A_2 as shown in figure

Clearly $OA_2 = OB_1 = \sqrt{6}$

Thus, A_2 represents $\sqrt{6}$ on the number line.

Also, we know that $\sqrt{6}$ is not a rational number.

Thus, A_2 is a point on the number line not representing a rational number

Continuing in this manner, we can represent $\sqrt{7}$ and $\sqrt{8}$ also on the number lines as shown in the figure

Thus, $OA_3 = OB_2 = \sqrt{7}$ and $OA_4 = OB_3 = \sqrt{8}$

3.

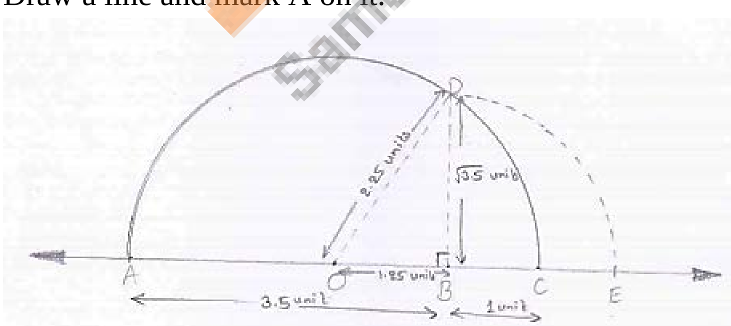
Sol:

Given to represent $\sqrt{3\cdot 5}, \sqrt{9\cdot 4}, \sqrt{10\cdot 5}$ on the real number line

Representation of $\sqrt{3\cdot 5}$ on real number line:

Steps involved:

(i) Draw a line and mark A on it.



(ii) Mark a point B on the line drawn in step - (i) such that $AB = 3\cdot 5$ units

(iii) Mark a point C on AB produced such that BC = 1unit

(iv) Find mid-point of AC. Let the midpoint be O

$$\Rightarrow AC = AB + BC = 3\cdot 5 + 1 = 4\cdot 5$$

$$\Rightarrow AO = OC = \frac{AC}{2} = \frac{4\cdot 5}{2} = 2\cdot 25$$

(v) Taking O as the center and $OC = OA$ as radius drawn a semi-circle. Also draw a line passing through B perpendicular to OB. Suppose it cuts the semi-circle at D.
Consider triangle OBD, it is right angled at B

$BD^2 = OD^2 - OB^2$

$\Rightarrow BD^2 = OC^2 - (OC - BC)^2$ $\quad [\because OC = OD = radius]$

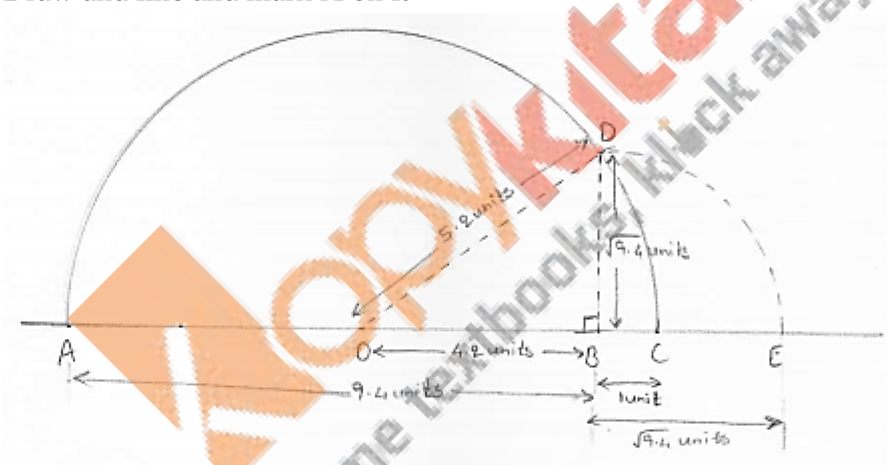
$\Rightarrow BD^2 = 2OC \cdot BC - (BC)^2$

$\Rightarrow BD = \sqrt{2 \times 2 \cdot 25 \times 1 - (1)^2} \Rightarrow BD = \sqrt{35}$

(vi) Taking B as the center and BD as radius draw an arc cutting OC produced at E. point E so obtained represents $\sqrt{3 \cdot 5}$ as $BD = BE = \sqrt{3 \cdot 5} =$ radius
Thus, E represents the required point on the real number line.

Representation of $\sqrt{9 \cdot 4}$ on real number line steps involved:

(i) Draw and line and mark A on it



(ii) Mark a point B on the line drawn in step (i) such that $AB = 9 \cdot 4$ units

(iii) Mark a point C on AB produced such that $BC = 1$ unit.

(iv) Find midpoint of AC. Let the midpoint be O.

$\Rightarrow AC = AB + BC = 9 \cdot 4 + 1 = 10 \cdot 4$ units

$\Rightarrow AD = OC = \frac{AC}{2} = \frac{10 \cdot 4}{2} = 5 \cdot 2$ units

(v) Taking O as the center and $OC = OA$ as radius draw a semi-circle. Also draw a line passing through B perpendicular to OB. Suppose it cuts the semi-circle at D.
Consider triangle OBD, it is right angled at B

$\Rightarrow BD^2 = OD^2 - OB^2$

$\Rightarrow BD^2 = OC^2 - (OC - BC)^2$ $\quad [\because OC = OD = radius]$

$\Rightarrow BD^2 = OC^2 - \left(OC^2 - 2OC \cdot BC + (BC)^2\right)$

$\Rightarrow BD^2 = 2OC \cdot \left(BC - \left(BC^2\right)\right)$

$\Rightarrow BD^2 = \sqrt{2 \times (5 \cdot 2) \times 1 - 1^2} \Rightarrow BD = \sqrt{9 \cdot 4}$ *units*

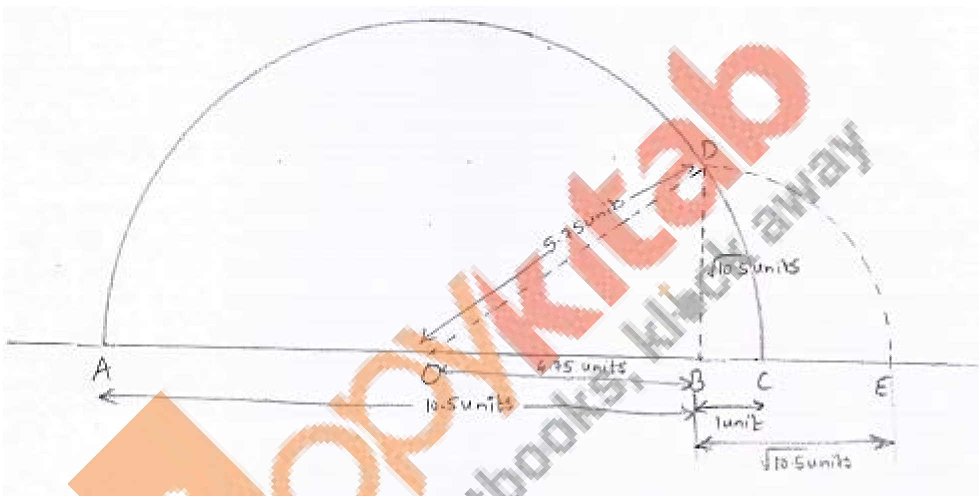
(vi) Taking B as center and BD as radius draw an arc cutting OC produced at E so obtained represents $\sqrt{9 \cdot 4}$ as $BD = BE = \sqrt{9 \cdot 4} = radius$

Thus, E represents the required point on the real number line.

Representation of $\sqrt{10 \cdot 5}$ on the real number line:

Steps involved:

(i) Draw a line and mark A on it



(ii) Mark a point B on the line drawn in step (i) such that $AB = 10 \cdot 5$ *units*

(iii) Mark a point C on AB produced such that $BC = 1 unit$

(iv) Find midpoint of AC. Let the midpoint be 0.

$\Rightarrow AC = AB + BC = 10 \cdot 5 + 1 = 11 \cdot 5$ *units*

$\Rightarrow AO = OC = \dfrac{AC}{2} = \dfrac{11 \cdot 5}{2} = 5 \cdot 75$ *units*

(v) Taking O as the center and $OC = OA$ as radius, draw a semi-circle. Also draw a line passing through B perpendicular to DB. Suppose it cuts the semi-circle at D. consider triangle OBD, it is right angled at B

$\Rightarrow BD^2 = OD^2 - OB^2$

$\Rightarrow BD^2 = OC^2 - (OC - BC)^2 \qquad [\because OC = OD = radius]$

$\Rightarrow BD^2 = OC^2 - \left[OC^2 - 2OC \cdot BC + (BC)^2\right]$

$\Rightarrow BD^2 = 2OC \cdot BC - BC^2$

$\Rightarrow BC^2 = OD^2 - OB^2$

$$\Rightarrow BD^2 = OC^2 - (OC - BC)^2 \qquad [\because OC = OD = radius]$$

$$\Rightarrow BD^2 = OC^2 - \left[OC^2 - 2OC \cdot BC + (BC)^2\right]$$

$$\Rightarrow BD^2 = 2OC \cdot BC - BC^2$$

$$\Rightarrow BD = \sqrt{2 \times 575 \times 1 - (1)^2} \Rightarrow BD = \sqrt{10 \cdot 5}$$

(vi) Taking B as the center and BD as radius draw on arc cutting OC produced at E. point E so obtained represents $\sqrt{10 \cdot 5}$ as $BD = BE = \sqrt{10 \cdot 5} =$ radius arc

Thus, E represents the required point on the real number line

4.

Sol:

(i) True

As we know that rational and irrational numbers taken together from the set of real numbers

(ii) True

As, π is ratio of the circumference of a circle to its diameter, it is an irrational number

$$\Rightarrow \pi = \frac{2\pi r}{2r}$$

(iii) False

Irrational numbers can be represented by points on the number line.