# Exercise -1.4

**1.**

**Sol:**

A number which can neither be expressed as a terminating decimal nor as a repeating decimal is called an irrational number. For example,

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A number which can neither be expressed as a terminating decimal nor as a repeating decimal is called an irrational number, For example,

0.33033003300033...

On the other hand, every rational number is expressible either as a terminating decimal or as a repeating decimal. For examples, $3.\overline{24}$ and 6.2876 are rational numbers

**2.**

**Sol:**

$\sqrt{7}$ is not a perfect square root, so it is an irrational number.

We have,

$$\sqrt{4} = 2 = \frac{2}{1}$$

$\sqrt{4}$ can be expressed in the form of $\frac{p}{q}$, so it is a rational number.

The decimal representation of $\sqrt{4}$ is $2.0$.

2 is a rational number, whereas $\sqrt{3}$ is an irrational number.

Because, sum of a rational number and an irrational number is an irrational number, so $2+\sqrt{3}$ is an irrational number.

$\sqrt{2}$ is an irrational number. Also $\sqrt{3}$ is an irrational number.

The sum of two irrational numbers is irrational.

$\therefore \sqrt{3}+\sqrt{2}$ is an irrational number.

$\sqrt{5}$ is an irrational number. Also $\sqrt{3}$ is an irrational number.

The sum of two irrational numbers is irrational.

$\therefore \sqrt{3}+\sqrt{5}$ is an irrational number.

We have,

$$\left(\sqrt{2}-2\right)^2 = \left(\sqrt{2}\right)^2 - 2\times\sqrt{2}\times 2 + (2)^2$$

$$= 2 - 4\sqrt{2} + 4$$

$$= 6 - 4\sqrt{2}$$

Now, 6 is a rational number, whereas $4\sqrt{2}$ is an irrational number.
The difference of a rational number and an irrational number is an irrational number.
So, $6-4\sqrt{2}$ is an irrational number.

$\therefore \left(\sqrt{2}-2\right)^2$ is an irrational number.

We have,

$\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)=\left(2\right)^2-\left(\sqrt{2}\right)^2$ $\qquad \left[\because \left(a-b\right)\left(a+b\right)=a^2-b^2\right]$

$=4-2$

$=2=\frac{2}{1}$

Since, 2 is a rational number.

$\therefore \left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)$ is a rational number.

We have,

$\left(\sqrt{2}+\sqrt{3}\right)^2=\left(\sqrt{2}\right)^2+2\times\sqrt{2}\times\sqrt{3}+\left(\sqrt{3}\right)^2$

$=2+2\sqrt{6}+3$

$=5+2\sqrt{6}$

The sum of a rational number and an irrational number is an irrational number, so $5+2\sqrt{6}$ is an irrational number.

$\therefore \left(\sqrt{2}+\sqrt{3}\right)^2$ is an irrational number.

The difference of a rational number and an irrational number is an irrational number

$\therefore \sqrt{5}-2$ is an irrational number.

$\sqrt{23}=4.79583152331........$

$\sqrt{225}=15=\frac{15}{1}$

Rational number as it can be represented in $\frac{p}{q}$ form.

0.3796
As decimal expansion of this number is terminating, so it is a rational number.

$7.478478...........=7.\overline{478}$
As decimal expansion of this number is non-terminating recurring so it is a rational number.

**3.**

**Sol:**

We have

$$\sqrt{4} = 2 = \frac{2}{1}$$

$\sqrt{4}$ can be written in the form of $\frac{p}{q}$, so it is a rational number.

Its decimal representation is 2.0.

We have,

$$3\sqrt{18} = 3\sqrt{2 \times 3 \times 3}$$
$$= 3 \times 3\sqrt{2}$$
$$= 9\sqrt{2}$$

Since, the product of a rations and an irrational is an irrational number.

$\therefore 9\sqrt{2}$ is an irrational

$\Rightarrow 3\sqrt{18}$ is an irrational number.

We have,

$$\sqrt{1 \cdot 44} = \sqrt{\frac{144}{100}}$$
$$= \frac{12}{10}$$
$$= 1.2$$

Every terminating decimal is a rational number, so 1.2 is a rational number.

Its decimal representation is 1.2.

We have,

$$\sqrt{\frac{9}{27}} = \frac{3}{\sqrt{27}} = \frac{3}{\sqrt{3 \times 3 \times 3}}$$
$$= \frac{3}{3\sqrt{3}}$$
$$= \frac{1}{\sqrt{3}}$$

Quotient of a rational and an irrational number is irrational numbers so $\frac{1}{\sqrt{3}}$ is an irrational number.

$\Rightarrow \sqrt{\frac{9}{27}}$ is an irrational number.

We have,

$$-\sqrt{64} = -\sqrt{8 \times 8}$$

$= -8$

$= -\frac{8}{1}$

$-\sqrt{64}$ can be expressed in the form of $\frac{p}{q}$, so $-\sqrt{64}$ is a rotational number.

Its decimal representation is $-8.0$.

We have,

$\sqrt{100} = 10$

$= \frac{10}{1}$

$\sqrt{100}$ can be expressed in the form of $\frac{p}{q}$. so $\sqrt{100}$ is a rational number.

The decimal representation of $\sqrt{100}$ is 10.0.

**4.**

**Sol:**

(i) We have

$x^2 = 5$

Taking square root on both sides.

$\Rightarrow \sqrt{x^2} = \sqrt{5}$

$\Rightarrow x = \sqrt{5}$

$\sqrt{5}$ is not a perfect square root, so it is an irrational number.

(ii) We have

$y^2 = 9$

$\Rightarrow y = \sqrt{9}$

$= 3$

$= \frac{3}{1}$

$\sqrt{9}$ can be expressed in the form of $\frac{p}{q}$, so it a rational number.

(iii) We have

$z^2 = 0.04$

Taking square root on the both sides, we get,

$\sqrt{z^2} = \sqrt{0.04}$

$\Rightarrow z = \sqrt{0.04}$

$= 0.2$

$= \frac{2}{10}$

$= \frac{1}{5}$

z can be expressed in the form of $\frac{p}{q}$, so it is a rational number.

(iv) We have

$u^2 = \frac{17}{4}$

Taking square root on both sides, we get,

$\sqrt{u^2} = \sqrt{\frac{17}{4}}$

$\Rightarrow u = \sqrt{\frac{17}{2}}$

Quotient of an irrational and a rational number is irrational, so u is an irrational number.

(v) We have

$v^2 = 3$

Taking square root on both sides, we get,

$\sqrt{v^2} = \sqrt{13}$

$\Rightarrow v = \sqrt{3}$

$\sqrt{3}$ is not a perfect square root, so y is an irrational number.

(vi) We have

$w^2 = 27$

Taking square root on both des, we get,

$\sqrt{w^2} = \sqrt{27}$

$\Rightarrow w == \sqrt{3 \times 3 \times 3}$

$= 3\sqrt{3}$

Product of a rational and an irrational is irrational number, so w is an irrational number.

(vii) We have

$t^2 = 0.4$

Taking square root on both sides, we get

$\sqrt{t^2} = \sqrt{0.4}$

$$\Rightarrow t = \sqrt{\frac{4}{10}}$$

$$= \frac{2}{\sqrt{10}}$$

Since, quotient of a rational and an irrational number is irrational number, so $t$ is an irrational number.

**5.**

**Sol:**

(i) $\sqrt{3}$ is an irrational number.

Now, $\left(\sqrt{3}\right)-\left(\sqrt{3}\right)=0$

0 is the rational number.

(ii) Let two irrational numbers are $5\sqrt{2}$ and $\sqrt{2}$

Now, $\left(5\sqrt{2}\right)-\left(\sqrt{2}\right)=4\sqrt{2}$

$4\sqrt{2}$ is the rational number.

(iii) Let two irrational numbers are $\sqrt{11}$ and $-\sqrt{11}$

Now, $\left(\sqrt{11}\right)+\left(-\sqrt{11}\right)=0$

0 is the rational number.

(iv) Let two irrational numbers are $4\sqrt{6}$ and $\sqrt{6}$

Now, $\left(4\sqrt{6}\right)+\left(\sqrt{6}\right)=5\sqrt{6}$

$5\sqrt{6}$ is the rational number.

(v) Let two irrational numbers are $2\sqrt{3}$ and $\sqrt{3}$

Now, $2\sqrt{3}\times\sqrt{3}=2\times 3$

$=6$

6 is the rational number.

(vi) Let two irrational numbers are $\sqrt{2}$ and $\sqrt{5}$

Now, $2\sqrt{3}\times\sqrt{5}=\sqrt{10}$

$\sqrt{10}$ is the rational number.

(vii) Let two irrational numbers are $3\sqrt{6}$ and $\sqrt{6}$

Now, $\frac{3\sqrt{6}}{\sqrt{6}}=3$

3 is the rational number.

(viii) Let two irrational numbers are $\sqrt{6}$ and $\sqrt{2}$

Now, $\frac{\sqrt{6}}{\sqrt{2}} = \frac{\sqrt{3+2}}{\sqrt{2}}$

$= \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{2}}$

$= \sqrt{3}$

$\sqrt{3}$ is an irrational number.

**6.**

**Sol:**

Let, 2 = 0.212112111211112

And, b = 0.232332333233332...

Clearly, $a < b$ because in the second decimal place a has digit 1 and b has digit 3

If we consider rational numbers in which the second decimal place has the digit 2, then they will lie between a and b.

Let,

$x$ = 0.22

$y$ = 0.22112211...

Then,

$a < x < y < b$

Hence, $x$, and $y$ are required rational numbers.

**7.**

**Sol:**

Let, a = 0.515115111511115...

And, b = 0.5353353335...

We observe that in the second decimal place a has digit 1 and b has digit 3, therefore, $a < b.$ So if we consider rational numbers

$x = 0.52$

$y$ = 0.52052052...

We find that,

$a < x < y < b$

Hence $x$, and $y$ are required rational numbers.

**8.**

**Sol:**

Let, a = 0.2101

And, b = 0.2222...

We observe that in the second decimal place a has digit 1 and b has digit 2, therefore $a < b.$ in the third decimal place a has digit 0. So, if we consider irrational numbers

$x = 0.211011001100011....$

We find that

$a < x < b$

Hence, $x$ is required irrational number.

**9.**

**Sol:**

Let, $a = 0.3010010001$

And, $b = 0.3030030003...$

We observe that in the third decimal place a has digit 1 and b has digit 3, therefore $a < b.$ in the third decimal place a has digit 1. so, if we consider rational and irrational numbers

$x = 0.302$

$y = 0.302002000200002.....$

We find that

$a < x < b$

And, $a < y < b$

Hence, x and y are required rational and irrational numbers respectively.



**10.**

**Sol:**

Let $a = 0.5 = 0.50$

And, $b = 0.55$

We observe that in the second decimal place a has digit 0 and b has digit 5, therefore $a < b.$ so, if we consider irrational numbers

$x = 0.51051005100051...$

$y = 0.530535305353530...$

We find that

$a < x < y < b$

Hence, $x$ and $y$ are required irrational numbers.

**11.**

**Sol:**

Let, $a = 0.1 = 0.10$

And, $b = 0.12$

We observe that in the second decimal place a has digit 0 and b has digit 2, Therefore $a < b.$ So, if we consider irrational numbers

$x = 0.11011001100011...$

$y = 0.111011110111110...$

We find that,

$a < x < y < b$

Hence, $x$ and $y$ are required irrational numbers.

**12.**

**Sol:**

If possible, let $\sqrt{3}+\sqrt{5}$ be a rational number equal to $x$. Then,

$$x = \sqrt{3}+\sqrt{5}$$

$$\Rightarrow x^2 = \left(\sqrt{3}+\sqrt{5}\right)^2$$

$$\Rightarrow x^2 = \left(\sqrt{3}\right)^2 + \left(\sqrt{5}\right)^2 + 2\times\sqrt{3}\times\sqrt{5}$$

$$= 3+5+2\sqrt{15}$$

$$= 8+2\sqrt{15}$$

$$\Rightarrow x^2 - 8 = 2\sqrt{15}$$

$$\Rightarrow \frac{x^2-8}{2} = \sqrt{15}$$

Now, $x$ is rational

$\Rightarrow x^2$ is rational

$\Rightarrow \frac{x^2-8}{2}$ is rational

$\Rightarrow \sqrt{15}$ is rational

But, $\sqrt{15}$ is rational

Thus, we arrive at a contradiction. So, our supposition that $\sqrt{3}+\sqrt{5}$ is rational is wrong.

Hence, $\sqrt{3}+\sqrt{5}$ is an irrational number.

**13.**

**Sol:**

$$\frac{5}{7} = 0.\overline{714285}$$

$$\frac{9}{11} = 0.\overline{81}$$

3 irrational numbers are

0.73073007300073000073.......

0.75075007500075000075.......

0.79079007900079000079.......