Exercise 4.8

Q1

Evaluate the following:

$\sin\left(\sin^{-1}\frac{7}{25}\right)$

Solution

$\sin\left(\sin^{-1}\frac{7}{25}\right)$

$= \frac{7}{25}\quad\left[\because \sin\left(\sin^{-1}x\right) = x \text{ for all } x \in [-1,\ 1]\right]$

Q2

Evaluate the following:

$\sin\left(\cos^{-1}\frac{5}{13}\right)$

Solution

$\sin\left(\cos^{-1}\frac{5}{13}\right)$

$= \sin\left(\sin^{-1}\frac{12}{13}\right)\ldots\ldots\ldots\left[\because \cos^{-1}x = \sin^{-1}\sqrt{1-x^2}\right]$

$= \frac{12}{13}$

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Q3

Evaluate the following:

$\sin\left(\tan^{-1}\frac{24}{7}\right)$

Solution

$\sin\left(\tan^{-1}\frac{24}{7}\right)$

$= \sin\left(\sin^{-1}\frac{24}{25}\right)\ldots\ldots\left[\because \tan^{-1}\left(\frac{b}{p}\right) = \sin^{-1}\left(\frac{b}{h}\right)\right]$

$= \frac{24}{25}$

Q4

Evaluate the following:

$\sin\left(\sec^{-1}\frac{17}{8}\right)$

Solution

$\sin\left(\sec^{-1}\frac{17}{8}\right)$

$= \sin\left(\sin^{-1}\frac{15}{17}\right) \ldots\ldots \left[\because \sec^{-1}\left(\frac{h}{p}\right) = \sin^{-1}\left(\frac{b}{h}\right)\right]$

$= \frac{15}{17}$

Q5

Evaluate the following:

$\operatorname{cos ec}\left(\cos^{-1}\frac{3}{5}\right)$

Solution

$\operatorname{cosec}\left(\cos^{-1}\frac{3}{5}\right)$

$= \operatorname{cosec}\left(\operatorname{cosec}^{-1}\frac{5}{4}\right) \ldots\ldots \left[\because \cos^{-1}\left(\frac{p}{h}\right) = \operatorname{cosec}^{-1}\left(\frac{h}{b}\right)\right]$

$= \frac{5}{4}$

Q6

Evaluate the following:

$\sec\left(\sin^{-1}\frac{12}{13}\right)$

Solution

$\sec\left(\sin^{-1}\frac{12}{13}\right)$

$= \sec\left(\sec^{-1}\frac{13}{5}\right)$

$= \frac{13}{5}$

Q7

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Evaluate $\tan\left(\cos^{-1}\frac{8}{17}\right)$

Solution

$\tan\left(\cos^{-1}\frac{8}{17}\right)$

$$= \tan\left(\tan^{-1}\frac{\sqrt{1-\left(\frac{8}{17}\right)^2}}{\frac{8}{17}}\right) \qquad \left\{\text{Since } \cos^{-1}x = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right\}$$

$$= \tan\left(\tan^{-1}\frac{\frac{15}{17}}{\frac{8}{17}}\right)$$

$$= \tan\left(\tan^{-1}\frac{15}{8}\right)$$

$$= \tan\left(\tan^{-1}\frac{15}{8}\right)$$

$$= \frac{15}{8} \qquad \left\{\text{Since } \tan\left(\tan^{-1}x\right) = x \text{ if } x \in R\right\}$$

Hence,

$$\tan\left(\cos^{-1}\frac{8}{17}\right) = \frac{15}{8}$$

Q8

Evaluate the following:

$\cot\left(\cos^{-1}\frac{3}{5}\right)$

Solution

$\cot\left(\cos^{-1}\frac{3}{5}\right)$

$= \cot\left(\cot^{-1}\frac{3}{4}\right)$

$= \frac{3}{4}$

Q9

Evaluate the following:

$\cos\left(\tan^{-1}\frac{24}{7}\right)$

Solution

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$\cos\left(\tan^{-1}\frac{24}{7}\right)$

$= \frac{1}{\sqrt{1+\left(\frac{24}{7}\right)^2}}$ .... $\left[\because \cos\left(\tan^{-1}x\right) = \frac{1}{\sqrt{1+x^2}}\right]$

$= \frac{7}{25}$

Q10

Prove the following result:

$\tan\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right) = \frac{17}{6}$

Solution

$\tan\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right)$

$= \tan\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right)$ .... $\left[\cos^{-1}\left(\frac{b}{h}\right) = \tan^{-1}\left(\frac{p}{b}\right)\right]$

$= \tan\left(\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right)\right)$ .... $\left[\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$= \tan\left(\tan^{-1}\left(\frac{\frac{17}{12}}{\frac{1}{2}}\right)\right)$

$= \tan\left(\tan^{-1}\left(\frac{17}{6}\right)\right)$

$= \frac{17}{6}$

Q11

Prove the following result:

$\cos\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right) = \frac{6}{5\sqrt{13}}$

Solution

$$\cos\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$$

$$=\cos\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right)\ldots\ldots\left[\begin{matrix}\sin^{-1}\left(\frac{p}{h}\right)=\tan^{-1}\left(\frac{p}{b}\right)\\ \cot^{-1}\left(\frac{b}{p}\right)=\tan^{-1}\left(\frac{p}{b}\right)\end{matrix}\right]$$

$$=\cos\left(\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right)\right)\ldots\ldots\left[\tan^{-1}(x)+\tan^{-1}(y)=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$=\cos\left(\tan^{-1}\left(\frac{\frac{17}{12}}{\frac{1}{2}}\right)\right)$$

$$=\cos\left(\tan^{-1}\left(\frac{17}{6}\right)\right)$$

$$=\cos\left(\cos^{-1}\left(\frac{6}{5\sqrt{13}}\right)\right)\ldots\ldots\left[\tan^{-1}\left(\frac{p}{b}\right)=\cos^{-1}\left(\frac{b}{h}\right)\right]$$

$$=\frac{6}{5\sqrt{13}}$$

Q12

Evaluate the following:

$$\tan\left(\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}\right)=\frac{63}{16}$$

Solution

$$\tan\left(\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}\right)$$

$$=\tan\left(\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{4}{3}\right)\ldots\ldots\left[\begin{matrix}\sin^{-1}\left(\frac{p}{h}\right)=\tan^{-1}\left(\frac{p}{b}\right)\\ \cos^{-1}\left(\frac{b}{h}\right)=\tan^{-1}\left(\frac{p}{b}\right)\end{matrix}\right]$$

$$=\tan\left(\tan^{-1}\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\times\frac{4}{3}}\right)\right)\ldots\ldots\left[\tan^{-1}(x)+\tan^{-1}(y)=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$=\tan\left(\tan^{-1}\left(\frac{\frac{21}{12}}{\frac{4}{9}}\right)\right)$$

$$=\tan\left(\tan^{-1}\left(\frac{63}{16}\right)\right)$$

$$=\frac{63}{16}$$

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Q13

Evaluate the following:

$$\sin\left(\cos^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}\right)=\frac{63}{65}$$

Solution

$$\sin\left(\cos^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}\right)$$

$$=\sin\left(\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{5}{12}\right)\ldots\ldots\ldots\begin{bmatrix}\sin^{-1}\left(\frac{p}{h}\right)=\tan^{-1}\left(\frac{p}{b}\right)\\ \cos^{-1}\left(\frac{b}{h}\right)=\tan^{-1}\left(\frac{p}{b}\right)\end{bmatrix}$$

$$=\sin\left(\tan^{-1}\left(\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\times\frac{5}{12}}\right)\right)\ldots\ldots\ldots\left[\tan^{-1}(x)+\tan^{-1}(y)=\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$=\sin\left(\tan^{-1}\left(\frac{\frac{21}{12}}{\frac{4}{9}}\right)\right)$$

$$=\sin\left(\tan^{-1}\left(\frac{63}{16}\right)\right)$$

$$=\sin\left(\sin^{-1}\left(\frac{63}{65}\right)\right)\ldots\ldots\ldots\left[\tan^{-1}\left(\frac{p}{b}\right)=\sin^{-1}\left(\frac{p}{h}\right)\right]$$

$$=\frac{63}{65}$$

Q14

Solve:

$$\cos\left(\sin^{-1}x\right)=\frac{1}{6}$$

Solution

$$\frac{1}{6}=\cos\left(\sin^{-1}x\right)$$

$$\frac{1}{6}=\cos\left(\cos^{-1}\sqrt{1-x^2}\right)\ldots\ldots\ldots\left[\sin^{-1}x=\cos^{-1}\sqrt{1-x^2}\right]$$

$$\frac{1}{6}=\sqrt{1-x^2}$$

$$\frac{1}{36}=1-x^2$$

$$x^2=\frac{35}{36}$$

$$x=\pm\frac{\sqrt{35}}{6}$$

Q15

Solve:

$\cos\{2\sin^{-1}(-x)\} = 0$

Solution

$0 = \cos\{2\sin^{-1}(-x)\}$

$0 = \cos\left(\sin^{-1}\left(-2x\sqrt{1-x^2}\right)\right)$ .... $\left[2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)\right]$

$0 = \cos\left(\cos^{-1}\sqrt{1-\left(4x^2-4x^4\right)}\right)$ .... $\left[\sin^{-1}x = \cos^{-1}\sqrt{1-x^2}\right]$

$0 = \sqrt{1-\left(4x^2-4x^4\right)}$

$0 = 1-\left(4x^2-4x^4\right)$

$4x^4-4x^2+1=0$

$4x^4-2x^2-2x^2+1=0$

$2x^2\left(2x^2-1\right)-1\left(2x^2-1\right)=0$

$\left(2x^2-1\right)^2=0$

$2x^2-1=0$

$x = \pm\frac{1}{\sqrt{2}}$

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