Binary Operations Ex 3.1 Q1(i)

We have,

$$a * b = a^b \text{ for all } a, b \in N$$

Let $a \in N$ and $b \in N$

$\Rightarrow \quad a^b \in N$

$\Rightarrow \quad a * b \in N$

The operation $*$ defines a binary operation on N

Binary Operations Ex 3.1 Q1(ii)

We have,

$a \circ b = a^b$ for all $a, b \in Z$

Let $a \in Z$ and $b \in Z$

$\Rightarrow \quad a^b \notin Z \qquad \Rightarrow a \circ b \notin Z$

For example, if $\quad a = 2, \quad b = -2$

$\Rightarrow \quad a^b = 2^{-2} = \frac{1}{4} \notin Z$

$\therefore$ The operation '$\circ$' does not define a binary operation on Z.

Binary Operations Ex 3.1 Q1(iii)

We have,

$a * b = a + b - 2$ for all $a, b \in N$

Let $a \in N$ and $b \in N$

Then, $a + b - 2 \notin N$ for all $a, b \in N$

$\Rightarrow \quad a * b \notin N$

For example $\quad a = 1, \; b = 1$

$\Rightarrow \quad a + b - 2 = 0 \notin N$

$\therefore$ The operation $*$ does not define a binary operation on N

Binary Operations Ex 3.1 Q1(iv)

We have,

$S = \{1, 2, 3, 4, 5\}$

and, $\quad a \times_6 b$ = Remainder when ab is divided by 6

Let $a \in S$ and $b \in S$

$\Rightarrow \quad a \times_6 b \notin S$ for all $a, b \in S$

For example, $a = 2, \; b = 3$

$\Rightarrow \quad 2 \times_6 3$ = Remainder when 6 is divided by 6 = $0 \notin S$

$\therefore \quad \times_6$ does not define a binary oparation on S

Binary Operations Ex 3.1 Q1(v)

We have,

$$S = \{0, 1, 2, 3, 4, 5\}$$

and, $$a +_6 b = \begin{cases} a+b; \text{ if } a+b < 6 \\ a+b-6; \text{ if } a+b \geq 6 \end{cases}$$

Let $a \in S$ and $b \in S$ such that $a + b < 6$

Then $a +_6 b = a + b \in S$ $\quad [\because\ a + b < 6 = 0, 1, 2, 3, 4, 5]$

Let $a \in S$ and $b \in S$ such that $a + b > 6$

Then $a +_6 b = a + b - 6 \in S$ $\quad [\because \text{if } a + b \geq 6 \text{ then } a + b - 6 \geq 0 = 0, 1, 2, 3, 4, 5]$

$\therefore$ $a +_6 b \in S$ for $a, b \in S$

$\therefore$ $+_6$ defines a binary oparation on S

Binary Operations Ex 3.1 Q1(vi)

We have,

$$a \circ b = a^b + b^a \text{ for all } a, b \in N$$

Let $a \in N$ and $b \in N$

$\Rightarrow$ $a^b \in N$ and $b^a \in N$

$\Rightarrow$ $a^b + b^a \in N$

$\Rightarrow$ $a \circ b \in N$

Thus, the operation '$\circ$' defines a binary relation on N

Binary Operations Ex 3.1 Q1(vii)

We have,

$$a * b = \frac{a-1}{b+1} \text{ for all } a, b \in Q$$

Let $a \in Q$ and $b \in Q$

Then $\dfrac{a-1}{b+1} \notin Q$ for $b = -1$

$\Rightarrow$ $a * b \notin Q$ for all $a, b \in Q$

Thus, the operation $*$ does not define a binary operation on Q

Binary Operations Ex 3.1 Q2

(i) On $\mathbf{Z}^+$, * is defined by $a * b = a - b$.
It is not a binary operation as the image of (1, 2) under * is $1 * 2 = 1 - 2 = -1 \notin \mathbf{Z}^+$.

(ii) On $\mathbf{Z}^+$, * is defined by $a * b = ab$.
It is seen that for each $a, b \in \mathbf{Z}^+$, there is a unique element ab in $\mathbf{Z}^+$.
This means that * carries each pair (a, b) to a unique element $a * b = ab$ in $\mathbf{Z}^+$.
Therefore, * is a binary operation.

(iii) On $\mathbf{R}$, * is defined by $a * b = ab^2$.
It is seen that for each $a, b \in \mathbf{R}$, there is a unique element ab^2 in $\mathbf{R}$.
This means that * carries each pair (a, b) to a unique element $a * b = ab^2$ in $\mathbf{R}$.
Therefore, * is a binary operation.

(iv) On $\mathbf{Z}^+$, * is defined by $a * b = |a - b|$.
It is seen that for each $a, b \in \mathbf{Z}^+$, there is a unique element $|a - b|$ in $\mathbf{Z}^+$.
This means that * carries each pair (a, b) to a unique element $a * b = |a - b|$ in $\mathbf{Z}^+$.
Therefore, * is a binary operation.

(v) On $\mathbf{Z}^+$, * is defined by $a * b = a$.
* carries each pair (a, b) to a unique element $a * b = a$ in $\mathbf{Z}^+$.
Therefore, * is a binary operation.

(vi) on R, * is defined by $a * b = a + 4b^2$
it is seen that for each element $a, b \in R$, there is unique element $a + 4b^2$ in R
This means that * carries each pair (a, b) to a unique element $a * b = a + 4b^2$ in R.
Therefore, * is a binary operation.

Binary Operations Ex 3.1 Q3

It is given that, $a * b = 2a + b - 3$
Now

$$3 * 4 = 2 \times 3 + 4 - 3$$
$$= 10 - 3$$
$$= 7$$

Binary Operations Ex 3.1 Q4

The operation * on the set A = {1, 2, 3, 4, 5} is defined as
$a * b$ = L.C.M. of a and b.
2*3 = L.C.M of 2 and 3 = 6. But 6 does not belong to the given set.
Hence, the given operation * is not a binary operation.

Binary Operations Ex 3.1 Q5

We have,

$$S = \{a, b, c\}$$

We know that the total number of binary operation on a set S with n element is n^{n^2}

$\Rightarrow$ Total number of binary operation on $S = \{a, b, c\} = 3^{3^2} = 3^9$

Binary Operations Ex 3.1 Q6

We have,

$$S = \{a, b\}$$

The total number of binary operation on $S = \{a, b\}$ in $2^{2^2} = 2^4 = 16$

Binary Operations Ex 3.1 Q7

We have,

$$M = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \in R - \{0\} \right\} \text{ and}$$

$$A * B = AB \text{ for all } A, B \in M$$

Let $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \in M$ and $B = \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} \in M$

Now, $AB = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} = \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix}$

$\therefore \quad a \in R,\ b \in R,\ c \in R,\ \&\ d \in R$

$\Rightarrow \quad ac \in R$ and $bd \in R$

$\Rightarrow \quad \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix} \in M$

$\Rightarrow \quad A * B \in M$

Thus, the operater $*$ difines a binary operation on M

Binary Operations Ex 3.1 Q8

S = set of rational numbers of the form $\frac{m}{n}$ where $m \in Z$ and $n = 1, 2, 3$

Also, $a * b = ab$

Let $a \in S$ and $b \in S$

$\Rightarrow \quad ab \notin S$

For example $a = \frac{7}{3}$ and $b = \frac{5}{2}$

$\Rightarrow \quad ab = \frac{35}{6} \notin S$

$\therefore \quad a * b \notin S$

Hence, the operater $*$ does not define a binary operation on S

Binary Operations Ex 3.1 Q9

It is given that, a*b = 2a + b

Now

$(2*3) = 2 \times 2 + 3$

$= 4 + 3$

$= 7$

$(2*3)*4 = 7*4 = 2 \times 7 + 4$

$= 14 + 4$

$= 18$

Binary Operations Ex 3.1 Q10

It is given that, a*b = LCM (a, b)

Now

5*7 = LCM (5, 7)

= 35