# RD Sharma Solutions
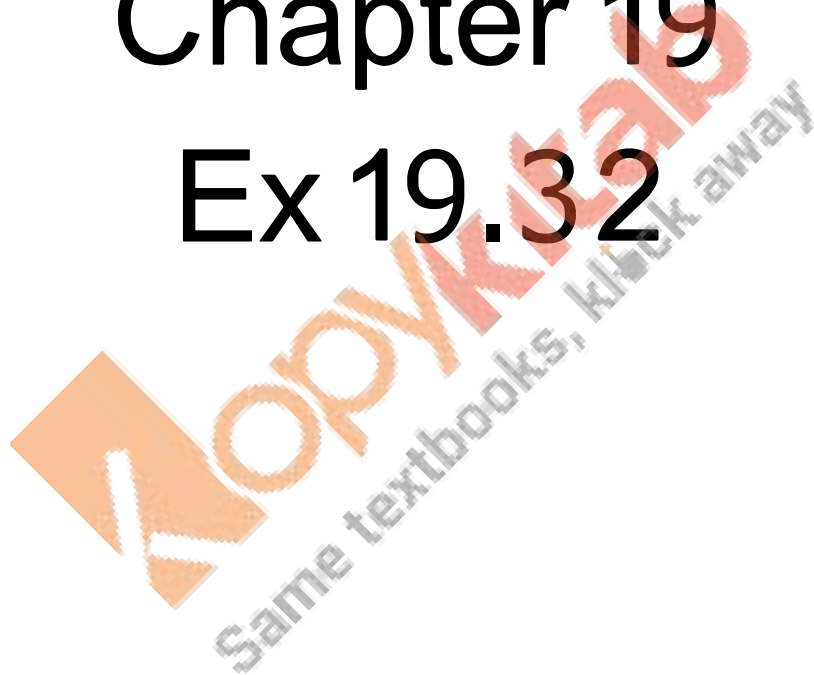
# Class 12 Maths

# Chapter 19

# Ex 19.32

Let $I = \int \dfrac{1}{(x-1)\sqrt{2x+3}}\,dx$

Let $I = \int \dfrac{1}{(x-1)\sqrt{2x+3}}\,dx$

Let $2x + 3 = t^2$

$\Rightarrow \qquad 2dx = 2t\,dt$

$\therefore \qquad I = \int \dfrac{t\,dt}{\left(\dfrac{t^2-3}{2}-1\right)t}$

$\qquad\qquad = 2\int \dfrac{dt}{t^2-5}$

$\qquad\qquad = \dfrac{2}{2\sqrt{5}}\,log\left|\dfrac{t-\sqrt{5}}{t+\sqrt{5}}\right| + c$

Thus,

$$I = \dfrac{1}{\sqrt{5}}\,log\left|\dfrac{\sqrt{2x+3}-\sqrt{5}}{\sqrt{2x+3}+\sqrt{5}}\right| + c$$

**Indefinite Integrals Ex 19.32 Q3**

Let $I = \int \dfrac{1}{(x-1)\sqrt{x+2}}\,dx$

$$I = \int \dfrac{(x-1)+2}{(x-1)\sqrt{x+2}}\,dx$$

$$I = \int \dfrac{dx}{\sqrt{x+2}} + 2\int \dfrac{dx}{(x-1)\sqrt{x+2}} \qquad\qquad \text{---(A)}$$

Now,

$$\int \dfrac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_1$$

and,

$$\int \dfrac{dx}{(x-1)\sqrt{x+2}}$$

Let $x+2 = t^2$

$\Rightarrow \quad dx = 2t\,dt$

$\therefore \quad \int \dfrac{dx}{(x-1)\sqrt{x+2}} = 2\int \dfrac{t\,dt}{(t^2-3)t} = 2\int \dfrac{dt}{t^2-3}$

$$= \dfrac{2\times 1}{2\sqrt{3}} \log\left|\dfrac{t-\sqrt{3}}{t+\sqrt{3}}\right| + c_2$$

$$= \dfrac{1}{\sqrt{3}} \log\left|\dfrac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + 2$$

Thus, form (A),

$$I = 2\sqrt{x+2} + c_1 + \dfrac{2}{\sqrt{3}} \log\left|\dfrac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c_2$$

Hence,

$$I = 2\sqrt{x+2} + \dfrac{2}{\sqrt{3}} \log\left|\dfrac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c$$

**Indefinite Integrals Ex 19.32 Q4**

Let $I = \int \dfrac{x^2}{(x-1)\sqrt{x+2}}\,dx$

$$= \int \dfrac{(x^2 - 1 + 1)}{(x-1)\sqrt{x+2}}\,dx$$

$$= \int \dfrac{(x+1)(x-1)}{(x-1)\sqrt{x+2}}\,dx + \int \dfrac{dx}{(x-1)\sqrt{x+2}}$$

$$= \int \dfrac{(x+1)}{\sqrt{x+2}}\,dx + \int \dfrac{dx}{(x-1)\sqrt{x+2}}$$

$$= \int \dfrac{(x+2) - 1}{\sqrt{x+2}}\,dx + \int \dfrac{dx}{(x-1)\sqrt{x+2}}$$

$$I = \int \sqrt{x+2}\,dx - \int \dfrac{dx}{\sqrt{x+2}} + \int \dfrac{dx}{(x-1)\sqrt{x+2}} \qquad \text{---(A)}$$

Now,

$$\int \sqrt{x+2}\,dx = \dfrac{2}{3}(x+2)^{\frac{3}{2}} + c_1$$

and,

$$\int \dfrac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_2$$

$$\int \dfrac{dx}{(x-1)\sqrt{x+2}}$$

Let $x + 2 = t^2$

$\Rightarrow \qquad dx = 2t\,dt$

$\therefore \qquad 2\int \dfrac{t\,dt}{(t^2 - 3)t} = 2\int \dfrac{dt}{t^2 - 3}$

$$= \dfrac{2}{2\sqrt{3}} \log\left|\dfrac{t - \sqrt{3}}{t + \sqrt{3}}\right| + c_3$$

$\therefore \qquad \int \dfrac{dx}{(x-1)\sqrt{x+2}} = \dfrac{1}{\sqrt{3}} \log\left|\dfrac{\sqrt{x+2} - \sqrt{3}}{\sqrt{x+2} + \sqrt{3}}\right| + c_3$

Thus, form (A)

$$I = \dfrac{2}{3}(x+2)^{\frac{3}{2}} - 2\sqrt{x+2} + \dfrac{1}{\sqrt{3}} \log\left|\dfrac{\sqrt{x+2} - \sqrt{3}}{\sqrt{x+2} + \sqrt{3}}\right| + c \qquad [\text{when } c = c_1 + c_2 + c_3]$$

**Indefinite Integrals Ex 19.32 Q5**

Let $I = \int \dfrac{x}{(x-3)\sqrt{x+1}} dx$

$= \int \dfrac{(x-3)+3}{(x-3)\sqrt{x+1}} dx$

$I = \int \dfrac{dx}{\sqrt{x+1}} + 3\int \dfrac{dx}{(x-3)\sqrt{x+1}}$       ---(A)

Now,

$$\int \dfrac{dx}{\sqrt{x+1}} = 2\sqrt{x+1} + c_1$$

and,

$$\int \dfrac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_2$$

$$\int \dfrac{dx}{(x-3)\sqrt{x+1}}$$

Let $x + 1 = t^2$

$\Rightarrow$     $dx = 2t\,dt$

$\therefore$     $\int \dfrac{dx}{(x-3)\sqrt{x+1}} = 2\int \dfrac{t\,dt}{(t^2-4)t}$

$= 2\left|\dfrac{dt}{t^2-4}\right|$

$= \dfrac{2}{2\times 2} \log\left|\dfrac{t-2}{t+2}\right| + c_2$

$\therefore$     $\int \dfrac{dx}{(x-3)\sqrt{x+1}} = \dfrac{1}{2} \log\left|\dfrac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right| + c_2$

Thus, form (A)

$$I = 2\sqrt{x+1} + \dfrac{3}{2}\log\left|\dfrac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right| + c$$        [when $c = c_1 + c_2$]

**Indefinite Integrals Ex 19.32 Q6**

Let $I = \int \dfrac{x}{\left(x^2+1\right)\sqrt{x}}\,dx$

Let $x = t^2$

$\Rightarrow \quad dx = 2t\,dt$

$\therefore \quad 2\int \dfrac{t\,dt}{\left(t^2+1\right)t}$

$= 2\left|\dfrac{dt}{t^4+1}\right|$

Dividing numerator and denominator by $t^2$

$I = 2\int \dfrac{\dfrac{t}{t^2}}{\left(t^2+\dfrac{1}{t^2}\right)}\,dt$

$= \int \dfrac{\left(1+\dfrac{1}{t^2}\right)-\left(1-\dfrac{1}{t^2}\right)}{\left(t^2+\dfrac{1}{t^2}\right)}\,dt$

$= \int \dfrac{\left(1+\dfrac{1}{t^2}\right)}{\left(t-\dfrac{1}{t}\right)^2+2}\,dt - \int \dfrac{\left(1-\dfrac{1}{t^2}\right)}{\left(1+\dfrac{1}{t}\right)^2-2}\,dt$

Let $t-\dfrac{1}{t} = z$

$\Rightarrow \quad \left(1+\dfrac{1}{t^2}\right)dt = dz \qquad$ [For Ist part]

and,

$t+\dfrac{1}{t} = y$

$\Rightarrow \quad \left(1-\dfrac{1}{t^2}\right)dt = dy \qquad$ [For IInd part]

$\therefore \quad I = \int \dfrac{dz}{z^2+2} - \int \dfrac{dy}{y^2-2}$

$= \dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{z}{\sqrt{2}}\right) - \dfrac{1}{2\sqrt{2}}\log\left|\dfrac{y-\sqrt{2}}{y+\sqrt{2}}\right| + c$

$= \dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{t^2-1}{\sqrt{2}\,t}\right) - \dfrac{1}{2\sqrt{2}}\log\left|\dfrac{x+1-\sqrt{2}\,x}{x+1+\sqrt{2x}}\right| + c$

**Indefinite Integrals Ex 19.32 Q7**

Let $\quad I = \int \dfrac{x}{\left(x^2 + 2X + 2\right)\sqrt{x+1}}\, dx$

Let $x + 1 = t^2$

$\Rightarrow \qquad dx = 2t\, dt$

$$= 2\int \dfrac{\left(t^2 - 1\right) t\, dt}{\left(t^4 + 1\right) t}$$

$$= 2\int \dfrac{\left(t^2 - 1\right) dt}{\left(t^4 + 1\right)}$$

$$= 2\int \dfrac{\left(1 - \dfrac{1}{t^2}\right) dt}{t^2 + \dfrac{1}{t^2}}$$

$$= 2\int \dfrac{\left(1 - \dfrac{1}{t^2}\right) dt}{\left(t + \dfrac{1}{t}\right)^2 - 2}$$

Let $t + \dfrac{1}{t} = y$

$$\Rightarrow \qquad \left(1 - \dfrac{1}{t^2}\right) dt = dy$$

$$\therefore \qquad I = 2\int \dfrac{dy}{y^2 - 2}$$

$$= \dfrac{2}{2\sqrt{2}} \log \left| \dfrac{y - \sqrt{2}}{y + \sqrt{2}} \right| + c$$

Thus,

$$I = \dfrac{1}{\sqrt{2}} \log \left| \dfrac{t^2 + 1 - \sqrt{2}t}{t^2 + 1 + \sqrt{2}t} \right| + c$$

Hence,

$$I = \dfrac{1}{\sqrt{2}} \log \left| \dfrac{x + 2 - \sqrt{2(x+1)}}{x + 2 + \sqrt{2(x+1)}} \right| + c$$

**Indefinite Integrals Ex 19.32 Q8**

Let $I = \int \dfrac{1}{(x-1)\sqrt{x^2+1}}\,dx$

Let $x - 1 = \dfrac{1}{t}$

$\Rightarrow \qquad dx = -\dfrac{1}{t^2}\,dt$

$\therefore \qquad I = -\int \dfrac{\dfrac{1}{t^2}dt}{\dfrac{1}{t}\sqrt{\left(\dfrac{1}{t}+1\right)^2+1}}$

$\qquad\qquad = -\int \dfrac{dt}{\sqrt{2t^2+2t+1}}$

$\qquad\qquad = -\dfrac{1}{\sqrt{2}}\int \dfrac{dt}{\sqrt{t^2+t+\dfrac{1}{2}}}$

$\qquad\qquad = -\dfrac{1}{\sqrt{2}}\int \dfrac{dt}{\sqrt{\left(t+\dfrac{1}{2}\right)^2+\dfrac{1}{4}}}$

$\therefore \qquad I = -\dfrac{1}{\sqrt{2}}\,\log\left|\left(t+\dfrac{1}{2}\right)+\sqrt{\left(t+\dfrac{1}{2}\right)^2+\dfrac{1}{4}}\right|+c \left[\text{When } t = \dfrac{1}{x-1}\right]$

**Indefinite Integrals Ex 19.32 Q9**

Let $\quad I = \int \dfrac{1}{(x+1)\sqrt{x^2+x+1}}\,dx$

Let $x + 1 = \dfrac{1}{t}$

$\Rightarrow \qquad dx = -\dfrac{1}{t^2}\,dt$

$\therefore \qquad I = -\int \dfrac{\dfrac{1}{t^2}\,dt}{\dfrac{1}{t}\sqrt{\left(\dfrac{1}{t^2}+\dfrac{1}{t}-1\right)}}$

$\qquad\qquad = -\int \dfrac{dt}{\sqrt{1+t-t^2}}$

$\qquad\qquad = -\int \dfrac{dt}{\sqrt{\dfrac{5}{4}-\left(\dfrac{1}{4}-t+t^2\right)}}$

$\qquad\qquad = -\int \dfrac{dt}{\sqrt{\dfrac{5}{4}-\left(t-\dfrac{1}{2}\right)^2}}$

$\qquad\qquad = -\sin^{-1}\left(\dfrac{t-\dfrac{1}{2}}{\dfrac{\sqrt{5}}{2}}\right)+c$

$\therefore \qquad I = -\sin^{-1}\left(\dfrac{2t-1}{\sqrt{5}}\right)+c \qquad\qquad\qquad \left[\text{When } t = \dfrac{1}{x+1}\right]$

**Indefinite Integrals Ex 19.32 Q11**

$$I = \int \frac{x}{\left(x^2 + 4\right) \sqrt{x^2 + 1}} \, dx$$

Let $x^2 + 1 = u^2$

$\Rightarrow \quad 2x\,dx = 2u\,du$

$\therefore \quad I = \int \frac{u}{\left(u^2 + 3\right) u} \, du$

$$= \int \frac{1}{u^2 + 3} \, du$$

$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{u}{\sqrt{3}}\right) + C$$

$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\sqrt{\frac{x^2 + 1}{3}}\right) + C$$

**Indefinite Integrals Ex 19.32 Q12**

Let $\quad I = \int \dfrac{1}{\left(1+x^2\right)\sqrt{1-x^2}}\,dx$

Let $x = \dfrac{1}{t}$

$\Rightarrow \qquad dx = -\dfrac{1}{t^2}\,dt$

$\therefore \qquad I = \int \dfrac{-\dfrac{1}{t^2}\,dt}{\left(\dfrac{1}{t^2}+1\right)\sqrt{\left(1-\dfrac{1}{t^2}\right)}}$

$\qquad\qquad = -\int \dfrac{t\,dt}{\left(t^2+1\right)\sqrt{t^2-1}}$

Let $t^2 - 1 = u^2$

$\Rightarrow \qquad 2t\,dt = 2u\,du$

$\qquad I = -\int \dfrac{u\,du}{\left(u^2+2\right)u}$

$\qquad\qquad = -\int \dfrac{du}{u^2+2}$

$\qquad\qquad = -\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{u}{\sqrt{2}}\right)+c$

$\qquad\qquad = -\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{\sqrt{t^2-1}}{\sqrt{2}}\right)+c$

Thus,

$\qquad I = -\dfrac{1}{\sqrt{2}}\tan^{-1}\left|\sqrt{\dfrac{1-x^2}{2x^2}}\right|+c$

**Indefinite Integrals Ex 19.32 Q13**

Let $\quad I = \int \dfrac{1}{\left(2x^2 + 3\right)\sqrt{x^2 - 4}}\, dx$

Let $x = \dfrac{1}{t}$

$\Rightarrow \qquad dx = -\dfrac{1}{t^2}\, dt$

$\therefore \qquad I = \int \dfrac{-\dfrac{1}{t^2}\, dt}{\left(\dfrac{2}{t^2} + 3\right)\sqrt{\left(\dfrac{1}{t^2} - 4\right)}}$

$\qquad\qquad = -\int \dfrac{t\, dt}{\left(2 + 3t^2\right)\sqrt{1 - 4t^2}}$

Let $1 - 4t^2 = u^2$

$\Rightarrow \qquad -8t\, dt = 2u\, du$

$\therefore \qquad I = \dfrac{1}{4} \int \dfrac{u\, du}{\dfrac{\left(11 - 3u^2\right)}{4}\, u}$

$\qquad\qquad = \dfrac{1}{3} \int \dfrac{du}{\dfrac{11}{3} - u^2}$

$\qquad\qquad = \dfrac{1}{2\sqrt{33}} \log \left| \dfrac{u - \sqrt{\dfrac{11}{3}}}{u + \sqrt{\dfrac{11}{3}}} \right| + c$

$\qquad\qquad = \dfrac{1}{2\sqrt{33}} \log \left| \dfrac{\sqrt{1 - 4t^2} - \sqrt{\dfrac{11}{3}}}{\sqrt{1 - 4t^2} + \sqrt{\dfrac{11}{3}}} \right| + c$

Hence,

$\qquad I = \dfrac{1}{2\sqrt{33}} \log \left| \dfrac{\sqrt{11}\,x + \sqrt{3x^2 - 12}}{\sqrt{11}\,x - \sqrt{3x^2 - 12}} \right| + c$

**Indefinite Integrals Ex 19.32 Q14**

$$I = \int \frac{x}{\left(x^2 + 4\right)\sqrt{x^2 + 9}}\, dx$$

Let $x^2 + 9 = u^2$

$\Rightarrow \qquad 2x\, dx = 2u\, du$

$\therefore \qquad I = \int \frac{u}{\left(u^2 - 5\right)u}\, du$

$$= \int \frac{du}{u^2 - 5}$$

$$= \frac{1}{2\sqrt{5}} \log\left(\frac{u - \sqrt{5}}{u + \sqrt{5}}\right) + C$$

$$= \frac{1}{2\sqrt{5}} \log\left(\frac{\sqrt{x^2 + 9} - \sqrt{5}}{\sqrt{x^2 + 9} + \sqrt{5}}\right) + C$$