RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.25

Indefinite Integrals Ex 19.25 Q1

Let $I = \int x \cos x \, dx$

Using integration by parts,

$$I = x\int \cos x \, dx - \int\left(1 \times \int \cos x \, dx\right)dx + c$$

$$= x \sin x - \int \sin x \, dx + c$$

$$I = x \sin x + \cos x + c$$

Indefinite Integrals Ex 19.25 Q2

Let $I = \int \log(x+1)\, dx$

$= \int 1 \times \log(x+1)\, dx$

Using integration by parts,

$$I = \log(x+1)\int 1\, dx - \int\left(\frac{1}{x+1} \times \int 1\, dx\right)dx + c$$

$$= x \log(x+1) - \int\left(\frac{x}{x+1}\right)dx + c$$

$$= x \log(x+1) - \int\left(1 - \frac{1}{x+1}\right)dx + c$$

$$I = x \log(x+1) - x + \log(x+1) + c$$

Indefinite Integrals Ex 19.25 Q3

Let $I = \int x^3 \log x \, dx$

Using integration by parts,

$$I = \log x \int x^3 dx - \int \left(\frac{1}{x} \times \int x^3 dx \right) dx + c$$

$$= \frac{x^4}{4} \log x - \int \frac{x^4}{4x} dx + c$$

$$= \frac{x^4}{4} \log x - \frac{1}{4} \int x^3 dx + c$$

$$= \frac{x^4}{4} \log x - \frac{1}{4} \int \frac{x^4}{4} dx + c$$

$$I = \frac{x^4}{4} \log x - \frac{1}{16} x^4 + c$$

Indefinite Integrals Ex 19.25 Q4

Take first function as x and second function as e^x. The integral of the second function is e^x.
Therefore, $\int xe^x \, dx = xe^x - \int 1.\, e^x \, dx = xe^x - e^x + C.$

Indefinite Integrals Ex 19.25 Q5

Let $I = \int xe^{2x} dx$

Using integration by parts,

$$I = x \int e^{2x} dx - \int \left(1 \times \int e^{2x} dx \right) dx + c$$

$$= \frac{xe^{2x}}{2} - \int \left(\frac{e^{2x}}{2} \right) dx + c$$

$$= \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + c$$

$$I = \left(\frac{x}{2} - \frac{1}{4} \right) e^{2x} + c$$

Indefinite Integrals Ex 19.25 Q6

Let $I = \int x^2 e^{-x} dx$

Using integration by parts,

$$I = x^2 \int e^{-x}\, dx - \int \left(2x \int e^{-x}\, dx\right)$$

$$= -x^2 e^{-x} - \int (2x)\left(-e^{-x}\right)$$

$$= -x^2 e^{-x} + 2\int x\, e^{-x} dx$$

$$= -x^2 e^{-x} + 2\left[x \int e^{-x} dx - \int \left(1 \times \int e^{-x} dx\right) dx\right]$$

$$= -x^2 e^{-x} + 2\left[x\left(-e^{-x}\right) - \int \left(-e^{-x}\right) dx\right]$$

$$= -x^2 e^{-x} - 2xe^{-x} + 2\int e^{-x} dx$$

$$I = -x^2 e^{-x} - 2xe^{-x} - 2e^{-x} + c$$

$$I = -e^{-x}\left(x^2 + 2x + 2\right) + c$$

Indefinite Integrals Ex 19.25 Q7

Let $I = \int x^2 \cos x\, dx$

Using integration by parts,

$$I = x^2 \int \cos x\, dx - \int \left(2x \int \cos x\, dx\right) dx$$

$$= x^2 \sin x - 2\int (x)(\sin x) dx$$

$$= x^2 \sin x - 2\left[x \int \sin x\, dx - \int \left(1 \times \int \sin x\, dx\right) dx\right]$$

$$= x^2 \sin x - 2\left[x(-\cos x) - \int (-\cos x) dx\right]$$

$$= x^2 \sin x + 2x \cos x - 2\int (\cos x) dx$$

$$I = x^2 \sin x + 2x \cos x - 2 \sin x + c$$

Indefinite Integrals Ex 19.25 Q8

Let $I = \int x^2 \cos 2x \, dx$

Using integration by parts,

$$I = x^2 \int \cos 2x \, dx - \int \left(2x \int \cos 2x \, dx\right) dx$$

$$= x^2 \frac{\sin 2x}{2} - 2\int x \left(\frac{\sin 2x}{2}\right) dx$$

$$= \frac{1}{2} x^2 \sin 2x - \int x \sin 2x \, dx$$

$$= \frac{1}{2} x^2 \sin 2x - \left[x \int \sin 2x \, dx - \int \left(1 \int \sin 2x \, dx\right) dx\right]$$

$$= \frac{1}{2} x^2 \sin 2x - \left[x \left(\frac{-\cos 2x}{2}\right) - \int \left(-\frac{\cos 2x}{2}\right) dx\right]$$

$$= \frac{1}{2} x^2 \sin 2x + \frac{x}{2} \cos 2x - \frac{1}{2} \int (\cos 2x) \, dx$$

$$I = \frac{1}{2} x^2 \sin 2x + \frac{x}{2} \cos 2x - \frac{1}{4} \sin 2x + c$$

Indefinite Integrals Ex 19.25 Q9

Let $I = \int x \sin 2x \, dx$

Using integration by parts,

$$I = x \int \sin 2x \, dx - \int \left(1 \int \sin 2x \, dx\right) dx$$

$$= x \left(-\frac{\cos 2x}{2}\right) - \int \left(-\frac{\cos 2x}{2}\right) dx$$

$$= -\frac{x}{2} \cos 2x + \frac{1}{2} \int \cos 2x \, dx$$

$$= -\frac{x}{2} \cos 2x + \frac{1}{2} \frac{\sin 2x}{2} + c$$

$$I = -\frac{x}{2} \cos 2x + \frac{1}{4} \sin 2x + c$$

Indefinite Integrals Ex 19.25 Q10

Let $I = \int \frac{\log(\log x)}{x} dx$

$= \int \left(\frac{1}{x}\right)(\log(\log x)) dx$

Using integration by parts,

$I = \log \log x \int \frac{1}{x} dx - \int \left(\frac{1}{x \log x} \int \frac{1}{x} dx\right) dx$

$= \log x \times \log(\log x) - \int \left(\frac{1}{x \log x} \log x\right) dx$

$= \log x \times \log(\log x) - \int \frac{1}{x} dx$

$= \log x \times \log(\log x) - \log x + c$

$I = \log x\,(\log\log x - 1) + c$

Indefinite Integrals Ex 19.25 Q11

Let $I = \int x^2 \cos x \, dx$

Using integration by parts,

$I = x^2 \int \cos x \, dx - \int \left(2x \int \cos x \, dx\right) dx$

$= x^2 \sin x - 2 \int x \sin x \, dx$

$= x^2 \sin x - 2\left[x \int \sin x \, dx - \int \left(1 \int \sin x \, dx\right) dx\right]$

$= x^2 \sin x - 2\left[x(-\cos x) - \int (-\cos x) \, dx\right]$

$= x^2 \sin x + 2x \cos x - 2 \int (\cos x) \, dx$

$I = x^2 \sin x + 2x \cos x - 2 \sin x + c$

Indefinite Integrals Ex 19.25 Q12

Let $I = \int x \, \text{cosec}^2 x \, dx$

Using integration by parts,

$I = x \int \text{cosec}^2 x \, dx - \int \left(\int \text{cosec}^2 x \, dx\right) dx$

$= -x \cot x + \int \cot x \, dx$

$= -x \cot x + \log|\sin x| + c$

Indefinite Integrals Ex 19.25 Q13

Let $I = \int x \cos^2 x\, dx$

Using integration by parts,

$$I = x\int \cos^2 x\, dx - \int \left(1\int \cos^2 x\, dx\right) dx$$

$$= x\int\left(\frac{\cos 2x + 1}{2}\right)dx - \int\left(\int\left(\frac{1+\cos 2x}{2}\right)dx\right)dx$$

$$= \frac{x}{2}\left[\frac{\sin 2x}{2} + x\right] - \frac{1}{2}\int\left(x + \frac{\sin 2x}{2}\right)dx$$

$$= \frac{x}{4}\sin 2x + \frac{x^2}{2} - \frac{1}{2}\times\frac{x^2}{2} - \frac{1}{4}\left(-\frac{\cos 2x}{2}\right) + c$$

$$I = \frac{x}{4}\sin 2x + \frac{x^2}{4} + \frac{1}{8}\cos 2x + c$$

Indefinite Integrals Ex 19.25 Q14

Let $I = \int x^n \log x\, dx$

Using integration by parts,

$$I = \log x\int x^2 dx - \int\left(\frac{1}{x}\int x^2 dx\right)dx$$

$$= \frac{x^{n+1}}{n+1}\log x - \int\left(\frac{1}{x}\times\frac{x^{n+1}}{n+1}\right)dx$$

$$= \frac{x^{n+1}}{n+1}\log x - \int\left(\frac{x^n}{n+1}\right)dx$$

$$I = \frac{x^{n+1}}{n+1}\log x - \frac{1}{(n+1)^2}x^{n+1} + c$$

Indefinite Integrals Ex 19.25 Q15

$$\int\frac{\log x}{x^n}dx = \int(\log x)\left(\frac{1}{x^n}\right)dx$$

by integration by parts

$$\int(\log x)\left(\frac{1}{x^n}\right)dx = \log x\int\left(\frac{1}{x^n}\right)dx - \int\left(\frac{d(\log x)}{dx}\right)\left(\int\left(\frac{1}{x^n}\right)dx\right)dx$$

$$= \log x\left(\frac{x^{1-n}}{1-n}\right) - \int\frac{1}{x}\left(\frac{x^{1-n}}{1-n}\right)dx = \log x\left(\frac{x^{1-n}}{1-n}\right) - \int\left(\frac{x^{-n}}{1-n}\right)dx$$

$$= \log x\left(\frac{x^{1-n}}{1-n}\right) - \left(\frac{1}{1-n}\right)\left(\frac{x^{1-n}}{1-n}\right) = \log x\left(\frac{x^{1-n}}{1-n}\right) - \left(\frac{x^{1-n}}{[1-n]^2}\right) + C$$

Indefinite Integrals Ex 19.25 Q16

Let $I = \int x^2 \sin^2 x\, dx$

$$= \int x^2 \left(\frac{1-\cos 2x}{2}\right) dx$$

$$= \int \frac{x^2}{2} dx - \int \left(\frac{x^2 \cos 2x}{2}\right) dx$$

$$= \frac{x^3}{6} - \frac{1}{2}\left[\int x^2 \cos 2x\, dx\right]$$

$$= \frac{x^3}{6} - \frac{1}{2}\left[x^2 \int \cos 2x\, dx - \int \left(2x \int \cos 2x\, dx\right) dx\right]$$

$$= \frac{x^3}{6} - \frac{1}{2}\left(x^2 \frac{\sin 2x}{2}\right) + \frac{1}{2} \times 2\int \left(x \frac{\sin 2x}{2}\right) dx$$

$$= \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x + \frac{1}{2}\left[x \int \sin 2x\, dx - \int \left(1 \int \sin 2x\, dx\right) dx\right]$$

$$= \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x + \frac{1}{2}\left[x\left(-\frac{\cos 2x}{2}\right) - \int \left(-\frac{\cos 2x}{2}\right) dx\right]$$

$$= \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x - \frac{1}{4} x \cos 2x + \frac{1}{4} \frac{\sin 2x}{2} + c$$

$$I = \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x - \frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x + c$$

Indefinite Integrals Ex 19.25 Q17

Let $I = \int 2x^3 e^{x^2} x\, dx$

Let $x^2 = t$

$2x\, dx = dt$

$I = \int t \times e^t dt$

Using integration by parts,

$$= t \int e^t dt - \int \left(1 \times \int e^t dt\right) dt$$

$$= te^t - \int e^t dt$$

$$= te^t - e^t + c$$

$$= e^t (t-1) + c$$

$$I = e^{x^2} (x^2 - 1) + c$$

Indefinite Integrals Ex 19.25 Q18

Let $I = \int x^3 \cos x^2 \, dx$

Let $x^2 = t$

$2x \, dx = dt$

$I = \frac{1}{2} \int t \cos t dt$

Using integration by parts,

$$= \frac{1}{2}\left[t \int \cos t \, dt - \int \left(1 \times \int \cos t dt\right) dt\right]$$

$$= \frac{1}{2}\left[t \times \sin t - \int \sin t \, dt\right]$$

$$= \frac{1}{2}\left[t \sin t + \cos t\right] + c$$

$$I = \frac{1}{2}\left[x^2 \sin x^2 + \cos x^2\right] + c$$

Indefinite Integrals Ex 19.25 Q19

Let $I = \int x \sin x \cos x \, dx$

$$= \int \frac{x}{2}\left(2 \sin x \cos x\right) dx$$

$$= \frac{1}{2} \int x \sin 2x \, dx$$

Using integration by parts,

$$= \frac{1}{2}\left[x \int \sin 2x \, dx - \int \left(1 \times \int \sin 2x \, dx\right) dx\right]$$

$$= \frac{1}{2}\left[x\left(\frac{-\cos 2x}{2}\right) - \int \left(\frac{-\cos 2x}{2}\right) dx\right]$$

$$= -\frac{1}{4} x \cos 2x + \frac{1}{4} \int \cos 2x \, dx$$

$$I = -\frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x + c$$

Indefinite Integrals Ex 19.25 Q20

Let $\quad I = \int \sin x \left(\log \cos x\right) dx$

Let $\quad \cos x = t$

$-\sin x \, dx = dt$

$I = -\int \log t \, dt$

$= -\int 1 \times \log t \, dt$

Using integration by parts,

$$= -\left[\log t \int dt - \int \left(\frac{1}{t} \times \int dt\right) dt\right]$$

$$= -\left[t \log t - \int \frac{1}{t} \times t \, dt\right]$$

$$= -\left[t \log t - \int dt\right]$$

$$= -\left[t \log t - t + c_1\right]$$

$$= t\left(1 - t \log t\right) + c$$

$$I = \cos x \left(1 - \log \cos x\right) + c$$

Indefinite Integrals Ex 19.25 Q21

Let $\quad I = \int \left(\log x\right)^2 x \, dx$

Using integration by parts,

$$= \left(\log x\right)^2 \int x \, dx - \int \left(2\left(\log x\right)\left(\frac{1}{x}\right) \int x \, dx\right) dx$$

$$= \frac{x^2}{2}\left(\log x\right)^2 - 2\int \left(\log x\right)\left(\frac{1}{x}\right)\left(\frac{x^2}{2}\right) dx$$

$$= \frac{x^2}{2}\left(\log x\right)^2 - \int x \left(\log x\right) dx$$

$$= \frac{x^2}{2}\left(\log x\right)^2 - \left[\log x \int x \, dx - \int \left(\frac{1}{x} \int x \, dx\right) dx\right]$$

$$= \frac{x^2}{2}\left(\log x\right)^2 - \left[\frac{x^2}{2} \log x - \int \left(\frac{1}{x} \times \frac{x^2}{2}\right) dx\right]$$

$$= \frac{x^2}{2}\left(\log x\right)^2 - \frac{x^2}{2} \log x + \frac{1}{2} \int x \, dx$$

$$= \frac{x^2}{2}\left(\log x\right)^2 - \frac{x^2}{2} \log x + \frac{1}{4} x^2 + c$$

$$I = \frac{x^2}{2}\left[\left(\log x\right)^2 - \log x + \frac{1}{2}\right] + c$$

Indefinite Integrals Ex 19.25 Q22

Let $I = \int e^{\sqrt{x}}\, dx$

Let $\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

$I = 2\int e^t t\,dt$

$I = 2\left[t\int e^t dt - \int\left(1\int e^t dt\right)dt\right]$

$I = 2\left[te^t - \int e^t dt\right]$

$= 2\left[te^t - e^t\right] + c$

$= 2e^t\left(t - 1\right) + c$

$I = 2e^{\sqrt{x}}\left(\sqrt{x} - 1\right) + c$

Indefinite Integrals Ex 19.25 Q23

Let $I = \int \frac{\log\left(x+2\right)}{\left(x+2\right)^2} dx$

Let $\frac{1}{x+2} = t$

$-\frac{1}{\left(x+2\right)^2} dx = dt$

$I = -\int \log\left(\frac{1}{t}\right) dt$

$= -\int \log t^{-1}\, dt$

$= -\int 1 \times \log t\, dt$

Using integration by parts,

$I = \log t \int dt - \int\left(\frac{1}{t}\int dt\right)dt$

$= t\log t - \int\left(\frac{1}{t} \times t\right)dt$

$= t\log t - \int dt$

$= t\log t - t + c$

$= \frac{1}{x+2}\left(\log\left(x+2\right)^{-1} - 1\right) + c$

$I = \frac{-1}{x+2} - \frac{\log\left(x+2\right)}{x+2} + c$

Indefinite Integrals Ex 19.25 Q24

Let $I = \int \frac{x + \sin x}{1 + \cos x} dx$

$$= \int \frac{x}{2\cos^2 \frac{x}{2}} dx + \int \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} dx$$

$$= \frac{1}{2} \int x \sec^2 \frac{x}{2} dx + \int \tan \frac{x}{2} dx$$

Using integration by parts,

$$= \frac{1}{2}\left[x \int \sec^2 \frac{x}{2} dx - \int \left(1 \int \sec^2 \frac{x}{2} dx\right) dx\right] + \int \tan \frac{x}{2} dx$$

$$= \frac{1}{2}\left[2x \tan \frac{x}{2} - 2\int \tan \frac{x}{2} dx\right] + \int \tan \frac{x}{2} dx + c$$

$$= x \tan \frac{x}{2} - \int \tan \frac{x}{2} dx + \int \tan \frac{x}{2} dx + c$$

$$I = x \tan \frac{x}{2} + c$$

Indefinite Integrals Ex 19.25 Q25

Let $I = \int \log_{10} x \, dx$

$$= \int \frac{\log x}{\log 10} dx$$

$$= \frac{1}{\log 10} \int 1 \times \log x \, dx$$

Using integration by parts,

$$= \frac{1}{\log 10}\left[\log x \int dx - \int \left(\frac{1}{x} \int dx\right) dx\right]$$

$$= \frac{1}{\log 10}\left[x \log x - \int \left(\frac{x}{x}\right) dx\right]$$

$$= \frac{1}{\log 10}\left[x \log x - x\right]$$

$$I = \frac{x}{\log 10}(\log x - 1)$$

Indefinite Integrals Ex 19.25 Q26

Let $I = \int \cos\sqrt{x}\, dx$

$\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

$= \int 2t \cos t\, dt$

$I = 2\int t \cos t\, dt$

$I = 2\left[t\int \cos t\,dt - \int \left(1\int \cos t\,dt\right) dt\right]$

$= 2\left[t \sin t - \int \sin t\, dt\right]$

$= 2\left[t \sin t + \cos t\right] + c$

$I = 2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right] + c$

Indefinite Integrals Ex 19.25 Q27

Let $I = \int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$

Let us substitute, $t = \cos^{-1}x$

$\Rightarrow dt = \frac{-1}{\sqrt{1-x^2}}dx$

Also, $\cos t = x$

Thus,

$I = -\int t \cos t\, dt$

Now let us solve this by the 'by parts' method.

Let $u = t$; $du = dt$

$\int \cos t\, dt = \int dv$

$\Rightarrow \sin t = v$

Thus, $I = -\left[t\sin t - \int \sin t\, dt\right]$

$\Rightarrow I = -\left[t\sin t + \cos t\right] + C$

Substituting the value $t = \cos^{-1}x$, we have,

$I = -\left[\cos^{-1}x \sin t + x\right] + C$

$\Rightarrow I = -\left[\cos^{-1}x\sqrt{1-x^2} + x\right] + C$

Indefinite Integrals Ex 19.25 Q29

Let $I = \int \operatorname{cosec}^3 dx$

$= \int \operatorname{cosec}x - \operatorname{cosec}^2 x dx$

Using integration by parts,

$$= \operatorname{cosec}x \times \int \operatorname{cosec}^2 x dx + \int \left(\operatorname{cosec}x \cot x \int \operatorname{cosec}^2 x dx\right) dx$$

$$= \operatorname{cosec}x \times (-\cot x) + \int \operatorname{cosec}x \cot x (-\cot x) dx$$

$$= -\operatorname{cosec}x \cot x - \int \operatorname{cosec}x \cot^2 x dx$$

$$= -\operatorname{cosec}x \cot x - \int \operatorname{cosec}x \left(\operatorname{cosec}^2 x - 1\right) dx$$

$$= -\operatorname{cosec}x \cot x - \int \operatorname{cosec}^3 x dx + \int \operatorname{cosec}x dx$$

$$I = -\operatorname{cosec}x \cot x - I + \log\left|\tan\frac{x}{2}\right| + c_1$$

$$2I = -\operatorname{cosec}x \cot x + \log\left|\tan\frac{x}{2}\right| + c_1$$

$$I = -\frac{1}{2}\operatorname{cosec}x \cot x + \frac{1}{2}\log\left|\tan\frac{x}{2}\right| + c$$

Indefinite Integrals Ex 19.25 Q30

Let $I = \int \sec^{-1}\sqrt{x}\, dx$

Let $\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

$$I = \int 2t \sec^{-1} t\, dt$$

$$= 2\left[\sec^{-1} t \int t\, dt - \int \left(\frac{1}{t\sqrt{t^2 - 1}} \int t\, dt\right) dt\right]$$

$$= 2\left[\frac{t^2}{2}\sec^{-1} - \int \left(\frac{t}{2t\sqrt{t^2 - 1}}\right) dt\right]$$

$$= t^2 \sec^{-1} t - \int \frac{t}{\sqrt{t^2 - 1}} dt$$

$$= t^2 \sec^{-1} t - \frac{1}{2}\int \frac{2t}{\sqrt{t^2 - 1}} dt$$

$$= t^2 \sec^{-1} t - \frac{1}{2} \times 2\sqrt{t^2 - 1} + c$$

$$I = x \sec^{-1}\sqrt{x} - \sqrt{x - 1} + c$$

Indefinite Integrals Ex 19.25 Q31

$\int \sin^{-1}\sqrt{x}\, dx =$

let $x = t^2 \rightarrow dx = 2t\,dt$

$\int \sin^{-1}\sqrt{x}\, dx = \int \sin^{-1}\sqrt{t^2}\, 2t\,dt = \int \sin^{-1} t\, 2t\,dt$

$= \sin^{-1} t \int 2t\,dt - \left(\int \frac{d \sin^{-1} t}{dt} \left(\int 2t\,dt \right) dt \right)$

$= \sin^{-1} t\,(t^2) - \int \frac{1}{\sqrt{1-t^2}} (t^2)\,dt$

Lets solve $\int \frac{1}{\sqrt{1-t^2}} (t^2)\,dt$

$\int \frac{1}{\sqrt{1-t^2}} (t^2)\,dt = \int \frac{t^2 - 1 + 1}{\sqrt{1-t^2}} dt = \int \frac{t^2 - 1}{\sqrt{1-t^2}} dt + \int \frac{1}{\sqrt{1-t^2}} dt$

we know that, value of $\int \frac{1}{\sqrt{1-t^2}} dt = \sin^{-1} t$

Remaining integral to evaluate is $\int \frac{t^2 - 1}{\sqrt{1-t^2}} dt = \int -\sqrt{1-t^2}\, dt$

sub t=sin u, dt= cos$u\, du$

$\int -\sqrt{1-t^2}\, dt = \int -\cos^2 u\, du = -\int \left[\frac{1+\cos 2u}{2} \right] du$

$= -\frac{u}{2} - \frac{\sin 2u}{4}$

Substitute back u=sin^{-1}t and $t = \sqrt{x}$

$= -\frac{\sin^{-1}\sqrt{x}}{2} - \frac{\sin(2\sin^{-1}\sqrt{x})}{4}$

$\int \sin^{-1}\sqrt{x}\, dx = x \sin^{-1}\sqrt{x} - \frac{\sin^{-1}\sqrt{x}}{2} - \frac{\sin(2\sin^{-1}\sqrt{x})}{4}$

$\sin(2\sin^{-1}\sqrt{x}) = 2\sqrt{x}\sqrt{1-x}$

$\int \sin^{-1}\sqrt{x}\, dx = x \sin^{-1}\sqrt{x} - \frac{\sin^{-1}\sqrt{x}}{2} - \frac{\sqrt{x(1-x)}}{2}$

Indefinite Integrals Ex 19.25 Q32

Let $I = \int x \tan^2 x\, dx$

$= \int x \left(\sec^2 x - 1\right) dx$

$= \int x \sec^2 x\, dx - \int x\, dx$

$= \left[x \int \sec^2 x\, dx - \int \left(1 \int \sec^2 x\, dx \right) dx \right] - \frac{x^2}{2}$

$= x \tan x - \int \tan x\, dx - \frac{x^2}{2}$

$I = x \tan x - \log \sec x - \frac{x^2}{2} + c$

Indefinite Integrals Ex 19.25 Q33

Let $I = \int x \left(\frac{\sec 2x - 1}{\sec 2x + 1}\right) dx$

$= \int x \left(\frac{1 - \cos 2x}{1 + \cos 2x}\right) dx$

$= \int x \left(\frac{\sec^2 x}{\cos^2 x}\right) dx$

$= \int x \tan^2 x dx$

$= \int x \left(\sec^2 x - 1\right) dx$

$= \int x \sec^2 x dx - \int dx$

$= \left[x \int \sec^2 x\, dx - \int \left(1 \int \sec^2 x\, dx\right) dx\right] - \frac{x^2}{2}$

$= x \tan x - \int \tan x\, dx - \frac{x^2}{2}$

$I = x \tan x - \log \sec x - \frac{x^2}{2} + c$

Indefinite Integrals Ex 19.25 Q34

Let $I = \int (x + 1) e^x \log\left(xe^x\right) dx$

Let $xe^x = t$

$\left(1 \times e^x + xe^x\right) dx = dt$

$(x + 1) e^x dx = dt$

$I = \int \log t\, dt$

$= \int 1 \times \log t\, dt$

$= \log t \int dt - \int \left(\frac{1}{t} \int dt\right) dt$

$= t \log t - \int \left(\frac{1}{t} t\right) dt$

$= t \log t - \int dt$

$= t \log t - t + c$

$= t\left(\log t - 1\right) + c$

$I = xe^x \left(\log xe^x - 1\right) + c$

Indefinite Integrals Ex 19.25 Q35

Let $I = \int \sin^{-1}\left(3x - 4x^3\right)dx$

Let $x = \sin\theta$

$dx = \cos\theta\, d\theta$

$= \int \sin^{-1}\left(3\sin\theta - 4\sin^3\theta\right)\cos\theta\, d\theta$

$= \int \sin^{-1}\left(\sin 3\theta\right)\cos\theta\, d\theta$

$= \int 3\theta \cos\theta\, d\theta$

$= 3\left[\theta\int \cos\theta\, d\theta - \int\left(1\int\cos\theta\, d\theta\right)d\theta\right]$

$= 3\left[\theta\sin\theta - \int\sin\theta\, d\theta\right]$

$= 3\left[\theta\sin\theta + \cos\theta\right] + c$

$I = 3\left[x\sin^{-1}x + \sqrt{1-x^2}\right] + c$

Indefinite Integrals Ex 19.25 Q36

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$

$\Rightarrow \int\sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = \int 2\theta\cdot\sec^2\theta\, d\theta = 2\int\theta\cdot\sec^2\theta\, d\theta$

Integrating by parts, we obtain

$2\left[\theta\cdot\int\sec^2\theta d\theta - \int\left\{\left(\frac{d}{d\theta}\theta\right)\int\sec^2\theta d\theta\right\}d\theta\right]$

$= 2\left[\theta\cdot\tan\theta - \int\tan\theta d\theta\right]$

$= 2\left[\theta\tan\theta + \log\left|\cos\theta\right|\right] + C$

$= 2\left[x\tan^{-1}x + \log\left|\frac{1}{\sqrt{1+x^2}}\right|\right] + C$

$= 2x\tan^{-1}x + 2\log\left(1+x^2\right)^{-\frac{1}{2}} + C$

$= 2x\tan^{-1}x + 2\left[-\frac{1}{2}\log\left(1+x^2\right)\right] + C$

$= 2x\tan^{-1}x - \log\left(1+x^2\right) + C$

Indefinite Integrals Ex 19.25 Q37

Let $I = \int \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right) dx$

Let $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$I = \int \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2 x}\right)\sec^2\theta\, d\theta$

$= \int \tan^{-1}(\tan 3\theta)\sec^2\theta\, d\theta$

$= \int 3\theta \sec^2\theta\, d\theta$

$= 3\left[\theta \int \sec^2\theta\, d\theta - \int\left(1 \int \sec^2\theta\, d\theta\right) d\theta\right]$

$= 3\left[\theta \tan\theta - \int \tan\theta\, d\theta\right]$

$= 3\left[\theta \tan\theta + \log\sec\theta\right] + c$

$= 3\left[x\tan^{-1}x - \log\sqrt{1 + x^2}\right] + c$

$I = 3x\tan^{-1}x - \frac{3}{2}\log\left|1 + x^2\right| + c$

Indefinite Integrals Ex 19.25 Q38

Let $I = \int x^2 \sin^{-1} x\, dx$

$I = \sin^{-1} x \int x^2 dx - \int\left(\frac{1}{\sqrt{1 - x^2}} \int x^2 dx\right) dx$

$= \frac{x^3}{3}\sin^{-1} x - \int \frac{x^3}{3\sqrt{1 - x^2}} dx$

$I = \frac{x^3}{3}\sin^{-1} x - \frac{1}{3} I_1 + c_1 \text{ - - - - - - } (1)$

$I_1 = \int \frac{x^3}{\sqrt{1 - x^2}} dx$

Let $1 - x^2 = t^2$

$-2x\, dx = 2t\, dt$

$-x\, dx = t\, dt$

$I_1 = -\int \frac{(1 - t^2)\, t\, dt}{t}$

$= \int (t^2 - 1)\, dt$

$= \frac{t^3}{3} - t + c_2$

$= \frac{(1 - x^2)^{\frac{3}{2}}}{3} - (1 - x^2)^{\frac{1}{2}} + c_2$

Now,

$I = \frac{x^3}{3}\sin^{-1} x - \frac{1}{9}(1 - x^2)^{\frac{3}{2}} + \frac{1}{3}(1 - x^2)^{\frac{1}{2}} + c$

Indefinite Integrals Ex 19.25 Q39

Let $I = \int \frac{\sin^{-1} x}{x^2} dx$

$= \int \left(\frac{1}{x^2}\right)\left(\sin^{-1} x\right) dx$

$I = \left[\sin^{-1} x \int \frac{1}{x^2} dx - \int \left(\frac{1}{\sqrt{1-x^2}} \int \frac{1}{x^2} dx\right) dx\right]$

$= \sin^{-1} x \left(-\frac{1}{x}\right) - \int \frac{1}{\sqrt{1-x^2}} \left(-\frac{1}{x}\right) dx$

$I = -\frac{1}{x} \sin^{-1} x + \int \frac{1}{x\sqrt{1-x^2}} dx$

$I = -\frac{1}{x} \sin^{-1} x + I_1 \text{ - - - - - - -}(1)$

Where,

$I_1 = \int \frac{1}{x\sqrt{1-x^2}} dx$

Let $1 - x^2 = t^2$

$-2x\, dx = 2t\, dt$

$I_1 = \int \frac{x}{x^2\sqrt{1-x^2}} dx$

$= -\int \frac{t\, dt}{\left(1-t^2\right)\sqrt{t}}$

$= -\int \frac{dt}{\left(1-t^2\right)}$

$= \int \frac{1}{t^2-1} dt$

$= \frac{1}{2} \log \left|\frac{t-1}{t+1}\right|$

$= \frac{1}{2} \log \left|\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right| + c_1$

Now,

$$I = -\frac{\sin^{-1}x}{x} + \frac{1}{2}\log\left|\left(\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right)\left(\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}-1}\right)\right| + c$$

$$= -\frac{\sin^{-1}x}{x} + \frac{1}{2}\log\left|\frac{\left(\sqrt{1-x^2}-1\right)^2}{1-x^2-1}\right| + c$$

$$= -\frac{\sin^{-1}x}{x} + \frac{1}{2}\log\left|\frac{\left(\sqrt{1-x^2}-1\right)^2}{-x^2}\right| + c$$

$$= -\frac{\sin^{-1}x}{x} + \log\left|\frac{\sqrt{1-x^2}-1}{-x}\right| + c$$

$$I = -\frac{\sin^{-1}x}{x} + \log\left|\frac{1-\sqrt{1-x^2}}{x}\right| + c$$

Indefinite Integrals Ex 19.25 Q40

Let $I = \int \frac{x^2\tan^{-1}x}{1+x^2}dx$

Let $\tan^{-1}x = t$

$x = \tan t$

$\frac{1}{1+x^2}dx = dt$

$I = \int t\tan^2 t\,dt$

$= \int t\left(\sec^2 t - 1\right)dt$

$= \int\left(t\sec^2 t - t\right)dt$

$= \int t\sec^2 t\,dt - \int t\,dt$

$= \left[t\int\sec^2 t\,dt - \int\left(1\int\sec^2 t\,dt\right)dt\right] - \frac{t^2}{2}$

$= \left[t\tan t - \int\tan t\,dt\right] - \frac{t^2}{2}$

$= t\tan t - \log\sec t - \frac{t^2}{2} + c$

$= x\tan^{-1}x - \log\sqrt{1+x^2} - \frac{\tan^2 x}{2} + c$

$$I = x\tan^{-1}x - \frac{1}{2}\log\left|1+x^2\right| - \frac{\tan^2 x}{2} + c$$

Indefinite Integrals Ex 19.25 Q41

Let $I = \int \cos^{-1}\left(4x^3 - 3x\right) dx$

Let $x = \cos\theta$

$dx = -\sin\theta\, d\theta$

$I = -\int \cos^{-1}\left(4\cos^3\theta - 3\cos\theta\right)\sin\theta\, d\theta$

$= -\int \cos^{-1}\left(\cos 3\theta\right)\sin\theta\, d\theta$

$= -\int 3\theta \sin\theta\, d\theta$

$= -3\left[\theta\int \sin\theta\, d\theta - \int\left(1\int \sin\theta\, d\theta\right) d\theta\right]$

$= -3\left[-\theta\cos\theta + \int \cos\theta\, d\theta\right]$

$= 3\theta\cos\theta - 3\sin\theta + c$

$I = 3x\cos^{-1}x - 3\sqrt{1 - x^2} + c$

Indefinite Integrals Ex 19.25 Q42

Let $I = \int \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right) dx$

Let $x = \tan t$

$dx = \sec^2 t\, dt$

$I = \int \cos^{-1}\left(\frac{1 - \tan^2 t}{1 + \tan^2 t}\right)\sec^2 t\, dt$

$= \int \cos^{-1}\left(\cos 2t\right)\sec^2 t\, dt$

$= \int 2t \sec^2 t\, dt$

$= 2\left[t\int sce^2 t\, dt - \int\left(1\int \sec^2 t\, dt\right) dt\right]$

$= 2\left[t\tan^2 t - \int \tan t\, dt\right]$

$= 2\left[t\tan^2 t - \log\sec t\right] + c$

$= 2\left[x\tan^{-1}x - \log\sqrt{1 + x^2}\right] + c$

$I = 2x\tan^{-1}x - \log\left|1 + x^2\right| + c$

Indefinite Integrals Ex 19.25 Q43

Let $I = \int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$

Let $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$I = \int \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\sec^2\theta\, d\theta$

$= \int \tan^{-1}(\tan 2\theta)\sec^2\theta\, d\theta$

$= \int 2\theta \sec^2\theta\, d\theta$

$= 2\left[\theta\int \sec^2\theta\, d\theta - \int\left(1\int \sec^2\theta\, d\theta\right)d\theta\right]$

$= 2\left[\theta\tan\theta - \int \tan\theta\, d\theta\right]$

$= 2\left[\theta\tan\theta - \log\sec\theta\right] + c$

$= 2\left[x\tan^{-1}x - \log\sqrt{1+x^2}\right] + c$

$I = 2x\tan^{-1}x - \log\left|1+x^2\right| + c$

Indefinite Integrals Ex 19.25 Q44

Let $I = \int (x+1)\log x\, dx$

$= \log x\int (x+1)dx - \int\left(\frac{1}{x}\int(x+1)\,dx\right)dx$

$= \left(\frac{x^2}{2}+x\right)\log x - \int\frac{1}{x}\left(\frac{x^2}{2}+x\right)dx$

$= \left(\frac{x^2}{2}+x\right)\log x - \frac{1}{2}\int x\,dx - \int dx$

$= \left(x+\frac{x^2}{2}\right)\log x - \frac{1}{2}\times\frac{x^2}{2} - x + c$

$I = \left(x+\frac{x^2}{2}\right)\log x - \left(\frac{x^2}{4}+x\right) + c$

Indefinite Integrals Ex 19.25 Q45

Let $I = \int x^2 \tan^{-1} x \, dx$

$$= \tan^{-1} x \int x^2 dx - \int \left(\frac{1}{1+x^2} \int x^2 dx \right)$$

$$= \tan^{-1} x \left(\frac{x^3}{3} \right) - \frac{1}{3} \int \frac{x^3}{1+x^2} dx$$

$$= \frac{1}{3} x^3 \tan^{-1} x - \frac{1}{3} \int \left(x - \frac{x}{1+x^2} \right) dx$$

$$= \frac{1}{3} x^3 \tan^{-1} x - \frac{1}{3} \times \frac{x^2}{2} + \frac{1}{3} \int \frac{x}{1+x^2} dx$$

$$I = \frac{1}{3} x^3 \tan^{-1} x - \frac{1}{6} x^2 + \frac{1}{6} \log \left| 1 + x^2 \right| + c$$

Indefinite Integrals Ex 19.25 Q46

Let $I = \int \left(e^{\log x} + \sin x \right) \cos x \, dx$

$$= \int \left(x + \sin x \right) \cos x \, dx$$

$$= \int x \cos x \, dx + \int \sin x \cos x \, dx$$

$$= \left[x \int \cos x \, dx - \int \left(1 \int \cos x \, dx \right) dx \right] + \frac{1}{2} \int \sin 2x \, dx$$

$$= \left[x \sin x - \int \sin x \, dx \right] + \frac{1}{2} \left(-\frac{\cos 2x}{2} \right) + c$$

$$I = x \sin x + \cos x - \frac{1}{4} \cos 2x + c$$

$$I = x \sin x + \cos x - \frac{1}{4} \left[1 - 2 \sin^2 x \right] + c$$

$$I = x \sin x + \cos x - \frac{1}{4} + \frac{1}{2} \sin^2 x + c$$

$$I = x \sin x + \cos x + \frac{1}{2} \sin^2 x + c - \frac{1}{4}$$

$$I = x \sin x + \cos x + \frac{1}{2} \sin^2 x + k, \text{ where } k = c - \frac{1}{4}$$

Indefinite Integrals Ex 19.25 Q47

Let $I = \int \frac{\left(x \tan^{-1} x\right)}{\left(1+x^2\right)^{\frac{3}{2}}} dx$

Let $\tan^{-1} x = t$

$\frac{1}{1+x^2} dx = dt$

$I = \int \frac{t \tan t}{\sqrt{1+\tan^2 t}} dt$

$= \int \frac{t \tan t}{\sec t} dt$

$= \int t \frac{\sin t}{\cos t} \cos t \, dt$

$= \int t \sin t \, dt$

$= \left[t \int \sin t dt - \int \left(1 \int \sin t \, dt\right) dt\right]$

$= \left[-t \cos t + \int \cos t \, dt\right]$

$= \left[-t \cos t + \sin t\right] + c$

$$I = -\frac{\tan^{-1} x}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$$

Indefinite Integrals Ex 19.25 Q48

Let $I = \int \tan^{-1}\left(\sqrt{x}\right) dx$

Let $x = t^2$

$dx = 2t \, dt$

$I = \int 2t \tan^{-1} t \, dt$

$= 2\left[\tan^{-1} t \int t \, dt - \int \left(\frac{1}{1+t^2} \int t \, dt\right) dt\right]$

$= 2\left[\frac{t^2}{2} \tan^{-1} t - \int \frac{t^2}{2\left(1+t^2\right)} dt\right]$

$= t^2 \tan^{-1} t - \int \frac{t^2 + 1 - 1}{1+t^2} dt$

$= t^2 \tan^{-1} t - \int \left(1 - \frac{1}{1+t^2}\right) dt$

$= t^2 \tan^{-1} t - t + \tan^{-1} t + c$

$= \left(t^2 + 1\right) \tan^{-1} t - t + c$

$$I = \left(x+1\right) \tan^{-1} \sqrt{x} - \sqrt{x} + c$$

Indefinite Integrals Ex 19.25 Q49

$$\int x^3 \tan^{-1} x\, dx =$$

$$\int x^3 \tan^{-1} x\, dx = \tan^{-1} x \int x^3 dx - \left(\int \frac{d \tan^{-1} x}{dx} \left(\int x^3 dx \right) dx \right)$$

$$= \tan^{-1} x \frac{x^4}{4} - \left(\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx \right)$$

$$= \tan^{-1} x \frac{x^4}{4} - \left(\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx \right)$$

$$\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx = \frac{1}{4} \left[\int \frac{1}{1+x^2} dx + (x^2 - 1) dx \right]$$

$$\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx = \frac{1}{4} \left[\tan^{-1} x + \frac{x^3}{3} - x \right]$$

$$\int x^3 \tan^{-1} x\, dx = \frac{x^4}{4} \tan^{-1} x - \frac{1}{4} \left[\tan^{-1} x + \frac{x^3}{3} - x \right] + C$$

Indefinite Integrals Ex 19.25 Q50

Let $I = \int x \sin x \cos 2x\, dx$

$$= \frac{1}{2} \int x \left(2 \sin x \cos 2x \right) dx$$

$$= \frac{1}{2} \int x \left(\sin(x + 2x) - \sin(2x - x) \right) dx$$

$$= \frac{1}{2} \int x \left(\sin 3x - \sin x \right) dx$$

$$= \frac{1}{2} \left[x \int \left(\sin 3x - \sin x \right) dx - \int \left(1 \int \left(\sin 3x - \sin x \right) dx \right) dx \right]$$

$$= \frac{1}{2} \left[x \left(\frac{-\cos 3x}{3} + \cos x \right) - \int \left(-\frac{\cos 3x}{3} + \cos x \right) dx \right]$$

$$I = \frac{1}{2} \left[-x \frac{\cos 3x}{3} + x \cos x + \frac{1}{9} \sin 3x - \sin x \right] + c$$

Indefinite Integrals Ex 19.25 Q52

Let first function be $\sin^{-1} x$ and second dunction be $\dfrac{x}{\sqrt{1 - x^2}}$.

First we find the intergral of the second function, i.e., $\int \dfrac{x dx}{\sqrt{1 - x^2}}$.

Put $t = 1 - x^2$. Then $dt = -2x \, dx$

Therefore, $$\int \frac{x dx}{\sqrt{1 - x^2}} = -\frac{1}{2}\int \frac{dt}{\sqrt{t}} = -\sqrt{t} = -\sqrt{1 - x^2}$$

Hence, $$\int \frac{x \sin^{-1} x}{\sqrt{1 - x^2}} dx = \left(\sin^{-1} x\right)\left(-\sqrt{1 - x^2}\right) - \int \frac{1}{\sqrt{1 - x^2}}\left(-\sqrt{1 - x^2}\right) dx$$

$$= -\sqrt{1 - x^2}\sin^{-1} x + x + C = x - \sqrt{1 - x^2}\sin^{-1} x + C$$

Indefinite Integrals Ex 19.25 Q53

Let $I = \int \sin^3 \sqrt{x}\, dx$

$\sqrt{x} = t$

$x = t^2$

$dx = 2t\, dt$

$I = 2\int t \sin^3 t\, dt$

$= 2\int t\left(\frac{3\sin t - \sin 3t}{4}\right)dt$

$= \frac{1}{2}\int t\left(3\sin t - \sin 3t\right)dt$

Using integration by parts,

$I = \frac{1}{2}\left[t\left(-3\cos t + \frac{1}{3}\cos 3t\right) - \int\left(-3\cos t + \frac{\cos 3t}{3}\right)dt\right]$

$= \frac{1}{2}\left[\frac{-9t\cos t + t\cos 3t}{3} - \left\{-3\sin t + \frac{\sin 3t}{9}\right\}\right] + c$

$= \frac{1}{2}\left[\frac{-9t\cos t + t\cos 3t}{3} + \frac{27\sin t - 3\sin 3t}{9}\right] + c$

$= \frac{1}{18}\left[-27t\cos t + 3t\cos 3t + 27\sin t - 3\sin 3t\right] + c$

$I = \frac{1}{18}\left[3\sqrt{x}\cos 3\sqrt{x} + 27\sin\sqrt{x} - 27\sqrt{x}\cos\sqrt{x} - 3\sin 3\sqrt{x}\right] + c$

Indefinite Integrals Ex 19.25 Q54

Let $I = \int x\sin^3 x\, dx$

$= \int x\left(\frac{3\sin x - \sin 3x}{4}\right)dx$

$= \frac{1}{4}\int x\left(3\sin x - \sin 3x\right)dx$

Using integration by parts,

$I = \frac{1}{4}\left[x\int\left(3\sin x - \sin 3x\right)dx - \int\left(1\int\left(3\sin x - \sin 3x\right)dx\right)dx\right]$

$= \frac{1}{4}\left[x\left(-3\cos x + \frac{\cos 3x}{3}\right) - \int\left(-3\cos x + \frac{\cos 3x}{3}\right)dx\right]$

$= \frac{1}{4}\left[-3x\cos x + \frac{x\cos 3x}{3} + 3\sin x - \frac{\sin 3x}{9}\right] + c$

$I = \frac{1}{36}\left[3x\cos 3x - 27x\cos x + 27\sin x - \sin 3x\right] + c$

indefinite integrals Ex 19.25 Q56

Let $I = \int x \cos^3 x \, dx$

$$= \int x \left(\frac{3\cos x + \cos 3x}{4} \right) dx$$

$$= \frac{1}{4} \int x \left(3\cos x + \cos 3x \right) dx$$

Using integration by parts,

$$I = \frac{1}{4} \left[x \int \left(3\cos x + \cos 3x \right) dx - \int \left(1 \int \left(3\cos x + \cos 3x \right) dx \right) dx \right]$$

$$= \frac{1}{4} \left[x \left(3\sin x + \frac{\sin 3x}{3} \right) - \int \left(3\sin x + \frac{\sin 3x}{3} \right) dx \right]$$

$$= \frac{1}{4} \left[3x \sin x + \frac{x \sin 3x}{3} + 3\cos x + \frac{\cos 3x}{9} \right] + c$$

$$I = \frac{3x \sin x}{4} + \frac{x \sin 3x}{12} + \frac{3\cos x}{4} + \frac{\cos 3x}{36} + c$$

Indefinite Integrals Ex 19.25 Q58

Let $I=\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$

Let $x=a\tan^2\theta$

$$dx=2a\tan\theta\sec^2\theta\,d\theta$$

$$I=\int\left(\sin^{-1}\sqrt{\frac{a\tan^2\theta}{a+a\tan^2\theta}}\right)\left(2a\tan\theta\sec^2\theta\right)d\theta$$

$$=\int\left(\sin^{-1}\sqrt{\frac{\tan^2\theta}{\sec^2\theta}}\right)\left(2a\tan\theta\sec^2\theta\right)d\theta$$

$$=\int\sin^{-1}\left(\sin\theta\right)\left(2a\tan\theta\sec^2\theta\right)d\theta$$

$$=\int 2\theta a\tan\theta\sec^2\theta\,d\theta$$

$$=2a\int\theta\left(\tan\theta\sec^2\theta\right)d\theta$$

$$=2a\left[\theta\int\tan\theta\sec^2\theta\,d\theta-\int\left(\int\tan\theta\sec^2\theta\,d\theta\right)d\theta\right]$$

$$=2a\left[\theta\frac{\tan^2\theta}{2}-\int\frac{\tan^2\theta}{2}d\theta\right]$$

$$=a\theta\tan^2\theta-\frac{2a}{2}\int\left(\sec^2\theta-1\right)d\theta$$

$$=a\theta\tan^2\theta-a\tan\theta+a\theta+c$$

$$=a\left(\tan^{-1}\sqrt{\frac{x}{a}}\right)\frac{x}{a}-a\sqrt{\frac{x}{a}}+a\tan^{-1}\sqrt{\frac{x}{a}}+c$$

$$I=x\tan^{-1}\sqrt{\frac{x}{a}}-\sqrt{ax}+a\tan^{-1}\sqrt{\frac{x}{a}}+c$$

Indefinite Integrals Ex 19.25 Q59

Let $\quad I = \int \frac{x^3 \sin^{-1} x^2}{\sqrt{1-x^4}} dx$

Let $\quad \sin^{-1} x^2 = t$

$$\frac{1}{\sqrt{1-x^4}} (2x)\, dx = dt$$

$$I = \int \frac{x^2 \sin^{-1} x^2}{\sqrt{1-x^2}} x\, dx$$

$$= \int (\sin t)\, t \frac{dt}{2}$$

$$= \frac{1}{2} \int t \sin t\, dt$$

$$= \frac{1}{2} \left[t \int \sin t\, dt - \int \left(1 \int \sin t\, dt\right) dt \right]$$

$$= \frac{1}{2} \left[t\left(-\cos t\right) - \int \left(-\cos t\right) dt \right]$$

$$= \frac{1}{2} \left[-t \cos t + \sin t \right] + c$$

$$I = \frac{1}{2} \left[x^2 - \sqrt{1-x^4} \sin^{-1} x^2 \right] + c$$

Indefinite Integrals Ex 19.25 Q60

Let $I = \int \frac{x^2 \sin^{-1} x}{\left(1-x^2\right)^{\frac{3}{2}}} dx$

Let $\sin^{-1} x = t$

$$\frac{1}{\sqrt{1-x^2}} dx = dt$$

$$I = \int \frac{\sin^2 t \times t}{\left(1-\sin^2 t\right)} dt$$

$$= \int \frac{t \sin^2 t}{\cos^2 t} dt$$

$$= \int t \tan^2 t dt$$

$$= \int t \left(\sec^2 t - 1\right) dt$$

$$= \int t \sec^2 t \, dt - \frac{t^2}{2}$$

$$= t \int \sec^2 t \, dt - \int \left(1 \int \sec^2 t \, dt\right) dt - \frac{t^2}{2}$$

$$= t \tan t - \int \tan t \, dt - \frac{t^2}{2}$$

$$= t \tan t - \log \sec t - \frac{t^2}{2} + c$$

$$I = \frac{x}{\sqrt{1-x^2}} \sin^{-1} x + \log\left|1-x^2\right| - \frac{1}{2}\left(\sin^{-1} x\right)^2 + c$$