

Exercise – 16.2

Question 1:

A die is rolled. Let E be the event “die shows 4” and F be the event “die shows even number”. Are E and F mutually exclusive?

Solution 1:

When a die is rolled, the sample space is given by

$$S = \{1, 2, 3, 4, 5, 6\}$$

Accordingly, $E = \{4\}$ and $F = \{2, 4, 6\}$

It is observed that $E \cap F = \{4\} \neq \phi$

Therefore, E and F are not mutually exclusive events.

Question 2:

A die is thrown. Describe the following events:

(i) A: a number less than 7

(ii) B: a number greater than 7

(iii) C: a multiple of 3

(iv) D: a number less than 4

(v) E: an even number greater than 4

(vi) F: a number not less than 3

Also find $A \cup B$, $A \cap B$, $B \cup C$, $E \cap F$, $D \cap E$, $A - C$, $D - E$, $E \cap F'$, F'

Solution 2:

When a die is thrown, the sample space is given by $S = \{1, 2, 3, 4, 5, 6\}$.

Accordingly:

(i) $A = \{1, 2, 3, 4, 5, 6\}$

(ii) $B = \emptyset$

(iii) $C = \{3, 6\}$

(iv) $D = \{1, 2, 3\}$

(v) $E = \{6\}$

(vi) $F = \{3, 4, 5, 6\}$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}, \quad A \cap B = \phi$$

$$B \cup C = \{3, 6\}, \quad E \cap F = \{6\}$$

$$D \cap E = \phi, \quad A - C = \{1, 2, 4, 5\}$$

$$D - E = \{1, 2, 3\}, \quad F' = \{1, 2\}, \quad E \cap F' = \emptyset$$

Question 3:

An experiment involves rolling a pair of dice and recording the number that comes up. Describe the following events.

A: the sum is greater than 8, B: 2 occurs on either die

C: The sum is at least 7 and multiple of 3.

Which pairs of these events are mutually exclusive?

Solution 3:

When a pair of dice is rolled, the sample space is given by

$$S = \{(x, y): x, y = 1, 2, 3, 4, 5, 6\}$$

$$= \left\{ \begin{array}{l} (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) \\ (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) \\ (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6) \\ (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) \\ (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) \\ (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \end{array} \right\}$$

Accordingly,

$$A = \{(3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)\}$$

$$B = \{(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (1, 2), (3, 2), (4, 2), (5, 2), (6, 2)\}$$

$$C = \{(3, 6), (4, 5), (5, 4), (6, 3), (6, 6)\}$$

It is observed that

$$A \cap B = \phi$$

$$B \cap C = \phi$$

$$C \cap A = \{(3, 6), (4, 5), (5, 4), (6, 3), (6, 6)\} \neq \emptyset$$

Hence, events A and B and events B and C are mutually exclusive.

Question 4:

Three coins are tossed once. Let A denote the event “three heads show”, B denote the event “two heads and one tail show”. C denote the event “three tails show” and D denote the event ‘a head shows on the first coin’. Which events are (i) mutually exclusive? (ii) simple? (iii) compound?

Solution 4:

When three coins are tossed, the sample space is given by

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Accordingly,

$$A = \{HHH\}$$

$$B = \{HHT, HTH, THH\}$$

$$C = \{TTT\}$$

$$D = \{HHH, HHT, HTH, HTT\}$$

We now observe that

$$A \cap B = \phi, A \cap C = \phi, A \cap D = \{HHH\} \neq \phi$$

$$B \cap C = \phi, B \cap D = \{HHT, HTH\} \neq \phi$$

$$C \cap D = \phi$$

(i) Event A and B; event A and C; event B and C; and event C and D are all mutually exclusive.

(ii) If an event has only one sample point of a sample space, it is called a simple event. Thus, A and C are simple events.

(iii) If an event has more than one sample point of a sample space, it is called a compound event. Thus, B and D are compound events.

Question 5:

Three coins are tossed. Describe

- (i) Two events which are mutually exclusive.
- (ii) Three events which are mutually exclusive and exhaustive.
- (iii) Two events, which are not mutually exclusive.
- (iv) Two events which are mutually exclusive but not exhaustive.
- (v) Three events which are mutually exclusive but not exhaustive.

Solution 5:

When three coins are tossed, the sample space is given by

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

(i) Two events that are mutually exclusive can be

A: getting no heads and B: getting no tails

This is because sets $A = \{TTT\}$ and $B = \{HHH\}$ are disjoint.

(ii) Three events that are mutually exclusive and exhaustive can be

A: getting no heads

B: getting exactly one head

C: getting at least two heads

i.e.,

$$A = \{TTT\}$$

$$B = \{HTT, THT, TTH\}$$

$$C = \{HHH, HHT, HTH, THH\}$$

This is because $A \cap B = B \cap C = C \cap A = \phi$ and $A \cup B \cup C = S$

(iii) Two events that are not mutually exclusive can be

A: getting three heads

B: getting at least 2 heads

i.e.,

$$A = \{HHH\}$$

$$B = \{HHH, HHT, HTH, THH\}$$

This is because $A \cap B = \{HHH\} \neq \phi$

(iv) Two events which are mutually exclusive but not exhaustive can be

A: getting exactly one head

B: getting exactly one tail

i.e.,

$$A = \{HTT, THT, TTH\}$$

$$B = \{HHT, HTH, THH\}$$

This is because $A \cap B = \phi$, but $A \cup B \neq S$

(v) Three events that are mutually exclusive but not exhaustive can be

A: getting exactly three heads

B: getting one head and two tails

C: getting one tail and two heads

i.e.,

$$A = \{HHH\}$$

$$B = \{HTT, THT, TTH\}$$

$$C = \{HHT, HTH, THH\}$$

This is because $A \cap B = B \cap C = C \cap A = \phi$, but $A \cup B \cup C \neq S$

Question 6:

Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice ≤ 5

Describe the events

- (i) A' (ii) not B (iii) A or B (iv) A and B (v) A but not C (vi) B or C
 (vii) B and C (viii) $A \cap B' \cap C'$

Solution 6:

When two dice are thrown, the sample space is given by

$$S = \{(x, y) : x, y = 1, 2, 3, 4, 5, 6\}$$

$$= \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\}$$

Accordingly,

$$A = \left\{ \begin{array}{l} (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3) \\ (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\}$$

$$B = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3) \\ (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \end{array} \right\}$$

$$C = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)\}$$

$$(i) \quad A' = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), \\ (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \end{array} \right\} = B$$

$$(ii) \quad \text{Not B} = B' = \left\{ \begin{array}{l} (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), \\ (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\} = B$$

$$(iii) \quad A \text{ or } B = A \cup B = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\} = S$$

$$(iv) \quad A \text{ and } B = A \cap B = \emptyset$$

$$(v) \quad A \text{ but not } C = A - C$$

$$= \left\{ (2, 4), (2, 5), (2, 6), (4, 2), (4, 3), (4, 4), (4, 5), \right. \\ \left. (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \right\}$$

(vi) $B \text{ or } C = B \cup C$

$$= \left\{ (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), \right. \\ \left. (2, 3), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), \right. \\ \left. (4, 1), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) \right\}$$

(vii) $B \text{ and } C = B \cap C = \{(1, 1), (1, 2), (1, 3), (1, 4), (3, 1), (3, 2)\}$

(viii) $C' = \left\{ (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6), (4, 2) \right. \\ \left. (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) \right. \\ \left. (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \right\}$

$$\therefore A \cap B' \cap C' = A \cap A \cap C' = A \cap C'$$

$$= \left\{ (2, 4), (2, 5), (2, 6), (4, 2), (4, 3), (4, 4), (4, 5), \right. \\ \left. (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \right\}$$

Question 7:

Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice ≤ 5

State true or false: (give reason for your answer)

- A and B are mutually exclusive
- A and B are mutually exclusive and exhaustive
- $A = B'$
- A and C are mutually exclusive
- A and B' are mutually exclusive
- A', B', C are mutually exclusive and exhaustive.

Solution 7:

$$A = \left\{ (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), \right. \\ \left. (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \right\}$$

$$B = \left\{ (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (3, 1), (3, 2), (3, 3), \right. \\ \left. (3, 4), (3, 5), (3, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) \right\}$$

$$C = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)\}$$

- (i) It is observed that $A \cap B = \phi$

\therefore A and B are mutually exclusive.

Thus, the given statement is true.

- (ii) It is observed that $A \cap B = \phi$ and $A \cup B = S$

\therefore A and B are mutually exclusive and exhaustive.

Thus, the given statement is true.

- (iii) It is observed that

$$B' = \left\{ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), \right. \\ \left. (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \right\} = A$$

Thus, the given statement is true.

(iv) It is observed that $A \cap C = \{(2,1), (2,2), (2,3), (4,1)\} \neq \emptyset$

\therefore A and C are not mutually exclusive.

Thus, the given statement is false.

(v) $A \cap B' = A \cap A = A$

$\therefore A \cap B' = \emptyset$

\therefore A and B' are not mutually exclusive.

Thus, the given statement is false.

(vi) It is observed that $A' \cup B' \cup C = S$.

However,

$$B' \cap C = \{(2,1), (2,2), (2,3), (4,1)\} \neq \emptyset$$

Therefore, events A' , B' and C are not mutually exclusive and exhaustive.

Thus, the given statement is false.

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