Exercise 15.3

Question 1:

From the data given below state which group is more variable, A or B?

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Group A	9	17	32	33	40	10	9
Group B	10	20	30	25	43	15	7

Solution 1:

Firstly, the standard deviation of group A is calculated as follows.

Marks	Group A f_i	Mid-point x_i	$y_i = \frac{x_i - 45}{10}$	y_i^2	$f_i\, y_i$	$f_i\, y_i^2$
10-20	9	15	-3	9	-27	81
20-30	17	25	-2	4	-34	68
30-40	32	35	-1	1	-32	32
40-50	33	45	0	0	0	0
50-60	40	55	1	1	40	40
60-70	10	65	2	4	20	40
70-80	9	75	3	9	27	81
	150				-6	342

Here, h = 10, N = 150, A = 45

$$\text{Mean} = A + \frac{\sum_{i=1}^{7} x_i}{N} \times h = 45 + \frac{(-6)\times 10}{150} \times 45 - 0.4 = 44.6$$

$$\sigma_1^2 = \frac{h^2}{N^2}\left[N\sum_{i=1}^{7} f_i y_i^2 - \left(\sum_{i=1}^{7} f_i y_i\right)^2\right]$$

$$= \frac{100}{22500}\left[150 \times 342 - (-6)^2\right]$$

$$= \frac{1}{225}(51264)$$

$$= 227.84$$

$$\therefore \text{Standard deviation } (\sigma_1) = \sqrt{227.84} = 15.09$$

The standard deviation of group B is calculated as follows.

Marks	Group B f_i	Mid-point x_i	$y_i = \frac{x_i - 45}{10}$	y_i^2	$f_i\, y_i$	$f_i\, y_i^2$
10-20	10	15	-3	9	-30	90
20-30	20	25	-2	4	-40	80
30-40	30	35	-1	1	-30	30
40-50	25	45	0	0	0	0
50-60	43	55	1	1	43	43
60-70	15	65	2	4	30	60
70-80	7	75	3	9	21	63
	150				-6	366

$$\text{Mean} = A + \frac{\sum_{i=1}^{7} f_i y_i}{N} \times h = 45 + \frac{(-6)\times 10}{150} \times 45 - 0.4 = 44.6$$

$$\sigma_2^2 = \frac{h^2}{N^2}\left[N\sum_{i=1}^{7} f_i y_i^2 - \left(\sum_{i=1}^{7} f_i y_i\right)^2\right]$$

$$= \frac{100}{22500}\left[150 \times 366 - (-6)^2\right]$$

$$= \frac{1}{225}(54864) = 243.84$$

$\therefore$ Standard deviation $(\sigma_1) = \sqrt{243.84} = 15.61$

Since the mean of both the groups is same, the group with greater standard deviation will be more variable.
Thus, group B has more variability in the marks.

Question 2:

From the prices of shares X and Y below, find out which is more stable in value:

X	35	54	52	53	56	58	52	50	51	49
Y	108	107	105	105	106	107	104	103	104	101

Solution 2:

The prices of the shares X are
35, 54, 52, 53, 56, 58, 52, 50, 51, 49
Here, the number of observations, N = 10

$$\text{Mean, } \bar{x} = \frac{1}{N}\sum_{i=1}^{10} x_i = \frac{1}{10} \times 510 = 51$$

The following table is obtained corresponding to shares X.

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
35	−16	256
54	3	9
52	1	1
53	2	4
56	5	25
58	7	49
52	1	1
50	−1	1
51	0	0
49	−2	4
		350

$$\text{Variance}(\sigma_1^2) = \frac{1}{N}\sum_{i=1}^{10}(x_i - \bar{x})^2 = \frac{1}{10} \times 350 = 35$$

$\therefore$ Standard deviation $(\sigma_1) = \sqrt{35} = 5.91$

$$\text{C.V.(Shares X)} = \frac{\sigma_1}{\bar{X}} \times 100 = \frac{5.91}{51} \times 100 = 11.58$$

The prices of share Y are
108, 107, 105, 105, 106, 107, 104, 103, 104, 101

$$\text{Mean, } \bar{y} = \frac{1}{N}\sum_{i=1}^{10} y_i = \frac{1}{10}\times 1050 = 105$$

The following table is obtained corresponding to shares Y.

x_i	$(y_i - \bar{y})$	$(y_i - \bar{y})^2$
108	3	9
107	2	4
105	0	0
105	0	0
106	1	1
107	2	4
104	−1	1
103	−2	4
104	−1	1
101	−4	16
		40

$$\text{Variance}\left(\sigma_1^2\right) = \frac{1}{N}\sum_{i=1}^{10}(y_i - \bar{y})^2 = \frac{1}{10}\times 40 = 4$$

$$\therefore \text{Standard deviation } (\sigma_2) = \sqrt{4} = 2$$

$$\text{C.V.(Shares Y)} = \frac{\sigma_2}{y}\times 100 = \frac{2}{105}\times 100 = 1.9 = 11.58$$

C.V. of prices of shares X is greater than the C.V. of prices of shares Y.
Thus, the prices of shares Y are more stable than the prices of shares X.

Question 3:
An analysis of monthly wages paid to workers in two firms A and B, belonging to the same industry, gives the following results:

	Firm A	Firm B
No. of wages earners	586	648
Mean of monthly wages	Rs. 5253	Rs. 5253
Variance of the distribution of wages	100	121

(i) Which firm A or B pays larger amount as monthly wages?
(ii) Which firm, A or B, shows greater variability in individual wages?

Solution 3:
(i) Monthly wages of firm A = Rs. 5253
Number of wage earners in firm A = 586
∴ Total amount paid = Rs. 5253 × 586
Monthly wages of firm B = Rs. 5253
Number of wage earners in firm B = 648
∴ Total amount paid = Rs. 5253 × 648

Thus, firm B pays the larger amount as monthly wages as the number of wage earners in firm B are more than the number of wage earners in firm A.

(ii) Variance of the distribution of wages in firm A $\left(\sigma_1^2\right)$ = 100

$\therefore$ Standard deviation of the distribution of wages in firm A $\left(\sigma_1\right) = \sqrt{100} = 10$

Variance of the distribution of wages in firm B$\left(\sigma_2^2\right)$ = 121

$\therefore$ Standard deviation of the distribution of wages in firm B$\left(\sigma_2^2\right) = \sqrt{121} = 11$

The mean of monthly wages of both the firms is same i.e., 5253. Therefore, the firm with greater standard deviation will have more variability.

Thus, firm B has greater variability in the individual wages.

Question 4:

The following is the record of goals scored by team A in a football session:

No. of goals scored	0	1	2	3	4
No. of matches	1	9	7	5	3

For the team B, mean number of goals scored per match was 2 with a standard deviation 1.25 goals. Find which team may be considered more consistent?

Solution 4:

The mean and the standard deviation of goals scored by team A are calculated as follows.

No. of goals scored	No. of matches	$f_i\ x_i$	x_i^2	$f_i\ x_i^2$
0	1	0	0	0
1	9	9	1	9
2	7	14	4	28
3	5	15	9	45
4	3	12	16	48
	25	50		130

$$\text{Mean} = \sum_{i=1}^{15} f_i x_i = \frac{50}{25} = 2$$

Thus, the mean of both the teams is same.

$$\sigma = \frac{1}{N}\sqrt{N\sum f_i x_i^2 - \left(\sum f_i x_i\right)^2}$$

$$= \frac{1}{25}\sqrt{25 \times 130 - (50)^2}$$

$$= \frac{1}{25}\sqrt{750}$$

$$= \frac{1}{25} \times 27.38$$

$$= 1.09$$

The standard deviation of team B is 1.25 goals.

The average number of goals scored by both the teams is same i.e., 2. Therefore, the team with lower standard deviation will be more consistent.

Thus, team A is more consistent than team B.

Question 5:

The sum and sum of squares corresponding to length *x* (in cm) and weight *y* (in gm) of 50 plant products are given below:

$$\sum_{i=1}^{50} x_i = 212,\ \sum_{i=1}^{50} x_i^2 = 902.8,\ \sum_{i=1}^{50} y_i = 261,\ \sum_{i=1}^{50} y_i^2 = 1457.6$$

Which is more varying, the length or weight?

Solution 5:

$$\sum_{i=1}^{50} x_i = 212,\ \sum_{i=1}^{50} x_i^2 = 902.8$$

Here, N = 50

$$\text{Mean, } \bar{x} = \frac{\sum_{i=1}^{50} y_i}{N} = \frac{212}{50} = 4.24$$

$$\text{Variance}\left(\sigma_1^2\right) = \frac{1}{N}\sum_{i=1}^{50}\left(x_i - \bar{x}\right)^2$$

$$= \frac{1}{50}\sum_{i=1}^{50}\left(x_i - 4.24\right)^2$$

$$= \frac{1}{50}\sum_{i=1}^{50}\left[x_i^2 - 8.48x_i + 17.97\right]$$

$$= \frac{1}{50}\left[\sum_{i=1}^{50} x_i^2 - 8.48\sum_{i=1}^{50} x_i + 17.97 \times 50\right]$$

$$= \frac{1}{50}\left[902.8 - 8.48 \times (212) + 898.5\right]$$

$$= \frac{1}{50}\left[1801.3 - 1797.76\right]$$

$$= \frac{1}{50} \times 3.54$$

$$= 0.07$$

$$\therefore \text{Standard deviation } \sigma_1 (\text{Length}) = \sqrt{0.07} = 0.26$$

$$\text{C.V.(Length)} = \frac{\text{Standard deviation}}{\text{Mean}} \times 100 = \frac{0.26}{4.24} \times 100 = 6.13$$

$$\sum_{i=1}^{50} y_i = 261,\ \sum_{i=1}^{50} y_i^2 = 1457.6$$

$$\text{Mean, } \bar{y} = \frac{1}{N}\sum_{i=1}^{50} y_i = \frac{1}{50} \times 261 = 5.22$$

$$\text{Variance}\left(\sigma_1^2\right) = \frac{1}{N}\sum_{i=1}^{50}\left(y_i - \bar{y}\right)^2$$

$$= \frac{1}{50}\sum_{i=1}^{50}\left(y_i\text{-}5.22\right)^2$$

$$= \frac{1}{50}\sum_{i=1}^{50}\left[y_i^2 - 10.44y_i + 27.24\right]$$

$$= \frac{1}{50}\left[\sum_{i=1}^{50}y_i^2 - 10.44\sum_{i=1}^{50}y_i + 27.24\times 50\right]$$

$$= \frac{1}{50}\left[1457.6 - 10.44\times(261) + 1362\right]$$

$$= \frac{1}{50}\left[2819.6 - 2724.84\right]$$

$$= \frac{1}{50}\times 94.76$$

$$= 1.89$$

$\therefore$ Standard deviation $\sigma_2\left(\text{Weight}\right) = \sqrt{1.89} = 1.37$

C.V.(Weight) = $\dfrac{\text{Standard deviation}}{\text{Mean}}\times 100 = \dfrac{1.37}{5.22}\times 100 = 26.24$

Thus, C.V. of weights is greater than the C.V. of lengths. Therefore, weights vary more than the lengths

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