Exercise 15.1

Question 1:
Find the mean deviation about the mean for the data 4, 7, 8, 9, 10, 12, 13, 17

Solution 1:
The given data is
4, 7, 8, 9, 10, 12, 13, 17

Mean of the data, $\overline{x} = \dfrac{4+7+8+9+10+12+13+17}{8} = \dfrac{80}{8} = 10$

The deviations of the respective observations from the mean $\overline{x}$, i.e. $x_i - \overline{x}$, are

- 6, - 3, - 2, - 1, 0, 2, 3, 7

The absolute values of the deviations, i.e. $|x_i - \overline{x}|$, are

6, 3, 2, 1, 0, 2, 3, 7

The required mean deviation about the mean is

$$\text{M.D.}(\overline{x}) = \frac{\sum_{i=1}^{8}|x_i - \overline{x}|}{8} = \frac{6+3+2+1+0+2+3+7}{8} = \frac{24}{8} = 3$$

Question 2:
Find the mean deviation about the mean for the data
38, 70, 48, 40, 42, 55, 63, 46, 54, 44

Solution 2:
The given data is
38, 70, 48, 40, 42, 55, 63, 46, 54, 44
Mean of the given data,

$$\overline{x} = \frac{38+70+48+40+42+55+63+46+54+44}{10} = \frac{500}{10} = 50$$

The deviations of the respective observations from the mean $\overline{x}$, i.e., $x_i - \overline{x}$, are

- 12, 20, - 2, - 10, - 8, 5, 13, - 4, 4, - 6

The absolute values of the deviations, i.e. $|x_i - \overline{x}|$, are

12, 20, 2, 10, 8, 5, 13, 4, 4, 6

The required mean deviation about the mean is

$$\text{M.D.}(\overline{x}) = \frac{\sum_{i=1}^{8}|x_i - \overline{x}|}{10}$$

$$= \frac{12+20+2+10+8+5+13+4+4+6}{10}$$

$$= \frac{84}{10}$$

$$= 8.4$$

Question 3:
Find the mean deviation about the median for the data.
13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Solution 3:
The given data is
13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17
Here, the numbers of observations are 12, which is even.
Arranging the data in ascending order, we obtain
10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18

$$\text{Median, M} = \frac{\left(\frac{12}{2}\right)^{th} \text{observation} + \left(\frac{12}{2}+1\right)^{th} \text{observation}}{2}$$

$$= \frac{6^{th} \text{ observation} + 7^{th} \text{ observation}}{2}$$

$$= \frac{13+14}{2} = \frac{27}{2} = 13.5$$

The deviations of the respective observations from the median, i.e. $x_i - M$, are
- 3.5, - 2.5, - 2.5, - 1.5, - 0.5, - 0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5
The absolute values of the deviations, $|x_i - M|$ are
3.5, 2.5, 2.5, 1.5, 0.5, 0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5
The required mean deviation about the median is

$$M.D.(M) = \frac{\sum_{i-1}^{12} |x_i - M|}{12}$$

$$= \frac{3.5+2.5+2.5+1.5+0.5+0.5+0.5+2.5+2.5+3.5+3.5+4.5}{12}$$

$$= \frac{28}{12} = 2.33$$

Question 4:
Find the mean deviation about the median for the data
36, 72, 46, 42, 60, 45, 53, 46, 51, 49

Solution 4:
The given data is
36, 72, 46, 42, 60, 45, 53, 46, 51, 49
Here, the number of observations is 10, which is even.
Arranging the data in ascending order, we obtain
36, 42, 45, 46, 46, 49, 51, 53, 60, 72

$$\text{Median M}=\frac{\left(\frac{10}{2}\right)^{th}\text{ observation}+\left(\frac{10}{2}+1\right)^{th}\text{ observation}}{2}$$

$$=\frac{5^{th}\text{ observation}+6^{th}\text{ observation}}{2}$$

$$=\frac{46+49}{2}=\frac{95}{2}=47.5$$

The deviations of the respective observations from the median, i.e. $x_i-\text{M}$ are

- 11.5, - 5.5, - 2.5, - 1.5, - 1.5, 1.5, 3.5, 5.5, 12.5, 24.5

The absolute values of the deviations, $|x_i-\text{M}|$, are

11.5, 5.5, 2.5, 1.5, 1.5, 1.5, 3.5, 5.5, 12.5, 24.5

Thus, the required mean deviation about the median is

$$\text{M.D.(M)}=\frac{\sum_{i=1}^{10}|x_i-\text{M}|}{10}=\frac{11.5+5.5+2.5+1.5+1.5+1.5+3.5+5.5+12.5+24.5}{10}$$

$$=\frac{70}{10}=7$$

Question 5:

Find the mean deviation about the mean for the data.

	5	10	15	20	25
f_i	7	4	6	3	5

Solution 5:

	f_i	f_i	$\lvert x_i-\bar{x}\rvert$	$f_i\lvert x_i-\bar{x}\rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	25	350		158

$$\text{N}=\sum_{i=1}^{5}\text{f}_\text{i}=25$$

$$\sum_{i=1}^{5}\text{f}_\text{i}\text{x}_\text{i}=350$$

$$\therefore\bar{\text{x}}=\frac{1}{\text{N}}\sum_{i=1}^{5}\text{f}_\text{i}\text{x}_\text{i}=\frac{1}{25}\times350=14$$

$$\therefore\text{MD}(\bar{\text{x}})=\frac{1}{\text{N}}\sum_{i=1}^{5}\text{f}_\text{i}|\text{x}_\text{i}-\bar{\text{x}}|=\frac{1}{25}\times158=6.32$$

Question 6:

Find the mean deviation about the mean for the data

	10	30	50	70	90
f_i	4	24	28	16	8

Solution 6:

	f_i	f_i	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	80	4000		1280

$$N = \sum_{i=1}^{5} f_i = 80, \ \sum_{i=1}^{5} f_i x_i = 4000$$

$$\therefore \bar{x} = \frac{1}{N} \sum_{i=1}^{5} f_i x_i = \frac{1}{80} \times 4000 = 50$$

$$\therefore MD(\bar{x}) = \frac{1}{N} \sum_{i=1}^{5} f_i \lvert x_i - \bar{x} \rvert = \frac{1}{80} \times 1280 = 16$$

Question 7:

Find the mean deviation about the median for the data.

	5	7	9	10	12	15
f_i	8	6	2	2	2	6

Solution 7:

The given observations are already in ascending order.

Adding a column corresponding to cumulative frequencies of the given data, we obtain the following table.

	f_i	$c.f.$
5	8	8
7	6	14
9	2	16
10	2	18
12	2	20
15	6	26

Here, N = 26, which is even.

Median is the mean of 13th and 14th observations. Both of these observations lie in the cumulative frequency 14, for which the corresponding observation is 7.

$$\therefore \text{Median} = \frac{13^{\text{th}}\text{observation} + 14^{\text{th}}\text{observation}}{2} = \frac{7+7}{2} = 7$$

The absolute values of the deviations from median, i.e. $\lvert x_i - M \rvert$, are

$\lvert x_i - M \rvert$	2	0	2	3	5	8
f_i	8	6	2	2	2	6

$f_i\lvert x_i - M\rvert$	16	0	4	6	10	48

$$\sum_{i=1}^{6} f_i = 26, \textit{and} \sum_{i=1}^{6} f_i\lvert x_i - M\rvert = 84$$

$$M.D.(M) = \frac{1}{N}\sum_{i=1}^{6} f_i\lvert x_i - M\rvert = \frac{1}{26}\times 84 = 3.23$$

Question 8:

Find the mean deviation about the median for the data

	15	21	27	30	35
f_i	3	5	6	7	8

Solution 8:

The given observations are already in ascending order.

Adding a column corresponding to cumulative frequencies of the given data, we obtain the following table.

	f_i	$c.f.$
15	3	3
21	5	8
27	6	14
30	7	21
35	8	29

Here, N = 29, which is odd.

$$\therefore \text{Median} = \left(\frac{7+7}{2}\right)^{th} \text{observation} = 15^{th} \text{ observation}$$

This observation lies in the cumulative frequency 21, for which the corresponding observation is 30.

$\therefore$ Median = 30

The absolute values of the deviations from median, i.e. $\lvert x_i - M\rvert$, are

$\lvert x_i - M\rvert$	15	9	3	0	5
f_i	3	5	6	7	8
$f_i\lvert x_i - M\rvert$	45	45	18	0	40

$$\sum_{i=1}^{5} f_i = 29, \sum_{i=1}^{5} f_i\lvert x_i - M\rvert = 148$$

$$\therefore M.D.(M) = \frac{1}{N}\sum_{i=1}^{5} f_i\lvert x_i - M\rvert = \frac{1}{29}\times 148 = 5.1$$

Question 9:

Find the mean deviation about the mean for the data.

Income per day	Number of persons
0-100	4
100-200	8

200-300	9
300-400	10
400-500	7
500-600	5
600-700	4
700-800	3

Solution 9:

The following table is formed.

Income per day	Number of persons f_i	Mid-point	f_i	$\lvert x_i - \overline{x}\rvert$	$f_i \lvert x_i - \overline{x}\rvert$
0 100	4	50	200	308	1232
100 200	8	150	1200	208	1664
200 300	9	250	2250	108	972
300 400	10	350	3500	8	80
400 500	7	450	3150	92	644
500 600	5	550	2750	192	960
600 700	4	650	2600	292	1168
700 800	3	750	2250	392	1176
	50		17900		7896

$$\text{Here, } N = \sum_{i=1}^{8} f_i = 50, \ \sum_{i=1}^{8} f_i x_i = 17900$$

$$\therefore \overline{x} = \frac{1}{N}\sum_{i=1}^{8} f_i x_i = \frac{1}{50} \times 17900 = 358$$

$$\text{M.D.}(\overline{x}) = \frac{1}{N}\sum_{i=1}^{8} f_i \lvert x_i - \overline{x}\rvert = \frac{1}{50} \times 7896 = 157.92$$

Question 10:

Find the mean deviation about the mean for the data

Height in cms	Number of boys
95-105	9
105-115	13
115-125	26
125-135	30
135-145	12
145-155	10

Solution 10:

The following table is formed.

Height in cms	Number of boys f_i	Mid-point	f_i	$\lvert x_i - \overline{x}\rvert$	$f_i \lvert x_i - \overline{x}\rvert$
95-105	9	100	900	25.3	227.7
105-115	13	110	1430	15.3	198.9
115-125	26	120	3120	5.3	137.8
125-135	30	130	3900	4.7	141

135-145	12	140	1680	14.7	176.4
145-155	10	150	1500	24.7	247

Here, $N = \sum_{i=1}^{6} f_i = 100, \ \sum_{i=1}^{6} f_i x_i = 12530$

$\therefore \bar{x} = \frac{1}{N}\sum_{i=1}^{6} f_i x_i = \frac{1}{100} \times 12530 = 125.3$

$\text{M.D.}(\bar{x}) = \frac{1}{N}\sum_{i=1}^{6} f_i |x_i - \bar{x}| = \frac{1}{100} \times 1128.8 = 11.28$

Question 11:

Calculate the mean deviation about median age for the age distribution of 100 persons given below:

Age	Number
16-20	5
21-25	6
26-30	12
31-35	14
36-40	26
41-45	12
46-50	16
51-55	9

Solution 11:

The given data is not continuous. Therefore, it has to be converted into continuous frequency distribution by subtracting 0.5 from the lower limit and adding 0.5 to the upper limit of each class interval.

The table is formed as follows.

Age	Number f_i	Cumulative frequency (c.f)	Mid-point	$\lvert x_i - \text{Med.}\rvert$	$f_i \lvert x_i - \text{Med.}\rvert$
15.5-20.5	5	5	18	20	100
20.5-25.5	6	11	23	15	90
25.5-30.5	12	23	28	10	120
30.5-35.5	14	37	33	5	70
35.5-40.5	26	63	38	0	0
40.5-45.5	12	75	43	5	60
45.5-50.5	16	91	48	10	160
50.5-55.5	9	100	53	15	135
	10				735

The class interval containing the $\frac{N^{th}}{2}$ or 50th item is 35.5 40.5.

Therefore, 35.5 40.5 is the median class.

It is known that,

$$\text{Median} = l + \frac{\frac{N}{2} - C}{f} \times h$$

Here, l = 35.5, C = 37, f = 26, h = 5, and N = 100

$$\therefore \text{Median} = 35.5 + \frac{50-37}{26} \times 5 = 35.5 + \frac{13 \times 5}{26} = 35.5 + 2.5 = 38$$

Thus, mean deviation about the median is given by,

$$\text{M.D.}(\text{M}) = \frac{1}{\text{N}} \sum_{i=1}^{8} \text{f}_\text{i} |\text{x}_\text{i} - \text{M}| = \frac{1}{100} \times 735 = 7.35$$
