Exercise 14.5

Question 1: Show that the statement
p: 'If x is a real number such that $x^3 + 4x = 0$, then x is 0' is true by
(i) direct method
(ii) method of contradiction
(iii) method of contrapositive

Solution 1: p: 'If x is a real number such that $x^3 + 4x = 0$, then x is 0'.
Let q: x is a real number such that $x^3 + 4x = 0$
r: x is 0.
(i) To show that statement p is true, we assume that q, is true and then show that r is true.
Therefore, let statement q be true.
$x^3 + 4x = 0$
$x(x^2 + 4) = 0$
$x = 0$ or $x^2 + 4 = 0$
However, since x is real, it is 0.
Thus, statement r is true.
Therefore, the given statement is true.
(ii) To show statement p to be true by contradiction, we assume that p is not true.
Let x be a real number such that $x^3 + 4x = 0$ and let x is not 0.
Therefore, $x^3 + 4x = 0$
$x(x^2 + 4) = 0$
$x = 0$ or $x^2 + 4 = 0$
$x = 0$ or $x^2 = -4$
However, x is real. Therefore, $x = 0$, which is a contradiction since we have assumed that x is not 0.

Thus, the given statement p is true.
(iii)To prove statement p to be true by contrapositive method, we assume that r is false and prove that q must be false.
Here, r is false implies that it is required to consider the negation of statement r. This obtains the following statement.
~ r: x is not 0
I can be seen that $(x^2 + 4)$ will always be positive
x = 0 implies that the product of any positive real number with x is not zero.
Let us consider the product of x with $(x^2 + 4)$
$\therefore x(x^2 + 4) = 0$
$x^3 + 4x = 0$
This shows that statement q is not true.
Thus, it has been proved that
$\sim r \Rightarrow \sim q$
Therefore, the given statement p is true.

Question 2: Show that the statement 'For any real numbers a and b, $a^2 = b^2$ implies that a = b' is not true by giving a counter-example.

Solution 2: The given statement can be written in the form of 'if-then' as follows.
If a and b are real numbers such that $a^2 = b^2$, then a = b.
Let p: a and b are real numbers such that $a^2 = b^2$.
q: a = b
The given statement has to be proved false. For this purpose, it has to be proved that if p, then ~ q To sow this, two real numbers, a and b, with $a^2 = b^2$ are required such that $a \neq b$.
Let a = 1 and b = –1
$a^2 = (1)^2$ and $b^2 = (-1)^2 = 1$
$a^2 = b^2$
However, a = b
Thus, it can be concluded that the given statement is false.

Question 3: Show that the following statement is true by the method of contrapositive.
p: If x is an integer and x^2 is even, then x is also even.

Solution 3: p: If x is an integer and x^2 is even, then x is also even.
Let q: x is an integer and x^2 is even.
r: x is even.
To prove that p is true by contrapositive method, we assume that r is false, and prove that q is also false.
Let x is not even.
To prove that q is false, it has to be proved that x is not an integer or x^2 is not even.
x is not even implies that x^2 is also not even.
Therefore, statement q is false.
Thus, the given statement p is true.

Question 4: By giving a counter example, show that the following statements are not true.
(i) p: If all the angles of a triangle are equal, then the triangle is an obtuse angled triangle.
(ii) q: The equation $x^2 – 1 = 0$ does not have a root lying between 0 and 2.

Solution 4: (i) The given statement is of the form 'if q then r'.
q: All the angles of a triangle are equal.
r: The triangle is an obtuse-angled triangle.
The given statement p has to be proved false. For this purpose, it has to be proved that if q, then
~ r.
To show this, angles of a triangle are required such that none of them is an obtuse angle.
It is known that the sum of all angles of a triangle is 180°. Therefore, if all the three angles are equal, then each of them is of measure 60°, which is not an obtuse angle.
In an equilateral triangle, the measure of all angles is equal. However, the triangle is not an obtuse-angled triangle.
Thus, it can be concluded that the given statement p is false.
(ii) The given statement is as follows.
q: The equation $x^2 – 1 = 0$ does not have a root lying between 0 and 2.
This statement has to be proved false. To show this, a counter example is required.
Consider $x^2 – 1 = 0$
$x^2 = 1$
$x = \pm 1$
One root of the equation $x^2 – 1 = 0$, i.e. the root $x = 1$, lies between 0 and 2.
Thus, the given statement is false.

Question 5: Which of the following statements are true and which are false? In each case give a valid reason for saying so.
(i) p: Each radius of a circle is a chord of the circle.
(ii) q: The centre of a circle bisects each chord of the circle.

iii) r: Circle is a particular case of an ellipse.
(iv) s: If x and y are integers such that $x > y$, then $– x < – y$.
(v) t: $\sqrt{11}$ is a rational number.

Solution 5: (i) The given statement p is false.
According to the definition of chord, it should intersect the circle at two distinct points.
(ii) The given statement q is false.
If the chord is not the diameter of the circle, then the centre will not bisect that chord.
In other words, the centre of a circle only bisects the diameter, which is the chord of the circle.
(iii) The equation of an ellipse is,
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$
If we put $a = b = 1$, then we obtain
$x^2 + y^2 =1$,which is an equation of a circle
Therefore, circle is a particular case of an eclipse.
Thus, statement r is true.

(iv) $x > y$

$\Rightarrow -x < -y$ (By a rule of inequality)

Thus, the given statement s is true.

(v) 11 is a prime number and we know that the square root of any prime number is an irrational number.

Therefore, $\sqrt{11}$ is an irrational number.

Thus, the given statement *t* is false.