Exercise – 4A

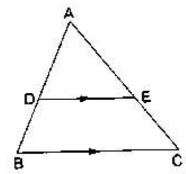
1. D and E are points on the sides AB and AC respectively of a ΔABC such that DE ∥ BC.

(i) If AD = 3.6cm, AB = 10cm and AE = 4.5cm, find EC and AC.

(ii) If AB = 13.3cm, AC = 11.9cm and EC = 5.1cm, find AD.

(iii) If $\frac{AD}{DB} = \frac{4}{7}$ and AC = 6.6cm, find AE.

(iv) If $\frac{AD}{AB} = \frac{8}{15}$ and EC = 3.5cm, find AE.



Sol:

(i) In Δ ABC, it is given that DE ∥ BC.

Applying Thales' theorem, we get:

$\frac{AD}{DB} = \frac{AE}{EC}$

∵ AD = 3.6 cm , AB = 10 cm, AE = 4.5cm

∴ DB = 10 − 3.6 = 6.4cm

Or, $\frac{3.6}{6.4} = \frac{4.5}{EC}$

Or, EC = $\frac{6.4 \times 4.5}{3.6}$

Or, EC= 8 cm

Thus, AC = AE + EC

= 4.5 + 8 = 12.5 cm

(ii) In Δ ABC, it is given that DE ∥ BC.

Applying Thales' Theorem, we get :

$\frac{AD}{DB} = \frac{AE}{EC}$

Adding 1 to both sides, we get :

$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$

$\Rightarrow \frac{AB}{DB} = \frac{AC}{EC}$

$\Rightarrow \frac{13.3}{DB} = \frac{11.9}{5.1}$

$\Rightarrow DB = \frac{13.3 \times 5.1}{11.9} = 5.7$ cm

Therefore, AD=AB-DB=13.5-5.7=7.6 cm

(iii) In Δ ABC, it is given that DE ∥ BC.

Applying Thales' theorem, we get :

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{4}{7} = \frac{AE}{EC}$

Adding 1 to both the sides, we get :

$\frac{11}{7} = \frac{AC}{EC}$

$\Rightarrow$ EC = $\frac{6.6 \times 7}{11}$ = 4.2 cm

Therefore,

AE = AC – EC = 6.6 – 4.2 = 2.4 cm

(iv) In Δ ABC, it is given that DE ∥ BC.

Applying Thales' theorem, we get:

$\frac{AD}{AB} = \frac{AE}{AC}$

$\Rightarrow \frac{8}{15} = \frac{AE}{AE+EC}$

$\Rightarrow \frac{8}{15} = \frac{AE}{AE+3.5}$

⟹ 8AE + 28 = 15AE

⟹ 7AE = 28

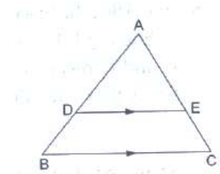
⟹ AE = 4cm

2. D and E are points on the sides AB and AC respectively of a ΔABC such that DE ∥ BC. Find the value of x, when

(i) AD = x cm, DB = (x – 2) cm, AE = (x + 2) cm and EC = (x – 1) cm.

(ii) AD = 4cm, DB = (x – 4) cm, AE = 8cm and EC = (3x – 19) cm.

(iii) AD = (7x – 4) cm, AE = (5x – 2) cm, DB = (3x + 4) cm and EC = 3x cm.



Sol:

(i) In Δ ABC, it is given that DE || BC.

Applying Thales' theorem, we have :

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

⟹X(x-1) = (x-2) (x+2)

$\Rightarrow x^2 - x = x^2 - 4$

⟹x= 4 cm

(ii) In Δ ABC, it is given that DE ∥ BC.

Applying Thales' theorem, we have :

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{4}{x-4} = \frac{8}{3x-19}$

⟹ 4 (3x-19) = 8 (x-4)

⟹12x – 76 = 8x – 32

⟹4x = 44

⟹ x = 11 cm

(iii) In Δ ABC, it is given that DE || BC.

Applying Thales' theorem, we have :

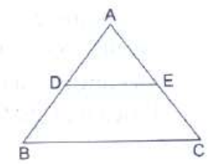
$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{7x-4}{3x+4} = \frac{5x-2}{3x}$

⟹3x (7x-4) = (5x-2) (3x+4)

$\Rightarrow 21x^2 - 12x = 15x^2 + 14x - 8$
$\Rightarrow 6x^2 - 26x + 8 = 0$
$\Rightarrow (x-4)(6x-2) = 0$
$\Rightarrow x = 4, \frac{1}{3}$
$\because x \neq \frac{1}{3}$ (as if $x = \frac{1}{3}$ $then\ AE\ will\ become\ negative$)
$\therefore x = 4$ cm

3. D and E are points on the sides AB and AC respectively of a ΔABC. In each of the following cases, determine whether DE ∥ BC or not.



(i) AD = 5.7cm, DB = 9.5cm, AE = 4.8cm and EC = 8cm.
(ii) AB = 11.7cm, AC = 11.2cm, BD = 6.5cm and AE = 4.2cm.
(iii) AB = 10.8cm, AD = 6.3cm, AC = 9.6cm and EC = 4cm.
(iv) AD = 7.2cm, AE = 6.4cm, AB = 12cm and AC = 10cm.

Sol:

(i) We have:
$\frac{AD}{DE} = \frac{5.7}{9.5} = 0.6\ cm$
$\frac{AE}{EC} = \frac{4.8}{8} = 0.6\ cm$
Hence, $\frac{AD}{DB} = \frac{AE}{EC}$
Applying the converse of Thales' theorem,
We conclude that DE || BC.

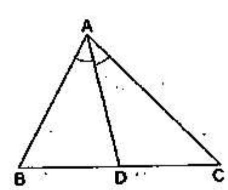
(ii) We have:
AB = 11.7 cm, DB = 6.5 cm
Therefore,
AD = 11.7 -6.5 = 5.2 cm
Similarly,
AC = 11.2 cm, AE = 4.2 cm
Therefore,
EC = 11.2 – 4.2 = 7 cm
Now,
$\frac{AD}{DB} = \frac{5.2}{6.5} = \frac{4}{5}$
$\frac{AE}{EC} = \frac{4.2}{7}$
Thus, $\frac{AD}{DB} \neq \frac{AE}{EC}$
Applying the converse of Thales' theorem,
We conclude that DE is not parallel to BC.

(iii) We have:
AB = 10.8 cm, AD = 6.3 cm
Therefore,

DB = 10.8 – 6.3 = 4.5 cm
Similarly,
AC = 9.6 cm, EC = 4cm
Therefore,
AE = 9.6 – 4 = 5.6 cm
Now,
$\frac{AD}{DB} = \frac{6.3}{4.5} = \frac{7}{5}$
$\frac{AE}{EC} = \frac{5.6}{4} = \frac{7}{5}$
$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$
Applying the converse of Thales' theorem,
We conclude that DE || BC.

(iv) We have :
AD = 7.2 cm, AB = 12 cm
Therefore,
DB = 12 – 7.2 = 4.8 cm
Similarly,
AE = 6.4 cm, AC = 10 cm
Therefore,
EC = 10 – 6.4 = 3.6 cm
Now,
$\frac{AD}{DB} = \frac{7.2}{4.8} = \frac{3}{2}$
$\frac{AE}{EC} = \frac{6.4}{3.6} = \frac{16}{9}$
This, $\frac{AD}{DB} \neq \frac{AE}{EC}$
Applying the converse of Thales' theorem,
We conclude that DE is not parallel to BC.

4. In a ΔABC, AD is the bisector of ∠A.
(i) If AB = 6.4cm, AC = 8cm and BD = 5.6cm, find DC.
(ii) If AB = 10cm, AC = 14cm and BC = 6cm, find BD and DC.
(iii) If AB = 5.6cm, BD = 3.2cm and BC = 6cm, find AC.
(iv) If AB = 5.6cm, AC = 4cm and DC = 3cm, find BC.



Sol:
(i) It is give that AD bisects ∠A.
Applying angle – bisector theorem in Δ ABC, we get:
$\frac{BD}{DC} = \frac{AB}{AC}$
$\Rightarrow \frac{5.6}{DC} = \frac{6.4}{8}$
$\Rightarrow DC = \frac{8 \times 5.6}{6.4} = 7\ cm$

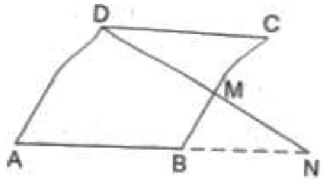
(ii) It is given that AD bisects $\angle A$.
Applying angle – bisector theorem in Δ ABC, we get:
$\frac{BD}{DC} = \frac{AB}{AC}$
Let BD be x cm.
Therefore, DC = (6- x) cm
$\Rightarrow \frac{x}{6-x} = \frac{10}{14}$
$\Rightarrow 14x = 60-10x$
$\Rightarrow 24x = 60$
$\Rightarrow x = 2.5$ cm
Thus, BD = 2.5 cm
DC = 6-2.5 = 3.5 cm

(iii) It is given that AD bisector $\angle A$.
Applying angle – bisector theorem in Δ ABC, we get:
$\frac{BD}{DC} = \frac{AB}{AC}$
BD = 3.2 cm, BC = 6 cm
Therefore, DC = 6- 3.2 = 2.8 cm
$\Rightarrow \frac{3.2}{2.8} = \frac{5.6}{AC}$
$\Rightarrow AC = \frac{5.6 \times 2.8}{3.2} = 4.9\ cm$

(iv) It is given that AD bisects $\angle A$.
Applying angle – bisector theorem in Δ ABC, we get:
$\frac{BD}{DC} = \frac{AB}{AC}$
$\Rightarrow \frac{BD}{3} = \frac{5.6}{4}$
$\Rightarrow BD = \frac{5.6 \times 3}{4}$
$\Rightarrow BD = 4.2$ cm
Hence, BC = 3+ 4.2 = 7.2 cm

5. M is a point on the side BC of a parallelogram ABCD. DM when produced meets AB produced at N. Prove that

(i) $\frac{DM}{MN} = \frac{DC}{BN}$ (ii) $\frac{DN}{DM} = \frac{AN}{DC}$



Sol:
(i) Given: ABCD is a parallelogram

To prove:

(i) $\frac{DM}{MN} = \frac{DC}{BN}$

(ii) $\frac{DN}{DM} = \frac{AN}{DC}$

Proof: In Δ DMC and Δ NMB

∠DMC = ∠NMB (Vertically opposite angle)

∠DCM = ∠NBM (Alternate angles)

By AAA- Similarity

ΔDMC ~ ΔNMB

$\therefore \frac{DM}{MN} = \frac{DC}{BN}$

NOW, $\frac{MN}{DM} = \frac{BN}{DC}$

Adding 1 to both sides, we get

$\frac{MN}{DM} + 1 = \frac{BN}{DC} + 1$

$\Rightarrow \frac{MN+DM}{DM} = \frac{BN+DC}{DC}$

$\Rightarrow \frac{MN+DM}{DM} = \frac{BN+AB}{DC}$ [∵ ABCD is a parallelogram]

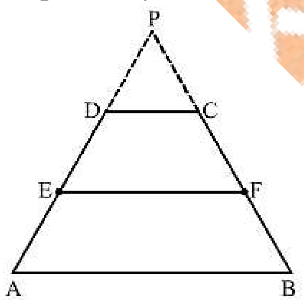
$\Rightarrow \frac{DN}{DM} = \frac{AN}{DC}$

6. Show that the line segment which joins the midpoints of the oblique sides of a trapezium is parallel sides

Sol:

(i)

Let the trapezium be ABCD with E and F as the mid Points of AD and BC, Respectively Produce AD and BC to Meet at P.



In Δ PAB, DC || AB.

Applying Thales' theorem, we get

$\frac{PD}{DA} = \frac{PC}{CB}$

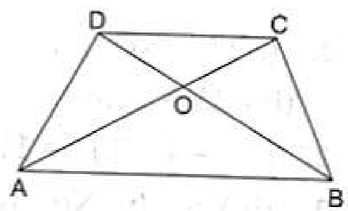
Now, E and F are the midpoints of AD and BC, respectively.

$\Rightarrow \frac{PD}{2DE} = \frac{PC}{2CF}$

$\Rightarrow \frac{PD}{DE} = \frac{PC}{CF}$

Applying the converse of Thales' theorem in Δ PEF, we get that DC
Hence, EF || AB.
Thus. EF is parallel to both AB and DC.
This completes the proof.

7. In the given figure, ABCD is a trapezium in which AB ∥ DC and its diagonals intersect at O. If AO = (5x – 7), OC = (2x + 1) , BO = (7x – 5) and OD = (7x + 1), find the value of x.



Sol:
In trapezium ABCD, AB ∥ CD and the diagonals AC and BD intersect at O.
Therefore,

$\frac{AO}{OC} = \frac{BO}{OD}$

$\Rightarrow \frac{5x-7}{2x+1} = \frac{7x-5}{7x+1}$

$\Rightarrow$ (5x-7) (7x+1) = (7x-5) (2x+1)

$\Rightarrow 35x^2 + 5x - 49x - 7 = 14x^2 - 10x + 7x - 5$

$\Rightarrow 21x^2 - 41x - 2 = 0$

$\Rightarrow 21x^2 - 42x + x - 2 = 0$

$\Rightarrow$ 21x (x-2) +1 (x-2) =0

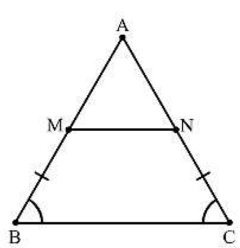
$\Rightarrow$ (x-2) (21x + 1) =0

$\Rightarrow x = 2, -\frac{1}{21}$

$\because x \neq -\frac{1}{21}$

$\therefore$ x = 2

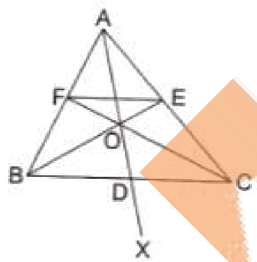
8. In ΔABC, M and N are points on the sides AB and AC respectively such that BM= CN. If $\angle B = \angle C$ then show that MN||BC
Sol:



In ΔABC, $\angle B = \angle C$
$\therefore$ AB = AC (Sides opposite to equal angle are equal)
Subtracting BM from both sides, we get

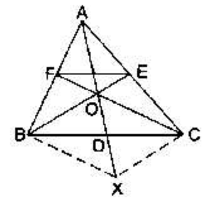
AB – BM = AC – BM
⟹AB – BM = AC – CN (∵BM =CN)
⟹AM =AN
∴∠AMN =∠ ANM (Angles opposite to equal sides are equal)
Now, in ΔABC,
$\angle A + \angle B + \angle C = 180^0$ ----(1)
(Angle Sum Property of triangle)
Again In In ΔAMN,
$\angle A + \angle AMN + \angle ANM = 180^0$ ----(2)
(Angle Sum Property of triangle)
From (1) and (2), we get
$\angle B + \angle C = \angle AMN + \angle ANM$
$\Rightarrow 2\angle B = 2\angle AMN$
$\Rightarrow \angle B = \angle AMN$
Since, ∠B and ∠ AMN are corresponding angles.
∴ MN || BC.

9. ΔABC and ΔDBC lie on the same side of BC, as shown in the figure. From a point P on BC, PQ||AB and PR||BD are drawn, meeting AC at Q and CD at R respectively. Prove that QR||AD.



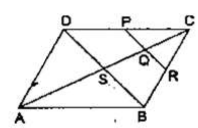
Sol:
In Δ CAB, PQ || AB.
Applying Thales' theorem, we get:
$\frac{CP}{PB} = \frac{CQ}{QA}$ …(1)
Similarly, applying Thales theorem in ΔBDC, Where PR||DM we get:
$\frac{CP}{PB} = \frac{CR}{RD}$ …(2)
Hence, from (1) and (2), we have :
$\frac{CQ}{QA} = \frac{CR}{RD}$
Applying the converse of Thales' theorem, we conclude that QR ∥ AD in Δ ADC.
This completes the proof.

10. In the given figure, side BC of a ΔABC is bisected at D and O is any point on AD. BO and CO produced meet AC and AB at E and F respectively, and AD is produced to X so that D is the midpoint of OX.
Prove that AO : AX = AF : AB and show that EF ∥ BC.



Sol:
It is give that BC is bisected at D.
∴ BD = DC
It is also given that OD =OX
The diagonals OX and BC of quadrilateral BOCX bisect each other.
Therefore, BOCX is a parallelogram.
∴ BO || CX and BX || CO
BX || CF and CX || BE
BX || OF and CX || OE
Applying Thales' theorem in Δ ABX, we get:
$\frac{AO}{AX} = \frac{AF}{AB}$ …(1)
Also, in Δ ACX, CX || OE.
Therefore by Thales' theorem, we get:
$\frac{AO}{AX} = \frac{AE}{AC}$ …(2)
From (1) and (2), we have:
$\frac{AO}{AX} = \frac{AE}{AC}$
Applying the converse of Theorem in Δ ABC, EF || CB.
This completes the proof.

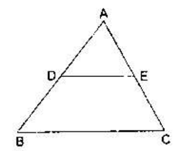
11. ABCD is a parallelogram in which P is the midpoint of DC and Q is a point on AC such that CQ = $\frac{1}{4}$ AC. If PQ produced meets BC at R, prove that R is the midpoint of BC.



Sol:
We know that the diagonals of a parallelogram bisect each other.
Therefore,
CS = $\frac{1}{2}$ AC …(i)
Also, it is given that CQ = $\frac{1}{4}$ AC …(ii)
Dividing equation (ii) by (i), we get:
$\frac{CQ}{CS} = \frac{\frac{1}{4}AC}{\frac{1}{2}AC}$
Or, CQ = $\frac{1}{2}$ CS
Hence, Q is the midpoint of CS.

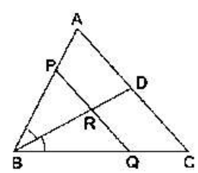
Therefore, according to midpoint theorem in ΔCSD
PQ || DS
If PQ || DS, we can say that QR || SB
In Δ CSB, Q is midpoint of CS and QR ∥ SB.
Applying converse of midpoint theorem , we conclude that R is the midpoint of CB.
This completes the proof.

12. In the adjoining figure, ABC is a triangle in which AB = AC. IF D and E are points on AB and AC respectively such that AD = AE, show that the points B, C, E and D are concyclic.



Sol:
Given:
AD = AE …(i)
AB = AC …(ii)
Subtracting AD from both sides, we get:
⟹ AB – AD = AC – AD
⟹ AB – AD = AC - AE (Since, AD = AE)
⟹ BD = EC …(iii)
Dividing equation (i) by equation (iii), we get:
$\frac{AD}{DB} = \frac{AE}{EC}$
Applying the converse of Thales' theorem, DE∥BC
⟹ ∠DEC + ∠ECB = 180^0 (Sum of interior angles on the same side of a Transversal Line is 0^0 .)
⟹ ∠DEC + ∠CBD = 180^0 (Since, AB = AC ⟹ ∠B = ∠C)
Hence, quadrilateral BCED is cyclic.
Therefore, B,C,E and D are concylic points.

13. In ΔABC, the bisector of ∠B meets AC at D. A line OQ ∥ AC meets AB, BC and BD at O, Q and R respectively. Show that BP × QR = BQ × PR



Sol:
In triangle BQO, BR bisects angle B.
Applying angle bisector theorem, we get:
$\frac{QR}{PR} = \frac{BQ}{BP}$
⟹BP × QR = BQ × PR
This completes the proof.